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PORTFOLIO OPTIMIZATION WITH SEMI-VARIANCE MODEL: AN APPLICATION ON BIST-100 INDEX*

YARI-VARYANS MODELİ İLE PORTFÖY OPTİMİZASYONU: BIST-100 ENDEKSİ ÜZERİNDE BİR UYGULAMA

Serdar Ramazan KAHRAMAN**

Kartal SOMUNCU***

Abstract

Aim: The aim of the study is to compare the performance of portfolios constructed based on variance and semi-variance using data obtained from the BIST-100 Index.

Method: In the study, using the return data obtained from the adjusted weighted average price data of securities in the BIST-100, variance and covariance matrices were constracted to generate optimal portfolios, and the returns of two different portfolios were calculated and compared.

Findings: The findings of the study indicate that, despite securities within the BIST-100 Index generally yielding negative returns during the 2018-2019 period, portfolios constructed based on semi-variance protected investors from the risk of negative returns. It was observed that as the levels of risk tolerance increased, the returns of portfolios also increased.

Conclusions: It has been concluded that portfolios created according to semi-variance offer better protection for investors with low risk tolerance against the risk of unexpected negative returns.

Keywords: Semi-Variance, Investment, Risk, Portfolio Optimization, BIST-100.

Öz

Aim: Araştırmanın amacı, varyans ve yarı varyansa göre oluşturulacak portföylerin performanslarını, BIST-100 Endeksi'nden elde edilen verileri kullanarak karşılaştırmaktır.

Yöntem: Çalışmada, BIST-100'de yer alan menkul kıymetlerin düzeltilmiş ağırlıklı ortalama fiyat verileri kullanılarak elde edilen getiri verileri yardımıyla varyans ve kovaryans matrisleri oluşturularak optimal portföyler elde edilmiş ve iki farklı portföyün getirileri hesaplanarak karşılaştırılmıştır.

Bulgular: Araştırmanın bulguları, 2018 - 2019 döneminde BIST - 100 endeksinde yer alan menkul kıymetler genel olarak negatif getiri sağlamasına rağmen, yarı varyansa göre oluşturulan portföylerin yatırımcıyı negatif getiri riskinden koruduğunu göstermektedir. Risk toleransı düzeyleri arttıkça portföylerin getirilerinin de arttığı gözlenmiştir.

Sonuç: Yarı varyansa göre oluşturulan portföylerin, düşük risk toleranslarında yatırımcıyı beklenmedik negatif getiri riskinden daha iyi koruduğu sonucuna ulaşılmıştır.

Anahtar Kelimeler: Yarı Varyans, Yatırım, Risk, Portföy Optimizasyonu, BIST-100.

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** Afyon Kocatepe University, sramazankahraman@hotmail.com, Orcid: 0000-0003-3895-5390

*** Afyon Kocatepe University, ksomuncu@gmail.com, Orcid: 0000-0002-5087-414X

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I. Introduction

The aim in portfolio optimization is to create an investment portfolio using various metrics and methods in a way that makes it more efficient compared to other alternatives and to make decisions regarding the selection of financial assets for investment. One of the primary goals of investors is to select the best investment opportunities to maximize portfolio returns. This selection can be quite challenging and complex for investors seeking acceptable risk for high return opportunities. There are various financial instruments available, such as stocks, bonds, gold, and more; however, opting for a single investment instrument and tying the entire investment to the risk of that single instrument is not a good choice. This situation prompts investors to seek an appropriate combination, known as diversification, to spread the risk of loss. A diversified portfolio shows less variability in expected returns and possesses a more reasonable risk behavior. Risk arises from the uncertainty of the future returns of the anticipated investment. It's common for investors to accept more risk for potentially higher returns, or for those seeking less risk to have lower return expectations. Therefore, investors can be categorized as risk-averse, neutral, or lover, depending on their risk preferences. A rational investor needs an acceptable level of return, despite taking risks. Thus, regardless of the category they fall into, an investor's expected return depends on the risk they assume. This means that the primary decision point for an investor is to determine the level of risk they are willing to undertake.

Under current conditions, both individual and institutional investors are engaging in stock investments with the aim of maximizing their returns. The return on a stock investment is contingent upon the level of risk undertaken by the investor. Investors aim to invest at the lowest tolerable risk level and achieve the highest possible return during the investment period. The notion that stock market investments yield higher returns, combined with ease of trading and relatively lower costs compared to other investments, has attracted both individual and institutional investors to stocks (Halıcı, 2008). The preference for investing in stocks has led to the development of the concept of a portfolio. Originally meaning 'wallet', the term 'portfolio' refers to a collection of securities.

Investors should invest in stocks that are suitable for their investment goals and are expected to appreciate in value over the investment horizon in order to achieve their investment objectives and maximize returns. However, there is a certain level of risk associated with bearing the risk in order to achieve the expected returns. To mitigate this risk, diversification of the portfolio by increasing the number of stocks is necessary. At this point, the need for investing in multiple stocks arises, leading to the creation of a portfolio (Halıcı, 2008).

A portfolio represents the total value generated by an investor through investing in multiple instruments with similar or different characteristics. To put it differently, portfolios are compilations of various financial instruments, usually including stocks, bonds, and derivatives, put together by an individual or a group (Ceylan and Korkmaz, 1998). The composition and management of portfolios for individuals or institutions are contingent on their return expectations, liquidity preferences, and the risk levels associated with financial instruments. The concept of portfolio selection was introduced by Harry M. Markowitz in his 1952 article titled "Portfolio Selection." The Markowitz Mean-Variance theory, widely used to this day, has come under criticism for its potential to lead to incorrect decisions in portfolios that do not conform to a normal distribution, thus spurring the quest for alternative methods.

In portfolios composed of multiple securities, the risk and return characteristics change depending on the variations in the portfolio's composition. A portfolio that is expected to provide maximum return at a specific risk level, or alternatively, a portfolio with minimum risk at a certain level of return, is called an optimal portfolio. In the finance literature, it was previously accepted that systematic risk could not be completely eliminated in any investment, and therefore, a portfolio could not be fully diversified. However, in Modern Portfolio Theory (MPT) introduced to the financial literature by Markowitz, it is suggested that the reduction of risk is possible through diversification.

An investor looking to invest in securities must choose securities to include in their portfolio from various financial assets. This selection is determined based on the risk and return characteristics of the securities. At this point, it is noted that the risk and return of the created portfolio may differ from the risk and return of the assets selected when forming the portfolio. When planning which securities to include in the portfolio, the relationship between securities also plays a significant role. If all the securities included in the portfolio move in the same direction, in other words, if there is a positive relationship in terms of price movements among them, then in the event of a market downturn, all the securities will lose value. Therefore, a portfolio composed of securities that move in the same direction has a higher level of risk. However, the purpose of portfolio construction is to diversify risk and minimize it. To achieve this, securities with minimum correlation to each other should be included together in the portfolio (Gurrib and Alshahrani, 2012:445).

The selection of the optimal portfolio is formed by taking into account various factors such as risk, expected return, liquidity, or transaction costs by the investor, and determining asset weights. The return and risk characteristics of a portfolio can change as the market environment evolves, and the portfolio may cease to be optimal. Therefore, it is necessary to restructure the portfolio to ensure the consistency of its return and risk characteristics. This adjustment can be achieved through periodic review of the portfolio structure, and this process is referred to as portfolio revision. By monitoring the changes that occur through revision, investors can maintain the optimal portfolio (Boda and Kanderova, 2018).

The final step in the investment management process involves periodically assessing the return and risk of the portfolio. To evaluate the performance of the portfolio in terms of return and risk, appropriate criteria and comparisons are necessary. In this context, the comparison criterion can be the index of relevant assets (such as stock indices, bond indices). Comparison criteria are commonly used by institutional investors to assess the performance of their portfolios. Throughout the investment management process, portfolio performance is influenced by changes in the investment environment and changes in investor decisions Focardi and Fabozzi, 2004. Globalization of the market provides investors with new investment opportunities but at the same time makes investment management more challenging.

Diversifying investment instruments offer options to investors but also make investment decisions and the investment process more complex. One of the significant factors in choosing investment instruments is the investor's risk tolerance. The composition of the portfolio is determined by the investor's risk tolerance. However, in traditional portfolio theory, risk is determined qualitatively rather than quantitatively and is classified as high, medium, and low. Investors who compose a significant portion of their portfolio with low-risk assets such as bonds and treasury bills are classified as conservative investors (Korkmaz et al., 2013). Investors who evenly divide their portfolio between stocks and bonds or treasury bills are considered balanced investors, while those who make up a significant portion of their portfolio with stocks are labeled as aggressive investors.

For the investor, risk is more important than expected return. When minimizing risk becomes the primary objective, investors tend to lean towards using risk-based portfolio theories. Among these methods, one of the oldest but at the same time least utilized is semi-variance. Since it was first proposed by Markowitz in 1959, the number of studies conducted on it has not been very high. Until the 1990s, the semi-variance method was not widely favored due to the high costs associated with computerized trading. However, since the early 2000s, computer technologies have experienced a significant leap, and parallel to this, transaction costs have reduced to very low levels. Nevertheless, the utilization of semi-variance as a risk measure has remained limited.

The most significant aspect that sets semi-variance apart from variance is its consideration of only returns that fall below the mean return. Returns that exceed the mean return are generally not considered a risk from the investor's perspective because they represent a desirable outcome. In this regard, semi-variance, compared to variance, provides a foundation for creating portfolios that are more in line with the investor's risk perception in risk measurement. Additionally, in the mean-variance method, it is assumed that returns follow a normal distribution. If the return distribution is not normal, the optimal portfolios obtained will not be considered optimal because they will not lie on the efficient frontier. This situation is a weakness of the mean-variance approach. Thus, investors may fall into the misconception of investing in the optimal portfolio. In the mean-semi-variance method, the assumption of normal distribution for returns is not made. Therefore, the most robust point of the method is that portfolios created according to semi-variance are on the efficient frontier, or in other words, they are considered optimal.

The weakest point of the semi-variance method is the difficulty of its implementation and the complexity of the required calculations. In this case, there is a danger for investors again, as suboptimal portfolios may be preferred.

Optimal portfolio, or portfolio optimization, fundamentally aims to either maximize return while keeping risk constant or minimize risk while keeping return constant. A portfolio that satisfies either of these assumptions is considered efficient (Akyer et al., 2018). In this case, regardless of which variable is kept constant, the optimal portfolio determines not only the composition of the portfolio in terms of securities but also the weights of these securities within the portfolio. To achieve this, the covariances or correlation coefficients between securities guide the investor in determining which securities should make up the portfolio. Moreover, means and variances also play a role in determining the weights of the securities within the portfolio.

The main objective of this study is to compare the performances of portfolios constructed based on variance and semi-variance using data obtained from the BIST-100 Index. According to the obtained results, portfolios constructed based on semi-variance appear to better protect investors with low risk tolerance from unexpected negative return risks.

II. Literature Review

In the Modern Portfolio Theory (MPT), Markowitz demonstrated that by calculating the expected return rate of a portfolio, the variation in the return rate is a significant measure of portfolio risk. Based on these assumptions, he developed a formula for calculating portfolio variance by computing efficient portfolio diversification. Markowitz's model is based on various assumptions about investor behavior. One of the most important of these is that investors assess risk based on the variability of the portfolio's expected return. Another crucial assumption is that investors will always prefer the highest return for a specific level of risk or the lowest risk for a specific level of return. More technically, MPT models an asset as a function of a normal distribution (or more generally, a random variable distributed elliptically) and defines risk as the standard deviation transformation. In other words, it models the portfolio as a weighted combination of assets. Thus, the portfolio is a weighted combination of asset returns. By combining different assets with imperfect positive correlations, MPT aims to reduce the total variance of the portfolio return. In this context, MPT provides the selection of a portfolio with the highest possible return rate for a specific amount of risk or the lowest possible risk for a specific expected return level.

Markowitz has three significant contributions to the traditional portfolio theory. The first one is proving that the risk of each asset that makes up the portfolio is not the same as the risk of the portfolio formed by these assets. Markowitz demonstrated that the portfolio risk can be less than the risk of the individual assets that make up the portfolio, and the portfolio's unsystematic risk can be zero under certain conditions (Ceylan and Korkmaz, 1998). The second one is the principle of dominance. He argued that investors would not prefer some portfolios because they are riskier even though they offer the same return, and they would not prefer others because they provide less return even though they have the same risk level. Therefore, he demonstrated that a portfolio chosen by the investor based on their preferences and sensitivity to risk is superior to others (Yayalar, 2016). His third contribution is the concept of the efficient frontier. Markowitz preferred quadratic programming, which uses the standard deviations, covariances, and expected returns of securities to calculate optimal portfolios. He named the line formed by the composition of optimal portfolios the efficient frontier. The efficient frontier represents the set of portfolios that provides the maximum return-to-risk ratio for a specific level of risk or the minimum risk-to-return ratio for a specific level of return. The optimal portfolio is the point where the efficient frontier and the investor's maximum utility curve are tangent to each other. Under this condition, all portfolios located on the efficient frontier are optimal. It has both a high return-to-risk ratio for equal risk levels and a low risk-to-return ratio for equal return levels. Depending on their risk tolerance and utility function, the investor should set a target on the efficient frontier. Because having a portfolio on the efficient frontier demonstrates that the portfolio is superior to other constructed portfolios.

Markowitz (1952) created a standard model in his research to address asset allocation (portfolio diversification) issues. This model is known as Mean-Variance Optimization or Mean-Variance Analysis, which is considered a cornerstone of Modern Portfolio Theory. In this model, the first criterion used is the expected return of assets, and the second is variance, which serves as a measure of risk. In this model, the goal is to minimize the portfolio variance when the expected return is set at a predetermined level. In the model, variance is used as a measure of risk. The principle of the model's application is a periodical, discrete-time optimization problem in which investors determine their investment decisions at the beginning of a period and cannot make any changes to these decisions until the end of the period.

According to Braga (2016), the finance literature and practice are based on two main ideas: the first is diversification, which is argued to be an ideal method for managing risk. Markowitz demonstrates that risk management through diversification is based on the fact that assets are not solely dependent on their individual risks but are influenced by the correlation/covariance between asset classes. This means that diversification cannot be achieved simply by increasing the number of asset classes in a single portfolio. The second concept is that investors make decisions for the assets used in the portfolio in a two-dimensional space. According to Markowitz (1952), expected return is a desired outcome for investors, while the variance of return is an undesired outcome. Based on this, it is emphasized that just as expected return is important, risk is also crucial for investors in portfolio construction.

Markowitz (1952) introduced the Mean-Variance model, in which variance is used as a measure of risk in portfolio optimization. The objective of the Mean-Variance model is to minimize the variance of a portfolio at a given expected return level. In this context, variance measures the deviation above and below the mean return (Schneeweis et al., 2010). The variance of the portfolio is calculated by computing the covariance matrix of asset returns. Variance may not be an appropriate risk measure because it includes both negative deviation (negative returns) and positive deviation (positive returns) in the model. However, while positive deviation is a desirable outcome for investors, negative deviation is considered undesirable. Therefore, variance may not be consistent with investors' actual perception of risk. Semi-variance, on the other hand, differs from variance by only considering observations below the mean. Additionally, semi-variance provides a way to mitigate downside risk in portfolio or asset analysis, making it a potentially valuable tool in this regard.

Semi-variance only takes into account the negative fluctuations of an asset, whereas standard deviation and variance provide measurements of overall volatility. Semi-variance can be used to calculate the average potential loss a portfolio may be exposed to by neutralizing all values above the mean or above a target return level of an investor. For risk-averse investors, minimizing semi-variance can help determine the optimal portfolio and mitigate risk. In other words, semi-variance is an appropriate risk measure because investors are more concerned about losses below the target return than they are about gains relative to the return. Markowitz (1959) proposed the Semi-Variance model, using semi-variance instead of variance, to overcome the weaknesses of the mean-variance model. Semi-variance is defined as follows:

$$\mathbf{SVar(R_p) = \frac{1}{N}\sum_{i=1}^{N} Max\ (0,\ E(R_i) - R_i)^2}$$

N: The total number of assets in the portfolio,

R_i: The observed value of it,

$E(R_i)$: Mean or target return

After the introduction of the semi-variance model by Markowitz in 1959, the variance measure continued to be used due to its simpler calculation. The main reasons for this are that the semi-variance model requires twice as much data input as the variance model, a lack of low-cost computer power, and the mathematical complexity of the variance model itself. Markowitz emphasized that an investor looking to allocate their investment to a group of securities should not only seek to maximize the profit associated with this allocation but also pay attention to reducing the associated risk. Markowitz (1952) has made significant contributions to the literature and proposed an average variance optimization model aimed at minimizing portfolio risk (measured by variance) for a given expected return level over a range of feasible portfolios. By altering the expected return level, the model establishes the efficient frontier as the set of efficient portfolios. However, as Markowitz (1959) noted, normal return distributions and second-degree utility functions are sufficient for the use of mean-variance analysis, but they are inadequate for non-normally distributed returns.

Roy (1952) was the first to emphasize the importance of a downside risk measure in portfolio selection, a principle he referred to as the "safety first" rule. Markowitz (1959, 1991) recognized the significance of Roy's work and the importance of measures of risk that assess the probability of falling below a predetermined target return. He argued that these measures provide more reasonable risk metrics compared to variance. To achieve this, he recommended using semi-variance as a part of his work.

Research on semi-variance continued in the 1960s and early 1970s. Quirk and Saposnik (1962) highlighted the theoretical superiority of semi-variance over variance in their study. Mao (1970) provided strong evidence that investors are primarily concerned with downside risk and, therefore, the use of semi-variance as a measure is necessary.

Many studies conducted by Mandelbrot (1963), Fama (1965), Campbell and Hentschel (1992), and Turner and Weigel (1992) have shown that portfolio returns generally do not follow a normal distribution. On the other hand, Rubinstein (1973), Kraus and Litzenberger (1976), and Harvey and Siddique (2000) in their studies have demonstrated that investors have skewness preferences and that utility functions are often not second-degree.

According to Nawrocki (1999), the use of the semi-variance measure in abnormal return distributions is more appropriate than the traditional use of variance. According to the study by Mut (2009), semi-variance allows for the selection of the asset with a lower probability of loss between two assets with the same mean, thereby more effectively addressing the investor's perception of risk.

Based on the information provided, the objective of this study is to investigate the effectiveness of Markowitz's 1959 proposed semi-variance model in portfolio optimization using historical data of stocks within the BIST-100 Index. The assumption is that risk is a primary factor in investment decisions. The study aims to explore how effective the semi-variance model could be in optimizing portfolios by considering the historical data of stocks within the index. In line with this objective, the aim is to present a new alternative to portfolio investment firms, portfolio managers, and researchers.

III. Method

The research encompasses the use of the Portfolio Selection Model, which earned Harry Markowitz the Nobel Prize. For this purpose, the mean-variance matrix was calculated using the average, variance, and covariance values. The model aims to investigate the minimum level of risk the investor needs to undertake in order to achieve the targeted level of return or, for a certain accepted level of risk, to maximize the expected returns of the portfolio structure (Ulucan, 2007).

Within the scope of the study, daily adjusted weighted average price data of stocks continuously listed on the BIST-100 index were used for a period spanning 4,377 days, from July 24, 2000, to December 29, 2017. This period includes business days and excludes weekends and holidays. During this period, stocks that were not continuously in the BIST-100 index - whether they were removed from the index during this time or added at a later stage - have been excluded from the study. The daily weighted average price data in Turkish Lira for the 24 stocks that continuously remained in the index throughout the entire period were obtained from Borsa İstanbul. The 24 stocks are collectively provided in Table 1.

Table 1. Securities Present in the BIST-100 Index between December 29, 2017, and July 24, 2000

AKBNK	AKBANK
AKSA	AKSA
ALARK	ALARKO HOLDİNG
ARCLK	ARÇELİK
ASELS	ASELSAN
AYGAZ	AYGAZ
DOHOL	DOĞAN HOLDİNG
ECILC	ECZACIBAŞI İLAÇ
AEFES	ANADOLU EFES
ENKAI	ENKA İNŞAAT
EREGL	EREĞLİ DEMİR ÇELİK
GARAN	GARANTİ BANKASI
ISCTR	İŞ BANKASI
KRDMD	KARDEMİR
KCHOL	KOÇ HOLDİNG
MIGRS	MİGROS
PETKM	PETKİM
SAHOL	SABANCI HOLDİNG
SISE	ŞİŞE CAM
TOASO	TOFAŞ
TRKCM	TRAKYA CAM
TUPRS	TÜPRAŞ
THYAO	TÜRK HAVA YOLLARI
YKBNK	YAPI VE KREDİ BANKASI

Source: Authors, 2019.

Using the price data obtained from Borsa Istanbul, 4376 return data points were calculated for each stock, and the mean, variance, and covariance values of the stock returns were determined. These values obtained from 4377 days of historical price data, such as mean, variance, and covariance, represent the actual parameters of the stocks' returns. For the calculation of semi-variance, the above-mentioned processes were repeated, excluding positive return values. The first step in calculating semi-variance is obtaining price data. Next, returns are calculated using these price data. The following step involves calculating the average of these returns. It is important to note that the process continues by considering only negative returns, excluding positive ones. The difference between each negative return and the average is calculated, and after obtaining these differences, their squares are summed up. This total sum is then divided by the total number of initial observations to obtain the semi-variance.

Using the variance-covariance matrix provided in Table 2, optimal portfolio structures based on different risk tolerances were computed, along with the portfolio's average return, risk, and objective function values.

Table 3 shows the weights of each security in the optimal portfolio according to different risk tolerances using the semi-variance covariance matrix, as well as the average return, risk, and objective function values obtained in portfolios based on these weights.

Table 2. Variance-Covariance Matrix

VARIANCE - COVARIANCE MATRIX																								
	AKBNK	**AKSA**	**ALARK**	**ARCLK**	**ASELS**	**AYGAZ**	**DOHOL**	**ECILC**	**AEFES**	**ENKAI**	**EREGL**	**GARAN**	**ISCTR**	**KRDMD**	**KCHOL**	**MIGRS**	**PETKM**	**SAHOL**	**SISE**	**TOASO**	**TRKCM**	**TUPRS**	**THYAO**	**YKBNK**
AKBNK	5.60	3.00	2.97	3.58	2.90	2.86	3.71	3.05	2.24	2.76	3.35	4.86	4.45	3.73	3.78	2.92	3.07	3.97	3.58	3.56	3.11	2.92	3.35	4.73
AKSA	3.00	4.92	2.64	2.82	2.60	2.57	3.03	2.75	2.02	2.32	2.78	3.32	3.07	3.22	2.92	2.47	2.71	2.84	2.95	2.93	2.68	2.45	2.79	3.33
ALARK	2.97	2.64	4.23	2.79	2.56	2.59	3.12	2.71	1.96	2.37	2.70	3.30	3.14	3.08	2.94	2.52	2.62	2.90	2.94	2.91	2.75	2.44	2.80	3.25
ARCLK	3.58	2.82	2.79	5.59	2.70	2.91	3.51	2.87	2.22	2.58	3.15	3.92	3.51	3.52	3.60	2.64	2.91	3.42	3.29	3.58	2.93	2.76	3.02	3.94
ASELS	2.90	2.60	2.56	2.70	6.85	2.56	3.00	2.90	1.84	2.23	2.64	3.32	3.05	3.40	2.87	2.45	2.60	2.72	2.87	2.85	2.66	2.44	2.81	3.36
AYGAZ	2.86	2.57	2.59	2.91	2.56	4.39	3.13	2.63	1.98	2.28	2.73	3.26	3.07	3.12	3.00	2.38	2.71	2.84	2.91	2.91	2.67	2.70	2.77	3.18
DOHOL	3.71	3.03	3.12	3.51	3.00	3.13	8.20	3.19	2.37	2.70	3.41	4.33	3.90	3.91	3.69	2.80	3.33	3.59	3.69	3.61	3.17	2.95	3.38	4.43
ECILC	3.05	2.75	2.71	2.87	2.90	2.63	3.19	5.30	1.92	2.41	2.80	3.44	3.19	3.41	2.96	2.56	2.77	2.93	3.02	2.94	2.84	2.52	2.95	3.44
AEFES	2.24	2.02	1.96	2.22	1.84	1.98	2.37	1.92	4.97	1.85	2.11	2.61	2.46	2.32	2.26	1.90	1.90	2.19	2.18	2.28	2.06	1.97	1.88	2.42
ENKAI	2.76	2.32	2.37	2.58	2.23	2.28	2.70	2.41	1.85	4.68	2.65	3.03	2.82	2.78	2.80	2.13	2.37	2.72	2.72	2.84	2.51	2.37	2.39	3.02
EREGL	3.35	2.78	2.70	3.15	2.64	2.73	3.41	2.80	2.11	2.65	5.25	3.69	3.44	3.86	3.34	2.50	3.00	3.24	3.30	3.34	2.83	2.90	2.91	3.78
GARAN	4.86	3.32	3.30	3.92	3.32	3.26	4.33	3.44	2.61	3.03	3.69	6.89	4.92	4.21	4.20	3.13	3.41	4.18	3.94	4.02	3.43	3.30	3.65	5.50
ISCTR	4.45	3.07	3.14	3.51	3.05	3.07	3.90	3.19	2.46	2.82	3.44	4.92	5.54	3.94	3.85	3.15	3.17	3.94	3.70	3.71	3.23	3.06	3.49	4.88
KRDMD	3.73	3.22	3.08	3.52	3.40	3.12	3.91	3.41	2.32	2.78	3.86	4.21	3.94	9.46	3.53	2.91	3.45	3.46	3.62	3.51	3.30	2.93	3.71	4.39
KCHOL	3.78	2.92	2.94	3.60	2.87	3.00	3.69	2.96	2.26	2.80	3.34	4.20	3.85	3.53	4.92	2.82	3.04	3.81	3.52	3.64	3.12	3.06	3.21	4.21
MIGRS	2.92	2.47	2.52	2.64	2.45	2.38	2.80	2.56	1.90	2.13	2.50	3.13	3.15	2.91	2.82	5.26	2.42	2.87	2.75	2.68	2.59	2.25	2.66	3.19
PETKM	3.07	2.71	2.62	2.91	2.60	2.71	3.33	2.77	1.90	2.37	3.00	3.41	3.17	3.45	3.04	2.42	5.18	3.02	3.04	3.02	2.75	2.70	3.35	3.51
SAHOL	3.97	2.84	2.90	3.42	2.72	2.84	3.59	2.93	2.19	2.72	3.24	4.18	3.94	3.46	3.81	2.87	3.02	4.92	3.47	3.49	3.08	2.97	3.16	4.18
SISE	3.58	2.95	2.94	3.29	2.87	2.91	3.69	3.02	2.18	2.72	3.30	3.94	3.70	3.62	3.52	2.75	3.04	3.47	5.16	3.56	3.56	2.89	3.20	3.98
TOASO	3.56	2.93	2.91	3.58	2.85	2.91	3.61	2.94	2.28	2.84	3.34	4.02	3.71	3.51	3.64	2.68	3.02	3.49	3.56	6.28	3.16	2.88	3.12	3.96
TRKCM	3.11	2.68	2.75	2.93	2.66	2.67	3.17	2.84	2.06	2.51	2.83	3.43	3.23	3.30	3.12	2.59	2.75	3.08	3.56	3.16	4.67	2.57	2.87	3.44
TUPRS	2.92	2.45	2.44	2.76	2.44	2.70	2.95	2.52	1.97	2.37	2.90	3.30	3.06	2.93	3.06	2.25	2.70	2.97	2.89	2.88	2.57	4.60	2.59	3.23
THYAO	3.35	2.79	2.80	3.02	2.81	2.77	3.38	2.95	1.88	2.39	2.91	3.65	3.49	3.71	3.21	2.66	3.35	3.16	3.20	3.12	2.87	2.59	6.04	3.65
YKBNK	4.73	3.33	3.25	3.94	3.36	3.18	4.43	3.44	2.42	3.02	3.78	5.50	4.88	4.39	4.21	3.19	3.51	4.18	3.98	3.96	3.44	3.23	3.65	7.38
AVR. RETURN	0.09	0.07	0.04	0.08	0.13	0.08	0.04	0.10	0.08	0.09	0.11	0.10	0.06	0.12	0.08	0.06	0.07	0.06	0.10	0.10	0.09	0.11	0.09	0.05
STD. DEVIATION	2.37	2.22	2.06	2.36	2.62	2.10	2.86	2.30	2.23	2.16	2.29	2.63	2.35	3.08	2.22	2.29	2.27	2.22	2.27	2.51	2.16	2.15	2.46	2.72

Source: Authors, 2019.

Table 3. Weights of Securities in the Optimal Portfolio According to the Variance-Covariance Matrix

PORTFOLIO		TOLERANCE	GOAL	WEIGHTINGS of SECURITIES in the OPTIMAL PORTFOLIO (%)																								
RETURN	RISK	RT	FUNCTION	AKBNK	AKSA	ALARK	ARCLK	ASELS	AYGAZ	DOHOL	ECILC	AEFES	ENKAI	EREGL	GARAN	ISCTR	KRDMD	KCHOL	MIGRS	PETKM	SAHOL	SISE	TOASO	TRKCM	TUPRS	THYAO	YKBNK	TOPLAM
0.09	2.59	**5**	-0.43	0.00	0.06	0.03	0.00	0.08	0.09	0.00	0.05	0.21	0.16	0.02	0.00	0.00	0.00	0.00	0.09	0.03	0.00	0.00	0.00	0.04	0.13	0.02	0.00	1
0.09	2.63	**10**	-0.17	0.00	0.04	0.00	0.00	0.11	0.08	0.00	0.06	0.21	0.15	0.04	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00	0.04	0.15	0.03	0.00	1
0.10	2.67	**15**	-0.08	0.00	0.02	0.00	0.00	0.13	0.07	0.00	0.07	0.21	0.15	0.06	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.04	0.17	0.03	0.00	1
0.10	2.72	**20**	-0.04	0.00	0.00	0.00	0.00	0.16	0.06	0.00	0.07	0.20	0.14	0.08	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.04	0.19	0.02	0.00	1
0.10	2.78	**25**	-0.01	0.00	0.00	0.00	0.00	0.18	0.04	0.00	0.08	0.20	0.14	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.21	0.02	0.00	1
0.10	2.83	**30**	0.01	0.00	0.00	0.00	0.00	0.20	0.02	0.00	0.08	0.19	0.13	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.23	0.01	0.00	1
0.11	2.89	**35**	0.02	0.00	0.00	0.00	0.00	0.22	0.00	0.00	0.07	0.18	0.12	0.12	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.24	0.00	0.00	1
0.11	2.95	**40**	0.03	0.00	0.00	0.00	0.00	0.24	0.00	0.00	0.07	0.17	0.10	0.13	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.26	0.00	0.00	1
0.11	3.01	**45**	0.04	0.00	0.00	0.00	0.00	0.26	0.00	0.00	0.06	0.16	0.09	0.14	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.00	1
0.11	3.07	**50**	0.05	0.00	0.00	0.00	0.00	0.28	0.00	0.00	0.05	0.14	0.08	0.15	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00	0.00	1
0.11	3.13	**55**	0.05	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.05	0.13	0.06	0.15	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00	0.00	1
0.11	3.21	**60**	0.06	0.00	0.00	0.00	0.00	0.31	0.00	0.00	0.04	0.12	0.05	0.16	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.00	1
0.11	3.29	**65**	0.06	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.03	0.10	0.03	0.16	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	1
0.12	3.37	**70**	0.07	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.02	0.09	0.02	0.17	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	1
0.12	3.81	**100**	0.08	0.00	0.00	0.00	0.00	0.43	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.00	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00	0.00	1
0.12	4.14	**150**	0.10	0.00	0.00	0.00	0.00	0.53	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.00	1
0.13	4.60	**200**	0.10	0.00	0.00	0.00	0.00	0.64	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00	0.00	1
0.13	5.83	**300**	0.11	0.00	0.00	0.00	0.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	1
0.13	6.38	**400**	0.12	0.00	0.00	0.00	0.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
0.13	6.82	**500**	0.12	0.00	0.00	0.00	0.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
0.13	6.85	**520**	0.12	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1

Source: Authors, 2019.

The equation Z(x) given below represents the expected utility of the investor's holdings. The section up to the negative sign represents the portfolio's expected return, while the part following the negative sign indicates the portfolio's expected risk.

$$Max\ Z(x) = \sum_{i=1}^{n} E[r_i] \cdot x_i - \frac{1}{R_T(w)} \cdot \sum_{i=1}^{n} \sum_{j \leq 1}^{n} x_i \cdot x_j \cdot E\left[\sigma_{ij}\right]$$

$\sum_{i=1}^{n} X_i = 1$ ve $X_i \geq 0$

i = 1, 2, …. $\mathrm{N}x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Z(x)	: investor's expected utility from the asset is represented by...
$E[r_i]$	: the expected return of the i^{th} security is represented by...
σ_{ij}	: the covariance of the expected returns between securities i and j is represented by...
R_T	: the investor's risk tolerance is represented by...
r_i	: the expected return of the i^{th} security is typically denoted by...
X_i	**:** the weight of the i^{th} security in the portfolio is represented by...
n	: the number of risky securities is usually denoted by...

The portfolio's expected return and the risk of the portfolio weighted by the risk tolerance jointly provide the value of the objective function. Hence, the objective function in the table showing the results of the calculations represents the value of the investor's utility function according to increasing risk tolerances. In the next stage, using the mean-variance matrix, the weights of securities in the optimal portfolio structure were calculated concerning the investor's portfolio, which varies based on the investor's risk tolerance increasing from 5 and increasing in increments of 5 units, according to the return and risk of the portfolio. Subsequently, the same procedures were applied to the mean-semi variance model, computing the expected value of the portfolio and the weights of securities within the portfolio for different risk tolerances. Performing the operations separately for both the mean-variance and mean-semi variance models is aimed at facilitating the comparison between these two models.

In the final stage, using the optimal portfolio structures obtained for different levels of risk tolerance from both the mean-variance and mean-semi variance models, the expected portfolio returns for the period between January 1, 2018, and January 21, 2019 were calculated, allowing for a comparison between the two models. In both the mean-variance and mean-semi variance models, the levels of risk tolerance were increased until the point where the optimal portfolio consists of only a single security. A risk tolerance level exceeding 100 indicates a scenario where the investor uses financial leverage, i.e., resorts to borrowing, to be able to invest more. The Solver tool in Excel was utilized for all the optimization calculations.

IV. Findings

During portfolio optimization, the aim is to minimize risk while simultaneously aiming to maximize returns. Various opinions and methods have been put forward on this subject. Over time, the increasing variety of investment instruments and the advancement in technology have made access to these instruments easier, leading to the emergence of new perspectives and methods. There are two aspects to an investment: risk and return. Risk arises from the uncertainty of data, such as the projected returns of an investment in the future. An investor requires a minimum investment return that is acceptable considering the risk undertaken. Hence, the expected return from every category of investor is dependent on the risk taken. This means that the primary decision-making stage for an investor is determining the level of risk they are willing to accept.

In this study, optimal portfolios have been created using return data from July 24, 2000, to December 29, 2017, based on both the variance-covariance matrix and the semi-variance-covariance matrix. Subsequently, optimal portfolios were constructed using return data from January 1, 2018, to January 21, 2019, and returns were calculated for each level of risk tolerance. These results are presented in Table 4.

Table 4. Comparison of Results

Risk Tolerance Levels	Variance – Covariance Matrix	Semi-Var. – Covariance Matrix	2018 – 19 Var.- Covariance Matrix
5	-0.0610798	0.0559348	0.0035341
10	-0.0514098	0.0667688	0.0337688
15	-0.0443146	0.0776029	0.0561394
20	-0.0373156	0.0816609	0.0777502
25	-0.0319508	0.0639496	0.0927043
30	-0.0307314	0.0432108	0.0954473
35	-0.0296319	0.0224720	0.0957731
40	-0.0288503	0.0017331	0.0960989
45	-0.0280490	-0.0190060	0.0964247
50	-0.0273577	-0.0397440	0.0967505
55	-0.0266683	-0.0604830	0.0970763
60	-0.0259786	-0.0812220	0.0974021
65	-0.0252889	-0.0930870	0.0977280
70	-0.0245990	-0.0930870	0.0980538

Source: Authors, 2019.

As seen in Table 4, for the optimal portfolios obtained according to the variance-covariance matrix and held between January 1, 2018, and January 21, 2019, there is no positive expected return for any level of risk tolerance. For all portfolios, the expected returns, expressed as a percentage, increase from -6.11% to -2.46% across risk tolerance levels from 5 to 70. According to these results, all portfolios obtained using the variance - covariance matrix would cause losses to the investor between January 1, 2018, and January 21, 2019. Indeed, this is the last thing an investor would want.

If an investor constructs their portfolios using the semi-variance-covariance matrix, for risk tolerance levels between 5 and 40, all portfolios will have positive expected returns. Additionally, between risk tolerance levels of 5 and 20, the portfolio's expected return increases from 5.60% to 8.17%; thereafter, it decreases. After a risk tolerance level of 45, the expected return of the portfolio becomes negative. The positive outcomes of portfolios created at low risk tolerance levels, such as 5 to 20, for the 2018-2019 period, indicate that portfolios formed based on semi-variance would potentially allow the investor to preserve their capital or even generate returns, in case the returns turn negative. Therefore, this result is consistent with previous studies on portfolios created based on semi-variance.

Another important finding is that the expected return values of portfolios created according to semi-variance up to 20 risk tolerance levels, using historical data, are even higher than those of portfolios formed based on 2018-2019 data. This indicates that portfolios created with low risk tolerance according to semi-variance may better protect investors from various unexpected risks compared to those with higher risk tolerance. As can be understood from the data, securities in the BIST-100 index generally provided negative returns during the period from January 2018 to January 2019. Nevertheless, portfolios formed according to semi-variance have successfully protected investors from the risk of negative returns.

The fourth column of Table 4 shows the average returns of portfolios that would be obtained by creating optimal portfolios at various risk tolerances based solely on the data between 2018 and 2019, without considering any historical data. When the third and fourth columns are evaluated together, a much more interesting result is obtained. The expected return values of portfolios created based on historical data using semi-variance up to a risk tolerance level of 20 are higher than the expected return values of portfolios created based on data from 2018 to 2019. This finding indicates that portfolios with low-risk tolerances created based on semi-variance would better protect the investor from various unexpected risks.

V. Result

The obtained findings are consistent with the results of studies by Quirk and Saposnik (1962), Mao (1970), Mandelbrot (1963), Fama (1965), Campbell and Hentschel (1992), Turner and Weigel (1992), Rubinstein (1973), Kraus and Litzenberger (1976), Nawrocki (1999), Harvey and Siddique (2000), and Mut (2009).

The purpose of this study is to compare the performance of portfolios created based on variance and semi-variance using data obtained from the BIST-100 Index. When the research findings are generally evaluated, it is observed that the securities in the BIST-100 Index generally provided negative returns during the 2018-2019 period. However, portfolios created according to semi-variance have protected the investor from the risk of negative returns. These findings demonstrate that as risk tolerance levels increase, the returns of the portfolios also increase.

In line with the findings of the study, it is recommended for investors and portfolio managers to consider using the Semi-Variance Model to obtain portfolios that are protected against risks. Determining the performance of this model during crisis periods and short-term periods could be a subject for future research endeavors. It is anticipated that investors and portfolio managers with a long-term investment horizon can protect their portfolios against risk using this model.

Despite the obtained findings, the primary reason for semi-variance not being widely preferred by investors is the complexity and difficulty in implementing the method. According to the results, portfolios that better protect investors with low risk tolerance from unexpected negative return risks have been constructed based on semi-variance.

ARTICLE INFORMATION FORM

Author Contributions:
Idea / Concept: Serdar Ramazan KAHRAMAN and Kartal SOMUNCU
Research Design: Serdar Ramazan KAHRAMAN and Kartal SOMUNCU
Article Writing: Serdar Ramazan KAHRAMAN and Kartal SOMUNCU
Data Collection: Serdar Ramazan KAHRAMAN and Kartal SOMUNCU
Analysis: Serdar Ramazan KAHRAMAN and Kartal SOMUNCU
Critical Reading: Serdar Ramazan KAHRAMAN and Kartal SOMUNCU
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