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Series Solution To The Plane Wave Scattering By A Perfectly Electric Conducting Half Plane

Mükemmel Elektrik İletkeni Yarım Düzlemle Düzlem Dalga Saçılımına Seri Çözüm

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Abstract

The scattering of plane waves by a half-plane of perfectly electric conducting (PEC) surface is taken into consideration by means of the Bessel series to express the total scattered field. First, total scattered field is obtained in terms of the Fresnel function by using the series addition of incident and reflected fields, and the solution of the Helmhotz equation. Next, the Fresnel function is decomposed into unit step function and Signum function times the Fresnel function. The obtained scattered, diffracted and geometrical optics (GO) fields are plotted numerically. The behavior of scattered, GO and diffracted fields are observed and interpreted by considering the electromagnetic scattering theory. Diffracted field which consists of incident diffracted, and reflected diffracted fields is plotted along with the scattered field. Diffracted field components come from the multiplication of the Signum function and the Fresnel function. Also, the GO field is expressed by the unit step function.

Key Words

"Diffraction, Helmholtz equation, Fresnel function, Scattering"

Öz

Toplam saçılan alanı ifade etmek için, düzlem dalgaların mükemmel elektrik iletken (PEC) yüzeyinin yarım düzlemi tarafından saçılması, Bessel serisi aracılığıyla dikkate alınır. İlk olarak, gelen ve yansıyan alanların seri toplamı ve Helmhotz denkleminin çözümü kullanılarak Fresnel fonksiyonu cinsinden toplam saçılmış alan elde edilir. Daha sonra, Fresnel fonksiyonu birim adım fonksiyonuna ve Signum fonksiyonu çarpı Fresnel fonksiyonuna ayrıştırılır. Elde edilen dağınık, kırınımlı ve geometrik optik (GO) alanları sayısal olarak çizilmiştir. Saçılan, GO ve kırılan alanların davranışları elektromanyetik saçılma teorisi dikkate alınarak gözlemlenir ve yorumlanır. Gelen kırınıma uğramış ve yansıtılmış kırınıma sahip alanlardan oluşan kırınımlı alan, saçılmış alanla birlikte çizilmiştir. Kırılan alan bileşenleri Signum fonksiyonu ile Fresnel fonksiyonunun çarpımından elde edilir. Ayrıca GO alanı birim adım fonksiyonuyla ifade edilir.

Anahtar Kelimeler

"Kırınım, Helmholtz denklemi, Fresnel fonksiyonu, Saçılma"

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1. Introduction

Electromagnetic wave scattering by perfectly conducting and imperfectly conducting surfaces has long been under investigation. Wave scattering by a PEC half-plane is a widely used canonical matter firstly solved by Sommerfeld (1896). Plane wave diffraction by an impedance half plane was firstly solved by Maliuzhinets (1958, 1960). Senior (1952) dealt with the same problem by using the Weiner-Hopf integral equations to find the electric and magnetic currents. Raman and Krishnan (1927) worked on the half-plane and wedge having finite conductivity by modifying the Sommerfeld's solution. Jones and Pidduck (1950) studied on the diffracted wave by a metallic wedge at large angles. Kouyoumjian and Pathak (1974) worked on the uniform diffraction theory for an edge. Bucci and Franceschetti (1976) studied scattering by half-plane. Tiberio et al. (1984) developed a uniform geometrical theory of diffraction (GTD) expression for the diffraction for a wedge. Their high-frequency solution formulation is compatible with the solution of uniform GTD solution for a perfectly conducting wedge as well. Sanyal and Bhattacharyya (1986) investigated the diffraction from a half-plane having two face impedances. In their study Van der Waerden's method is used to obtain the asymptotic expansion of Maliuzhinets' exact solution. Rojas (1988) presented an asymptotic solution for the plane wave diffraction by an impedance wedge. Büyükaksoy and Uzgören (1988) investigated the diffraction of waves with high-frequency by a curved surface, and proposed asymptotic expressions for the diffraction coefficients. Borghi et al. (1996) examined plane wave scattering by a perfectly conducting circular cylinder. Drawbacks of the impedance half plane diffraction solutions proposed by Senior and Maliuzhinets are shown by Umul (2009 a-b). Scattering of a line-sourced waves by a cylindrical parabolic reflector with impedance face is presented by Umul (2008) by using the surface integrals of the modified theory of physical optics (MTPO) he introduced earlier Umul (2004). Kara (2016) studied plane wave scattering by a cylindrical parabolic PEC reflector. Kara (2019) investigated inhomogeneous plane wave scattering of by a slit composed of two different half planes. Kara (2019) evaluated inhomogeneous plane wave scattering by a PEC half plane asymptotically. Kara (2020) examined diffraction of a line source originated waves by a parabolic reflector where the reflector is offset fed by the line source. Kara (2021) studied plane wave diffraction by an aperture with two half planes having different resistivities between isorefractive media. Kara and Mutlu (2023) investigated scattering by a truncated cylindrical conductive cap by using physical optics surface current on the cap. Umul (2009a) proposed a solution for an impedance half-plane having identical faces. Later, his method is used for the diffraction of waves by two different half planes between two dielectric media Umul and Yalçın (2010a,b). Umul (2013) presented a new series solution to the wave scattering by a half screen by means of Dirichlet and Neumann conditions. In this study a series solution, examined by James et al. (1986) and some other authors, of a plane wave scattering by a PEC half plane will be obtained. The obtained results will be plotted and interpreted.

2. Theory

In this section, we will take a whole perfectly electric conducting plane into account. An incident plane wave given in Equation (1), where k is the wavenumber and φ_0 is the angle of incidence, is considered. Geometry under consideration is indicated in Figure 1.

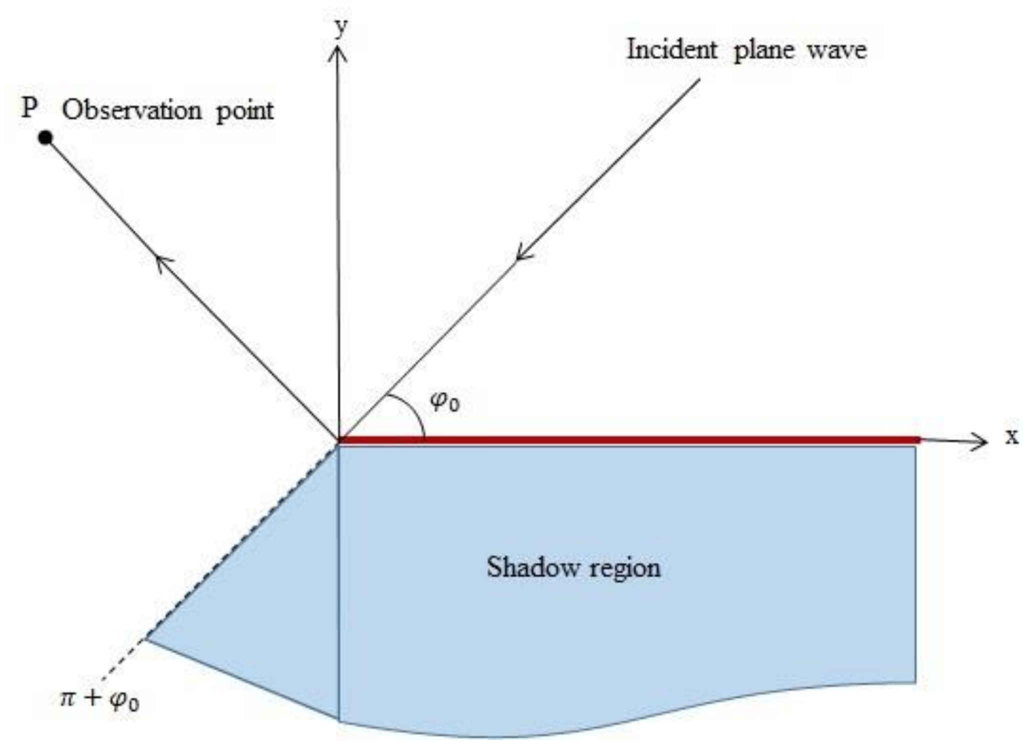

Figure 1. Scattering geometry

$E_i = \hat{e_z} e^{jk\rho cos(\varphi-\varphi_0)}$ (1)

For a whole plane reflected field is written as

$E_r = -\hat{e_z} e^{jk\rho cos(\varphi+\varphi_0)}$ (2)

By using the Bessel series given in Equation (3)

$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x) t^n$ (3)

$exp(jxsin\theta)$ can be written as

$e^{jxsin\theta} = \sum_{-\infty}^{\infty} J_n(x) e^{jn\theta}$ (4)

here $x = k\rho$, and $t = \exp(j\theta)$. Similarly by letting $t = e^{j\theta}$, and $\theta \to \frac{\pi}{2} - \theta$, we write,

$e^{jxcos\theta} = \sum_{n=-\infty}^{\infty} J_n(x) e^{jn\frac{\pi}{2}} e^{-jn\theta}$ (5)

Bessel equation can be expressed in terms of $e^{jxcos\theta}$ as,

$\sum_{n=-\infty}^{-1} J_n(x) e^{jn\frac{\pi}{2}} e^{-jn\theta} + J_0(x) + \sum_{n=1}^{\infty} J_n(x) e^{jn\frac{\pi}{2}} e^{-jn\theta}$ (6)

For the first summation term on the left, letting $n \to -n$ we write Equation (6) as,

$\sum_{n=\infty}^{1} J_{-n}(x) e^{j(-n)\frac{\pi}{2}} e^{jn\theta} + J_0(x) + \sum_{n=1}^{\infty} J_n(x) e^{jn\frac{\pi}{2}} e^{-jn\theta}$ (7)

By considering

$J_{-n}(x) = (-1)^n J_n(x)$ (8)

we rewrite Equation (7) as,

$\sum_{n=\infty}^{1} (-1)^n J_n(x) e^{-jn\frac{\pi}{2}} e^{jn\theta} + J_0(x) + \sum_{n=1}^{\infty} J_n(x) e^{jn\frac{\pi}{2}} e^{-jn\theta}$ (9)

which can be reduced to

$2\sum_{n=1}^{\infty} J_n(x)\, e^{\frac{jn\pi}{2}} cosn\theta + J_0(x)$ (10)

E_i and E_r will be summed up in series. For this purpose the expression of the form given in Equation (11) is used and Equation (12) and Equation (13) are obtained.

$e^{jxcos\theta} = J_0(x) + 2\sum_{n=1}^{\infty} J_n(x) e^{jn\frac{\pi}{2}} cosn\theta$ (11)

$E_i = \hat{e_z}\left[J_0(k\rho) + 2\sum_{n=1}^{\infty} J_n(k\rho) e^{jn\frac{\pi}{2}} cosn(\varphi - \varphi_0)\right]$ (12)

Similarly,

$E_r = -\hat{e_z}\left[J_0(k\rho) + 2\sum_{n=1}^{\infty} J_n(k\rho) e^{jn\frac{\pi}{2}} cosn(\varphi + \varphi_0)\right]$ (13)

If the incident and reflected fields are summed up, total scattered fields become as

$E_{tot} = E_i + E_r = \hat{e_z}\, 2\sum_{n=1}^{\infty} J_n(k\rho) e^{jn\frac{\pi}{2}} [cosn(\varphi - \varphi_0) - cosn(\varphi + \varphi_0)]$ (14)

which is reduced to

$E_{tot} = E_i + E_r = 4\sum_{n=1}^{\infty} J_n(k\rho) e^{jn\frac{\pi}{2}}\, sin(n\varphi) \sin(n\varphi_0)$ (15)

Here $x \in [0, \infty)$. The solution of Helmhotz equation $\nabla^2 E_z + k^2 E_z = 0$ in cylindrical coordinates is written as,

$E_z = J_v(k\rho)[A_v sin(v\varphi) + B_v cos(v\varphi)]$ (16)

φ=0 represents the top side, and φ=2π represents the bottom side of the half plane. At φ=0, $E_z = 0$,

$E_z = J_v(k\rho)[A_v sin(0) + B_v cos(0)] = 0$ (17)

As a result $B_v = 0$ is obtained. Thus E_z is reduced to,

$E_z = J_v(k\rho) A_v sin(v\varphi)$ (18)

At φ=2π,

$E_z = 0 = J_v(k\rho) A_v sin(v2\pi) = sin(n\pi)$ (19)

where $v = \frac{n}{2}$. E_z is written as,

$E_z = E_{tot} = \sum_{n=1}^{\infty} J_{\frac{n}{2}}(k\rho) A_{\frac{n}{2}} \sin\left(\frac{n}{2}\varphi\right)$ (20)

$A_{\frac{n}{2}}$ is obtained by equating Equation (15) and Equation (20) as,

$A_{\frac{n}{2}} = 2e^{j\frac{n\pi}{2\,2}} sin\left(\frac{n}{2}\varphi_0\right)$ (21)

If $A_{\frac{n}{2}}$ is substituted in Equation (20) total scattered field is written as,

$E_{tot} = 2\sum_{n=1}^{\infty} J_{\frac{n}{2}}(k\rho) e^{j\frac{n\pi}{2\,2}} sin\left(\frac{n}{2}\varphi_0\right) \sin\left(\frac{n}{2}\varphi\right)$ (22)

This is the total field comprising incident, reflected and diffracted fields. E_{tot} can also be expressed as

$E_{tot} = \frac{1}{2}J_0(k\rho) - \frac{1}{2}J_0(k\rho) + \sum_{n=1}^{\infty} J_{\frac{n}{2}}(k\rho)\left(e^{j\frac{\pi}{2}}\right)^{\frac{n}{2}} cos\left(\frac{n}{2}(\varphi - \varphi_0\right) - \sum_{n=1}^{\infty} J_{\frac{n}{2}}(k\rho)\left(e^{j\frac{\pi}{2}}\right)^{\frac{n}{2}} cos\left(\frac{n}{2}(\varphi + \varphi_0\right)$ (23)

where negative-coefficient terms belong to the reflected scattered field as the positive-coefficient terms comprise the incident scattered field. Incident scattered or reflected scattered fields in Equation (23) can be written in the form of

$\frac{1}{2}J_0(k\rho) + \sum_{n=1}^{\infty} j^{\frac{n}{2}} J_{\frac{n}{2}}(k\rho) \cos\left(\frac{n}{2}\theta\right)$ (24)

where $\theta = \varphi \pm \varphi_0$, Equation (24) is rewritten as,

$$\frac{1}{2}J_0(k\rho)+\sum_{n=1}^{\infty}j^n J_n(k\rho)cosn\theta+\sum_{n=0}^{\infty}j^{v_n}J_{v_n}(k\rho)cosv_n\theta \tag{25}$$

where

$$\frac{1}{2}J_0(k\rho)+\sum_{n=1}^{\infty}j^n J_n(k\rho)cosn\theta=\frac{1}{2}e^{jk\rho cos\theta} \tag{26}$$

and $v_n=n+\frac{1}{2}$ As a result, Equation (25) is reduced to

$$\frac{1}{2}e^{jk\rho cos\theta}+\sum_{n=0}^{\infty}j^{v_n}J_{v_n}(k\rho)cosv_n\theta \tag{27}$$

which can be expressed as

$$\frac{1}{2}e^{jk\rho cos\theta}+\frac{1}{2}\sum_{n=0}^{\infty}j^{v_n}J_{v_n}(k\rho)e^{jv_n\theta}+\frac{1}{2}\sum_{n=0}^{\infty}j^{v_n}J_{v_n}(k\rho)e^{-jv_n\theta}. \tag{28}$$

Finally, total scattered field given in Equation (23) is obtained as,

$$\frac{1}{2}J_0(k\rho)+\sum_{n=1}^{\infty}j^{\frac{n}{2}}J_{\frac{n}{2}}(k\rho)cos\left(\frac{n}{2}(\varphi-\varphi_0)\right)-\frac{1}{2}J_0(k\rho)-\sum_{n=1}^{\infty}j^{\frac{n}{2}}J_{\frac{n}{2}}(k\rho)cos\left(\frac{n}{2}(\varphi+\varphi_0)\right) \tag{29}$$

which is reduced to,

$$e^{jk\rho\cos(\varphi-\varphi_0)}F\left[-\sqrt{2k\rho}\cos\left(\frac{\varphi-\varphi_0}{2}\right)\right]-e^{jk\rho\cos(\varphi+\varphi_0)}F\left[-\sqrt{2k\rho}\cos\left(\frac{\varphi+\varphi_0}{2}\right)\right] \tag{30}$$

$F[a]$ is the Fresnel function which can be expressed as,

$$F[a]=u(-a)+sgn(a)F[|a|] \tag{31}$$

3. Numerical Results

In this section we will examine the behavior of diffracted, scattered and GO fields numerically. The wavelength λ is equal to 0.1 meter. ρ is 6λ, and the φ_0 is 60˚ (π/3). ρ is the observation distance. Figure 2 shows the total scattered field. It is seen that the value of the scattered field drops to 0.5 at 240˚ (π+ φ_0) where the diffracted component contribution takes place. Also, in Fig. 2, total scattered field continues to decrease smoothly thanks to the diffracted field expression containing terms yielding uniform field.

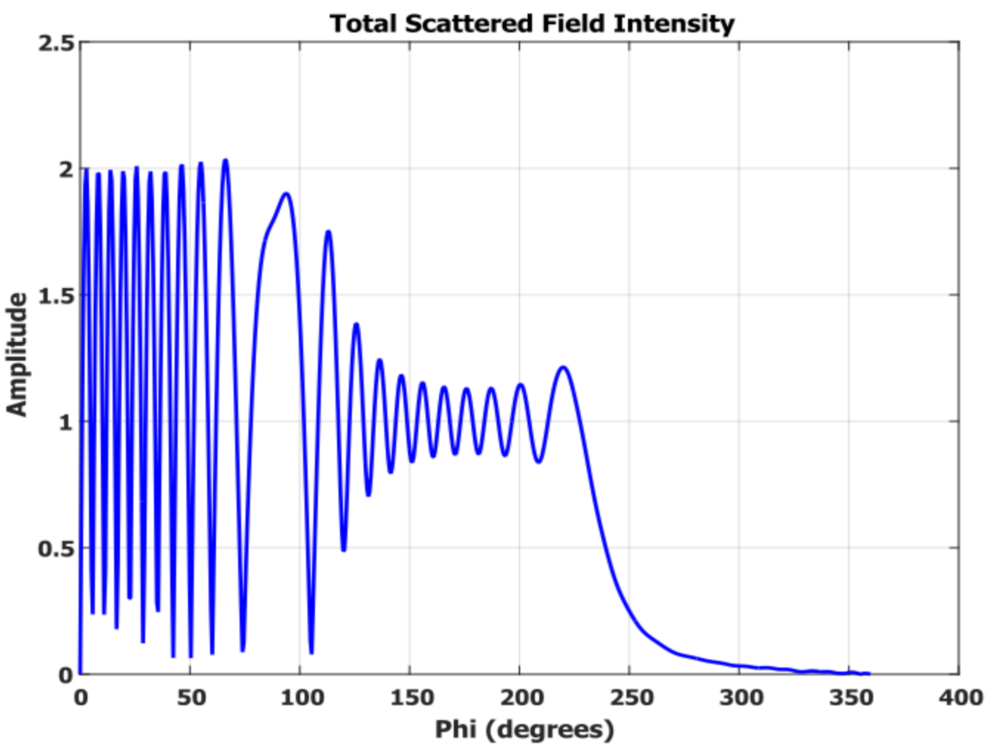

Figure 2. Total scattered field

In Figure 3 Incident diffracted and reflected diffracted field are given along with the total scattered field. Reflected diffracted and incident diffracted field peaks occur at 120˚ (π- φ_0) and 240˚ (π+φ_0) as expected. Because π- φ_0 is the reflection boundary, and π+ φ_0 is the shadow boundary. Transition points can be changed by changing the angle of incidence. For example, if the incident angle is chosen as π/6, reflection boundary will occur at 5 π/6 and the shadow boundary will be at 7 π/6. Diffracted field contributions come from the expression of Signum function times the Fresnel function, and unit step function determines the GO field.

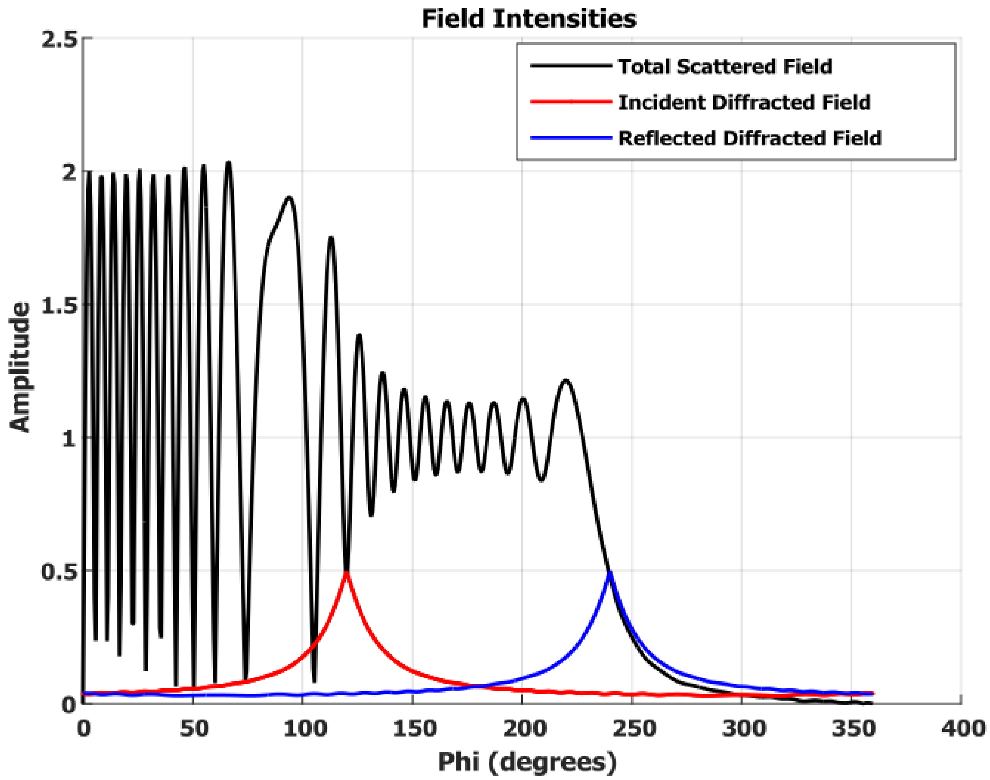

Figure 3. Variations of diffracted fields

Fig. 4 depicts the incident diffracted field and the GO field. Up to 240˚ ($\pi+\ \varphi_0$) the GO field exists, and after that point it cannot be seen. However the incident diffracted field intensity gradually decreases from that point.

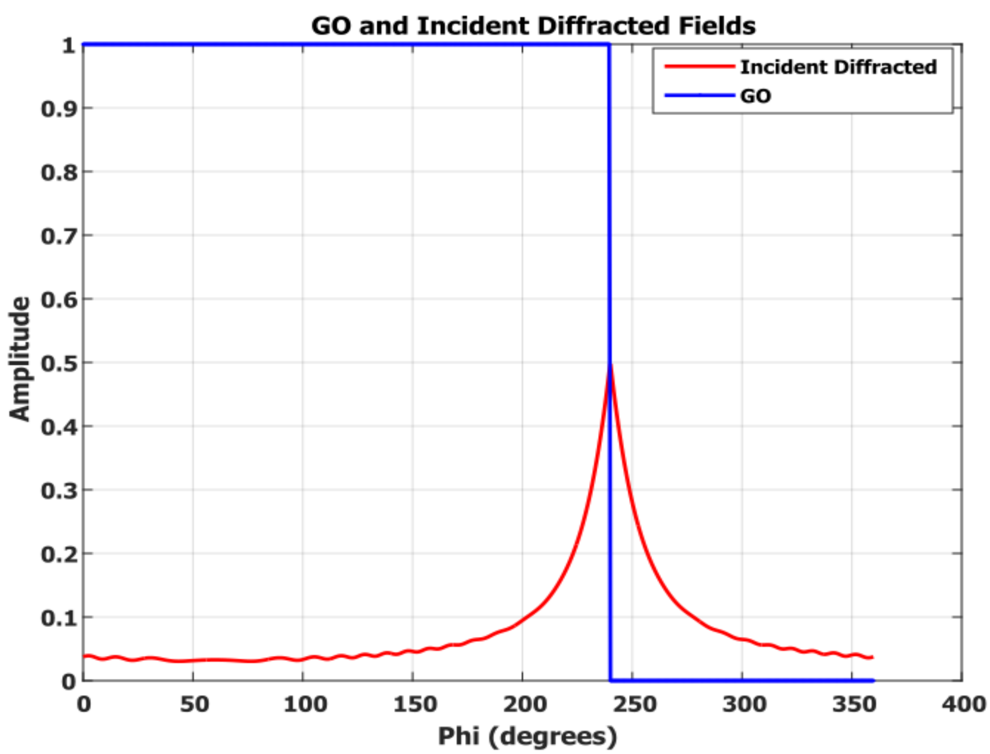

Figure 4. GO and incident diffracted field intensities

Fig. 5 depicts the reflected diffracted field and the GO field. The reflection boundary occurs at 120˚ ($\pi-\ \varphi_0$). The reflected diffracted field intensity has a peak at that point, and it starts to decrease above and below that angle.

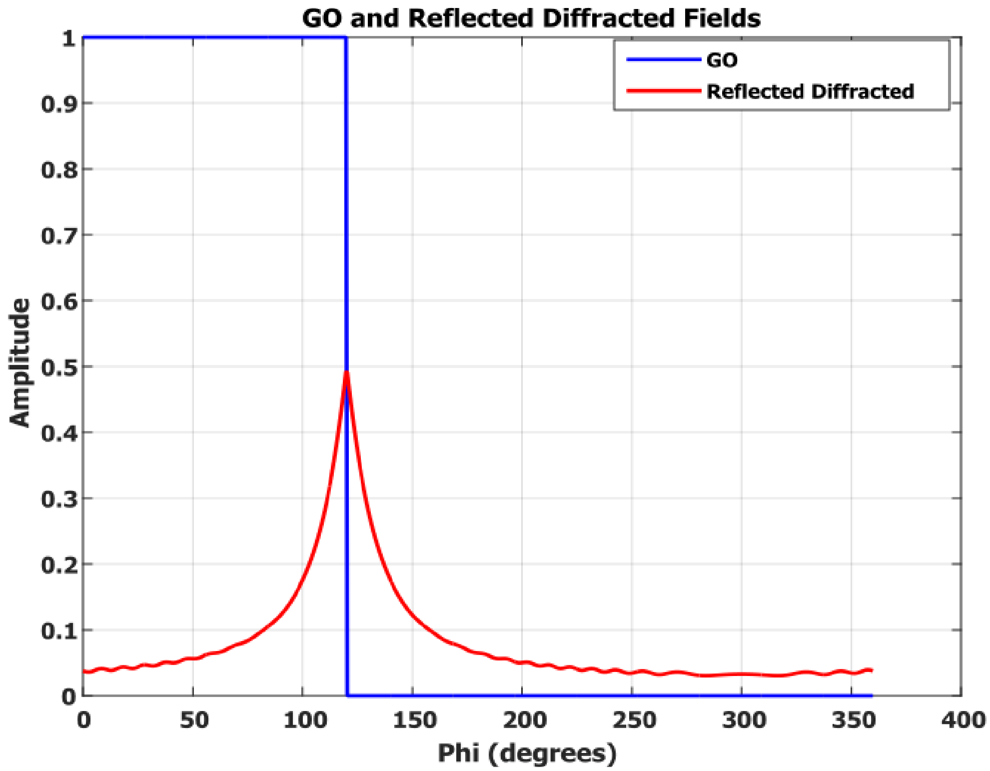

Figure 5. GO and reflected diffracted field intensities

4. Conclusion

In this study, scattering of plane waves by a perfectly electric conducting half-plane is investigated by employing the series solution method. Incident and reflected fields are added together to obtain the total scattered field. Scattered field components, incident diffracted and reflected diffracted fields, rewritten by means of the Fresnel function are obtained in another form by means of unit step function and Signum function and the Fresnel function itself. The aim of doing this was to obtain uniform diffracted fields, because Signum and Fresnel functions are used to express the field having abrupt changes in a uniform variation. It is observed that the first term in total scattered field given in Eq. (30) represents the reflected scattered field whereas the second term with negative sign depicts the incident scattered field.

Appendix

In this section, the expression given in Eq. (32) will be by means of Fresnel function.

$$\sum_{n=0}^{\infty}(jt)^{v_n}J_{v_n}(x) \tag{32}$$

which can be written as,

$$\sum_{n=0}^{\infty}\frac{(jt)^{v_n}n!}{n!}J_{v_n}(x), \tag{33}$$

where,

$$n! = \int_0^{\infty} e^{-u}u^n du. \tag{34}$$

Eq.(33) is rewritten as,

$$\sum_{n=0}^{\infty}\frac{(jt)^{v_n}n!}{n!}J_{v_n}(x) = \sum_{n=0}^{\infty}\frac{(jt)^{v_n}\int_0^{\infty} e^{-u}u^n du}{n!}J_{v_n}(x). \tag{35}$$

By writing $v_n = n + 1/2$, Eq. (35) becomes

$$\sum_{n=0}^{\infty}\frac{(jt)^n(jt)^{\frac{1}{2}}\int_0^{\infty} e^{-u}u^n du}{n!}J_{v_n}(x), \tag{36}$$

and finally we obtain,

$$\sum_{n=0}^{\infty}\frac{(jtu)^n}{n!}J_{v_n}(x) = \sqrt{\frac{2x}{\pi}}\sqrt{jt}\int_0^{\infty}\frac{e^{-u}sin\left(\sqrt{x^2-2jtux}\right)}{\sqrt{x^2-2jtux}} \tag{37}$$

Letting $-2jtux = v^2$, $du = -vdv/jtx$ is obtained.

$$\sqrt{\frac{2x}{\pi}}\sqrt{jt}\int_0^{\infty}\frac{e^{-u}sin\left(\sqrt{x^2-2jtux}\right)}{\sqrt{x^2-2jtux}} = \sqrt{\frac{2x}{\pi}}\sqrt{jt}\int_x^{\infty}\frac{e^{-\left(\frac{x^2-v^2}{2jtx}\right)}sinv\,dv}{jtx} \tag{38}$$

which is reduced to,

$$\frac{1}{j}\sqrt{\frac{1}{j2\pi tx}}e^{\frac{jx}{2t}}e^{jC^2}\left[\int_x^\infty e^{-j\left(\frac{v}{\sqrt{2tx}}-C\right)^2}dv-\int_x^\infty e^{-j\left(\frac{v}{\sqrt{2tx}}+C\right)^2}dv\right] \quad (39)$$

where $C=\sqrt{tx/2}$. Eq.(39) is arranged as,

$$\frac{1}{j}\sqrt{\frac{1}{j2\pi tx}}e^{j\frac{x}{2}\left(\frac{1}{t}+t\right)}\left[\int_x^\infty e^{-j\left(\frac{v}{\sqrt{2tx}}-\sqrt{\frac{tx}{2}}\right)^2}dv-\int_x^\infty e^{-j\left(\frac{v}{\sqrt{2tx}}+\sqrt{\frac{tx}{2}}\right)^2}dv\right] \quad (40)$$

By letting,

$$\frac{v}{\sqrt{2tx}}\pm\sqrt{\frac{tx}{2}}=-\beta, \quad (41)$$

Eq. (40) can be rewritten as,

$$\frac{-e^{j\frac{\pi}{4}}e^{j\frac{x}{2}\left(t+\frac{1}{t}\right)}}{\sqrt{\pi}}\left(\int_{\sqrt{\frac{x}{2}}\left(\sqrt{t}+\frac{1}{\sqrt{t}}\right)}^{\infty}e^{-j\beta^2}d\beta-\int_{-\sqrt{\frac{x}{2}}\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)}^{\infty}e^{-j\beta^2}d\beta\right), \quad (42)$$

where the F[a] is the Fresnel function given as,

$$F[a]=\frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}}\int_a^\infty e^{-jt^2}dt. \quad (43)$$

As a result we obtain,

$$\sum_{n=0}^{\infty}(jt)^{v_n}J_{v_n}(x)=-e^{j\frac{x}{2}\left(t+\frac{1}{t}\right)}\left\{F\left[\sqrt{\frac{x}{2}}\left(\sqrt{t}+\frac{1}{\sqrt{t}}\right)\right]-F\left[-\sqrt{\frac{x}{2}}\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)\right]\right\}. \quad (44)$$

Letting $x=k\rho$, and $t=\exp(j\theta)$ we obtain,

$$\sum_{n=0}^{\infty}(jt)^{v_n}J_{v_n}(x)=-e^{jk\rho cos\theta}\left(F\left[j\sqrt{2k\rho}\,sin\left(\frac{\theta}{2}\right)\right]-F\left[-\sqrt{2k\rho}\,cos\left(\frac{\theta}{2}\right)\right]\right) \quad (45)$$

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