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What Does Symmetry Look Like? A Qualitative Research Based on Mental Images

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ABSTRACT

Symmetry, located within the field of transformation geometry, is a mathematical concept that plays an extremely important role as a problem-solving technique. Despite this, it is rarely used in solving mathematical problems in secondary school. This research was carried out to reveal the mental images of secondary school students regarding the concept of symmetry in the secondary school mathematics curriculum. In the study conducted based on a qualitative perspective, the metaphors developed by the students regarding the concept of symmetry and the visuals they drew were examined. The study was conducted in the spring semester of the 2022-2023 academic year with 223 students studying in the 5th, 6th, and 7th grades at a public school in Ankara, selected according to the convenient sampling method. Content analysis was used to analyze and interpret the data obtained from the students. Looking at the results of the research; it turns out that secondary school students generally perceive the concept of symmetry as "having similar functionality", "visually overlapping" and "perfect and meeting in differences".

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INTRODUCTION

Mathematics, which we encounter in almost every aspect of life, enters the lives of children from a very young age. An example of this phenomenon is that children become acquainted with the concept of symmetry, although not consciously, through rope printing during the preschool period. Similarly, activities such as finding the similar shape of a given shape or correctly identifying the matching shoe or sock can also be shown as effective activities in the formation of the concept of symmetry at this age.

The symmetry we see in many fields, from art to nature, can be defined in different ways. For example, Leikin, Berman, and Zaslavsky (1997) define symmetry as a transformation that does not change the properties of the shape when applied. On the other hand, one of the most general definitions of symmetry is the mathematical meaning in which symmetry concerning the line is emphasized. In this definition, children form the concept of shape. Another definition is the meaning, which includes the concepts of balance and proportion and is used to indicate the harmonious combination of various parts within a whole. In this sense, efforts are made to develop the aesthetic feeling (Olkun, 2006; Öçal and Öçal, 2021). Since symmetry can be defined in diverse ways, it can be difficult to determine whether a shape has symmetry or not. Van de Walle, Karp, and Bay-Williams (2019) stated that to determine whether a shape has line or mirror symmetry, it is sufficient that the shape can be divided into two equal parts when folded along the line.

Symmetry, located within the field of transformation geometry, is a mathematical concept that plays an extremely important role as a problem-solving technique. Despite this, it is rarely used to solve mathematics problems in secondary school. Studies have revealed that teachers do not attach sufficient importance to using symmetry as a problem-solving tool (Leikin, Berman, & Zaslavsky, 2000). The concept of symmetry, which perhaps does not receive the necessary importance among other subjects of mathematics, has a very widespread application in many branches. It is used a lot, especially in the field of art (Cereci, 2012; Hickman and Huckstep, 2003). The symmetry that exists in nature, art, humans, and their daily use products is an element that reveals what is beautiful in works of art. Elements of proportion and symmetry have been used in many works of art since ancient times, and the beauty of the work of art is expressed with these elements (Karagözlü, 2015). While it is stated that the concept of symmetry comes to the fore in studies on the association of mathematics and mathematics teaching with art, it is emphasized that harmony, order, and beauty in works of art are achieved through symmetry (Atasay and Erdoğan, 2017). Regarding the subject, Köse and Özdaş (2009), in their study examining how 5th-grade students determined the symmetry line, looked at how the students defined the concept of symmetry. Students gave explanations about the concept of symmetry, such as "sameness," "harmony," "repetition of the same," "reflection," "similarity," and "inverted states." In addition, some students use expressions such as "when we divide a shape in half, there are the same shapes and equal shapes on both sides" and "when we fold it in the middle" when defining symmetry; this shows that they are aware of the line of symmetry. Studies have found that both teachers and students have some difficulties with the concept of symmetry and, as a result, make mistakes (Grenier, 1987; Köse, 2012; Küchemann, 1981; Orton, 1999). Examining the literature, it was determined that students did not have difficulty finding the symmetry of a shape when it was in a correct, vertical, or horizontal position but had difficulty when it was in an inclined position (Köse, 2012). It has been shown in different studies that supporting the subject of symmetry with different fields or activities has a positive effect on student success and attitude. In their study with 4th-grade students, Özkartal and Öçal (2021) determined that enriched learning activities caused a significant difference in students' symmetry success and perceptions but had no effect on permanence. In his study with 7th-grade students, Dağdelen (2012) observed that students' success in symmetry increased at the end of the course supported by origami. Durmuş (2017) found that teaching lessons with animations about symmetry significantly differentiated students' mathematics achievement.

Looking at the studies in the literature, they are generally aimed at teaching symmetry through a method, technique, strategy, or material. The current study differs from previous studies in that it was

conducted to reveal students' mental images of the concept of symmetry. Revealing perceptions about a concept is considered important in terms of providing clues for later activities and teaching this concept. In this context, the research was carried out to determine the mental images of secondary school students regarding the concept of symmetry.

METHOD

The present study investigates the mental images of symmetry among secondary school students, a crucial concept in the transformation geometry sub-field of mathematics education. A qualitative research design was employed to achieve the research objectives.

Working Group

The participant group of the study consists of 223 students (72 fifth graders, 65 sixth graders, 53 seventh graders, and 33 eighth graders) studying in two public secondary schools located in the central district of Ankara province in the 2022-2023 academic year and determined according to the easily accessible sampling method. The fact that one of the secondary schools where the study was conducted was an educational institution where the author had previously worked, and the mathematics teacher at the other secondary school had met the researcher before, enabled the research to be conducted by easily accessible sampling. In the literature, the convenience sampling method is seen as a frequently preferred sampling selection method as it provides economy (time, speed, and a practical process) for researchers (Yıldırım & Şimşek, 2006).

Data Collection Tools

In the literature, studies have been conducted to collect data with metaphors to determine the mental images of secondary school students regarding the meanings they attribute to the concept of symmetry (Ablak & Aksoy, 2018; 2021; Aydın, 2010; Günaydın, 2021; Güven & Güven, 2009; Kılcan & Akbaba, 2018; Saban, 2004; 2008; 2009) and the visuals/cartoons drawn by the students in this form were used. It is stated in the literature that visual materials (cartoons) are used to reflect the thoughts and feelings of the participants, emphasize skills in commenting on something and producing solutions (Ersoy & Türkkan, 2010), and can be used as a tool to determine the situation in some achievements (Ersoy, 2010).

Data Collection Process

Before collecting the data for the study, the researcher went to the educational institutions she had previously determined and informed the school administration and teachers about the purpose of the study and the implementation step of the study. Responding to some questions and requests from mathematics teachers, especially about the process after the implementation phase of the study (output of the study), the researcher met with the teachers of the appropriate courses to move on to the implementation phase of the research, went to the classrooms and informed the students for the application. At this stage, the metaphor form was first distributed to the participants and a brief explanation was made about the metaphor. Then, without touching on the concept of symmetry, the participants were given examples of developing metaphors on different topics (Ankara traffic is similar to blood circulation because they are both complicated. The sun is like the yolk of an egg because they are both yellow. It is like a math calculator because they both contain numbers, etc.) presented. After this stage, students were asked to develop a metaphor for the concept of symmetry, as in the examples presented, and to provide a justification for why they developed this metaphor, and if they wished, to draw a visual for the metaphor they developed or its justification. They were then told not to write descriptive information such as name, surname, and class number on the given metaphor forms, that the metaphors created would not be evaluated as true or false, and that these metaphor forms could not be seen by anyone other than the researcher. Participants were reminded that the developed metaphors could be anything such as concrete or abstract, animate, or inanimate, positive, or negative, and time was given to the participants to collect data. Participants completed the study within an average of one class hour. After the completed study forms were collected from the participants, the researcher thanked the participants for

participating in the application and left the application schools.

Analysis and Interpretation of Data

The analysis of the data obtained from the participant group was conducted based on a qualitative research perspective and content analysis, which is frequently used in this perspective, was used. Content analysis is explained as a technique that provides the opportunity to make indirect inferences about the different behaviors exhibited by people in the participant group (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz, & Demirel, 2010). On the other hand, in addition to the content analysis used in the study, the analysis was carried out by following the five stages followed by Saban (2009). The first phase mentioned is the coding and extraction phase. The metaphor forms obtained from the participants during the coding and sorting phase were numbered from 1 to 223. Then, the forms were examined and the forms in which the reason for the metaphor was not explained, only the definition was presented (Symmetry is similar to a mirror because……) and the forms that were left blank were excluded from the analysis, and the remaining 87 forms constituted the data set of the research. In the sample metaphor compilation phase, which is the second stage, the "metaphor analysis" methods mentioned by Saban (2009) and the "content analysis" methods mentioned by Yıldırım and Şimşek (2006) were used, the metaphors developed by the working group were re-examined and similar, likened and the bond between the like and the likened was evaluated. As a result of this evaluation, metaphors with weak imaginary structures were eliminated and excluded from the evaluation. In another stage, category creation, the mental images produced by the participants regarding the concept of symmetry were divided into categories according to the reasons for which they were created. In the next stage, the validity and reliability stage, another researcher (a researcher at the university who has similar studies) was asked to match the connection between the metaphor and the category to ensure that the metaphors in the conceptual categories reached by the researcher were able to represent the relevant category. As a result of the agreement between the matching made by the expert consulted by the researcher and the matching made by the researcher, the conceptual categories of nine metaphors were changed. Then, in the final stage of transferring the obtained data to the electronic environment, the findings section was created by calculating the mental images divided into categories, the frequency of the participants representing the category they belong to, presenting verbatim quotations of the reasons for the mental images and coding the participants (S23, S37, S127, …). While reporting this part, the mental images developed by the participants were scanned visually with the help of the word cloud program, and the images related to the developed mental images and their reasons were scanned verbatim to create the findings part of the research.

Ethic

I confirm that the study adheres to scientific ethical principles and protocols at all stages. All data and information beyond the study's scope are duly cited in the bibliography. The study's contributions to the field are objective and valuable. In addition, I agree to the terms and conditions of the Publication Ethics Committee (COPE) and attest that I uphold ethical duties and responsibilities.

FINDINGS

The mental images developed by the participants regarding the concept of symmetry (according to their frequency of development) are shown in the word clouds below.

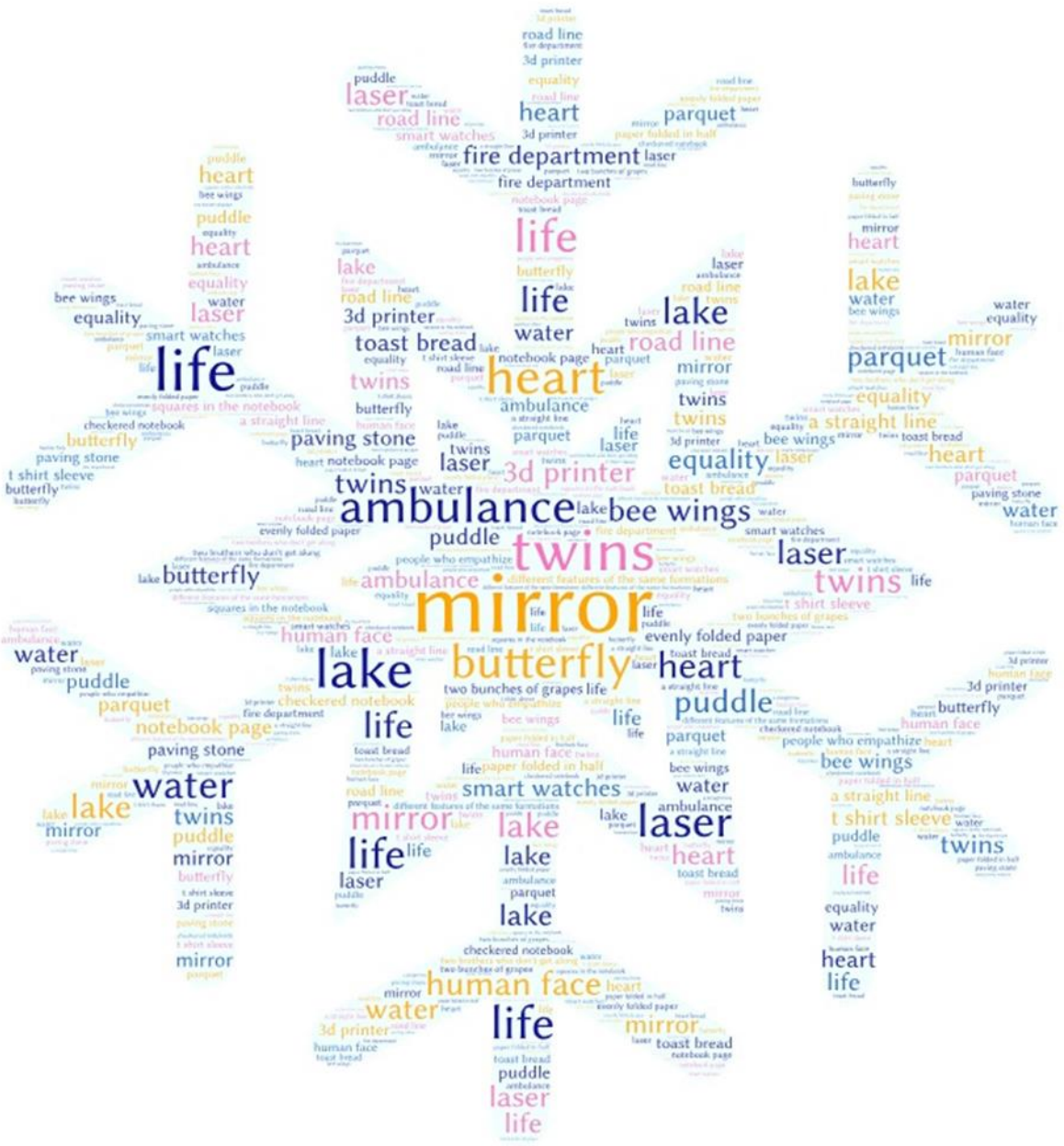

Figure 1. *Mental images developed by the participants regarding the concept of symmetry*

In the word cloud in Figure 1 above, the mental images developed by the students participating in the research regarding the concept of symmetry are shown, considering their frequencies. The students participating in the research developed a total of 87 metaphors related to symmetry. Among these mental images, the ones most developed by the students were "mirror", "ambulance" and "butterfly". Apart from these metaphors, "twins", "lake", "bee wings" and "human face" can also be shown among the frequently developed mental images. In addition, it was determined that 23 metaphors developed by the students were developed by only one student each.

Figure 2 below shows the metaphor categories created based on the mental images developed by the students participating in the study regarding the concept of symmetry.

Figure 2. *Metaphor categories created based on the mental images developed by the participants regarding the concept of symmetry*

The word cloud in Figure 2 above contains metaphor categories created by using the mental images developed by the students participating in the research for the concept of symmetry. The conceptual categories developed by the participants regarding symmetry are grouped under 3 headings in terms of their common features. Based on the mental images developed by the students participating in the research and the explanations they made, the thoughts that constitute the justification of the metaphor were grouped. Accordingly, it is the "having similar functionality" category that contains the most metaphors. This is followed by the "visually overlapping" and "perfect and meeting in differences" categories, respectively.

Having similar functionality. Examining Figure 2 above, it is understood that the category containing the most metaphors is the category of having similar functionality. Looking at the metaphors in this category, we see that the most common metaphors are "ambulance", "mirror" and "butterfly's wings". Looking at the reasons for the metaphors developed by the participants under this category and the drawings of some of these metaphors; *"Symmetry is like an ambulance sign because the word ambulance was written upside down and it was seen straight from the mirror, I drew it." (S, 24)*

Image 1. *Ambulance*

"Symmetry is like a mirror because when we look in the mirror, it gives a reverse movement. For example, if we raise our right hand, our image reflected in the mirror is left." (S, 164)

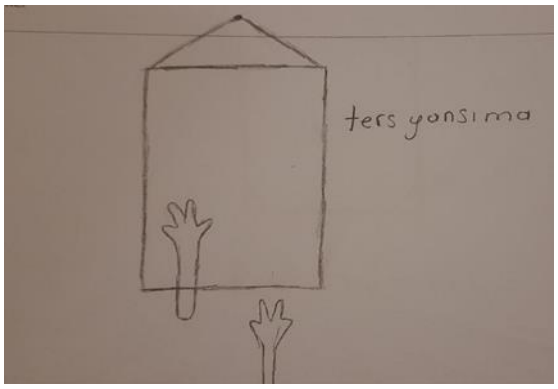

Image 2. *Mirror*

"Symmetry is like our reflection in a mirror because when we look in the mirror, we see the same version of ourselves, but with a changed direction. For example, when we put a watering can in front of the mirror, we see the same thing but in a changed direction." (S, 37)

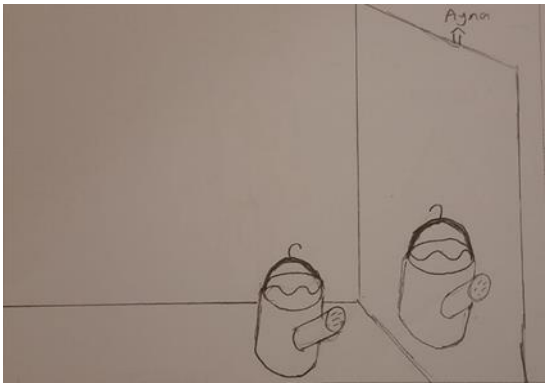

Image 3. *Reflection in the mirror*

"Symmetry is like a butterfly emerging from its cocoon because the opposite wings and antennas of a butterfly that has emerged from its cocoon are equal and symmetrical." (S, 143)

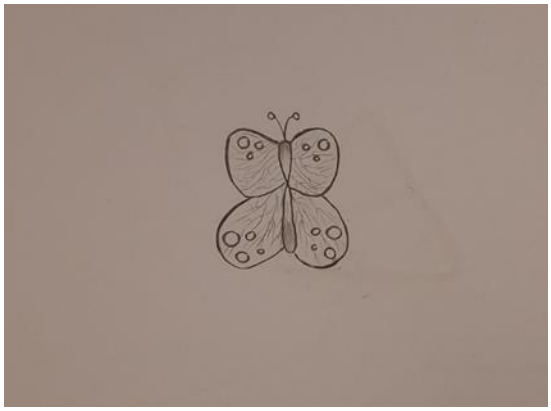

Image 4. *Butterfly wings*

Visually overlapping. Examining Figure 2, it is seen that the visual overlapping category is the second largest category in terms of containing metaphors. Examining the metaphors under this category, we see "lake", "life", "two brothers who don't get along", "two bunches of grapes", "paving stone", "checkered notebook", "laser", and "parquet". It was observed that the metaphors "puddle", "water", "t-shirt sleeve", "toast bread", and "paper folded in half" appeared only once in the category. Looking at the metaphors developed by the participants under this category, their justifications, and the drawings of these metaphors; *"Symmetry is like a lake because its reflection is equal to us." (S, 127)*

Image 5. *Lake*

"*Symmetry is like people who have twin siblings because in symmetry, things that are symmetrical to each other are identical to each other. Twin siblings also look alike. That's why symmetry is like twin brothers." (S, 36)*

Image 6. *Twins*

"Symmetry is like the human face because the human face is two equal parts." (S, 77)

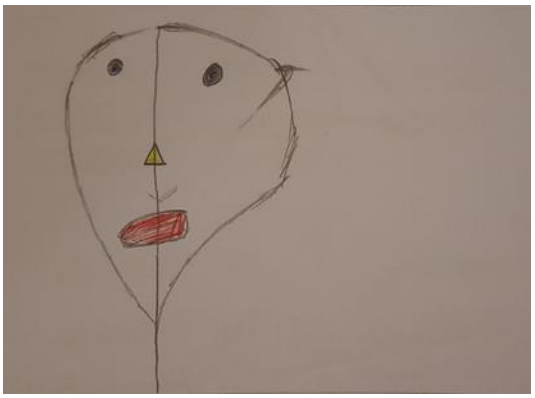

Image 7. *Human face*

"Symmetry is like a heart because the heart is symmetrical but not what humans have." (S, 113)

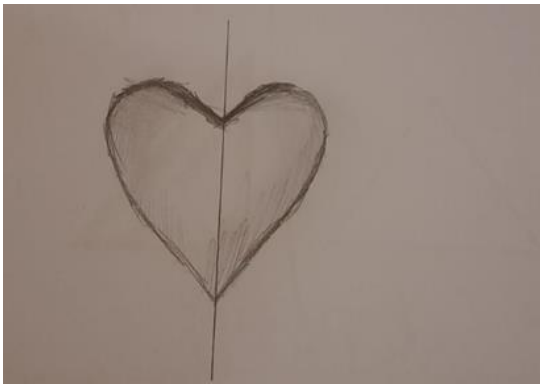

Image 8. *Heart*

Perfect and meeting in differences. Examining Figure 2 above, the Perfect and meeting in differences category is the third category containing the most metaphors. Examining the metaphors in this category, the metaphors "evenly folded paper", "equality" and "road line" appeared the same number of times and once each. Looking at the metaphor reasons and drawings of these metaphors of the participants who developed the metaphors under this category; *"Symmetry is like folding a sheet of paper evenly Because equality is ensured, and it is folded and regulated equally (S, 23).*

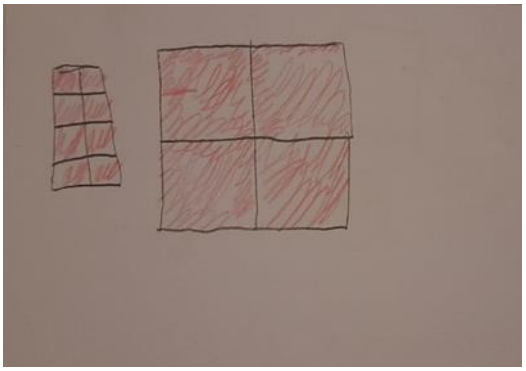

Image 9. *Evenly folded paper*

"Symmetry is like equality because everyone is equal." (S, 182)

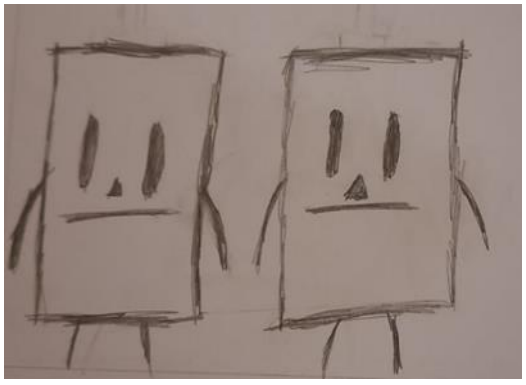

Image 10. *Equality*

"Symmetry is like a road line because their lines are the same." (P, 41)

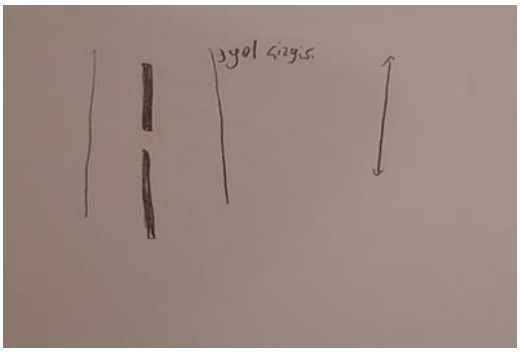

Image 11. *Road line*

DISCUSSION, CONCLUSION, RECOMMENDATIONS

In the results of this study, which was conducted to determine the mental images of secondary school students towards the concept of symmetry, the metaphors created by the participants for the concept of symmetry are generally in the "having similar functionality" category. It was determined that these were grouped under the categories of "visually overlapping" and "perfect and meeting in differences," respectively.

Looking at the research results, the category having similar functionality is the one containing the most metaphors. This can be considered evidence that most students think of symmetry as a reflection and create a mental image of it. In this category, the participants mostly created mental images such as mirrors and ambulances, which is similar to the findings of the study conducted by Köse and Özdaş (2009).

Looking at the metaphors in the "visually overlapping" category, the second category in which students produce the most metaphors, their justifications, and the drawings created by the students, it can be said that symmetry is perceived as an apparent similarity. The fact that the participants developed mental images in this way can be considered proof that they are at the "visualization" level stated by Van de Walle, Karp, and Bay-Williams (2019). It can be said that the reason for this is that the students participating in the research think of symmetry as the shape itself and its image after its symmetry is removed and compare the two images. From the mental images created by the students, "Symmetry is like twin brothers. Expressions such as "Because they are both similar to each other" can also be cited as examples of this.

In addition, another result of the research is that the participants touched on the principles of symmetry and equality in the mental images they created and the visuals they drew. These mental images are collected in the "perfect and meeting differences" category. Under this category, students emphasized that the shape and its appearance after the symmetry of the shape is taken are equal. According to the literature (Van de Walle, Karp, and Bay-Williams, 2019), this situation can be considered an indication that the participants are at the level of "informal inference." Individuals at this level think about discovering the relationships of geometric objects with each other from the characteristics of shapes. In this study, some participants' mental images about symmetry *(Symmetry is like equality because everyone is equal. Symmetry is like folding a piece of paper equally because equality is achieved, folded, and arranged equally*) supports the literature.

Looking at the results of the research in general, it shows that students think that their mental images of the concept of symmetry and the "shape and symmetry" of the drawings they deal with are similar. It is also supported in the literature (Grenier, 1987; Köse, 2012; Küchemann, 1981; Orton, 1999) that some drawings made by the participants regarding the concept of symmetry were made by considering the symmetry line only in vertical and horizontal positions. The fact that students mostly draw similar images

about symmetry and mostly use metaphors such as mirrors and ambulances may be because teachers generally use similar symmetry examples in teaching symmetry.

Based on the results given above, it is seen that the mental images of secondary school students regarding the concept of symmetry are generally similar, and they think of the symmetry line only in horizontal or vertical positions. Accordingly, while introducing the concept of symmetry to students, they should especially be taught that the symmetry line can be in different positions. In this study, students' mental images of the concept of symmetry and their drawings were examined from a qualitative research perspective. It may be recommended to conduct experimental studies by trying different teaching models for the relevant concepts in similar age groups.

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