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DIFFERENCE MATRIX AND SOME MULTIPLIER SEQUENCE SPACES

RAMAZAN KAMA AND BILAL ALTAY

Abstract. In this paper we show that completeness and barrelledness of a normed space can be characterized by means of sequence spaces obtained by a sequence in a normed space and difference matrix method. Other related results are established.

1. Introduction

By $\mathbb{N}$ and $\mathbb{R}$, we denote the sets of all natural and real numbers, respectively. Let $\mathbb{R}^{\mathbb{N}}$ be the space of all real sequences. Any vector subspace of $\mathbb{R}^{\mathbb{N}}$ is called a sequence space. By ℓ_∞, c and c_0, we write the spaces of all bounded, convergent and null sequences $x = (x_k)$, respectively, and we denote the norm on these spaces by $\|x\|_\infty = \sup_k | x_k |$. Also by bs, cs and ℓ_1, we denote the spaces of all bounded, convergent and absolutely convergent series, respectively.

Let $A = (a_{nk})$be an infinite matrix of real numbers a_{nk}, where $n,\ k \in \mathbb{N}$. Then, we write $Ax = \big((Ax)_n\big)$, the A-transform of $x \in \mathbb{R}^{\mathbb{N}}$, if $(Ax)_n = \sum_k a_{nk}x_k$ converges for each $n \in \mathbb{N}$. For a sequence space λ, the matrix domain λ_A of an infinite matrix A is defined by

$$\lambda_A = \left\{x = (x_k) \in \mathbb{R}^{\mathbb{N}} : Ax \in \lambda\right\},$$

which is a sequence space.

Let λ denote any one of the classical sequence spacesℓ_∞, c and c_0. λ_Δ consisting of the sequences $x = (x_k)$ is called as *the difference sequence space*, where Δ denotes the backward difference matrix $\Delta = (\Delta_{nk})$ and $\Delta^{(1)} = (\Delta^{(1)}_{nk})$ denotes the transpose of the matrix Δ, the forward difference matrix, which are defined by

$$\Delta_{nk} = \begin{cases} (-1)^{n-k} & , \quad \text{if } n-1 \leq k \leq n, \\ 0 & , \quad \text{if } 0 \leq k < n-1 \text{ or } k > n, \end{cases}$$

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and

$$\Delta^{(1)}_{nk} = \begin{cases} (-1)^{n-k} & , \quad \text{if } n \le k \le n+1, \\ 0 & , \quad \text{if } 0 \le k < n \text{ or } k > n+1 \end{cases}$$

for all $k, n \in \mathbb{N}$, respectively.

Several authors introduced and studied the domain of forward and backward difference matrices in classical sequence spaces [4, 5, 6, 9, 12, 13, 14].

A series $\sum_k x_k$ in a real Banach space X is called weakly unconditionally Cauchy series ($wuCs$) if $\sum_k |f(x_k)| < \infty$ for every $f \in X^*$(the dual space of X). We write the X-valued sequence spaces of weakly unconditionally Cauchy series, weakly convergent, bounded, absolutely convergent and convergent for $wuCs(X)$, $wcs(X)$, $bs(X)$, $\ell_1(X)$ and $cs(X)$, respectively.

It is well known (see [3] and [8]) that $x = (x_k) \in wuCs(X)$ if and only if $(a_k x_k) \in cs(X)$ for every $a = (a_k) \in c_0$, and also well know (see [7] and [15]) that X is a normed space then $x = (x_k) \in wuCs(X)$ if and only if the set

$$E = \left\{ \sum_{k=1}^{n} a_k x_k : |a_k| \le 1,\, k = 1, 2, \ldots, n;\, n \in \mathbb{N} \right\} \tag{1.1}$$

is bounded.

In [1, 16], authors used the space

$$S(x) \quad = \quad \{a = (a_k) \in \ell_\infty : (a_k x_k) \in cs(X)\}$$

of an arbitrary sequence (x_k) in a normed space X to characterize completeness and barrelledness of a normed space and weakly unconditionally Cauchy series. In [2], the space $BS(x)$, $LS(x)$ and $LS_w(x)$ were defined by the set of all sequences $a = (a_k) \in \mathbb{R}^{\mathbb{N}}$ such that $(a_k x_k) \in bs(X)$, $(a_k x_k) \in cs(X)$ and $(a_k x_k) \in wcs(X)$, respectively and some properties of these spaces were studied. In [11], these spaces are studied in the particular case of Cesáro summability.

In this paper we introduce some new sequence spaces of real sequences obtained by sequence in a normed space and backward difference method. We give some characterizations related to completeness and barrelledness of a normed space and some inclusion relations associated with these sequence spaces.

2. Main Results

In this section, we define some new sets of real sequences obtained by sequence in a normed space and backward difference method. Also, we give some characterizations the completeness and barrelledness of a normed space X by means of these spaces.

For a sequence $x = (x_k)$ in a normed space X, the sets $BS\Delta(x)$, $LS\Delta(x)$, $LS\Delta_w(x)$ and $LS\Delta_0(x)$ are defined by

$$\begin{aligned} BS\Delta(x) &= \left\{a = (a_k) \in \mathbb{R}^{\mathbb{N}} : (\Delta_k(a) x_k) \in bs(X)\right\} \\ LS\Delta_0(x) &= \{a = (a_k) \in BS\Delta(x) : (\Delta_k(a) x_k) \in w^* cs(X^{**})\} \\ LS\Delta(x) &= \{a = (a_k) \in BS\Delta(x) : (\Delta_k(a) x_k) \in cs(X)\} \\ LS\Delta_w(x) &= \{a = (a_k) \in BS\Delta(x) : (\Delta_k(a) x_k) \in wcs(X)\} \end{aligned}$$

where $\Delta_k(a) = (a_k - a_{k-1})$. The sets $BS\Delta(x)$, $LS\Delta(x)$, $LS\Delta_w(x)$ and $LS\Delta_0(x)$ are the normed spaces with the norm

$$\|a\|_{BS\Delta} = \sup_n \left\| \sum_{k=1}^{n} \Delta_k(a) x_k \right\|.$$

It is clear that $LS\Delta(x) \subset LS\Delta_w(x) \subset LS\Delta_0(x) \subset BS\Delta(x)$.

For a sequence $f = (f_k)$ in X^*, we define the set

$$LS\Delta_{w^*}(f) = \left\{ a = (a_k) \in \mathbb{R}^{\mathbb{N}} : (\Delta_k(a) x_k) \in w^* cs(X^*) \right\}.$$

It is clear that the inclusions $LS\Delta(f) \subset LS\Delta_w(f) \subset LS\Delta_{w^*}(f)$ and $S\Delta_{w^*}(f) = LS\Delta_{w^*}(f) \cap (l_\infty)_C$ are hold.

Theorem 2.1. *Let X be a normed space and $x = (x_k)$ be a sequence in X. Then, $BS\Delta(x)$ and $LS\Delta_0(x)$ are Banach spaces with the norm $\|.\|_{BS\Delta}$.*

Proof. Firstly, we shall prove the completeness of $BS\Delta(x)$. Let $(a^m) \subset BS\Delta(x)$ be a Cauchy sequence. Then, there exists $\epsilon > 0$ and $m_0 \in \mathbb{N}$ such that for $p, q > m_0$

$$\|a^p - a^q\|_{BS\Delta} < \epsilon \tag{2.1}$$

and thus

$$\begin{aligned} \|\Delta_i(a^p - a^q) x_i\| &= \left\| \sum_{k=1}^{i} \Delta_k(a^p - a^q) x_k - \sum_{k=1}^{i-1} \Delta_k(a^p - a^q) x_k \right\| \\ &\leq 2\|a^p - a^q\|_{BS\Delta} < \epsilon \end{aligned}$$

for every $i \in \mathbb{N}$. This means that $(\Delta_k(a^m))$ is a Cauchy sequence in $\mathbb{R}^{\mathbb{N}}$ for every $k \in \mathbb{N}$. We suppose that $\Delta_k(a^m) \to \Delta_k(a^0) \in \mathbb{R}$ for every $k \in \mathbb{N}$. We show that $a^0 \in BS\Delta(x)$. From (2.1) if we take limit as $q \to \infty$, then

$$\left\| \sum_{k=1}^{n} \Delta_k(a^p - a^0) x_k \right\| \leq \epsilon \tag{2.2}$$

for every $n \in \mathbb{N}$. Since $a^p \in BS\Delta(x)$ for each $p \in \mathbb{N}$ there exists $K_p > 0$ such that

$$\|a^p\|_{BS\Delta} \leq K_p. \tag{2.3}$$

From (2.2) and (2.3) for $\epsilon > 0$ and $p > m_0$ we have the inequality

$$\begin{aligned} \|a^0\|_{BS\Delta} &= \sup_n \left\| \sum_{k=1}^{n} \Delta_k(a^0) x_k \right\| \\ &\leq \sup_n \left\| \sum_{k=1}^{n} \Delta_k(a^0 - a^p) x_k \right\| + \sup_n \left\| \sum_{k=1}^{n} \Delta_k(a^p) x_k \right\| \\ &\leq \epsilon + K_p. \end{aligned}$$

Hence, this proof is complete.

Now, we show that the completeness of $LS\Delta_0(x)$. Let (a^m) be a Cauchy sequence in $LS\Delta_0(x)$. Since $LS\Delta_0(x) \subset BS\Delta(x)$, there exists a sequence $a^0 \in BS\Delta(x)$ such that $a^m \to a^0$. For $x^* \in X^*$ there exists $(y_m^{**}) \subset X^{**}$ and $n_0 \in \mathbb{N}$ such that for $\epsilon > 0$ and $n > n_0$

$$\left| \sum_{k=1}^{n} \Delta_k(a^m) x^*(x_k) - x^*(y_m^{**}) \right| < \frac{\epsilon}{3} \tag{2.4}$$

for $m \in \mathbb{N}$. On the other hand, since (a^m) be a Cauchy sequence there exists $\epsilon > 0$ and $m_0 \in \mathbb{N}$ such that for $p, q > m_0$

$$\|a^p - a^q\|_{BS\Delta} < \frac{\epsilon}{3}. \tag{2.5}$$

We can choose $x^* \in S_{X^*}$ (the unit sphere of X^*). From (2.4) for $\epsilon > 0$ and $p, q > m_0$

$$\begin{aligned} \|y_p^{**} - y_q^{**}\| &= |x^*(y_p^{**} - y_q^{**})| \\ &\leq \tfrac{2\epsilon}{3} + \|a^p - a^q\|_{BS\Delta} \\ &< \epsilon. \end{aligned} \tag{2.6}$$

Hence (y_m^{**}) is a Cauchy sequence in X^{**}. Thus there exists $y_0^{**} \in X^{**}$ such that $\lim_m y_m^{**} = y_0^{**}$. If we take limit as $q \to \infty$ from (2.5) and (2.6), then

$$\|a^p - a^0\|_{BS\Delta} < \frac{\epsilon}{3} \quad \text{and} \quad \|y_p^{**} - y_0^{**}\| < \frac{\epsilon}{3},$$

and also using (2.4), for $n > n_0$ we get

$$\begin{aligned} \left|\sum_{k=1}^n \Delta_k(a^0)x^*(x_k) - x^*(y_0^{**})\right| &\leq \left|\sum_{k=1}^n \Delta_k(a^0)x^*(x_k) - \sum_{k=1}^n \Delta_k(a^p)x^*(x_k)\right| \\ &\quad + \left|\sum_{k=1}^n \Delta_k(a^p)x^*(x_k) - x^*(y_p^{**})\right| \\ &\quad + |x^*(y_p^{**}) - x^*(y_0^{**})| \\ &< \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} \\ &= \epsilon. \end{aligned}$$

Then $a^0 \in LS\Delta_0(x)$. □

Theorem 2.2. *Let X be normed space. Then, X is a Banach space if and only if $LS\Delta(x)$ (or $LS\Delta_w(x)$) is a Banach space for every sequence $x = (x_k)$ in X with the norm $\|.\|_{BS\Delta}$.*

Proof. Let $x = (x_k)$ be a sequence in X and (a^m) be a Cauchy sequence in $LS\Delta(x)$ such that $a^m \to a^0 \in BS\Delta(x)$. Since the sequence (a^m) is in $LS\Delta(x)$, there exists $(y_m) \subset X$ and $n_0 \in \mathbb{N}$ such that for $\epsilon > 0$ and $n > n_0$

$$\left\|\sum_{k=1}^n \Delta_k(a^m)x_k - y_m\right\| < \frac{\epsilon}{3} \tag{2.7}$$

for $m \in \mathbb{N}$. Since (a^m) be a Cauchy sequence, there exists $\epsilon > 0$ and $m_0 \in \mathbb{N}$ such that for $p, q > m_0$

$$\|a^p - a^q\|_{BS\Delta} < \frac{\epsilon}{3}. \tag{2.8}$$

Then from (2.7) for $\epsilon > 0$ and $p, q > m_0$

$$\begin{aligned} \|y_p - y_q\| &\leq \tfrac{2\epsilon}{3} + \|a^p - a^q\|_{BS\Delta} \\ &< \epsilon. \end{aligned} \tag{2.9}$$

Therefore (y_m) is a Cauchy sequence in X, and by the completeness of X there exists $y_0 \in X$ such that $\lim_m y_m = y_0$. If we take limit as $q \to \infty$ from (2.8) and (2.9), then

$$\|a^p - a^0\|_{BS\Delta} < \frac{\epsilon}{3} \quad \text{and} \quad \|y_p - y_0\| < \frac{\epsilon}{3},$$

and also using (2.7), for $n > n_0$ we have that

$$\begin{aligned}\left\|\sum_{k=1}^{n}\Delta_k(a^0)x_k - y_0\right\| &\leq \left\|\sum_{k=1}^{n}\Delta_k(a^0 - a^p)x_k\right\| \\ &\quad + \left\|\sum_{k=1}^{n}\Delta_k(a^p)x_k - y_p\right\| + \|y_p - y_0\| \\ &< \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} \\ &= \epsilon.\end{aligned}$$

This means that $a^0 \in LS\Delta(x)$.

If X is not complete then there exists a sequence $x = (x_k) \in \ell_1(X) \setminus cs(X)$. Let suppose that $\|x_k\| < \frac{1}{k2^k}$ for every $k \in \mathbb{N}$. We denote the sequence $a^n \in \mathbb{R}^{\mathbb{N}}$ for every $n \in \mathbb{N}$ by

$$a_k^n = \begin{cases} n - k + 1, & \text{if } k \leq n, \\ 0, & \text{if } k > n, \end{cases} \quad (k \in \mathbb{N}).$$

We have that $a^n \in LS\Delta(x)$ for each $n \in \mathbb{N}$. If we consider $a^0 \in \mathbb{R}^{\mathbb{N}}$ such that $a_k^0 = 1$ for all $k \in \mathbb{N}$, then $a^0 \in BS\Delta(x) \setminus\ LS\Delta(x)$ and $\lim_n a^n = a^0$. Hence $LS\Delta(x)$ is not complete. □

Theorem 2.3. *If $f = (f_k)$ is a sequence in X^*, then $LS\Delta_{w^*}(f) \cap BS\Delta(f)$ is a Banach space.*

Proof. Let $f = (f_k)$ is a sequence in X^* and (a^m) be a Cauchy sequence in $LS\Delta_{w^*}(f)$ such that $a^m \to a^0 \in BS\Delta(f)$. Since $(a^m) \subset LS\Delta_{w^*}(f)$ there exists $(g_m) \subset X^*$ and $n_0 \in \mathbb{N}$ such that for $x \in S_X$, $\epsilon > 0$ and $n > n_0$

$$\left|\sum_{k=1}^{n}\Delta_k(a^m)f_k(x) - g_m(x)\right| < \frac{\epsilon}{3} \tag{2.10}$$

for $m \in N$. On the other hand, since (a^m) be a Cauchy sequence there exists $\epsilon > 0$ and $m_0 \in N$ such that for $p, q > m_0$

$$\|a^p - a^q\|_{BS\Delta} < \frac{\epsilon}{3}. \tag{2.11}$$

Then, from (2.10) for $\epsilon > 0$ and $p, q > m_0$

$$\begin{aligned}\|g_p - g_q\| &\leq \tfrac{2\epsilon}{3} + \|a^p - a^q\|_{BS\Delta} \\ &< \epsilon.\end{aligned} \tag{2.12}$$

So (g_m) is a Cauchy sequence in X^*. Hence there exists $g_0 \in X^*$ such that $\lim_m g_m = g_0$. We take limit as $q \to \infty$ from (2.11) and (2.12), and suppose that

$$\|a^p - a^0\|_{BS\Delta} < \frac{\epsilon}{3} \quad \text{and} \quad \|g_p - g_0\| < \frac{\epsilon}{3}, \tag{2.13}$$

and also using (2.10) for $n > n_0$

$$\begin{aligned}\left|\sum_{k=1}^{n}\Delta_k(a^0)f_k(x)-g_0(x)\right| &\leq \left|\sum_{k=1}^{n}\Delta_k(a^0-a^p)f_k(x)\right| \\ &+\left|\sum_{k=1}^{n}\Delta_k(a^p)f_k(x)-g_p(x)\right|+|g_p(x)-g_0(x)| \\ &< \frac{\epsilon}{3}+\frac{\epsilon}{3}+\frac{\epsilon}{3}=\epsilon,\end{aligned}$$

and hence $a^0 \in LS\Delta_{w^*}(f)$. □

Theorem 2.4. *The normed space X is a barrelled space if and only if $LS\Delta_{w^*}(f) \subset BS\Delta(f)$ for every sequence $f = (f_k)$ in X^*.*

Proof. We suppose that X is not a barrelled space. Then there exists a weak* bounded set $N \subset X^*$ which is unbounded. Therefore there exists $(g_k) \subset N$ and $C_x > 0$ such that

$$\|g_k\| > k^2 \quad \text{and} \quad \sup_k |g_k(x)| < C_x$$

for every $x \in X$. We define the sequence $(h_k) \subset X^*$ by

$$z_k = \begin{cases} g_1, & \text{if } k = 1, \\ \frac{1}{k}g_k - \frac{1}{k-1}g_{k-1}, & \text{if } k > 1. \end{cases}$$

We consider the sequence $x = (1, 1, 1, ...)$. Then $x \in LS\Delta_{w^*}(z) \setminus BS\Delta(z)$. □

In [10], we were defined the spaces $S\Delta(x)$ and $S\Delta_w(x)$ by

$$\begin{aligned} S\Delta(x) &= \{a = (a_k) \in (\ell_\infty)_\Delta : (\Delta_k(a)x_k) \in cs(X)\}, \\ S\Delta_w(x) &= \{a = (a_k) \in (\ell_\infty)_\Delta : (\Delta_k(a)x_k) \in wcs(X)\}. \end{aligned}$$

It is obvious that the inclusions $S\Delta(x) = LS\Delta(x) \cap (l_\infty)_\Delta$ and $S\Delta_w(x) = LS\Delta_w(x) \cap (l_\infty)_\Delta$ are hold. Now, we obtain some results associate with $LS\Delta(x)$, $S\Delta(x)$ and $(c_0)_\Delta$. This results are also valid if we take the spaces $S\Delta_w(x)$ and $LS\Delta_w(x)$ instead of $S\Delta(x)$ and $LS\Delta(x)$, respectively.

Theorem 2.5. *Let X be a normed space and $x = (x_k)$ be a sequence in X. If $\inf_k \|x_k\| > 0$, then $LS\Delta(x) = S\Delta(x)$.*

Proof. If $a = (a_k) \in LS\Delta(x)$, then we have

$$\|\Delta_n(a)x_n\| = \left\|\sum_{k=1}^{n}\Delta_k(a)x_k - \sum_{k=1}^{n-1}\Delta_k(a)x_k\right\| \to 0, n \to \infty.$$

Therefore $\Delta_n(a) \to 0$, and hence $(a_k) \in (c_0)_\Delta$. This shows that $(a_k) \in S\Delta(x)$. The inclusion $S\Delta(x) \subset LS\Delta(x)$ is obvious. □

Theorem 2.6. *If X is a Banach space and $x = (x_k)$ be a sequence in X, then $\inf_k \|x_k\| > 0$ if and only if $LS\Delta(x) = S\Delta(x)$.*

Proof. Necessity follows immediately from Theorem 2.5.

If $\inf_k \|x_k\| = 0$, then there exists a strictly increasing sequence (m_i) in $\mathbb{N}$ such that $\|x_{m_i}\| < \frac{1}{i^3}$. We define the sequence $a = (a_k)$ by

$$\Delta_k(a) = \begin{cases} i, & \text{if } k = m_i, \\ 0, & \text{if } k \neq m_i. \end{cases}$$

It is obvious that $(a_k) \notin S\Delta(x)$. Since the series $\sum_{k=1}^{\infty} \Delta_k(a)x_k$ is convergent by Cauchy criterion, we have $(a_k) \in LS\Delta(x)$. □

Theorem 2.7. *Let X be a normed space and $x = (x_k)$ be a sequence in X. If $\inf_k \|x_k\| > 0$, then $S\Delta(x) \subset (c_0)_\Delta$.*

Proof. If $(a_k) \in S\Delta(x)$,

$$\|\Delta_n(a)x_n\| = \left\| \sum_{k=1}^{n} \Delta_k(a)x_k - \sum_{k=1}^{n-1} \Delta_k(a)x_k \right\| \to 0, n \to \infty.$$

Thus $\Delta_n(a) \to 0$, and hence $(a_k) \in (c_0)_\Delta$. □

Theorem 2.8. *Let X be a Banach space and $x = (x_k)$ be a sequence in X. Then $\inf_k \|x_k\| > 0$ if and only if $S\Delta(x) \subset (c_0)_\Delta$.*

Proof. Necessity follows immediately from Theorem 2.7.

To prove the sufficiency it is enough to show $S\Delta(x) \backslash (c_0)_\Delta \neq \emptyset$. Let $\inf_k \|x_k\| = 0$. Then there exists a strictly increasing sequence (m_i) in $\mathbb{N}$ such that $\|x_{m_i}\| < \frac{1}{i^2}$. Let $a = (a_k)$ be the sequence defined by

$$\Delta_k(a) = \begin{cases} 1, & \text{if } k = m_i, \\ 0, & \text{if } k \neq m_i. \end{cases}$$

It can be easily seen that $a \notin (c_0)_\Delta$. Since $(\Delta_k(a)x_k) \in cs(X)$ by Cauchy criterion, $a \in S\Delta(x)$. □

Theorem 2.9. *If X be a Banach space, then the sequence $x = (x_k) \in wuCs(X)$ and $\inf_k \|x_k\| > 0$ if and only if $S\Delta(x) = (c_0)_\Delta$.*

Proof. Let $x = (x_k) \in wuCs(X)$ and $\inf_k \|x_k\| > 0$. Since $\inf_k \|x_k\| > 0$, the inclusion $S\Delta(x) \subset (c_0)_\Delta$ is obtained from Theorem 2.8. We take $b = (b_k) \in (c_0)_\Delta$. Since $x = (x_k) \in wuCs(X)$, the series $\sum_k \Delta_k(b)x_k$ is convergent. Thus $b = (b_k) \in S\Delta(x)$, and hence the inclusion $(c_0)_\Delta \subset S\Delta(x)$ is satisfied.

Since $(c_0)_\Delta \subset S\Delta(x)$, we have $x \in wuCs(X)$. Also, by Theorem 2.8, the inequality $\inf_k \|x_k\| > 0$ is obtained. □

3. Conclusion

In this paper, we introduced and studied the sets $BS\Delta(x)$, $LS\Delta(x)$, $LS\Delta_w(x)$ and $LS\Delta_0(x)$ ($LS\Delta_{w^*}(f)$) by means of a sequence $x = (x_k)$ in a normed space X ($f = (f_k)$ in X^*) and the difference matrix. We obtained the characterizations of completeness and barrelledness of the normed space X by means of these spaces.

Finally, we given some relations between these spaces. It is natural that the investigation of more general conclusion corresponding to the results of this paper can be studied by taking more general matrices instead of the difference matrix.

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Siirt University, Department of Mathematics, Siirt-TURKEY
E-mail address: ra.kama12@gmail.com

Inonu University, Faculty of Education, Malatya-TURKEY
E-mail address: bilal.altay@inonu.edu.tr