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ERRATUM: "UNIQUENESS OF MEROMORPHIC FUNCTIONS SHARING TWO FINITE SETS IN C WITH FINITE WEIGHT " [KONURALP J. MATH., 2(2)(2014), 42-52.]

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First of all the statements of *Theorem 1.1* and *Theorem 1.2* should be the following.

Theorem 1.1. Let $S_1 = \{0, -a\frac{n-1}{n}\}$, $S_2 = \{z : z^n + az^{n-1} + b = 0\}$ where $n(\geq 7)$ be an integer and a and b be two nonzero constants such that $z^n+az^{n-1}+b = 0$ has no multiple root. If f and g be two non-constant meromorphic functions having no simple pole such that $E_f(S_1, 0) = E_g(S_1, 0)$ and $E_f(S_2, 2) = E_g(S_2, 2)$, then $f \equiv g$.
Theorem 1.2. Let S_i, $i = 1$, 2 and f and g be taken as in *Theorem 1.1* where $n(\geq 8)$ is an integer. If $E_f(S_1, 0) = E_g(S_1, 0)$ and $E_f(S_2, 1) = E_g(S_2, 1)$, then $f \equiv g$.

Next by calculation it can be shown that in Lemma-2.2 we would always have $p = 0$. So in Lemma-2.2 we should replace $\overline{N}(r, 0; f \mid\geq p+1)+\overline{N}\left(r, -a\frac{n-1}{n}; f \mid\geq p+1\right)$ by $\overline{N}(r, 0; f) + \overline{N}\left(r, -a\frac{n-1}{n}; f\right)$. In that case the statement of the Lemma-2.2. should be replaced by

Lemma-2.2. Let S_1 and S_2 be defined as in *Theorem 1.1* and F, G be given by (2.1). If for two non-constant meromorphic functions f and g, $E_f(S_1, 0) = E_g(S_1, 0)$, $E_f(S_2, 0) = E_g(S_2, 0)$, where $H \not\equiv 0$ then

$$\begin{aligned} N(r, H) \quad \leq \quad & \overline{N}(r, 0; f) + \overline{N}\left(r, -a\frac{n-1}{n}; f\right) + \overline{N}_*(r, 1; F, G) \\ & + \overline{N}(r, \infty; f) + \overline{N}(r, \infty; g) + \overline{N}_0(r, 0; f^{'}) + \overline{N}_0(r, 0; g^{'}), \end{aligned}$$

where $\overline{N}_0(r, 0; f^{'})$ is the reduced counting function of those zeros of $f^{'}$ which are not the zeros of $f\left(f - a\frac{n-1}{n}\right)(F - 1)$ and $\overline{N}_0(r, 0; g^{'})$ is similarly defined.

Since throughout the paper we would have $p = 0$, so *Lemma-2.5* used in the paper is redundant.

There is also a gap in the analysis of the proof of *Lemma-2.7*. But the lemma can be proved in a more simpler way with the support of Corollary of Theorem

4.1, p. 216, { H.X. Yi and C.C. Yang, Uniqueness Theory of Meromorphic Functions, Science Press and Kluwer Academic Publishers (2003)}, when $n \geq 3$. As this supposition will not hamper the statement as well as the proof of the main theorem, we replace the old *Lemma-2.5* as used in the main paper by the following Corollary of Theorem 4.1, p. 216, { H.X. Yi and C.C. Yang, Uniqueness Theory of Meromorphic Functions, Science Press and Kluwer Academic Publishers (2003)}.

Lemma-2.5. Let f, g be two non-constant meromorphic functions. If f and g share four distinct values 0, 1, ∞, c CM and $c \neq -1, \frac{1}{2}, 2$, then $f \equiv g$.

In view of *Lemma-2.2*, *Lemma-2.6* will be changed which is given below in its corrected form.

Lemma-2.6. Let S_1, S_2 be defined as in *Theorem 1.1* and F, G be given by (2.1). If for two non-constant meromorphic functions f and g, $E_f(S_1, 0) = E_g(S_1, 0)$ and $E_f(S_2, m) = E_g(S_2, m)$, where $1 \leq m < \infty$ and $H \not\equiv 0$, then

$$\begin{aligned} & (n+1)\ \{T(r,f) + T(r,g)\} \\ \leq\ & 3\left\{\overline{N}(r,0;f) + \overline{N}\left(r, -a\frac{n-1}{n}; f\right)\right\} + 2\{\overline{N}(r,\infty;f) + \overline{N}(r,\infty;g)\} \\ & + \frac{1}{2}\left[N(r,1;F) + N(r,1;G)\right] - \left(m - \frac{3}{2}\right)\overline{N}_*(r,1;F,G) + S(r,f) + S(r,g). \end{aligned}$$

Proof. By the second fundamental theorem we get

$$\begin{aligned} (2.4)\qquad & (n+1)\{T(r,f) + T(r,g)\} \\ \leq\ & \overline{N}(r,1;F) + \overline{N}(r,0;f) + \overline{N}\left(r, -a\frac{n-1}{n}; f\right) + \overline{N}(r,\infty;f) + \overline{N}(r,1;G) + \overline{N}(r,0;g) \\ & + \overline{N}\left(r, -a\frac{n-1}{n}; g\right) + \overline{N}(r,\infty;g) - N_0(r,0;f') - N_0(r,0;g') + S(r,f) + S(r,g). \end{aligned}$$

Using *Lemmas 2.1*, *2.2*, *2.3* and *2.4* we note that

$$\begin{aligned} (2.5)\qquad & \overline{N}(r,1;F) + \overline{N}(r,1;G) \\ \leq\ & \frac{1}{2}\left[N(r,1;F) + N(r,1;G)\right] + N(r,1;F \mid= 1) - \left(m - \frac{1}{2}\right)\overline{N}_*(r,1;F,G) \\ \leq\ & \frac{1}{2}\left[N(r,1;F) + N(r,1;G)\right] + \overline{N}(r,0;f) + \overline{N}\left(r, -a\frac{n-1}{n}; f\right) \\ & + \overline{N}(r,\infty;f) + \overline{N}(r,\infty;g) - \left(m - \frac{3}{2}\right)\overline{N}_*(r,1;F,G) + \overline{N}_0(r,0;f') + \overline{N}_0(r,0;g') \\ & + S(r,f) + S(r,g). \end{aligned}$$

Using (2.5) in (2.4) and noting that $\overline{N}(r,0;f) + \overline{N}\left(r, -a\frac{n-1}{n}; f\right) = \overline{N}(r,0;g) + \overline{N}\left(r, -a\frac{n-1}{n}; g\right)$ the lemma follows. □

Corresponding to the *Lemmas 2.5*, corrected version of *Lemma-2.7* would be as follows.

Lemma-2.7. Let f, g be two non-constant meromorphic functions such that $E_f(\{0, -a\frac{n-1}{n}\}, 0) = E_g(\{0, -a\frac{n-1}{n}\}, 0)$ then $f^{n-1}(f+a) \equiv g^{n-1}(g+a)$ implies $f \equiv g$, where $n\ (\geq 3)$ is an integer and a is a nonzero finite constant.

Proof. Since $E_f(\{0,-a\frac{n-1}{n}\},0)=E_g(\{0,-a\frac{n-1}{n}\},0)$, so from

$$f^{n-1}(f+a)\equiv g^{n-1}(g+a)$$

we have f, g share $(0,\infty)$, $(-a,\infty)$ and (∞,∞). Again differentiating

$$f^{n-1}(f+a)\equiv g^{n-1}(g+a)$$

we have

$$nf^{n-2}(f+\frac{a(n-1)}{n})f^{'}\equiv ng^{n-2}(g+\frac{a(n-1)}{n})g^{'},$$

which implies f, g share $(-a\frac{n-1}{n},\infty)$. It follows that $f_1=\frac{f}{-a}$, $g_1=\frac{g}{-a}$ share $(0,\infty)$, $(\frac{n-1}{n},\infty)$, $(1,\infty)$ and (∞,∞). As $\frac{n-1}{n}\neq -1,\frac{1}{2},2$ when $n\geq 3$, so in view of *Lemma 2.5*, we have $f\equiv g$. □

In view of *Lemma-2.2*, *Lemma-2.6* and *Lemma-2.7*, the proof of the main theorems will be changed. Below the corrected forms are given.

Proof of Theorem 1.1. Let F, G be given by (2.1). Then F and G share $(1,3)$. We consider the following cases.
Case 1. Let $H\not\equiv 0$. Then using *Lemma 2.6* for $m=2$ and *Lemma 2.4* we obtain

$$\begin{aligned}
(3.1)\qquad & (n+1)\,\{T(r,f)+T(r,g)\}\\
\leq\ & 3\left\{\overline{N}(r,0;f)+\overline{N}\left(r,-a\frac{n-1}{n};f\right)\right\}+2\{\overline{N}(r,\infty;f)+\overline{N}(r,\infty;g)\}\\
& +\frac{1}{2}\left[N(r,1;F)+N(r,1;G)\right]-\ \frac{3}{2}\overline{N}_*(r,1;F,G)\\
& +S(r,f)+S(r,g)\\
\leq\ & 3[T(r,f)+T(r,g)]+2\left[\frac{1}{2}\{N(r,\infty;f)+N(r,\infty;g)\}\right]+\frac{1}{2}\left[N(r,1;F)+N(r,1;G)\right]\\
& +S(r,f)+S(r,g)\\
\leq\ & (\frac{n}{2}+4)\{T(r,f)+T(r,g)\}+S(r,f)+S(r,g),
\end{aligned}$$

which gives a contradiction for $n\geq 7$.
Case 2 Let $H\equiv 0$. Now the conclusion of the theorem can be obtained from *Lemmas 2.10*, *2.8* and *2.7*. □

Proof of Theorem 1.2. Let F, G be given by (2.1). Then F and G share $(1,3)$. We consider the following cases.
Case 1. Let $H\not\equiv 0$. Then using *Lemma 2.6*, *Lemma 2.9* for $m=1$ and *Lemma*

2.4 we obtain

$$
\begin{aligned}
(3.2)\qquad & (n+1)\left\{T(r,f)+T(r,g)\right\} \\
\leq\ & 3\left\{\overline{N}(r,0;f)+\overline{N}\left(r,-a\frac{n-1}{n};f\right)\right\}+2\{\overline{N}(r,\infty;f)+\overline{N}(r,\infty;g)\} \\
& +\frac{1}{2}\left[N(r,1;F)+N(r,1;G)\right]+\frac{1}{2}\overline{N}_*(r,1;F,G)+S(r,f)+S(r,g) \\
\leq\ & 3[T(r,f)+T(r,g)]+2[\frac{1}{2}\{N(r,\infty;f)+N(r,\infty;g)\}]+\frac{1}{2}\left[N(r,1;F)+N(r,1;G)\right] \\
& +\frac{1}{4}\left\{\overline{N}(r,0;f)+\overline{N}\left(r,-a\frac{n-1}{n};f\right)+\overline{N}(r,0;g)+\overline{N}\left(r,-a\frac{n-1}{n};g\right)\right\} \\
& +S(r,f)+S(r,g) \\
\leq\ & \left(\frac{n}{2}+4+\frac{1}{2}\right)\{T(r,f)+T(r,g)\}+S(r,f)+S(r,g),
\end{aligned}
$$

which leads to a contradiction for $n \geq 8$.

We now omit the rest of the proof since the same is similar to that of *Theorem 1.1.* □