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DESIGN OF VISCO-ELASTIC SUPPORTS FOR TIMOSHENKO CANTILEVER BEAMS

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Highlights

- This study investigates the most suitable support configuration for a cantilever beam, including viscoelastic supports across different vibration modes.
- The determination of the ideal stiffness and damping coefficients of the viscoelastic components is achieved by minimizing the absolute acceleration at the free end of the beam.
- Analytical derivative equations are formulated for both the stiffness and damping parameters.
- The present work introduces a concurrent optimization approach for both stiffness and damping.
- The effectiveness of viscoelastic supports in predicting ideal spring and damping coefficients and their ability to provide optimal support solutions for various vibration modes are demonstrated.

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ABSTRACT: The appropriate design of supports, upon which beams are usually placed as structural components in many engineering scenarios, has substantial significance in terms of both structural efficacy and cost factors. When beams experience various dynamic vibration effects, it is crucial to contemplate appropriate support systems that will effectively adapt to these vibrations. The present work investigates the most suitable support configuration for a cantilever beam, including viscoelastic supports across different vibration modes. Within this particular framework, a cantilever beam is simulated using beam finite elements. The beam is positioned on viscoelastic supports, which are represented by simple springs and damping elements. These supports are then included in the overall structural model. The equation of motion for the beam is first formulated in the temporal domain and then converted to the frequency domain via the use of the Fourier Transform. The basic equations used in the frequency domain are utilized to establish the dynamic characteristics of the beam by means of transfer functions. The determination of the ideal stiffness and damping coefficients of the viscoelastic components is achieved by minimizing the absolute acceleration at the free end of the beam. In order to minimize the objective function associated with acceleration, the nonlinear equations derived from Lagrange multipliers are solved using a gradient-based technique. The governing equations of the approach need partial derivatives with respect to design variables. Consequently, analytical derivative equations are formulated for both the stiffness and damping parameters. The present work introduces a concurrent optimization approach for both stiffness and damping. Passive constraints are established inside the optimization problem to impose restrictions on the lower and higher boundaries of the stiffness and damping coefficients. On the other hand, active constraints are used to ascertain the specific values of the overall stiffness and damping coefficients. The efficacy of the established approach in estimating the ideal spring and damping coefficients of viscoelastic supports and its ability to provide optimal support solutions for various vibration modes have been shown via comparative experiments with prior research.

Keywords: *Timoshenko beam, Transfer function, Visco-elastic support, Beam vibration, Damping, Spring*

1. INTRODUCTION

The support and bond conditions of beams, which are frequently employed in engineering problems, are crucial to their dynamic behavior. It can be observed in all areas of structural engineering design, in building-type structures during construction, and in welded or riveted machine element connections in marine and aircraft structures. Support conditions play a significant role in structure analysis for more realistic problem resolution. Changes in support positions and conditions can also substantially alter structural performance. It must be designed with attention. Not only are bearings anticipated to secure the structure, but they can also be redesigned to enhance structural performance. Unexplored is the optimization of support locations to reduce structural displacements. Timoshenko et al. [1] studied the unconstrained vibration of beams resting on variable Winkler spring foundations. Wei and Yida [2]

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examined the dynamic response of a viscoelastic Winkler foundation supported elastic beam. Chung et al. [3] proposed an analytical method for determining the natural frequencies of beams that are supported elastically at both extremities and constrained in the middle. Chen and Sheu [4] investigated a damped laminated Timoshenko beam axially laid on a viscoelastic base. [5] Metrikine and Dieterman studied the uniform motion of a mass along an axially compressed Euler-Bernoulli beam on a viscoelastic base. Using the Winkler model, Lee et al. [6] investigated the spontaneous vibrations of piles partially driven into elastic soil. The effect of point visco-elastic supports on the dynamic stability of visco-elastic piles was investigated by Zhen-Yu et al. [7]. Chen and Huang investigated the Timoshenko beam on a viscoelastic Winkler foundation for a range of beam, foundation, and loading conditions. Chen et al. examined the response of the Timoshenko beam on a viscoelastic base to a moving harmonic load [8,9]. Ansari et al. [10] investigated the inside-outside resonance vibration of a finite Euler-Bernoulli beam supported by a nonlinear viscoelastic base through which a moving load passes. Zhen et al. [11] studied the steady-state responses of an infinite Euler-Bernoulli beam supported by a nonlinear viscoelastic Winkler foundation and subjected to a harmonic moving load. Zheng et al. examined the instability analysis of a beam resting on a visco-elastic foundation and subjected to a moving mass-spring-damping system [12]. Vostroukhov and Metrikine [13] conducted a theoretical investigation into the steady-state dynamic response of a train traveling on a railway track supported by periodically placed viscoelastic supports. Metrikine examined the steady-state condition of an infinite spring on a nonlinear viscoelastic base subjected to a moving point load [14]. [15] Majorana and Pomaro investigated the dynamic stability of an elastic beam supported by viscoelastic translational and rotational supports. Basu and Rao devised analytical solutions for the steady-state response of an infinite beam resting on a viscoelastic foundation and subjected to a constant-speed concentrated load [16]. Froio and co. [17] They obtained numerically the Discontinuous Least Squares Finite Element Method formulation for the steady-state response of a tensioned spring on viscoelastic support under live load using the Discontinuous Least Squares Finite Element Method. Dimitrovová [18] investigated the dynamic interaction between two near masses on a Pasternak beam in a viscoelastic soil.

The optimal location of beam supports for elastic and plastic behaviour has been researched [19-21]. Akesson and Olhoff examined the minimum rigidity configuration that maximizes the fundamental natural frequency [22]. Hou and Chuang [23] determined the optimal support condition for a cantilever beam by deriving the sensitivity of the natural frequency to the support position. Wang derived the variation of frequency based on the support position in closed form for an Euler-Bernolli beam utilizing the conventional normal modal method [24]. In a separate study, Wang and Chen [25] used a genetic algorithm to determine the optimal beam support positions for various boundary conditions. Won and Park demonstrated the optimal location of a beam's supports based on the rigidity of the supports [26]. Aristizabal analyzed the free vibration of non-prismatic beams and columns and proposed a matrix method solution [27]. Sinha and Friswell formulated the location of the spring support and the global stiffness matrix in a beam element using element shape functions [28]. Aydin determined the optimal elastic support rigidity and location to minimize the end displacement or acceleration of a cantilever beam subjected to structural natural vibration. They solved the problem of optimizing the support conditions of stiffnesses and positions to minimize the shear force of a beam during various harmonic vibrations. Aydin et al. [29-31] investigated the determination of optimal elastic springs for cantilever beams supported by elastic foundations. Wang and Wen investigated the optimal position of viscoelastic supports for dampening beam vibration under harmonic load [32].

Certain researchers have investigated the optimization of column supports that maximize fracture load [33-35]. Lee and Co. Solutions were shown to find the free vibrations of beams for general boundary conditions and a new method was devised to find the buckling loads and natural harmonics of prismatic beams supported by an elastic spring at their center [36].

Huang and Huang [37] utilized the Laplace transform method to analyze the response and mechanical properties of viscoelastic Timoshenko beams. Liu et al. [38] derived closed form frequency sensitivities using the Lagrange Multipliers method based on the Rayleigh principle. Takewaki [39] presented a

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method using transfer functions to optimize the location and number of dampers in a built-up beam resting on viscous dampers. Sun analyzed the displacement of the beam on a viscoelastic base subjected to moving masses [40] using Green's function and Fourier transform in closed form. Kargarnovin et al. [41] studied the response of beams supported by nonlinear viscoelastic foundations to harmonic live loads. Çalım [42] analyzed the dynamic behavior of Pasternak-type beams subjected to time-dependent stresses on viscoelastic foundations. Mazilu [43] discussed the response of an infinite cable on viscoelastic support subjected to a moving harmonic load using the Green function method. Abdelghany et al. [44] investigated the dynamic response of a non-uniform beam subjected to a live load and supported by a non-linear viscoelastic foundation. Using integral transformation and contour integral methods, Dimitrovová [45] studied the dynamic response of an infinite beam resting on a classical Pasternak foundation and subjected to a moving mass, taking into consideration inhomogeneous initial conditions. Roy et al. [46] examined the interaction between an infinite spring supported by a homogeneous viscoelastic layer and a series of discrete mechanical systems all moving at the same constant speed. Dimitrovová developed a new semi-analytical solution for a mass moving uniformly on a beam, assuming homogeneous initial conditions, on a two-parameter visco-elastic basis. In another study [47, 48], Dimitrovová obtained a semi-analytical solution for a uniformly moving mass on a beam supported by a viscoelastic foundation with two parameters. Huang and Zou [49] studied the dynamic response of an elastic circular plate on a half-field viscoelastic Winkler foundation influenced by a moving rigid body with a decreased initial velocity. Aydin et al. modelled a Timoshenko-type cantilever beam resting on viscoelastic supports with finite elements, applied the Fourier transform to the equation of motion, derived governing equations in the frequency space, and demonstrated how they should be supported for minimum vibration behaviour with the displacement transfer function [50]. Cimellaro derived it to locate the optimal positioning of viscoelastic dampers using the transfer function vector of the absolute acceleration at any shear structure's natural frequency. The absolute acceleration transfer function vector is expressed for a cantilever beam supported by elastic springs [51].

This research aimed to evaluate the optimization of spring and damper positions and amounts with the objective of minimizing the amplitude of the transfer function for top absolute acceleration. The Fourier Transform was utilized to analyze the equation of motion represented in the time domain, enabling the expression of its behavior via transfer functions. The optimization issue is formally described using the Lagrange Multipliers technique, and the optimality criteria are then determined. The sensitivity equations are obtained by analytical derivation. Takewaki [39] has updated the Steepest Direction Search Algorithm (SDSA) approach to address the challenge of optimizing damper performance. This adaptation involves considering both optimal damping and stiffness distribution. The efficacy of the suggested strategy is shown via the presentation of a numerical example. The suggested approach involves doing frequency domain computations and temporal domain analysis to effectively reduce the vibration of the beam.

2. FORMULATION OF THE PROBLEM OF CANTILEVER BEAM SITTING ON VISCO-ELASTIC SUPPORTS

A cantilever beam resting on springs and viscous dampers, defined as visco-elastic supports, is seen in Figure 1 below. Here k_n and c_n are the stiffness and damping coefficients of the visco-elastic support at the n^{th} node. Considering that the cantilever beam is divided into n frame elements and a lumped mass is added to its end point, the equation of motion of a Timoshenko type cantilever beam in the absence of visco-elastic supports with 2n degrees of freedom is expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = -\boldsymbol{M}\boldsymbol{r}\ddot{u}_g(t) \quad (1)$$

In this equation, $\boldsymbol{u}(t)$ is displacement, $\dot{\boldsymbol{u}}(t)$ is velocity and $\ddot{\boldsymbol{u}}(t)$ is acceleration; $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ show the mass, damping and stiffness matrix of the beam. $\boldsymbol{r}$ refers to the influence vector whose elements showing

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the direction of ground motion are one, and $\ddot{u}_g(t)$ refers to the acceleration of the vertical support movement.

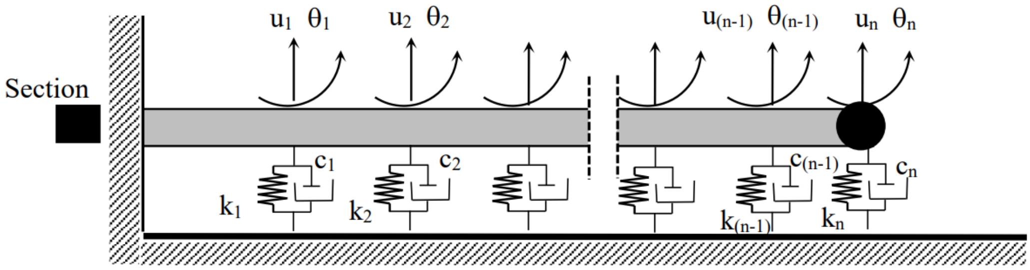

Figure 1. Cantilever beam resting on visco-elastic supports

If Fourier Transform is applied to Equation (1), $\boldsymbol{U(\omega)}$ and $\ddot{U}_g(\omega)$; Provided that $\boldsymbol{u(t)}$ and $\boldsymbol{\ddot{u}_g(t)}$ are Fourier transforms, Equation (1) becomes the following:

$$(\boldsymbol{K} + i\omega\boldsymbol{C} - \omega^2\boldsymbol{M})\boldsymbol{U}(\omega) = -\boldsymbol{Mr}\ddot{U}_g(\omega) \quad (2)$$

Here, ω refers to the circular frequency of the external effect, and $\boldsymbol{i}$ refers to $\sqrt{-\mathbf{1}}$. As seen in Figure 1, when the beam is supported from below with visco-elastic supports, Equation (2)

$$\big((\boldsymbol{K} + \boldsymbol{K_{ad}}) + i\omega(\boldsymbol{C} + \boldsymbol{C_{ad}}) - \omega^2\boldsymbol{M}\big)\boldsymbol{U_{ad}}(\omega) = -\boldsymbol{Mr}\ddot{U}_g(\omega) \quad (3)$$

Here, $\boldsymbol{K_{ad}}$ and $\boldsymbol{C_{ad}}$ denote additional stiffness and damping matrices with terms belonging to visco-elastic supports. $\boldsymbol{U_{ad}(\omega)}$ expresses the Fourier transform of the displacements after the springs are added. As a new parameter,

$$\widehat{\boldsymbol{U}}(\omega) = \frac{\boldsymbol{U_{ad}}(\omega)}{\ddot{U}_g(\omega)} \quad (4)$$

A transfer function can be defined as [39]. Here, if the frequency value is taken as equal to the frequency corresponding to the nth mode behavior of the beam ($\omega = \omega_n$n), the support movement will be defined as a harmonic movement with frequency ω_n. Using Equation (4), Equation (3) is rearranged as follows:

$$\boldsymbol{A}\widehat{\boldsymbol{U}}(\omega_n) = -\boldsymbol{Mr} \quad (5)$$

Here $\widehat{\boldsymbol{U}}(\boldsymbol{\omega_n})$ refers to the transfer function of the displacements calculated at the nth natural frequency of the structure. Matrix A, which contains the spring stiffness coefficients (k_1, k_2,…, k_n) and damping coefficients ($C_1, C_2, \dots, C_n$) which are design variables can be written as.

$$\boldsymbol{A} = (\boldsymbol{K}+\boldsymbol{K_{ad}}) + i\omega_n(\boldsymbol{C}+\boldsymbol{C_{ad}}) - \omega_n^2\boldsymbol{M} \quad (6)$$

In this equation, $\boldsymbol{K}$, $\boldsymbol{M}$ and $\boldsymbol{C}$ matrices are known. The coefficients in the $\boldsymbol{K_{ad}}$ and $\boldsymbol{C_{ad}}$ matrices containing the design variables will be found to be optimum. If equation (5) is rewritten as follows:

$$\widehat{\boldsymbol{U}}(\omega_n) = -\boldsymbol{A}^{-1}\boldsymbol{M}\boldsymbol{r} \tag{7}$$

The transfer function of absolute acceleration is found for the n^th^ mode. The transfer function for the absolute acceleration given below was derived by Cimellaro [51] and was used in this study to solve the problem of a cantilever beam resting on a visco-elastic foundation.

$$\widehat{\ddot{\boldsymbol{U}}}(\boldsymbol{\omega_n}) = \boldsymbol{\psi}(\boldsymbol{\omega_n}) = -\boldsymbol{M}^{-1}\big(\boldsymbol{K}+\boldsymbol{K_{ad}} + i\omega_n(\boldsymbol{C}+\boldsymbol{C_{ad}})\big)\widehat{\boldsymbol{U}} \tag{8}$$

2.1. Optimization Problem of Visco-Elastic Supports

While making optimum designs of structures or building elements, many different objective functions and constraints are defined depending on the problem. Weight, frequency, buckling load, specific displacements, accelerations, energy, etc. of the structural system. Quantities such as these can sometimes appear as objective functions in minimization and sometimes maximization problems. Apart from these, many objective functions can also be considered. The objective function appropriate to the nature of the problem can be selected by the designer. In this study, the transfer function amplitude of the end absolute acceleration of the beam was selected as the objective function for the optimization of visco-elastic supports supporting a built-in beam. Mathematical representation of the objective function used can be written in the form.

$$Min \quad f(\boldsymbol{K_{ad}}, \boldsymbol{C_{ad}}) = f(k_1, k_2, k_3, \dots, k_n, c_1, c_2, c_3, \dots, c_n) \tag{9}$$

Here, f the aim function is defined as the amplitude of the transfer function of the end absolute acceleration of the beam.

$$f(\boldsymbol{K_{ad}}, \boldsymbol{C_{ad}}) = |\boldsymbol{\psi}(\boldsymbol{\omega_n})| \qquad (i = 1, 2, \dots, n) \tag{10}$$

Here $|\boldsymbol{\psi}(\boldsymbol{\omega_n})|$ expresses the absolute value of the transfer function amplitude of the vertical acceleration at the end point of the beam. Additionally, for each spring and damping coefficient,

$$0 \le k_i \le \bar{k}_i \qquad (i = 1,2, \dots, n) \tag{11}$$

$$0 \le c_i \le \bar{c}_i \qquad (i = 1,2, \dots, n) \tag{12}$$

Passive restrictions can be given in this form. Here $\bar{k}_i$ indicates the upper limit of the spring coefficient and $\bar{c}_i$ indicates the upper limit of the damping coefficient. There is also an active restriction on the sum of the spring and damping coefficients. These can be written as

$$\sum_{i=1}^{n} k_i = \bar{K} \tag{13}$$

$$\sum_{i=1}^{n} c_i = \bar{C} \tag{14}$$

Here, $\bar{K}$ and $\bar{C}$ represent the sum of the spring and damping coefficients to be added.

2.2. Optimality Criteria

A gradient-based optimization approach is used here. Optimality criteria can be derived using the Lagrange Multipliers method. Depending on the generalized Lagrangian functional objective function, constraint functions and Lagrange multipliers (**λ**, **μ** and **ν**) can be written as

$$L(k_i, c_i, \lambda, \mu_i, \upsilon_i, \alpha_i, \beta_i) = f(k_i, c_i) + \lambda_1 \left(\sum_{i=1}^{n} (k_i - \bar{K})\right) + \lambda_2 \left(\sum_{i=1}^{n} (c_i - \bar{C})\right) + \sum_{i=1}^{n} \mu_i (0 - k_i) + \sum_{i=1}^{N} \upsilon_i (k_i - \bar{k}_i) + \sum_{i=1}^{n} \alpha_i (0 - c_i) + \sum_{i=1}^{N} \beta_i (c_i - \bar{c}_i) \tag{15}$$

If Equation (15) is differentiated according to the design variables (k_i and c_i) and Lagrange Multipliers,

$$\frac{\partial f}{\partial k_i} + \lambda_1 = 0 \quad (i = 1,2, \ldots, n) \quad 0 < k_i < \bar{k}_i \tag{16}$$

$$\sum_{i=1}^{N} k_i - \bar{K} = 0 \tag{17}$$

$$\frac{\partial f}{\partial c_i} + \lambda_2 = 0 \quad (i = 1,2, \ldots, n) \quad 0 < c_i < \bar{c}_i \tag{18}$$

$$\sum_{i=1}^{N} c_i - \bar{C} = 0 \tag{19}$$

Optimality criteria are derived as follows. Here, $\frac{\partial f}{\partial k_i}$ and $\frac{\partial f}{\partial c_i}$ express the partial derivative of the objective function with respect to the i[th] design variable k_i and c_i. For lower and upper constraints, Equation (16) and Equation (18) can be changed and written as follows:

$$\frac{\partial f}{\partial k_i} + \lambda \geq 0 \qquad k_i = 0 \tag{20}$$

$$\frac{\partial f}{\partial k_i} + \lambda \leq 0 \qquad k_i = \bar{k}_i \tag{21}$$

$$\frac{\partial f}{\partial c_i} + \lambda \geq 0 \qquad c_i = 0 \tag{22}$$

$$\frac{\partial f}{\partial c_i} + \lambda \leq 0 \qquad c_i = \bar{c}_i \tag{23}$$

These equations can be solved with a modified version of SDSA given by Takewaki [39].

2.2. Solution Algorithm

If the partial derivative of Equation (5) is taken according to the design variables,

$$\frac{\partial \boldsymbol{A}}{\partial \boldsymbol{k_j}}\widehat{\boldsymbol{U}} + \boldsymbol{A}\frac{\partial \widehat{\boldsymbol{U}}}{\partial \boldsymbol{k_j}} = \boldsymbol{0} \qquad (j = 1 \dots, n) \tag{24}$$

$$\frac{\partial \boldsymbol{A}}{\partial \boldsymbol{c_j}}\widehat{\boldsymbol{U}} + \boldsymbol{A}\frac{\partial \widehat{\boldsymbol{U}}}{\partial \boldsymbol{c_j}} = \boldsymbol{0} \qquad (j = 1 \dots, n) \tag{25}$$

Equations (24) and (25) derived by Takewaki [39]. The first order derivatives of absolute accelerations in Equation (8) were derived by Cimellaro and can be written as follows [51].

$$\frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{k_j}} = -\boldsymbol{A}^{-1}\frac{\partial \boldsymbol{A}}{\partial \boldsymbol{k_j}}\boldsymbol{\omega_s^2}\widehat{\boldsymbol{U}} \tag{26}$$

$$\frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{c_j}} = -\boldsymbol{A}^{-1}\frac{\partial \boldsymbol{A}}{\partial \boldsymbol{c_j}}\boldsymbol{\omega_s^2}\widehat{\boldsymbol{U}} \tag{27}$$

Transfer function values of the forces in Equation (5),

$$\psi_i = Re[\psi_i] + Im[\psi_i] \tag{28}$$

It is expressed in complex form. Each of the derived terms calculated from Equations (26)-(27) can also be shown as follows:

$$\frac{\partial \psi_i}{\partial k_j} = Re\left[\frac{\partial \psi_i}{\partial k_j}\right] + Im[\frac{\partial \psi_i}{\partial k_j}] \tag{29}$$

$$\frac{\partial \psi_i}{\partial c_j} = Re\left[\frac{\partial \psi_i}{\partial c_j}\right] + Im[\frac{\partial \psi_i}{\partial c_j}] \tag{30}$$

If the absolute value of ψ_i, which is the acceleration, is written as follows, it can be calculated as:

$$|\psi_i| = \sqrt{(Re[\psi_i])^2 + (Im[\psi_i])^2} \tag{31}$$

If the derivative of Equation (31) is taken with respect to the j[th] design variable (k_i and c_i),

$$\frac{\partial|\psi_i|}{\partial k_j} = \frac{1}{|\psi_i|}\left\{Re[\psi_i]\left(Re\left[\frac{\partial \psi_i}{\partial k_j}\right]\right) + Im[\psi_i]\left(Im\left[\frac{\partial \psi_i}{\partial k_j}\right]\right)\right\} \tag{32}$$

$$\frac{\partial|\psi_i|}{\partial c_j} = \frac{1}{|\psi_i|}\left\{Re[\psi_i]\left(Re\left[\frac{\partial \psi_i}{\partial c_j}\right]\right) + Im[\psi_i]\left(Im\left[\frac{\partial \psi_i}{\partial c_j}\right]\right)\right\} \tag{33}$$

In this way, the first order partial derivative of the objective function with respect to the j[th] design variable is found.

Solution Algorithm ($k_i \leq \bar{k}_i$ *and* $c_i \leq \bar{c}_i$):

Step1. Initially, take all the stiffness and damping coefficients of the visco-elastic supports as $k_i = 0$ and $c_i = 0$ $(j = 1,2, \dots, n)$. Assume the total stiffness and damping coefficient to be added in each step as $\Delta K = \frac{\bar{K}}{m}$ and $\Delta C = \frac{\bar{C}}{m}$ and choose the number of design steps (m).

Step2. Calculate $\frac{\partial f}{\partial k_j}$ and $\frac{\partial f}{\partial c_j}$ using Equations (29)-(30).

Step3. Find the indices p and r, satisfying the conditions $-\frac{\partial f}{\partial k_p} = Max(-\frac{\partial f}{\partial k_j})$ and $-\frac{\partial f}{\partial c_r} = Max(-\frac{\partial f}{\partial c_j})$.

Step4. Renew the objective function f as $f + \frac{\partial f}{\partial k_p}\Delta k_p + \frac{\partial f}{\partial c_r}\Delta c_r$, where $\Delta k_p = \Delta K$ and $\Delta c_r = \Delta C$. Δk_p and Δc_r are the amount of stiffness and damping coefficient to be added in that design step.

Step5. Repeat Step2 through Step5 until the constraints $\sum_{i=1}^{n} k_i = \bar{K}$ and $\sum_{i=1}^{n} c_i = \bar{C}$ are satisfied.

In this algorithm, the method intended by Takewaki [39] to find optimum damping coefficients is reduced to a state where only first-order derivatives are used. In order to apply the solution algorithm, the derivative of matrix A must be found according to the design variables (k_i) indicating stiffness.

Formulation of the eigenvalue-eigenvector problem in a vibrating mechanical system can be expressed as:

$$\boldsymbol{K_T}(k)\mathbf{\Phi_n} = \Omega_\mathrm{n}(\mathrm{k})\mathbf{M}\mathbf{\Phi_n} \tag{34}$$

It can be specified as $\boldsymbol{K_T} = (\boldsymbol{K} + \boldsymbol{K_{ad}})$. Here $\Omega_n = \omega_n^2$ and $\boldsymbol{\Phi_n}$ is n for the undamped state. Shows eigenvalues and eigenvectors. If equation (34) is multiplied by $\boldsymbol{\Phi_n}^T$ from the left, the following equation is obtained.

$$\mathbf{\Phi}_\mathrm{n}^\mathrm{T}\mathbf{K}_\mathrm{T}(\mathrm{k})\mathbf{\Phi}_\mathrm{n} = \Omega_\mathrm{n}(\mathrm{k})\mathbf{\Phi_n^T}\mathbf{M}\mathbf{\Phi_n} \tag{35}$$

Here $\bar{m}_n = \mathbf{\Phi_n^T}\mathbf{M}\mathbf{\Phi_n}$ is the modal mass for the n^th^ mode and $\bar{k}_n = \mathbf{\Phi_n^T}\mathbf{K_T}\mathbf{\Phi_n}$. If it is defined as modal stiffness for this mode and $\Omega_n(k)$ is written from Equation (35) is obtained as.

$$\Omega_\mathrm{n}(k) = \frac{\bar{k}_n}{\bar{m}_n} \tag{36}$$

The first order derivative of $\Omega_n(k)$ according to the design variable for the j^th^ stiffness is found as follows.

$$\frac{\partial \Omega_\mathrm{n}}{\partial \mathrm{k_j}} = \frac{1}{\bar{m}_n}\frac{\partial \bar{k}_n}{\partial \mathrm{k_j}} \tag{37}$$

Here $\frac{\partial \bar{k}_n}{\partial \mathrm{k_j}} = \mathbf{\Phi_n^T}\frac{\partial \boldsymbol{K_T}}{\partial \mathrm{k_j}}\mathbf{\Phi_n}$ is calculated with the help of the equation. If $\Omega_n = \omega_n^2$ is added into Equation (37) and arranged,

$$\frac{\partial \omega_\mathrm{n}}{\partial \mathrm{k_j}} = \frac{1}{2\bar{m}_n\omega_n}\frac{\partial \bar{k}_n}{\partial \mathrm{k_j}} \tag{38}$$

In this form, the first order derivative of the n^{th} undamped natural frequency with respect to the j^{th} stiffness coefficient is found. The structural damping matrix can be written as follows, proportional to the mass can be written as.

$$\boldsymbol{C} = \alpha(k)\boldsymbol{M} \tag{39}$$
$$\alpha(k) = 2\zeta\omega_n(k) \tag{40}$$

If Equation (40) is placed into Equation (39) and its derivative is taken according to the j[th] stiffness coefficient, the first order derivative of the structural damping matrix according to the design variable can be found as follows:

$$\frac{\partial \boldsymbol{C}}{\partial k_j} = 2\zeta \frac{\partial \omega_n}{\partial k_j} \mathbf{M} \tag{41}$$

Derivative of matrix A given by Equation (6) according to the j[th] stiffness design variable is found as.

$$\frac{\partial \boldsymbol{A}}{\partial k_j} = \frac{\partial \boldsymbol{K_T}}{\partial k_j} + i\frac{\partial \omega_n}{\partial k_j}(\boldsymbol{C} + \boldsymbol{C_{ad}}) + i\omega_n \frac{\partial(\boldsymbol{C} + \boldsymbol{C_{ad}})}{\partial k_j} - \frac{\partial \Omega}{\partial k_j}\boldsymbol{M} \tag{42}$$

2.3. Example Problem

The cantilever beam seen in Figure 2 has 6 m space and is divided into 1 m finite element parts. There are six nodes in total. A linear and an angular displacement were assumed at each node and modeled as a Timoshenko beam. Density of the beam material ρ=7.8 10^3 kg/m^3, modulus of elasticity E=2.06 10^{11} N/m^2, shear modulus G=7.94 10^{10} N/m^2, correction factor κ=5/6, cross-sectional area A=0.05 m^2, moment of inertia I= 2.08 10^{-4} m^4 and the total stiffness amount, $\overline{\mathrm{K}}$, and the total damping coefficient amount, $\overline{\mathrm{C}}$ are selected separately for the first three modes as in Table 1 below. $\Delta\mathrm{C} = \overline{\mathrm{C}}/300$ and $\Delta\mathrm{C} = \overline{\mathrm{C}}/300$. Additionally, a 100 kg mass was added to the end of the beam.

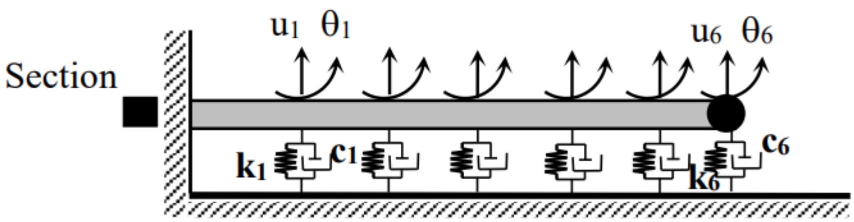

Figure 2. 12 degrees of freedom cantilever beam resting on visco-elastic supports

Table 1. Total stiffness and total damping amounts taken according to modes in the model

Modes	$\overline{\mathbf{K}}$ (N/m)	$\overline{\boldsymbol{C}}$ (Ns/m)
1st mode	$8.0\ 10^5$	$6.0\ 10^3$
2nd mode	$1.0\ 10^7$	$6.0\ 10^4$
3rd mode	$5.0\ 10^7$	$12.0\ 10^4$

In the solutions for the first three modes, Aydin et al. [50] considered displacement control in a previous study to obtain optimum design, and this study, which is considered acceleration control, is compared with this Figure 3 shows the mode shapes of the cantilever beam with and without optimum visco-elastic support.

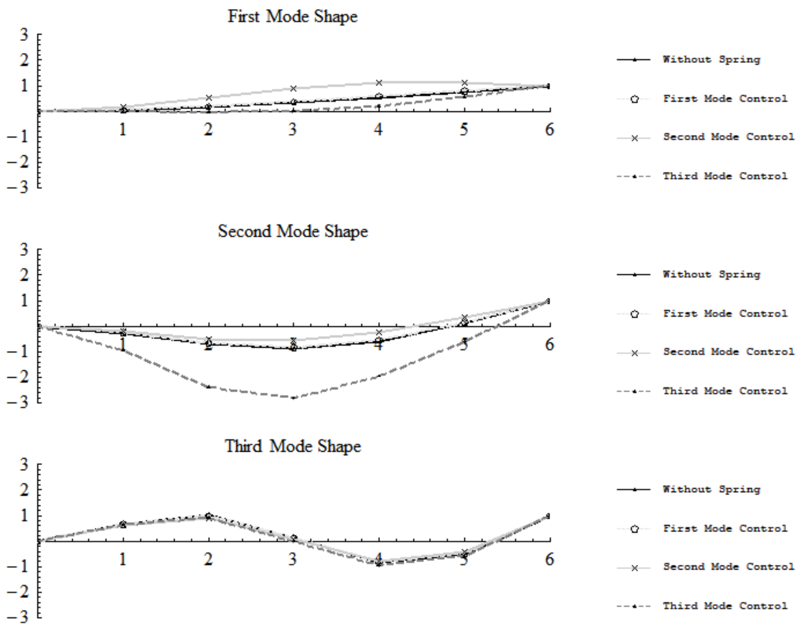

Figure 3. The first three mode shapes of the beam according to without both visco-elastic supports and optimal supports.

2.3.1 According to the first mode

To apply the optimization algorithm explained in the subject, $\omega = \omega_1$ was first selected, the selected total stiffness amount ($\bar{K} = 8.0\ 10^5\ N/m$) and the total damping amount ($\bar{C} = 6.0\ 10^3\ Ns/m$). is placed optimally according to the first mode of the structure.

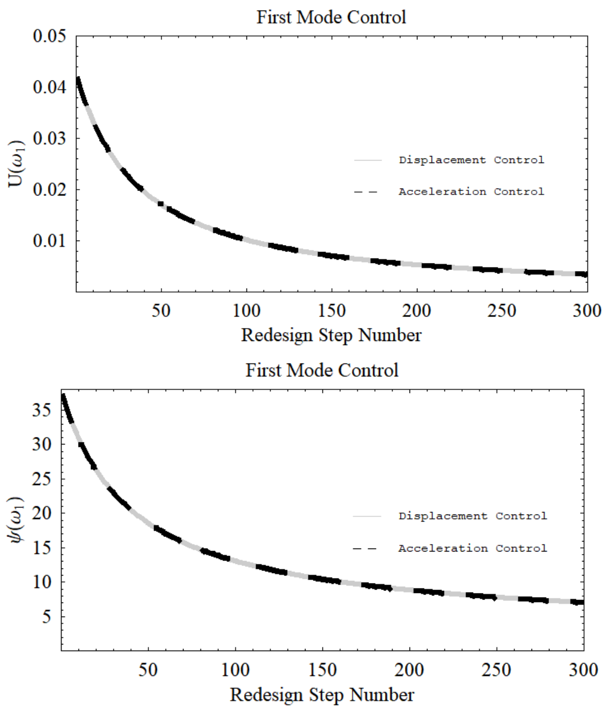

Figure 4. Change of objective function

Figure 4 shows the change in the transfer function amplitude of the end displacement [50] and absolute acceleration at the support, defined as the objective function, during the optimization phase. It is seen that the objective function amplitude, which is a positive value, is reduced in the design steps.

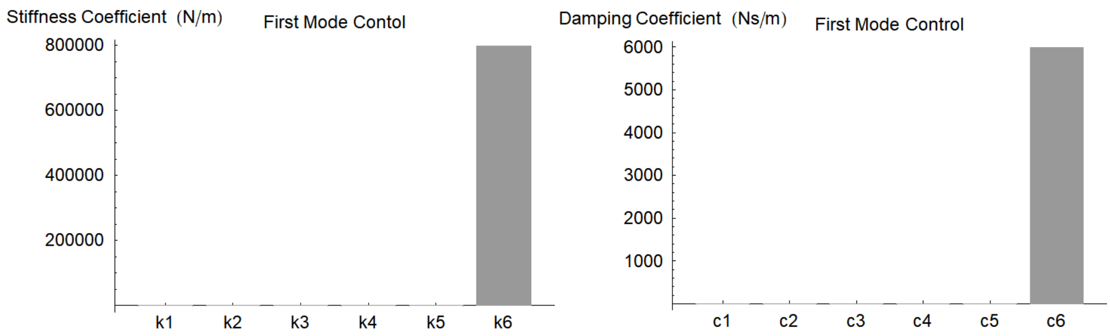

Figure 5. Distribution of optimum stiffness and damping coefficients with displacement control

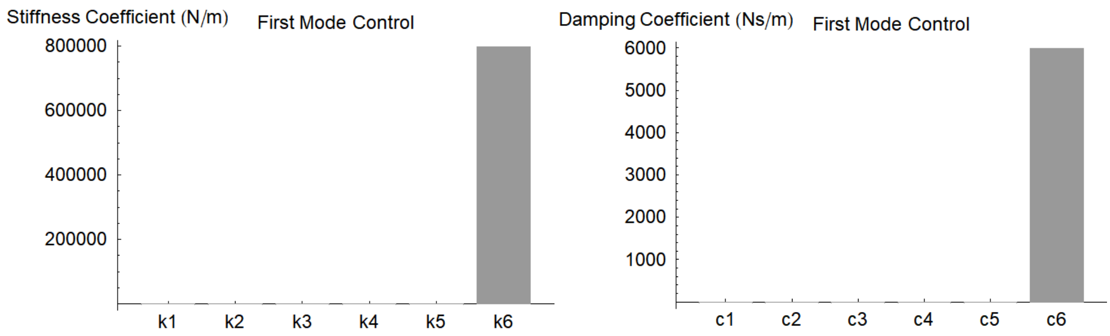

Figure 6. Distribution of optimum stiffness and damping coefficients with acceleration control

Displacement and acceleration controlled analyses were carried out, taking the first mode into consideration. The optimum stiffness and damping coefficients for the first mode found in the study where the objective function for displacement and absolute acceleration were used, were added to the 6th node as k_6=8.0 10^5 N/m and c_6=6.0 10^3 Ns/m. At the end of the design, the optimum stiffness and damping coefficients found for the first mode were added to the nodes at the end and are drawn in Figures 5 and 6.

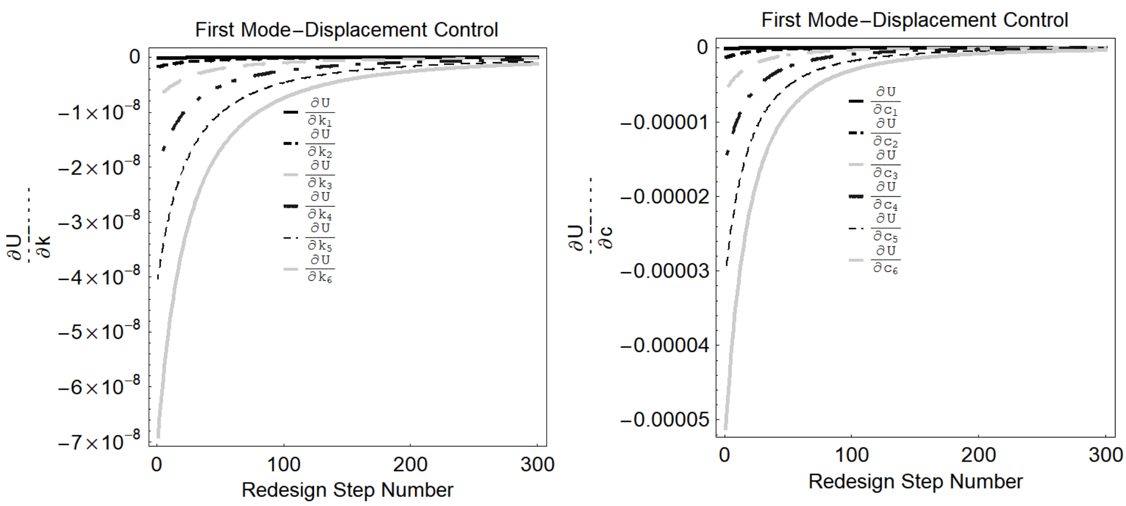

Figure 7. Variation of first order partial derivatives of the objective function for displacement according to stiffness and damping coefficients

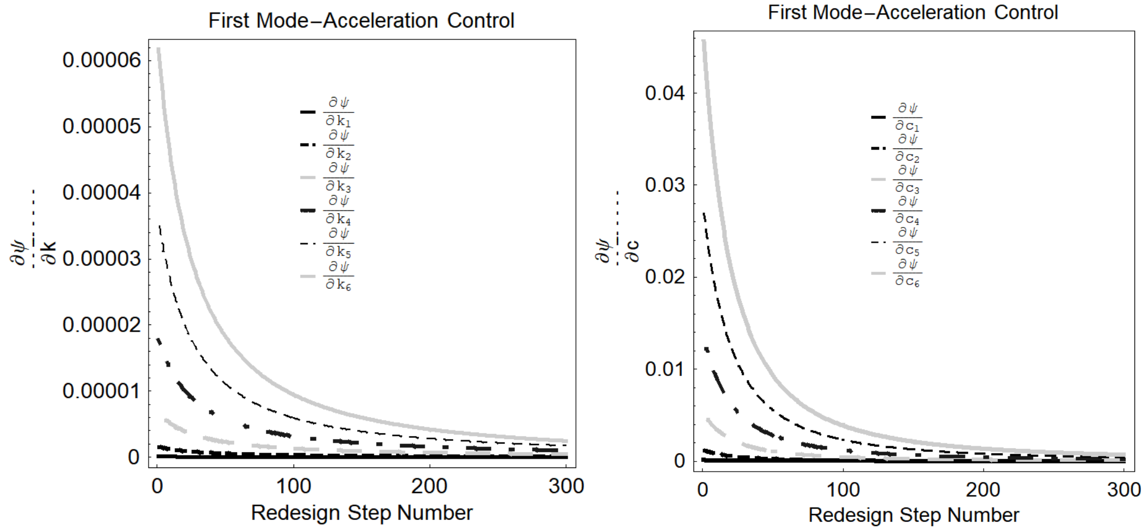

Figure 8. Variation of first order partial derivatives of the objective function for absolute acceleration according to stiffness and damping coefficients

Figures 7 and 8 show the changes of the first-order derivatives of the objective functions for displacement and absolute acceleration according to the design variables (stiffness and damping) in the design steps of the optimization according to the first mode. It can be seen from these graphs that the optimality criteria are met and convergence occurs.

2.3.2 According to the second mode

To apply the optimization algorithm explained in the subject, $\omega = \omega_2$ was first selected, the selected total stiffness amount ($\bar{K} = 1.0\ 10^7\ N/m$) and the total damping amount ($\bar{C} = 6.0\ 10^4\ Ns/m$) is placed optimally according to the second mode of the structure.

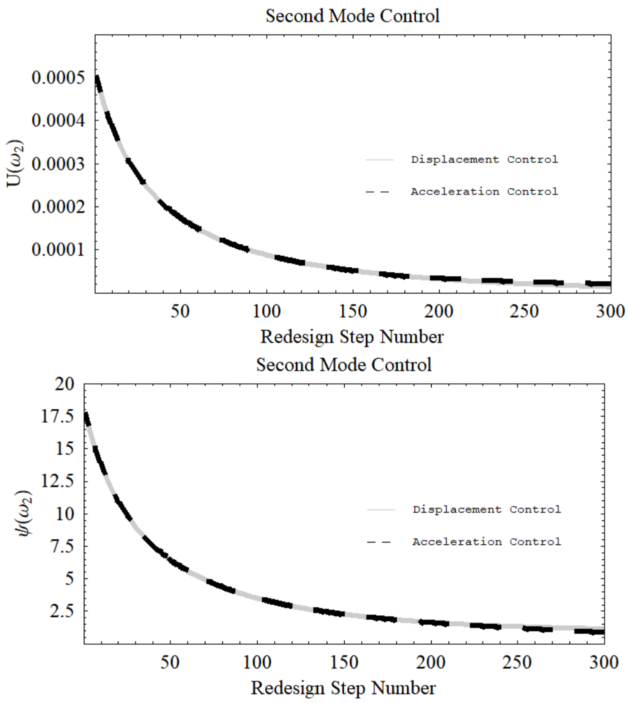

Figure 9. Change of objective function

Figure 9 shows the change in the transfer function amplitude of the tip displacement [50] and absolute acceleration at the support, defined as the objective function, in the optimization phase. It is seen that the objective function amplitude, which is a positive value, is reduced in the design steps.

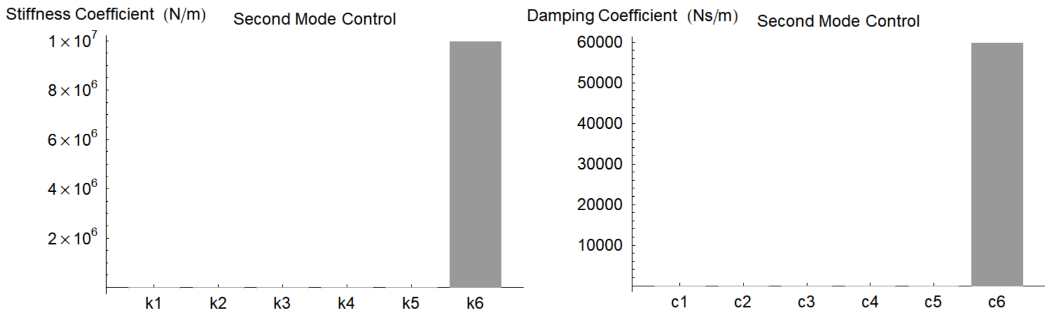

Figure 10. Distribution of optimum stiffness and damping coefficients with displacement control

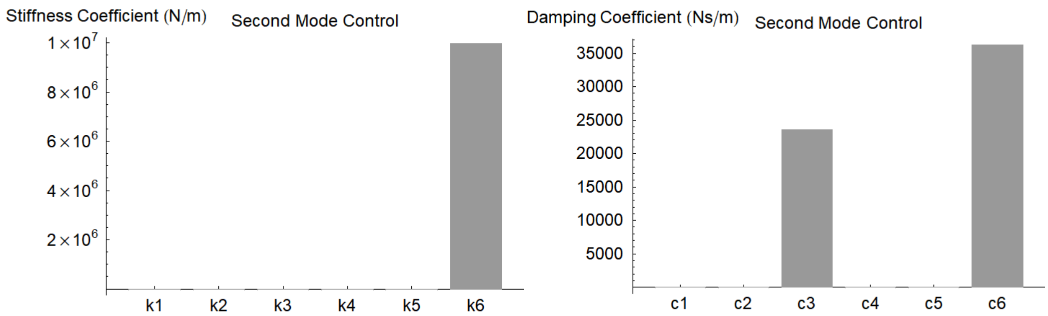

Figure 11. Distribution of optimum stiffness and damping coefficients with acceleration control

Displacement and acceleration controlled analyses were carried out, taking the second mode into account. The optimum stiffness and damping coefficients for the second mode found in the study where the objective function for displacement was used were added to the 6th node as k_6=1.0 10^7 N/m and c_6=6.0 10^4 Ns/m. Here, the optimum stiffness and damping coefficients for the second mode are added to the extreme node. In the study where the objective function for absolute acceleration was used, the optimum stiffness and damping coefficients for the second mode were added as c_3=2.38 10^4 Ns/m to the 3rd node, k_6=1.0 10^7 N/m and c_6=3.62 10^4 Ns/m to the 6th node. The optimum stiffness and damping coefficients found for the second mode at the end of the design are drawn in Figures 10 and 11.

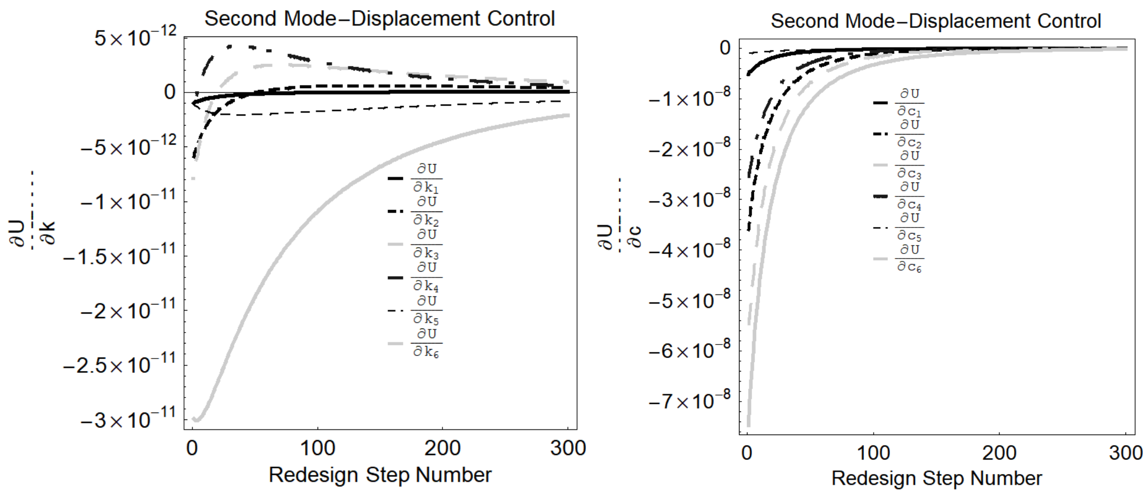

Figure 12. Variation of first order partial derivatives of the objective function for displacement according to stiffness and damping coefficients

The study was selected among the papers presented at the 23rd National Mechanics Congress of TUMTMK (04-08 September 2023 Konya, TURKIYE)

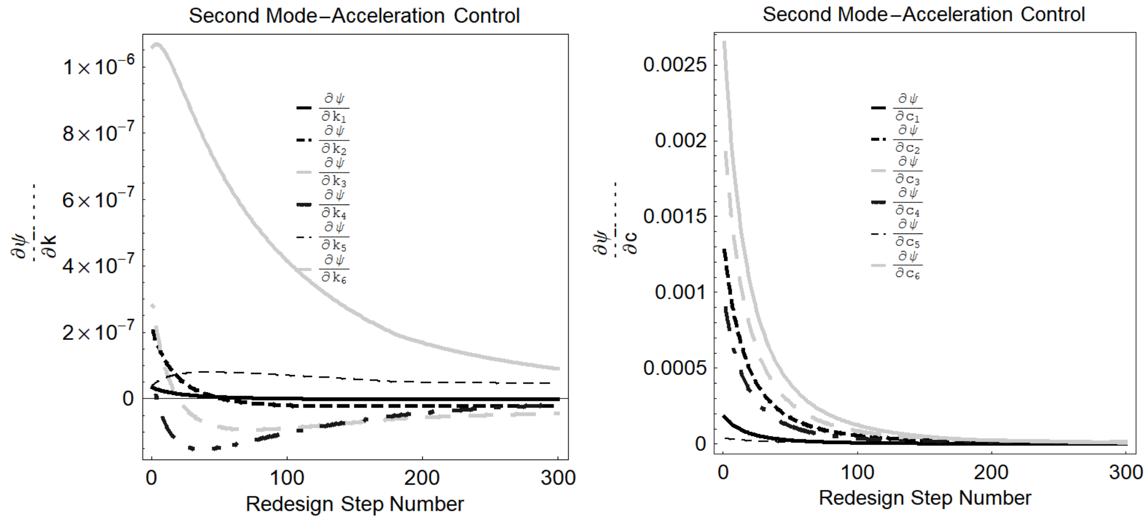

Figure 13. Variation of first order partial derivatives of the objective function for absolute acceleration according to stiffness and damping coefficients

Figures 12 and 13 show the changes of the first-order derivatives of the objective functions for displacement and absolute acceleration according to the design variables (stiffness and damping) in the design steps of the optimization according to the second mode. It can be seen from these graphs that the optimality criteria are met and convergence occurs. In the second mode, it has been observed that convergence based on absolute acceleration is more successful than convergence based on displacement.

2.3.3 According to the third mode

To apply the optimization algorithm explained in the subject, $\omega = \omega_1$ was first selected, the selected total stiffness amount ($\bar{K} = 5.0\ 10^7\ N/m$) and the total damping amount ($\bar{C} = 1.2\ 10^5\ Ns/m$). is placed optimally according to the third mode of the structure.

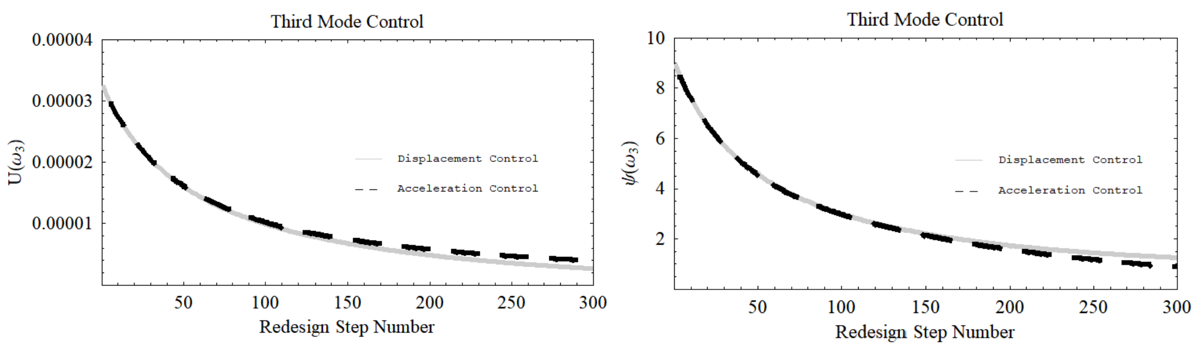

Figure 14. Change of objective function

Figure 14 shows the change in the transfer function amplitude of the tip displacement [50] and absolute acceleration at the support, defined as the objective function, during the optimization phase. It is seen that the objective function amplitude, which is a positive value, is reduced in the design steps.

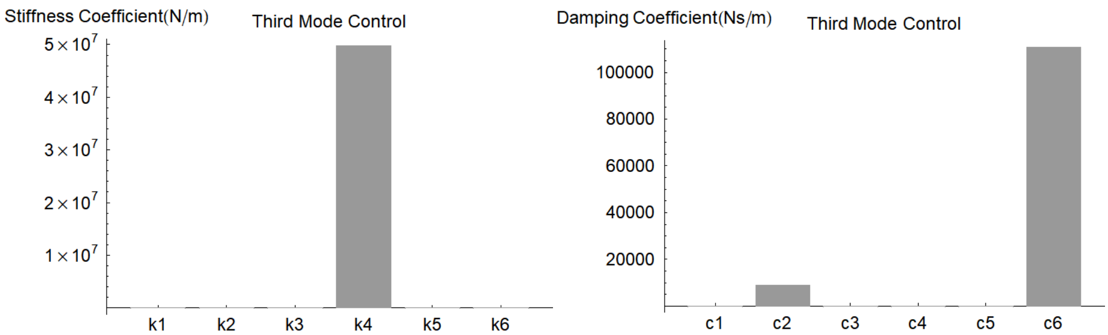

Figure 15. Distribution of optimum stiffness and damping coefficients with displacement control

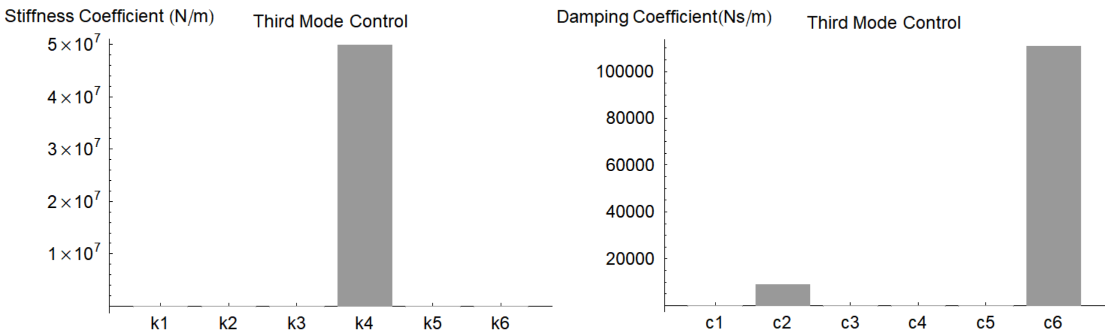

Figure 16. Distribution of optimum stiffness and damping coefficients with acceleration control

Displacement and acceleration controlled analyses were carried out, taking the third mode into account. The optimum stiffness and damping coefficients for the third mode found in the study where the objective function for displacement was used were added as c_2=8.8 10^3 Ns/m to the 2nd node, k_4=5.0 10^7 N/m to the 4th node and c_6=1.11 10^5 Ns/m to the 6th node. In the study where the objective function was used for absolute acceleration, the optimum stiffness and damping coefficients for the third mode were added as c_2=7.32 10^4 Ns/m to the 2nd node, k_6=5.0 10^7 N/m and c_6=4.68 10^4 Ns/m to the 4th node. The optimum stiffness and damping coefficients found for the third mode at the end of the design are drawn in Figures 15 and 16.

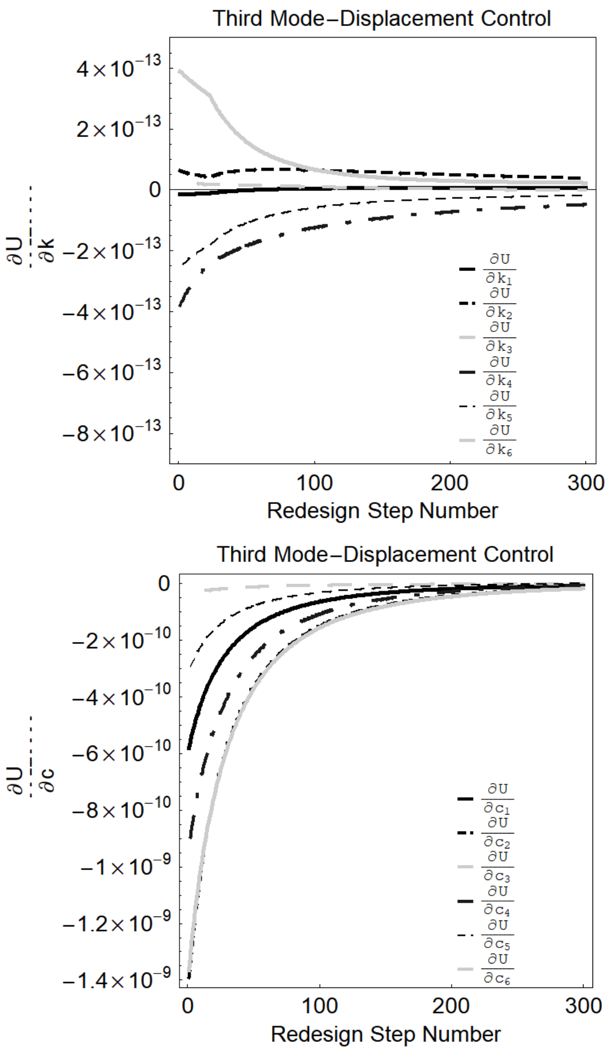

Figure 17. Variation of first order partial derivatives of the objective function for displacement according to stiffness and damping coefficients

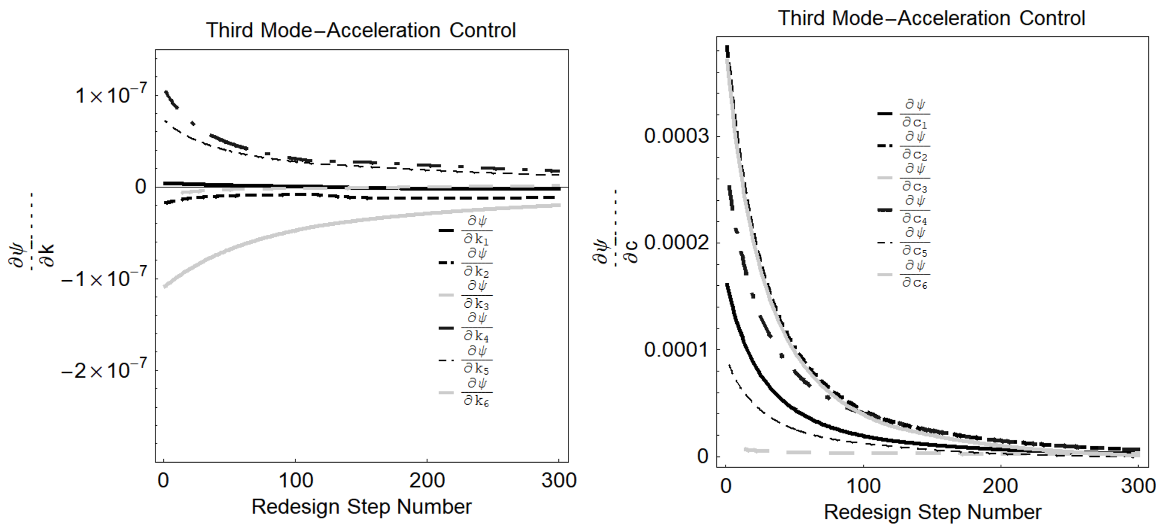

Figure 18. Variation of first order partial derivatives of the objective function for absolute acceleration according to stiffness and damping coefficients

Figures 17 and 18 show the changes in the first-order derivatives of the objective functions for displacement and absolute acceleration according to the design variables (stiffness and damping) in the design steps of the optimization considering the third mode. It can be seen from these figures that the optimality criteria are met and convergence occurs. In the third mode, it has been observed that convergence based on absolute acceleration is more successful than convergence based on displacement.

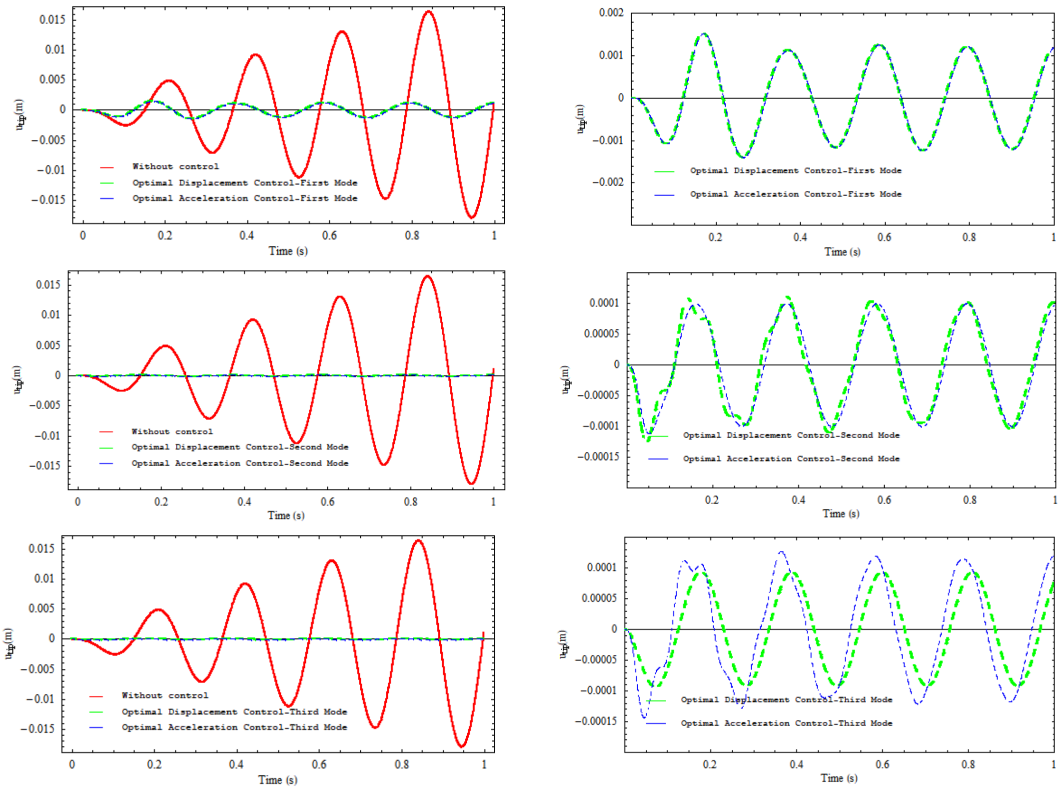

Figure 19. The time histories of the tip displacement under the harmonic base acceleration

The optimal design is examined in the frequency domain, and the time response of the tip displacement and absolute acceleration of the optimum designs found are also investigated. The optimum design for the first three modes is compared to the results found in the model without visco-elastic support. The resonance behavior of the beam with both optimal supports and without support is checked by using time history analyses under harmonic loads. Thus, the vertical support acceleration of excitation is chosen as $\ddot{u}_g = Sin(\omega_n t)$ $n = 1,2,\ldots.,6)$. As a result of the time history analyses, Figures 19 and 20 clearly show that the optimal stiffness and damping designs found with the investigated method can drastically decrease the tip displacement and absolute acceleration. It can be seen that the optimal support designs improve the behaviour of the beam in terms of displacement and acceleration. As can be seen from Figure 20, optimum designs perform better in the first three modes than without viscoelastic support cases. Figures 19 and 20 present the time histories of the tip displacements and the tip absolute accelerations during one second. The graphs (Figures 19 and 20) on the right in both figures are the subtracted versions of the graphs on the left, which have no support. That is, the graphs (Figures 19 and 20) on the right only show the behavior of the optimum designs, not the behavior of the visco-elastic supported case. The first mode response can be dominant in some engineering problems, while another mode can be a priority in other designs. Therefore, in this study, each mode of behavior is considered independently of each other and optimum designs are found accordingly.

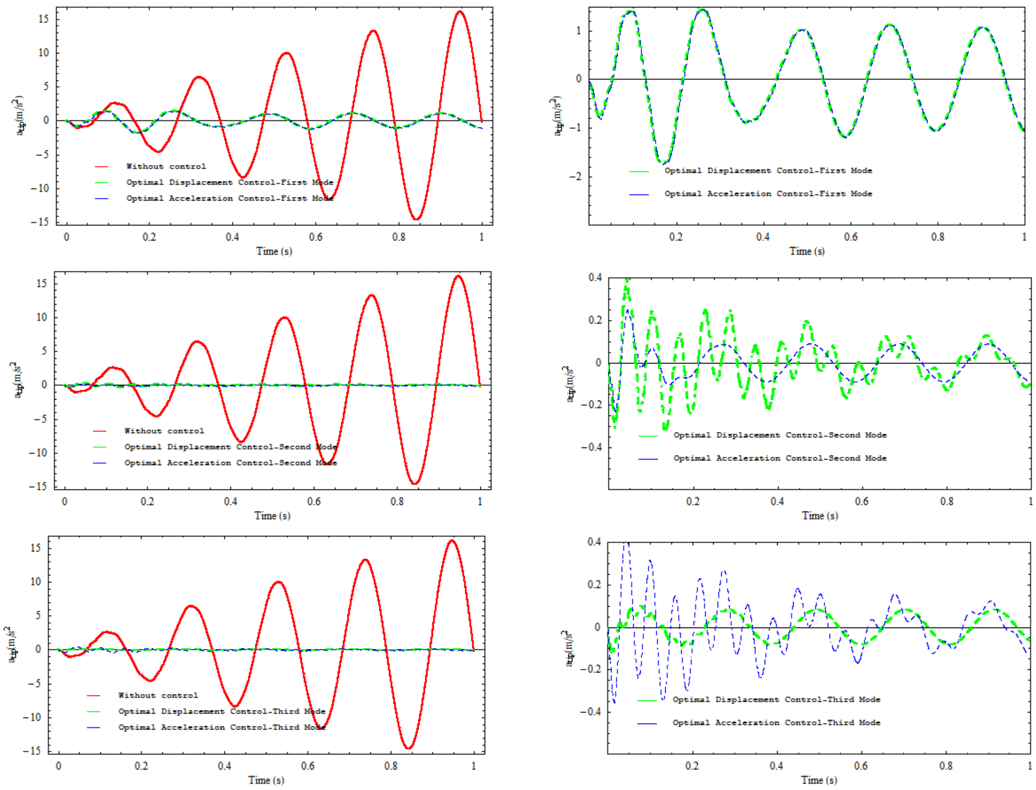

Figure 20. The time histories of the tip acceleration under the harmonic base acceleration

3. CONCLUSIONS

This article illustrates a technique for designing the supports of a Timoshenko cantilever beam supported by viscoelastic elements. As viscoelastic support, a damper and spring are utilized. Consequently, the primary focus of this research is the optimal configuration of the rigidity coefficient and simultaneous damping coefficient. The investigation focused on the determination of the optimal locations and quantities of dampers and springs, which were defined as viscoelastic supports positioned at the nodes, using finite elements to analyze a cantilever Timoshenko beam. In order to achieve this objective, transfer functions were implemented in the optimization problem, optimality criteria were established, and analytical sensitivity formulations were derived for their solution. In the prior displacement-controlled investigation, sole consideration was given to the initial mode. The objective functions were optimized with the absolute acceleration and displacement values for the beam end considered. An optimal viscoelastic support design is one in which the desirable mode behavior can be minimized, as demonstrated by numerical analysis. The optimal configuration for both objective functions, as determined by the first mode, yields the most favorable outcomes with regard to absolute acceleration and tip displacement. An analogous outcome was achieved when the displacement objective function was implemented in the second mode. The stiffness coefficient in the objective function for absolute acceleration is observed at the tip node, whereas the damping coefficient is appended to both the tip and intermediate nodes, as per the second mode. Ultimately, it was noted that the nodes in the third mode, which incorporated the damping and stiffness coefficients, exhibited a marginal distinction from those in the initial two modes. When examining the distribution of spring stiffness coefficients in the third mode control, the identical node is positioned in both objective functions. In displacement control, the damping coefficients are distributed to the second and sixth nodes; in acceleration control, they are distributed to the second and fourth nodes. Time history analyses conducted under harmonic loads reveal that acceleration-based designs exhibit superior acceleration reduction capabilities, while displacement-based designs demonstrate superior displacement reduction.

Declaration of Ethical Standards

The author declares that all ethical guidelines including authorship, citation, data reporting, and publishing original research are followed.

Declaration of Competing Interest

The author declares that there is no conflict of interest.

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