Düzce University Journal of Science & Technology

Research Article

Statistical Assessment of Counterflow Ranque-Hilsch Vortex Tube Performance

Selen Yılmaz IŞIKHAN[a], Murat KORKMAZ[b]*, Volkan KIRMACI[c]

[a] *Vocational and Higher School of Social Sciences, Hacettepe University, Ankara, Turkey.*
[b] *Başkent OSB Vocational Higher School of Technical Sciences, Department of Electrical and Energy, Hacettepe University, Ankara, Turkey*
[c] *Faculty of Engineering, Architecture, and Design, Mechanical Engineering, Bartın University, Bartın, Turkey*
* *Corresponding author's e-mail address:* mkorkmaz@hacettepe.edu.tr

DOI: 10.29130/dubited.1384832

ABSTRACT

The vortex tube, which consists of a simple tube, is a device that can simultaneously heat and cool thanks to environmentally friendly pressurized fluids (air, oxygen, nitrogen, etc.). Many studies have been included in the literature to evaluate the Ranque-Hilsch Vortex tube's performance and to reveal influential factors. Variance analysis, linear regression analysis, and the Taguchi method are primarily used in practice. This study aimed to compare the strengths and weaknesses of the factorial experimental design and Taguchi orthogonal array design in the statistical evaluation of the factors affecting the heat exchange of the Ranque-Hilsch Vortex tube. For this purpose, a detailed theory was created for the appropriate factorial ANOVA model to the data set (4 × 5 × 12 = 240 experiments) containing the Ranque-Hilsch Vortex tube and effective material type (polyamide, steel, brass, and aluminum), nozzle number (2,3,4,5 and 6), and input pressure parameters (1,5-7 bar). Following the factorial ANOVA solution, including all binary interactions, the findings were obtained according to the most suitable L16 Taguchi Orthogonal array, considering the four levels for each material, nozzle, and pressure. As a result of the ANOVA, all parameters were statistically significant on heat change ($p < 0.001$). On the other hand, the pressure was obtained as the only statistically significant factor according to the Taguchi analysis ($F = 35.17$, $p = 0.008$). The advantages and disadvantages of the two methods were compared regarding the test findings and graphical performances.

Keywords: *Factorial ANOVA Experimental Design, Taguchi Orthogonal Array, Ranque-Hilsch Vortex Tube, Optimization, Statistical Method Comparison.*

Karşıt Akışlı Ranque-Hilsch Vorteks Tüpü Performansının İstatistiksel Değerlendirmesi

Öz

Basit bir borudan oluşan vorteks tüp, çevre dostu basınçlı akışkanlar (hava, oksijen, azot vb.,) sayesinde aynı anda ısıtma ve soğutma yapabilen bir cihazdır. Ranque-Hilsch Vortex tüpünün performansını değerlendirmek ve etkileyen faktörleri ortaya çıkarmak için literatürde birçok çalışmaya yer verilmiştir. Uygulamada öncelikle varyans analizi, doğrusal regresyon analizi ve Taguchi yöntemi kullanılmaktadır. Bu çalışma, Ranque-Hilsch Vortex tüpünün ısı değişimini etkileyen faktörlerin istatistiksel değerlendirmesinde faktöriyel deney tasarımı ile Taguchi ortogonal dizi tasarımının güçlü ve zayıf yönlerini karşılaştırmayı amaçlamıştır. Bu amaçla Ranque-Hilsch Vortex tüpü ve etkin malzeme tipi (polyamid, çelik, pirinç ve alüminyum), nozül sayısı (2,3,4,5 ve 6) ve giriş basıncı parametrelerini (1,5-7 bar) içeren veri setine (4 x 5 x 12 = 240 deney) uygun faktöriyel ANOVA

Received: 04/12/2023, Revised: 10/01/2024, Accepted: 06/08/2024

modeli için detaylı bir teori oluşturulmuştur. Tüm ikili etkileşimleri içeren faktöriyel ANOVA çözümü sonrasında her malzeme, nozül ve basınç için dört seviye dikkate alınarak en uygun L16 Taguchi Ortogonal dizisine göre bulgular elde edilmiştir. ANOVA sonucunda tüm parametreler ısı değişimi açısından istatistiksel olarak anlamlı bulunmuştur (p < 0,001). Öte yandan Taguchi analizine göre istatistiksel olarak anlamlı olan tek faktörün basınç olduğu elde edilmiştir (F=35,17, p=0,008). Test bulguları ve grafik performansları dikkate alınarak iki yöntemin avantaj ve dezavantajları karşılaştırılmıştır.

Anahtar Kelimeler: *Faktöriyel ANOVA Deney Tasarımı, Taguchi Ortogonal Dizi, Ranque-Hilsch Vorteks Tüpü, Optimizasyon, İstatistiksel Yöntem Karşılaştırması.*

I. INTRODUCTION

Vortex tubes are systems first invented by Ranque in 1931 and developed by Hilsch in 1947. They can perform cooling and heating simultaneously and have no moving parts except the control valve [1]. The vortex tube is called the Ranque-Hilsch Vortex Tube (RHVT) because it is named after the people who invented and developed it. RHVT is preferred today because it is small and light, reaches the regime very quickly, is not harmful to the environment compared to other cooling and heating systems, and does not require much maintenance [2,3]. The principle of obtaining hot and cold fluids in Counterflow RHVTs is explained in Figure 1. Due to the conservation principle of angular momentum, which occurs with the pressurized fluid applied to the RHVT, the angular velocity of the flow in the center rises to higher values than the angular velocity of the flow of the tube wall. Therefore, two flows occur inside the tube, rotating at different speeds. Center flow at higher rates forces the wall flow to accelerate. Thus, energy transfer occurs from the center flow to the wall flow. The center flow with a decrease in mechanical energy is cold flow, and the flow at the tube wall is hot flow due to the effect of friction on the tube wall and the mechanical energy transferred from the center flow [4,5,6].

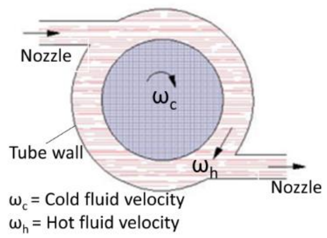

Figure 1. *Movement of hot and cold flow in counter flow RHVT*

There are many different experimental modeling and numerical studies about vortex tubes. Dutta et al. tried to develop a CFD model to analyze the pressurized flow in the pipe using the actual gas model in the vortex tube [7]. Bovand et al. have studied different RHVT types of vortex tubes at variable pressure values [8]. Han et al. have investigated how the vortex tube affects the heating and cooling performance using different types of inlet fluid pressures [9]. Shamsoddini and Nezhad investigated the effect of the number of nozzles in the vortex tube. They stated that there should be a higher number of nozzles for the vortex tubes' cooling capacity to be high [10]. S. Eiamsa-ard investigated the effects of the number, size, and shape of the nozzles in the vortex tube on hot and cold fluids. He also examined how the structural features of the nozzles affect the cooling performance of the RHVT [11]. Xue et al. made suggestions by conducting studies for the energy distribution and exergy density in the measured fluid values in the vortex tube. Also, another study analyzed and optimized the role of the cold liquid fraction, called the equal division of the rotating flows between the cold and hot outlets [12,13]. Khait et al. used the RSM turbulence model to construct a comprehensive CFD model to predict the flow distribution inside the vortex tube air separator [14]. Kandil and Abdelghany evaluated the effect of some geometric parameters, such as the ratio of the vortex tube length to tube diameter, on vortex tube performance [15].

Mohammadi and Farhadi used a mixture of hydrocarbons to analyze the separation process and gas fractionation effect in a vortex tube [16].

In the mentioned studies on Vortex tubes, it was found that statistical analyses remained superficial. Generally, Taguchi analysis was used to determine the influential factors on temperature or select the optimum level. This study aims to compare the Taguchi L16 orthogonal experimental design performance according to various statistical and visual analysis criteria with the Analysis of Variance performed according to the known factorial experimental design.

II. MATERIAL AND METHOD

The study used a counterflow RHVT with structural features with an internal diameter of 7 mm and a body length of 10 cm (Figure 2). Aluminum, steel, brass, and polyamide materials with two, three, four, five, and six nozzles were used in the experiments (Figure 3). While data were being taken in the experiments, digital thermometer probes were used to measure flow temperatures with $\pm 1^0$C accuracy and a manometer with 5% sensitivity was used to measure the inlet pressure. In the experimental study, the initial pressure of 1.5 bar was reached in the experiments by using the oxygen tube and the valve at the RHVT fluid inlet. Then, after getting a pressure of 1.5 bar, air at 1.5 bar pressure was sent to the cold and hot flow outlets of the RHVT until the temperature values read on the measuring instruments were stabilized, and the temperature values and volumetric flow rates of the hot and cold fluid coming out of the RHVT were measured. However, the same procedures were repeated for all nozzles made of different materials. All conditions that were first realized at 1.5 bar pressure in the experimental studies were also realized in all experiments carried out between 2 bar and 7 bar pressure values. During the experiments, the ambient temperature was set at 21^0C. The properties of the materials used in the experiments are shown in Table 1. At the same time, each experiment was repeated three times, and the average test results were calculated and used in the analysis.

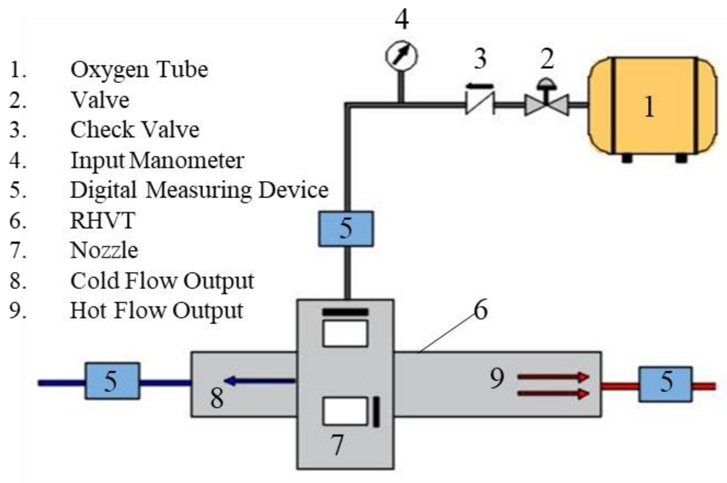

Figure 2. *Experimental study*

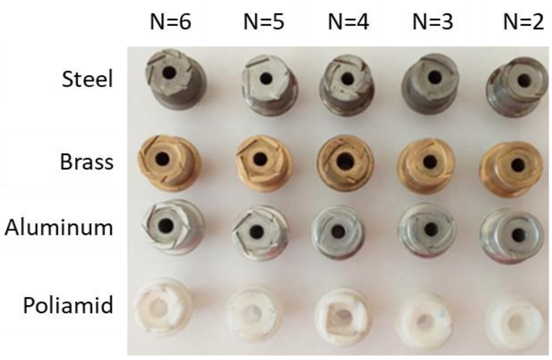

Figure 3. *Nozzles used in the experimental study*

Table 1. *Input and output parameters used in RHVT*

Input parameters					Output parameter
Fluid	**Material**	**Nozzle number**	**P_{in} (Inlet pressure) (bar)**	**k (Heat conduction coefficient) (W/mK)**	**Temperature differences (°K)**
Oxygen (1)	Poliamid(1)	2,3,4,5,6	1.5 – 7 bar	0.257	ΔT
	Brass (2)			117	
	Aluminum (3)			226	
	Steel (4)			23	

RHVTs are open systems with one input and two different outputs. The cold mass ratio (μc), which shows how much of the inlet fluid in the vortex tube is converted to the cold flow formed at the outlet of the vortex tube, is given in Equation 1.

$$\xi = \frac{mass\ flow\ rate\ of\ cold\ flow}{mass\ flow\ rate\ of\ inlet\ flow} \quad (1)$$

In RHVTs, cold flow temperature difference (ΔTc) and hot flow temperature difference (ΔTh) are expressed in equations 2-3.

$$\Delta T_c = T_{in} - T_c \quad (2)$$

$$\Delta T_h = T_h - T_{in} \quad (3)$$

RHVT performance is calculated as shown in Equation 4 [6,17].

$$\Delta T = T_h - T_c \quad (4)$$

The main hypotheses analyzed for oxygen gas are as follows.

A. 1. Hypotheses

For material type *(Rows):* H_0: µ1=µ2=µ3= µ4; H_a: At least two of the means differ.

If H_0 is rejected, then the level of material types is effective on Δt.
Nozzle number *(Columns):* H_0: µ1=µ2=µ3=µ4= µ5; H_a: At least two of the means differ.
If H_0 is rejected, nozzle number (namely column effect) is vital for Δt in this experiment.
Pressure category *(treatment):* H_0: µ1=µ2=µ3=… = µ12; H_a: At least two of the means differ.
If H_0 is rejected, this experiment's treatment effect (pressure here) is important.

The crosstabulation created to reveal the row-column and processing order of the experimental data set is shown in Appendix A. The data set conforms to the randomized block design, where each nozzle category provides an equal number of outcomes (here Δt) for each material type. This setup is repeated for 12 different pressure values.

A. 2. Modeling

Our experimental setup is not a p*p dimensional Latin square layout but conforms to a three-way randomized block layout in which row, column, and treatment effects affect the dependent variable. The other critical point is that the number of data in each block takes a single value or only two values here, just like in the Latin square format. Therefore, insufficient observations at each intersection (here 2) cause the test for interaction effects to be poor. Now, we can consider a randomized block design for an experiment with three factors A, B, and C. We can accept that every combination of A, B, and C levels is observed.

Statement: A has a levels, coded 1, 2, …, a
B has b levels, coded 1, 2, …, b
C has c levels, coded 1, 2,…, c
v = total number of treatments (=abc)
Variables: $\Delta t = t_{hot} - t_{cold}$
Material= 1: poliamid, 2: rice, 3: alüminum, 4: steel
Nozzle= 2, 3, 4, 5, 6
Pressure= 150, 200, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700

All material, nozzle, and pressure combinations were observed (giving 4×5×12=240). Since the number of samples in each sub-category in our model was insufficient to test the interaction between variables, the dataset was analyzed based on the model that includes the main effects and only possible two-way interactions [18]. In addition, both profile plots in variance analysis solutions and interaction graphs of Taguchi analysis will be examined for interactions. Now, if the standard Latin Square Design is implemented for the assignment of treatments:

Y_{ijk}'s are independently distributed as $N(\mu+\alpha_i+\beta_j+\gamma_k, \sigma^2)$

A linear model is;

$$\mathrm{Yijk} = \mu + \alpha i + \beta j + \gamma k + \varepsilon ijk,\ i = 1, 2, \ldots, u; \quad j = 1, 2, \ldots, v; \quad k = 1, 2, \ldots, r \quad (5)$$

ε_{ijk} are random errors independently and identically distributed like $N(0, \sigma^2)$.

Supposing $\sum_{i=1}^{u}\alpha_i = 0, \sum_{i=1}^{v}\beta_j = 0,\ \sum_{k=1}^{r}\gamma_k = 0$

α_i: Main effect of rows
β_j: Main effect of columns
γ_k: Main effect of treatments
The null hypotheses are taken into consideration:
H_{0row}: $\alpha_1=\alpha_2=\ldots=\alpha_u$
$H_{0column}$: $\beta_1=\beta_2=\ldots=\beta_v=0$
$H_{0treatment}$: $\gamma_1=\gamma_2=\ldots=\gamma_r=0$
The analysis of variance can be adjusted as below:
Minimizing $S=\sum_{i=1}^{u}\sum_{j=1}^{v}\sum_{k=1}^{r}\varepsilon_{ijk}^2$ concerning μ, α_i, β_j, and γ_k have given the least-squares estimate as:
$\mu = \bar{y}_{ooo}$

$\hat{\alpha}_i = \bar{y}_{ioo} - \bar{y}_{ooo}$ i=1, 2, …, u
$\hat{\beta}_j = \bar{y}_{ojo} - \bar{y}_{ooo}$ j=1, 2, …, v
$\hat{\gamma}_k = \bar{y}_{ook} - \bar{y}_{ooo}$ k=1, 2, …, r

The total sum of squares can be broken down into the common orthogonal sum of squares SSR, SSC, SSTr, and SSE utilizing the fitted model depending on these estimators:
TSS=SSR+SSC+SSTr+SSE

Where

$$TSS\text{: Total sum of squares} = \sum_{i=1}^{u}\sum_{j=1}^{v}\sum_{k=1}^{r}(y_{ijk} - \bar{y}_{ooo})^2$$
$$= \sum_{i=1}^{u}\sum_{j=1}^{v}\sum_{k=1}^{r} {y_{ijk}}^2 - \frac{G^2}{uvr} \quad (6)$$

$\frac{G^2}{uvr}$ = Correction factor

$$G = \sum_{i=1}^{u}\sum_{j=1}^{v}\sum_{k=1}^{r} y_{ijk} : \text{ Grant total of all observations} \quad (7)$$

$$\text{SSR: Sum of squares due to rows} = \mathbf{vr}\sum_{i=1}^{u}(\bar{y}_{ioo} - \bar{y}_{ooo})^2 = \frac{\sum_{i=1}^{u} R_i^2}{u} - \frac{G^2}{vr} \quad (8)$$

$R_i = \sum_{j=1}^{v}\sum_{k=1}^{r} y_{ijk}$: ith row total

$$\text{SSC: Sum of squares due to column} = \boldsymbol{ur}\sum_{j=1}^{v}(\bar{y}_{ojo} - \bar{y}_{ooo})^2 = \frac{\sum_{j=1}^{v} C_j^2}{v} - \frac{G^2}{ur} \quad (9)$$

Where $C_j = \sum_{i=1}^{u}\sum_{k=1}^{r} y_{ijk}$

$$\text{SSTr: Sum of squares due to treatment } = \boldsymbol{uv}\sum_{k=1}^{r}(\bar{y}_{ook} - \bar{y}_{ooo})^2 = \frac{\sum_{k=1}^{r} T_k^2}{r} - \frac{G^2}{uv} \quad (10)$$

Where $T_k = \sum_{i=1}^{u}\sum_{j=1}^{v} y_{ijk}$
Degrees of freedom carried by SSR, SSC, and SSTr are (u-1), (v-1), and (r-1), respectively.
Degrees of freedom carried by TSS is (uvr-1)
The degree of freedom carried by SSE is (u-1)(v-1)(r-1)
The expected values of the mean of squares are found as follows;

$$\mathrm{E(MSR)} = \mathrm{E}\left(\frac{SSR}{u-1}\right) = \sigma^2 + \frac{u}{u-1}\sum_{i=1}^{u}\alpha_i^2 \quad (11)$$

$$\mathrm{E(MSC)} = \mathrm{E}\left(\frac{SSC}{v-1}\right) = \sigma^2 + \frac{v}{v-1}\sum_{i=1}^{v}\beta_j^2 \quad (12)$$

$$\mathrm{E(MSTr)} = \mathrm{E}\left(\frac{SSTr}{r-1}\right) = \sigma^2 + \frac{r}{r-1}\sum_{k=1}^{r}\gamma_k^2 \quad (13)$$

$$\mathrm{E(MSE)} = \mathrm{E}\left(\frac{SSE}{(u-1)(v-1)(r-1)}\right) = \sigma^2 \quad (14)$$

Thus,

- ✓ Under H_{0R}, $F_R = \frac{MSR}{MSE} \sim F((u-1),(v-1)(r-1))$
- ✓ Under H_{0C}, $F_C = \frac{MSC}{MSE} \sim F((v-1),(u-1)(r-1))$
- ✓ Under H_{0T}, $F_T = \frac{MSTr}{MSE} \sim F((r-1),(u-1)(v-1))$

Decision Rules:

Reject H_{0R} at level α if $F_R > F_{1-\alpha}$; (u-1), (v-1)(r-1)
Reject H_{0C} at level α if $F_C > F_{1-\alpha}$; (v-1), (u-1)(r-1)
Reject H_{0T} at level α if $F_T > F_{1-\alpha}$; (r-1), (u-1)(v-1)

When any alternative hypothesis is accepted, the pairwise comparison test is used [19]. Variance analysis, as outlined in Table 2, is utilized for the purpose of analysis.

Table 2. *The analysis of the variance table*

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares	F
Rows	u-1	SSR	MSR	F_R
Columns	v-1	SSC	MSC	F_C

Treatments	r-1	SSTr	MSTr	F_T
Error	(u-1)(v-1)(r-1)	SSE	*MSE	
Total	(uvr-1)	TSS		

*MSE: mean squares error

III. RESULTS

The analysis of the variance model was tested by including the main effects and pairwise interaction terms for these three parameters (Table 3). According to the analysis of variance findings, the material type with the row factor, the number of nozzles as the column factor, and the pressure as the treatment factor were found to be influential factors on temperature difference (Δt) ($p<0.05$). Values expressing the degree of effect and showing the percentage contribution of each parameter on the total variability are in the last column of Table 3. Based on the contribution values, the pressure parameter had the highest effect on the total variability (78.80%), followed by the material (11.51%), the number of nozzles (5.27%), the material*nozzle interaction (2.31%) and other interactions. The model's success in explaining the variance in temperature variability was 98.79%.

Table 3. *Findings of Analysis of Variance*

Dependent Variable: Δt

Source	Sum of Squares	df	Mean Square of Error	F	p-value	Contribution (%)
Material	6575.58	3	2191.86	757.55	.000	11.51
Nozzle	3009.93	4	752.48	260.07	.000	5.27
Pressure	45021.04	11	4092.82	1414.56	.000	78.80
Material * Nozzle	1318.70	12	109.89	37.98	.000	2.31
Material * Pressure	463.10	33	14.03	4.85	.000	0.81
Nozzle * Pressure	363.47	44	8.26	2.85	.000	0.64
Error	381.92	132	2.89			
Corrected Total	57133.76	239				

R Squared = .993 (Adjusted R Squared = .988)

Intra-factor comparison results are obtained using the last sub-category of material, nozzle, and pressure as comparison criteria in Appendix B. The difference with the previous category decreases as the nozzle or pressure category increases. For example, although the difference between 2, 3, and 4 with 6 nozzles was significant, there was no significant difference between the means of nozzles 5 and 6 ($p=0.202$). Similarly, there is a statistically significant difference between pressure levels up to 600 P_i and 700 P_i, while there is no difference between 650 P_i and 700 P_i. As the pressure level increases, the temperature difference also increases significantly.

According to the results of multiple comparisons for the material type, the mean Δt score for brass was significantly higher than Polyamide, Aluminum, and steel ($p<0.001$, Table 4). However, no significant difference was found between the mean scores of Aluminum and Polyamide ($p=0.954$).

Table 4. *Multiple Comparisons of Process Parameters (Tukey HSD)*

(I) idmaterial	(J) idmaterial	Mean Difference (I-J)	Std. Error	p-value	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Poliamid	Brass	-12.97*	.617	.000	-14.576	-11.380
	Aluminum	-.32	.617	.954	-1.919	1.276
	Steel	-4.28*	.617	.000	-5.886	-2.690
Brass	Poliamid	12.97*	.617	.000	11.380	14.576
	Aluminum	12.65*	.617	.000	11.058	14.254
	Steel	8.69*	.617	.000	7.091	10.288
Aluminum	Poliamid	.32	.617	.954	-1.276	1.919
	Brass	-12.65*	.617	.000	-14.254	-11.058
	Steel	-3.96*	.617	.000	-5.564	-2.368
Steel	Poliamid	4.28*	.617	.000	2.690	5.886
	Brass	-8.69*	.617	.000	-10.288	-7.091
	Aluminum	3.96*	.617	.000	2.368	5.564

The error term is Mean Square(Error) = 10.79 based on observed means. *: The mean difference is significant at the 0.05 level.

According to the results of multiple comparisons for the number of nozzles, there was no difference in the mean score between 5 and 6 nozzles (Appendix C, Mean difference = -0.883, $p = 0.704$). Apart from that, up to 5 nozzles, as the number of nozzles increases, the mean of Δt increases significantly ($p<0.05$).

According to multiple comparisons for pressure levels (Appendix D), significant differences were found between all pressure levels up to 550 P. Accordingly, Δt also increases with every 50 P increase. Nevertheless, there was no statistically significant difference between 500-550, 550-600, 600-650, and 650-700 P pressure levels ($p\geq0.05$).

When the profile plot obtained to evaluate the nozzle and material interaction is examined, although the interaction effect is statistically significant, it cannot be distinguished formally. As the nozzle value increases for each material type, it is seen that the mean of Δt also increases (Figure 4). The decrease in Polyamide and Aluminum and the increase in brass and steel (as they appear parallel) are valid for each nozzle level.

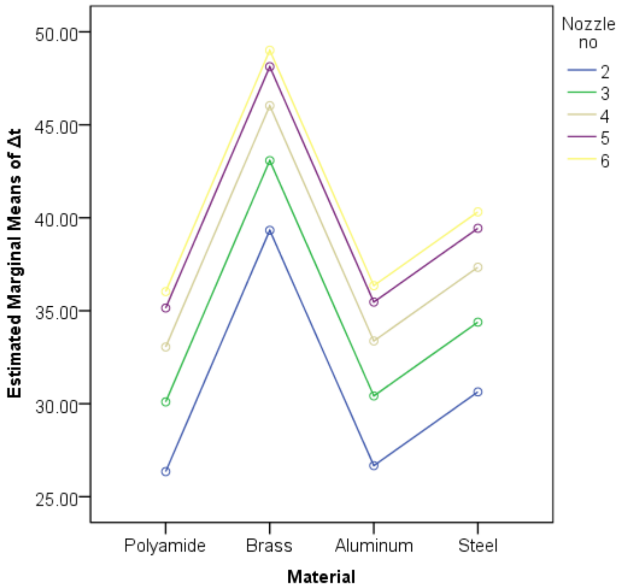

Figure 4. *Profile plot for material and nozzle on Δt of the vortex tube*

Suppose we evaluate the nozzle, material, and pressure together. In that case, it is evident in the profile plots in Figure 5 that as the pressure value increases in each material type, the mean Δt also increases. In the case of an increase from 2 nozzles to 6 nozzles, it is seen that the relative pressure value of each material is higher compared to the previous nozzle.

a **b**

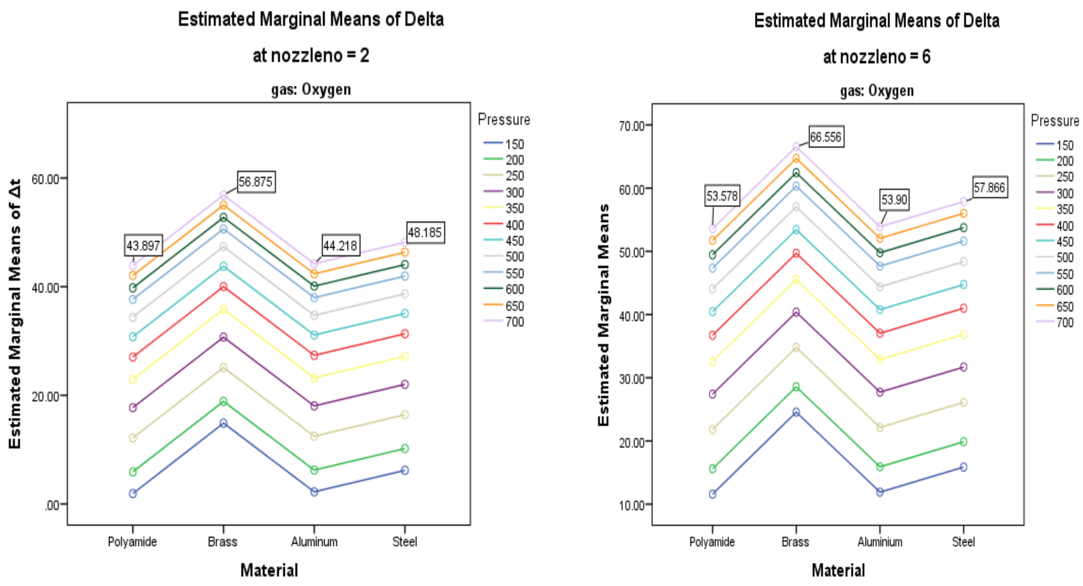

Figure 5. *Profile plot for material, nozzle, and pressure on Δt of the vortex tube, a) for two nozzles, b) for six nozzles*

Interaction graphs between parameters obtained from the analysis of variance and supporting the test results are shown in Figure 6.

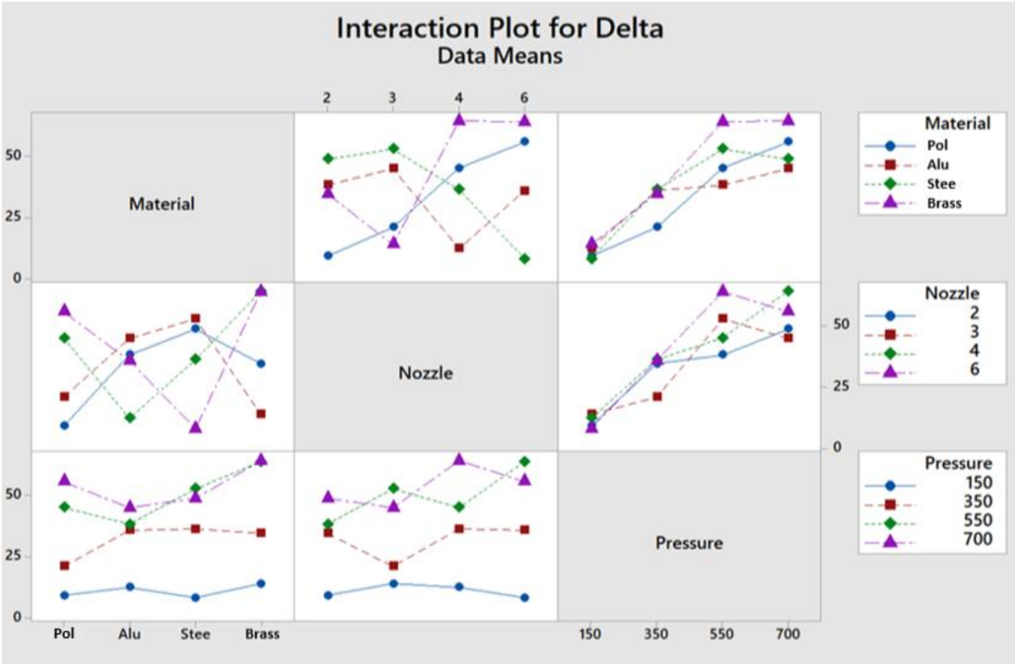

Figure 6. *Inter-Parameter Interaction Graph for Δt (from Analysis of Variance).*

The image of the L16 array where three factors with four levels and two interactions are planned is given in Table 5. As a result of the Taguchi analysis, the model, including the two interactions, could not be reached. The Taguchi L16 orthogonal array solution could only provide the test result, including the material*nozzle interaction. In addition, Taguchi restricted the interaction analysis and could not provide the interaction plot for the L16 layout. According to Taguchi's findings, only pressure was statistically significant ($p=0.005$). Material, nozzle number, and material and nozzle interaction were not statistically significant ($p\geq 0.05$). Taguchi analysis also provided S/N ratios and corresponding p values for all sub-levels of each parameter (Table 6). When the ranks in the response table for the signal-to-noise ratio findings were examined, the most influential variable was pressure on Δt (Delta=13.85), the material was in second place (Delta=3.04), nozzle number was in third place (Delta=1.72). The last was the material*nozzle interaction (Delta=1.47). The optimum levels for the parameters are: for pressure, the highest S/N ratio (with 34.51) was 700 n/m2, which is the 4th level; brass provided the highest S/N percentage with 31.54 for material, and four nozzles for nozzle type (with S/N=30.64) and 2×Polyamide for material*nozzle interaction (with S/N ratio= 30.30). The highest mean Δt estimate was 64.21, and the corresponding S/N ratio was 36.19. Table 5 includes mean Δt estimates and related S/N ratios in the last two columns.

Table 5. *L16 Orthogonal experimental design defined in Minitab and prediction findings*

Material	Nozzle	P_i(kPa)	Mat-Nozz	Nozz-Pres	Δt	S/N Ratio	Prediction
pol	2	150	2*pol	450	9.30	19.36	3.33
pol	3	350	Brass*3	1050	21.20	26.52	24.91
pol	4	550	Alu*4	2200	45.30	33.12	49.36
pol	6	700	Stee*6	4200	55.90	34.94	54.08
Brass	2	350	Alu*4	4200	34.80	30.83	35.86
Brass	3	150	Stee*6	2200	14.10	22.98	15.28
Brass	4	700	2*pol	1050	64.50	36.19	64.21

Brass	6	550	Brass*3	450	64.10	36.13	62.13
Alu	2	550	Stee*6	1050	38.40	31.68	42.61
Alu	3	700	Alu*4	450	45.10	33.08	46.51
Alu	4	150	Brass*3	4200	12.60	22.00	10.28
Alu	6	350	2*pol	2200	36.00	31.12	32.68
Steel	2	700	Brass*3	2200	49.00	33.80	49.68
Steel	3	550	2*pol	4200	53.10	34.50	46.78
Steel	4	350	Stee*6	450	36.50	31.24	35.03
Steel	6	150	Alu*4	1050	8.20	18.27	15.28

Taguchi Analysis Results for L16 Orthogonal Design:

Table 6. *Taguchi Analysis for SN ratios: Delta versus material; nozzle; pressure; material&nozzle*

Estimated Model Coefficients for SN ratios

Term	Coef	SE_Coef	T	P
Constant	29.740	0.538	55.271	0.000
Material pol	-1.248	0.932	-1.340	0.273
Material brass	1.795	0.932	1.927	0.150
Material Alu	-0.264	0.932	-0.284	0.795
Nozzle 2	-0.817	0.932	-0.877	0.445
Nozzle 3	-0.466	0.932	-0.500	0.651
Nozzle 4	0.901	0.932	0.967	0.405
Pressure 150	-9.080	0.932	-9.743	0.002
Pressure 350	0.192	0.932	0.206	0.850
Pressure 550	4.121	0.932	4.423	0.021
Mat-Nozz 2*pol	0.557	0.932	0.598	0.592
Mat-Nozz brass*3	-0.121	0.932	-0.130	0.905
Mat-Nozz Alu*4	-0.911	0.932	-0.978	0.400

S=2.152 R^2=97.4% $R_{adj}^2 = 87.0\%$

Analysis of Variance for SN Ratios

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Material	3	19.737	19.737	6.579	1.42	0.390
Nozzle	3	7.374	7.374	2.458	0.53	0.692
Pressure	3	488.822	488.822	162.941	35.17	0.008
Mat*Nozz	3	5.532	5.532	1.844	0.40	0.765
Residual Error	3	13.898	13.898	4.633		

Total		15	535.362	

Response Table for Signal-to-Noise Ratios

Larger is better

Level	Material	Nozzle	Pressure	Mat-Nozz
1	28.49	28.92	20.66	30.30
2	31.54	29.27	29.93	29.62
3	29.48	30.64	33.86	28.83
4	29.46	30.12	34.51	30.22
Delta	3.04	1.72	13.85	1.47
Rank	2	3	1	4

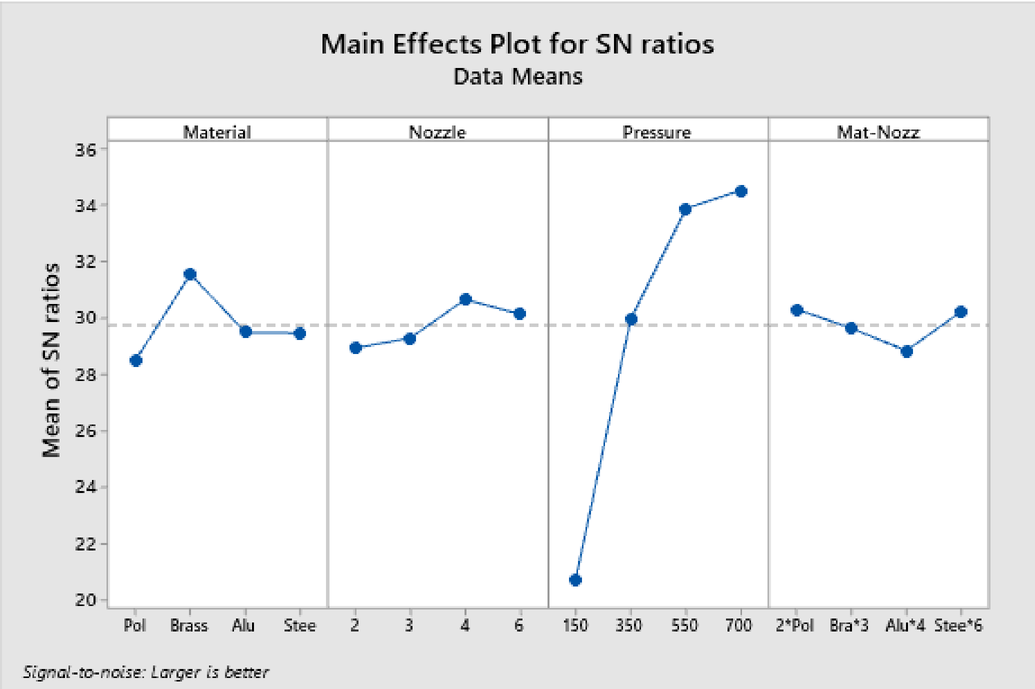

Figure 7. *Taguchi's main effects plot*

IV. CONCLUSION

Many publications have made various statistical analyses on the temperature change of the counterflow RHVTs based on specific process parameters [20]. This study is one of the rare publications that compare the performance of known statistical factorial analysis of variance (ANOVA) and optimum Taguchi orthogonal array analysis to detect the factors affecting the temperature change of the counterflow RHVTs. Their statistical pros and cons were also evaluated.

In their study [1], they determined the effects of process parameters and optimal factor levels based on Taguchi's L27 orthogonal array using signal-to-noise ratio (S/N) analysis, regression analysis, and analysis of variance (ANOVA) methods. The experiments were planned according to different inlet pressures, nozzle numbers, and fluid types. According to the ANOVA results from some of their analyses, the inlet pressure was the most dominant effect on the total change (p = 89.89%). Later, they stated that this situation was followed by the inlet pressure, the number of nozzles interactions (p=3.72%), and the number of nozzles alone (p=2.47%).

In the study of [3], they analyzed the performance of RHVTs connected in parallel using oxygen gas at different inlet pressures, using various materials and nozzles. The multiple linear regression techniques calculated the analysis. Then, a regression equation was obtained, and (S/N) ratios of the designed test results were found with the help of the Taguchi L16 array. As a result of the analyses, they emphasized that the factors affecting the temperature difference ΔT, in which the performance values are compared, are the inlet pressure, the number of nozzles, and the nozzle materials, respectively.

Another study used the Taguchi method to investigate the effect and optimization of temperature differences for process parameters in a counterflow vortex tube [21]. In their research, where they used the L27 orthogonal Taguchi array, 150, 400, and 650 kPa values were taken into account for Inlet pressure, 2, 4, and 6 for Nozzle number, and 0.5, 0.6, and 0.7 for the cold mass fraction. In their ANOVA results, all control factors were detected to have a statistically significant effect on the temperature difference. Inlet pressure (p = 58.11%) has the most significant contribution to total variation, followed by nozzle number (p = 24.78%) and a cold mass fraction (p = 4.431%). They found the optimum parameter levels maximizing the temperature difference as an inlet pressure of 650 kPa, a nozzle number of 2, and a cold mass fraction of 0.7. In this study, according to the analysis of variance, all the main effects and all the binary interactions of three parameters (4×5×12=240) were evaluated with a sufficient number of experiments. All exchanges, posthoc test results, the contribution of the parameters to the variance and the regression equation between the parameters, and the outcome variable could be obtained in the ANOVA findings. To reduce the number of experiments, considering the four levels of each process parameter (Material type polyamide, brass, aluminium, and steel; 2, 3, 4, and 6 for nozzle number; 150, 350, 550, and 700 for pressure), the most suitable orthogonal design, also including an interaction term, was determined as the L16 orthogonal array. Our findings are similar to (23); it was observed that the pressure parameter had the highest effect on the total variability (78.80%), followed by the material type (11.51%), the number of nozzles (5.27%), the material&nozzle interaction (2.31%) and other interactions depending on the contribution ratio. The optimum levels of the parameters that maximize the temperature difference were obtained as the 4th level for pressure "700 kPa", "brass" for the material, four nozzles for the number of nozzles, and 2×polyamide for the material*nozzle interaction.

When the traditional analysis of variance and the Taguchi method are compared in terms of both analysis performance and visual, both methods have advantages and disadvantages. In the analysis of variance results, there is no output in which the optimum levels of the parameters can be determined. However, the effect sizes of each parameter on the dependent variable are given as a percentage of contribution. However, the changes within the parameter levels can be tested. In addition, the analysis of variance options also shows post hoc comparison results of parameter levels. The advantages and disadvantages of these two methods considered in the study according to various criteria are summarized in Table 7. The L16 orthogonal array, the most suitable design for three factors, three interactions, and a 4-level structure, was chosen during the planning phase. In our study, the Taguchi analysis applied according to the L16 orthogonal array gave only one interaction (between the material*nozzle) between the factors and could not provide the interaction plots. The fact that only 16 experiments were used here instead of the 240 sample size in full factorial data analysis was also disadvantageous regarding the parameters, resulting in a significant (number of factors with $p<0.05$). This difficulty makes it obligatory to report the Analysis of Variance findings separately in all cases, in addition to the Taguchi array solutions, as in some studies [22,23].

Therefore, classical statistical analysis of variance methods should be preferred if reaching all levels of the parameters and the relevant outcome variable in the experimental design does not bring additional costs and if the interactions between the parameters used are also crucial for us.

Suppose the entire experimental setup is difficult and expensive to obtain, and only the main effects of the parameters are essential to the researcher. In that case, it is aimed to determine the best sublevels that provide the largest, smallest, or optimal outcome; the faster and less costly Taguchi method can be preferred. However, interaction effects and pairwise comparisons of parameter levels will be ignored in this method, unlike the analysis of variance. Consequently, factorial variance analysis provides more information on the outcome than machine learning methods or the non-parametric Taguchi method in case there are only a few factors in the data set, the size of the explanatory variable is not very large. The number of experiments is sufficient for parametric statistical methods such as factorial ANOVA.

Table 7. *Statistical assessment of Taguchi analysis compared to Analysis of Variance*

Criteria	Analysis of Variance	Taguchi Analysis
Testing the main effects	✓	✓
Testing interactions	✓	X (weak)
Graphical representation	✓	✓ (weak)
More robust prediction with S/N ratio	X	✓
Post-hoc comparison	✓	X
Reporting the contribution rate of each parameter to the model	✓	✓
Determining the optimum level for each parameter according to the S/N ratio	X	✓
Chance of statistical significance of parameters and their interactions	Higher	Lower

V. REFERENCES

[1] A. M. Pinar, O. Uluer, and V. Kırmacı, "Statistical assessment of counterflow vortex tube performance for different nozzle numbers, cold mass fractions, and inlet pressures via taguchi method," *Exp. Heat Transf.*, vol. 22, no. 4, pp. 271–282, 2009, doi: 10.1080/08916150903099058.

[2] H. Kaya, O. Uluer, E. Kocaoğlu, and V. Kirmaci, "Experimental analysis of cooling and heating performance of serial and parallel connected counterflow Ranquee–Hilsch vortex tube systems using carbon dioxide as a working fluid," *Int. J. Refrig.*, vol. 106, pp. 297–307, 2019, doi: 10.1016/j.ijrefrig.2019.07.004.

[3] H. Gökçe, "Optimization of Ranque–Hilsch vortex tube performances via Taguchi method," *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 42, no. 11, 2020, doi: 10.1007/s40430-020-02649-z.

[4] K. Dincer, S. Baskaya, B. Z. Uysal, and I. Ucgul, "Experimental investigation of the performance of a Ranque-Hilsch vortex tube with regard to a plug located at the hot outlet," *Int. J. Refrig.*, vol. 32, no. 1, pp. 87–94, 2009, doi: 10.1016/j.ijrefrig.2008.06.002.

[5] W. Fröhlingsdorf and H. Unger, "Numerical investigations of the compressible flow and the energy separation in the Ranque-Hilsch vortex tube," *Int. J. Heat Mass Transf.*, vol. 42, no. 3, pp. 415–422, 1998, doi: 10.1016/S0017-9310(98)00191-4.

[6] M. Korkmaz, A. Dogan, and V. Kırmacı, "Performance Analysis of Counterflow Ranque – Hilsch Vortex Tube with Linear Regression, Support Vector Machines and Gaussian Process Regression Method," *Gazi J. Eng. Sci.*, vol. 8, no. 2, pp. 361–370, 2022, doi: doi:10.30855/gmbd.0705015.

[7] T. Dutta, K. P. Sinhamahapatra, and S. S. Bandyopadhyay, "Numerical investigation of gas species and energy separation in the Ranque-Hilsch vortex tube using real gas model," *Int. J. Refrig.*, vol. 34, no. 8, pp. 2118–2128, 2011, doi: 10.1016/j.ijrefrig.2011.06.004.

[8] M. Bovand, M. S. Valipour, K. Dincer, and A. Tamayol, "Numerical analysis of the curvature effects on Ranque-Hilsch vortex tube refrigerators," *Appl. Therm. Eng.*, vol. 65, no. 1–2, pp. 176–183, 2014, doi: 10.1016/j.applthermaleng.2013.11.045.

[9] X. Han et al., "The influence of working gas characteristics on energy separation of vortex tube," *Appl. Therm. Eng.*, vol. 61, no. 2, pp. 171–177, 2013, doi: 10.1016/j.applthermaleng.2013.07.027.

[10] R. Shamsoddini and A. H. Nezhad, "Numerical analysis of the effects of nozzles number on the flow and power of cooling of a vortex tube," *Int. J. Refrig.*, vol. 33, no. 4, pp. 774–782, 2010, doi: 10.1016/j.ijrefrig.2009.12.029.

[11] S. Eiamsa-ard, "Experimental investigation of energy separation in a counterflow Ranque-Hilsch vortex tube with multiple inlet snail entries," *Int. Commun. Heat Mass Transf.*, vol. 37, no. 6, pp. 637–643, 2010, doi: 10.1016/j.icheatmasstransfer.2010.02.007.

[12] Y. Xue, M. Arjomandi, and R. Kelso, "Energy analysis within a vortex tube," *Exp. Therm. Fluid Sci.*, vol. 52, pp. 139–145, 2014, doi: 10.1016/j.expthermflusci.2013.09.004.

[13] Y. Xue, M. Arjomandi, and R. Kelso, "Experimental study of the thermal separation in a vortex tube," *Exp. Therm. Fluid Sci.*, vol. 46, pp. 175–182, 2013, doi: 10.1016/j.expthermflusci.2012.12.009.

[14] A. V. Khait, A. S. Noskov, A. V. Lovtsov, and V. N. Alekhin, "Semi-empirical turbulence model for numerical simulation of swirled compressible flows observed in Ranque-Hilsch vortex tube," *Int. J. Refrig.*, vol. 48, pp. 132–141, 2014, doi: 10.1016/j.ijrefrig.2014.09.006.

[15] H. A. Kandil and S. T. Abdelghany, "Computational investigation of different effects on the performance of the Ranque-Hilsch vortex tube," *Energy*, vol. 84, pp. 207–218, 2015, doi: 10.1016/j.energy.2015.02.089.

[16] S. Mohammadi and F. Farhadi, "Experimental and numerical study of the gas-gas separation efficiency in a Ranque-Hilsch vortex tube," *Sep. Purif. Technol.*, vol. 138, pp. 177–185, 2014, doi: 10.1016/j.seppur.2014.10.022.

[17] M. Korkmaz, A. Binal, H. Kaya, and V. Kırmacı, "ANN based ternary diagrams for thermal performance of a Ranque Hilsch vortex tube with different working fluids," *Therm. Sci. Eng. Prog.*, vol. 40, no. October 2022, p. 101803, 2023, doi: 10.1016/j.tsep.2023.101803.

[18] Bethea RM., Statistical methods for engineers and scientists. CRC Press, 2018.

[19] P. Valcheva, "Some combinatorial structures in experimental design: overview, statistical models and applications," *Biometrics Biostat. Int. J.*, vol. 7, no. 4, 2018, doi: 10.15406/bbij.2018.07.00228.

[20] H. Pouraria and M. R. Zangooee, "Numerical investigation of vortex tube refrigerator with a divergent hot tube," *Energy Procedia*, vol. 14, no. 2011, pp. 1554–1559, 2012, doi: 10.1016/j.egypro.2011.12.1132.

[21] A. M. Pinar, O. Uluer, and V. Kirmaci, "Optimization of counter flow Ranque-Hilsch vortex tube performance using Taguchi method," Int. J. Refrig., vol. 32, no. 6, pp. 1487–1494, 2009, doi: 10.1016/j.ijrefrig.2009.02.018.

[22] H. Gokce, "Evaluation and Optimization of O2 Used Ranque-Hilsch Vortex Tube Performance," Energy Sources, Part A Recover. Util. Environ. Eff., vol. 43, no. 13, pp. 1566–1576, 2021, doi: 10.1080/15567036.2020.1817188.

[23] T. Madani, M. Boukraa, M. Aissani, T. Chekifi, A. Ziadi, and M. Zirari, "Experimental investigation and numerical analysis using Taguchi and ANOVA methods for underwater friction stir welding of aluminium alloy 2017 process improvement," *Int. J. Press. Vessel. Pip.*, vol. 201, no. December 2022, p. 104879, 2023, doi: 10.1016/j.ijpvp.2022.104879.