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Is There a Relationship Between Number Sense and Algebraic Thinking?

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ABSTRACT

Understanding the relationship between number sense and other important skills provides a better understanding of the subject and shows what students should do to improve their number sense. It is very important for teachers to know the abilities of students and how these abilities relate to each other. The aim of this study is to determine the number sense and algebraic thinking levels of 7th and 8th-grade students and to examine the correlation between number sense and algebraic thinking levels. The correlational research method was used in the study. The research was conducted with 330 students from the 7th and 8th grades of two state secondary schools in Çumra district of Konya, Turkey. In the research, the 'Algebraic Thinking Test' developed by Hart et al. and adapted to Turkish by Altun was used. In addition, 'Number Sense Scale' developed by Kayhan Altay and Umay (2013) was used. It appears that the research findings indicate that the average score of students' number sense was relatively low. Additionally, the findings indicate that students tend to solve problems requiring number sense using rule-based methods. It seems that number sense does not show a significant difference according to grade level. Also, there was a significant relationship between the test scores of female students and male students in favor of female students. According to the findings of the algebraic thinking test, it was observed that there was a concentration in the answers of both 7th and 8th grade students at levels 0 and 1. As a result of the algebraic thinking level test, it was seen that the students could not perceive the letters as a variable or unknown and had difficulty in concluding the operations without assigning value to the letters. Additionally, the findings reveal a moderate level and significant positive relationship between students' number sense and their level in algebraic thinking.

Sayı Hissi ile Cebirsel Düşünme Becerisi Arasında Bir İlişki Var Mıdır?

Makale Bilgisi

Makale Geçmişi

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ÖZET

Sayı hissinin önemli diğer becerilerle olan ilişkisinin anlaşılması; bu konunun daha iyi kavranmasına ve öğrencilerde sayı hissinin geliştirilmesi için yapılması gerekenlere yol göstermektedir. Öğretmenlerin, öğrencilerinin sahip olduğu becerileri ve bu beceriler arasında nasıl bir ilişkinin var olduğunu bilmesi oldukça önemlidir. Bu araştırmanın amacı, 7. ve 8. sınıf öğrencilerinin sayı hissi ve cebirsel düşünme düzeylerini belirlemek ve sayı hissi ile cebirsel düşünme düzeyleri arasındaki ilişkiyi incelemektir. Araştırmada ilişkisel araştırma yöntemi kullanılmıştır. Araştırma, Konya ili Çumra ilçesinde bulunan iki devlet ortaokulunun 7. ve 8. sınıflarında öğrenim gören 330 öğrenci ile gerçekleştirilmiştir. Araştırmada, öğrencilerin cebirsel ifadeleri anlama ve düşünme düzeylerini belirlemek amacıyla Hart vd. tarafından geliştirilen ve Altun tarafından Türkçeye uyarlanan 'Cebirsel Düşünme Testi'

kullanılmıştır. Ayrıca öğrencilerin sayı hissini belirlemek amacıyla Kayhan Altay ve Umay (2013) tarafından geliştirilen 'Sayı Duyusu Ölçeği' veri toplama aracı olarak kullanılmıştır. Araştırma bulguları öğrencilerin sayı hissi puan ortalamalarının oldukça düşük olduğunu göstermektedir. Ayrıca bulgular, öğrencilerin sayı hissi gerektiren problemleri kural tabanlı yöntemler kullanarak çözme eğiliminde olduklarını göstermektedir. Sayı hissinin sınıf düzeyine göre anlamlı bir farklılık göstermediği görülmüştür. Ayrıca kız öğrenciler ile erkek öğrencilerin sınav puanları arasında kız öğrenciler lehine anlamlı bir ilişki bulunmuştur. Cebirsel düşünme testinin bulgularına göre hem 7. sınıf hem de 8. sınıf öğrencilerinin cevaplarında 0. ve 1. seviyelerde yığılma olduğu görülmüştür. Cebirsel düşünme düzeyi testi sonucunda öğrencilerin harfleri değişken veya bilinmeyen olarak algılayamadıkları, harflere değer vermeden işlemleri sonuçlandırmakta zorlandıkları görülmüştür. Ayrıca, bulgular öğrencilerin sayı hissi ile cebirsel düşünme düzeyleri arasında orta düzeyde ve anlamlı pozitif bir ilişki olduğunu ortaya koymaktadır.

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INTRODUCTION

Mathematics seems very difficult for some people and very easy for others. What should be done if elementary school students fall behind their classmates and get difficulty in mathematics? Is this only related to interest and effort or does it have a cognitive foundation? Is it because the child does not put any effort or he/she does not put any effort since it is difficult? We do not know the exact answers to these questions. However, we know that the answer is closely related to a kind of mathematical perception or perceptional ability called *number sense* (Olkun, 2015). In the standards set for the mathematics program in the New Jersey state of the United States (Rosenstein et al., 1996), number sense is stated as a characteristic of those who use mathematics successfully. In the research conducted by Kartal (2016), it was determined that only students with high academic success made statements involving number sense in the questions. Yang, Li, and Lin (2008) found a significant relationship between students' number sense and mathematics achievement.

The concept of number sense can have various definitions across the literature. Number sense generally refers to an individual's overall understanding and familiarity with numbers, mathematical operations, and their application in real-life situations. This ability requires to use of useful, resilient and efficient strategies such as mental calculation and estimation to understand numerical problems (McIntosh et al., 1992; Reys & Yang, 1998; Sowder, 1992; Yang et al., 2004). In another definition, number sense was given as flexibility and fluency in numbers, the ability to understand the meanings of numbers, to do mental calculations and to compare (Gersten & Chard, 1999). Hope (1989) described it as to make reasonable predictions related to using numbers, to notice arithmetic errors, to notice number patterns and the sense of choosing the most effective calculation method. Şengül and Gülbağcı (2013a) defined the number sense in the most basic form as the ability to use numbers and operations flexibly. Even though the definitions are different, all highlight the same points such as the ability of mental calculation and prediction, flexibility, comparison, and practical thinking.

Greeno (1991) stated that a theoretical analysis of the number sense should be made, and he put forward three components: flexible mental calculation, numerical calculation, and quantitative judgment. Markovits and Sowder (1994) described the number sense; and divided it into three components: number size, mental computation, and computational estimation. McIntosh et al. (1992) describe the components of number sense; number knowledge, operational knowledge, and the ability to use number and operational knowledge. Also, researchers agree that number sense has an important place in mathematics education. In ‘‘Principles and Standards for School Mathematics’’, the National Council of Mathematics Teachers [NCTM] states that the number sense is one of the basic ideas in mathematics (National Council of Teachers of Mathematics, 2000). Number sense also has reflections on mathematics education in Turkey. For example, one of the main purposes of mathematics education is defined as "Being able to use mental calculation and prediction skills effectively and efficiently" (Ministry of National Education [MONE], 2018). Acar and Peker (2021) examined the 2018 secondary school mathematics curriculum according to the number sense and its components. As a result of the study, it was determined that 62 of the 215 acquisitions in the curriculum were related to the number sense. Çetin and Öztürk (2020) examined the achievements in the primary school mathematics curriculum according to the number sense and its components. As a result of the research, 32 acquisitions in the learning field of numbers and operations and 7 in the measurement learning field were associated with the number sense.

In the analysis of the studies on number sense, it was seen that students generally have a low level of number sense (Facun & Nool, 2012; Harç, 2010; İymen, 2012; Kartal, 2016; Kayhan Altay, 2010; Mohamed & Johnny, 2010; Singh, 2009; Yang, 2005; Yapıcı, 2013). Besides, the number sense of teachers and teacher candidates are also low (Kaminski, 1997; Kayhan Altay & Umay, 2011; Şengül &

Gülbağcı Dede, 2014; Tsao & Lin, 2011; Yang, 2007). When some studies in the literature are examined; Birgin and Peker (2022) examined the number sense performances of 8th grade Turkish students. Most of the students performed poorly in the number sense test and its components, while there was no significant difference in between the number sense performances of girls and boys. However their number sense performances differed significantly according to the school location and the education level of the parents have reached the conclusion shown. Can and Yetkin Özdemir (2020) examined primary school fourth grade students' number sense in context-based and non-context-based questions. They concluded that the majority of the students used rule-based strategies in both tests. Yang and Sianturi (2021) examined the current performance and misconceptions of 308 Indonesian sixth graders in a three-stage test containing 40 questions about the number sense. The results showed that most of the students had poor number sense performance and exhibited significant misconceptions. Er and Artut (2022) examined the number sense skills of gifted individuals in terms of number sense components. As a result of the research, it was determined that the number sense of gifted secondary school students differed according to gender and this difference was in favor of male students. It was concluded that the highest number sense performance according to grade levels belonged to the eighth-grade students. In another study conducted with gifted students, it was determined that students preferred rule-based solutions instead of number sense (Altıntaş et al., 2023). In these studies, students' number sense skills were generally low, they focused on rule-based strategies, and the effects of gender and age on number sense were investigated.

There have been numerous studies exploring the relationship between number sense and various other skills. Reys and Yang (1998) examined the relationship between number sense and the written computation skills of students in Grade 6 and 8. As a result of the research, student performance in number sense test was lower compared to written computation test. Although the students were quite successful in their calculations using paper and pencil, they did not show the same success in the number sense test. Correlations between the “Written Computation Test” and “Number Sense Test” were found to be 0.53 and 0.69 for the sixth and eighth graders, respectively. Yang et al. (2008) examined the relationship between number sense and success in mathematics. At the end of the research, a significant relationship was found between the students' number sense and their success in mathematics. Louange and Bana (2010) investigated the relationship between students' number sense and problem-solving skills. The results indicating a strong relationship between number sense and problem-solving. Şengül and Gülbağcı (2013b) examined the relationship between number sense and self-efficacy in mathematics. As a result of the study, there was a moderate correlation between the number sense and self-efficacy in mathematics. Günkaya (2018) examined the correlation between the number sense and spatial abilities of 8th grade students. At the end of the research, indicating a positive and significant relationship between number sense and spatial abilities among 8th grade students. It was also found that students with higher mathematics success had positively and significantly higher number sense and spatial abilities. These studies concluded that number sense has a significant relationship with various skills such as written calculation skills, mathematics achievement, problem solving skills, self-efficacy and spatial ability.

There are many studies on number sense. In addition, it is also important to examine the relationship between number sense and different thinking styles. In this context, it is important to fill the gap in the literature. Number sense can also be associated with different thinking skills. Number sense refers to an intuitive understanding of numbers and their relationships, including concepts such as size, order, and relative quantities. On the other hand, thinking styles encompass various cognitive approaches that individuals use to process information, solve problems, and make decisions. The lack of studies investigating the relationship between number sense and different thinking styles may be an opportunity for researchers to enter a new field of research. Exploring how various thinking styles such

as analytical, creative, algebraic, probabilistic, holistic, and intuitive thinking work and examining their relationships with the number sense can provide valuable information on cognitive processes related to mathematical understanding and problem solving. Examination of interrelated skills together can offer great opportunities for the development of these skills.

Algebraic thinking is another skill that is essential for mathematics education. Algebraic thinking is a thinking style that includes essential skills for mathematics education (Kaf, 2007). This way of thinking encompasses skills such as reasoning, understanding variables, using notations and explaining the meaning of symbolic representations, working with models for the development of mathematical ideas, and transforming representations (Kaf, 2007). In another definition, algebraic thinking includes important ideas, representations, proportional reasoning, the meaning of variables, patterns and functions, inductive and deductive reasoning (Greenes & Findell, 1998). Developing algebraic thinking skills in students should be one of the most important purposes of mathematics programs. In the literature (Çağdaşer, 2008; Çakan Özbayar, 2017; Gülpek, 2006; Kaş, 2010; Kaya, 2017; Öner Sünkür et al., 2012) it is stated that the algebraic thinking levels of secondary school students develop up to the third level. In addition, it is known that algebraic thinking levels increase as a result of the education conducted according to the constructivist approach (Çağdaşer, 2008).

It is obvious that number sense is one of the basic skills in mathematics since numbers are in all parts of our lives. Therefore, number sense is one of the subjects which have been studied in recent years and its importance is frequently emphasized. Algebraic thinking is another way of thinking that includes the basic skills necessary for mathematics. When we consider skills such as basic properties of numbers, patterns, problem solving, flexible representation, reasoning and estimation, we can say that these skills are common to both number sense and algebraic thinking. For example, both number sense and algebraic thinking involve recognizing and analyzing patterns. Students with a well-developed number sense are better equipped to identify patterns in sequences, equations, and functions that are fundamental to algebra. Lawrence and Hennessy (2002) emphasized the estimation skill and the use of reference from the components of number sense in the lessons from designed to develop algebraic thinking in students. It is emphasized that such a course supports the development of algebraic thinking in students.

Significance of the Study

Researchers can better understand cases by examining possible relationships between variables. Identifying relationships allows people to make predictions. Understanding the relationship between number sense and other important skills provides a better understanding of the subject and shows what students should do to improve their number sense. It is very important for teachers to know the abilities of students and how these abilities relate to each other. There was no study found examining if there is a relationship between number sense and different ways of thinking in the literature review. It is thought that examining this relationship will be a guide for both mathematics teachers and researchers who are interested in this subject. It is thought that the existence of such a relationship will be very beneficial in terms of teaching with studies in which both of them are together in order to improve both the algebraic thinking levels and the number sense of the students**.**

Aim of the Study

This research aims to determine the level of algebraic thinking and number sense of 7th and 8th-grade students and to examine whether there is a relationship between them.

The most general research problem of the study is expressed as "Is there a significant relationship between secondary school students' number sense and algebraic thinking levels?" Sub-problems of the research,

a) How is the success of the students in the number sense test?

b) Does number sense show a significant difference according to grade level?

c) Does the number sense differ significantly by gender?

d) What is the distribution of students in terms of algebraic thinking levels?

e) Is there a relationship between number sense and algebraic thinking?

METHOD

Research Design

The correlational research method among the relational research methods was used in the study. Correlational research is effective in revealing the relationships between the variables without interfering with them and determining (Büyüköztürk et al., 2015).

Study Group

The research was implemented with 330 students from 7th and 8th grade students of the two state middle schools in Çumra district of Konya, Turkey. One of these schools consists entirely of female students and the other consists entirely of male students. Details about the students are presented in Table 1.

The convenience sampling method was used for the schools in the study. This method is a non-probability sampling technique where the researcher selects participants based on their convenience and accessibility. In this method, the researcher chooses individuals who are readily available and easy to reach (Büyüköztürk et al., 2015). Criterion sampling, which is one of the purposeful sampling methods, was used in choosing the class level to use in the study. In this method, observation units can be created from people, objects, events or situations with specific qualities (Büyüköztürk et al., 2015). According to Altun (2015), the development of algebraic thinking accelerates during the formal operational stage. Also, these class levels were selected since the test complied with the 7th and 8th grade middle school curriculum in Turkey.

Table 1
Gender and Class Distribution of the Students

		Gender					
		Male		Female		Total	
		f	%	f	%	f	%
Class	7th grade	86	53.4	89	52.7	175	53
	8th grade	75	46.6	80	47.3	155	47
	Total	161	48.8	169	51.2	330	100

When Table 1 is examined; A total of 161 male students, 86 students in the 7th grade and 75 students in the 8th grade, and a total of 169 female students, 89 students in the 7th grade and 80 students in the 8th grade, participated in the research. Students generally have good socioeconomic status and average mathematics achievement.

Data Collection Tools

As a data collection tool, the 'Number Sense Scale' developed by Kayhan Altay and Umay (2013) and the 'Algebraic Thinking Test' developed by Hart et al. (1998) and adapted to Turkish by Altun (2005) was used.

The Number Sense Scale consists of 17 multiple-choice and open-ended questions about numbers and operations, and the Cronbach-α reliability coefficient of the scale is 0.86. The sub-dimensions of the test were determined as flexibility in the calculation, conceptual thinking in fractions and the use of a reference point. In the scoring of the test, the points were given considering the use of number sense when solving the questions. Students who solved the question using number sense were given 1 point, and those who solved the question in a standard and routine way and those who could not reach the correct result were given 0 points. The minimum score was 0, and the maximum score was 17.

The Algebraic Thinking Test consists of 28 questions and is divided into four levels. Questions 1, 2 and 3 of the test were aimed at determining level 1, questions 4, 5 and 6 at level 2, questions 7, 8, 9, 10, 11 and 12 at level 3 and the remaining questions at level 4. Level 1 includes questions such as finding the value of a letter as a result of purely arithmetic operations, thinking of letters as objects and concluding a problem or concluding a process without valuing letters. At level 2, the questions are the same as those at level 1, but more complex when compared to them. At level 3, there are questions about perceiving the letters as unknown and using them in this way. In these questions, the letters represent the unknown and it is difficult for the student to recognize these unknowns as an object. At level 4, the generalizations are similar to the expressions in level 3 but include more complex generalizations. In these questions, students should recognize the letters as an unknown, use them in mathematical relations or equations and recognize that letters can represent multiple numbers. In determining the distribution of students' algebraic thinking levels, 2/3 of the questions of the respective level should be answered correctly. Secondly, considering that the algebraic thinking levels have a sequential structure, the student has to be successful in the previous levels to pass to a level. Also, students who fail to answer a sufficient number of questions correctly cannot be admitted levelling 1 and their level of algebraic thinking is considered 0 (Altun, 2005; Çağdaşer, 2008; Sayı, 2018).

Data Collection

The number sense test was applied to the students within one lesson time (40 minutes). Before starting the test, the researcher informed the class about the test. Students were warned about using shorter and practical solving methods rather than standard methods and asked to explain the methods they preferred to use. Algebraic thinking level test was applied to the students in the next lesson after the number sense test was applied. This test was also applied within one lesson time.

Data were collected by the first researcher in January 2019. The data collection process took 10 days. In order to prevent external interventions, the researcher was present in each class while solving the students' questions.

Data Analysis

The data obtained from the study was subjected to various statistical analyses using appropriate software. For data analysis, descriptive statistical methods (frequency, percentage calculation, average, standard deviation), independent samples t-test and correlation calculation, were performed. Also, coefficients of kurtosis and skewness were examined for the normality of distributions.

Before the statistical analysis of the obtained data, it was examined whether the distributions show normal distribution. Mean, mode and median values of the distributions were quite close to each other.

In addition, Tabachnick and Fidell (2013) accept that the distribution is a normal distribution when the skewness and kurtosis values are between ±1.50. Kurtosis and skewness values for the number sense test were 1.141 and 1.292, respectively; Kurtosis and skewness values for the algebraic thinking level test are .812 and 1.161, respectively. Since the kurtosis and skewness values are within the acceptable range, it was agreed that data were normally distributed.

Validity and Reliability

In quantitative research, validity and reliability are proven by numerical indicators. The Cronbach-α reliability coefficient of the Number Sense Scale and Algebraic Thinking Test was calculated respectively as 0.86 (Kayhan Altay & Umay, 2013) and 0.93 (Gülpek, 2006). In order to ensure the validity of the research, no intervention was made while the tests were applied to the students, and a similar environment was provided for each student. Moreover, a second researcher examined the tests to ensure reliability during the evaluation stage of the applied tests. The consistency rate between the coders was calculated as 93% according to the formula of Miles and Huberman (1994) and it was decided that there was a significant degree of agreement between the raters. Also, a mathematics education expert was consulted in the evaluation of student answers.

Ethic

This study was conducted with the permission of "Necmettin Erbakan University Social and Humanistic Sciences Scientific Research Ethics Committee (Date: 12 November 2021, Decision No: 2021/548).

FINDINGS

The findings of the research are presented in sub-headings according to the research questions.

How is the Success of the Students in the Number Sense Test?

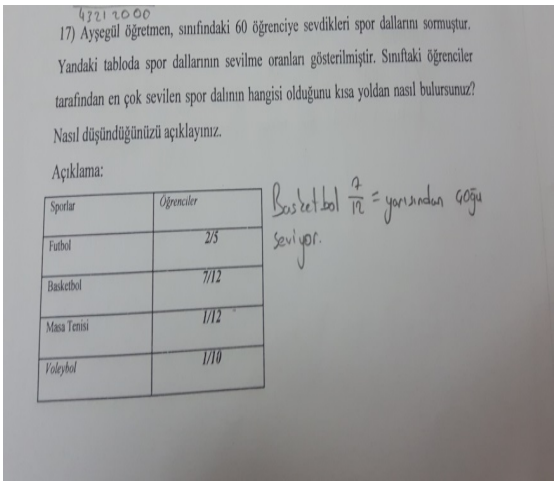
When analyzing the data, firstly the scores of the students from the number sense test were examined.

Table 2
Students' Scores from the Number Sense Test

Total students n	Mean *(M)*	Standard deviation *(SD)*
330	2.00	2.192

Table 2 shows that the students' average number sense score is 2.00. Considering the fact that the maximum score in the test would be 17, it could be said that the average score was quite low. The reason for this situation is that students solve problems using rule-based methods instead of number sense. Because when scoring the number sense test, only solutions using number sense were accepted as correct. Some student answers were given below as examples.

Figure 1
Student Answer to Question 17 (Student Using Number Sense)



17) Ayşegül teacher asked about their favorite sports. The table showed the rates of sport branches. Find out which sport is best loved by the students in the classroom most shortly. Explain how you reach that result.

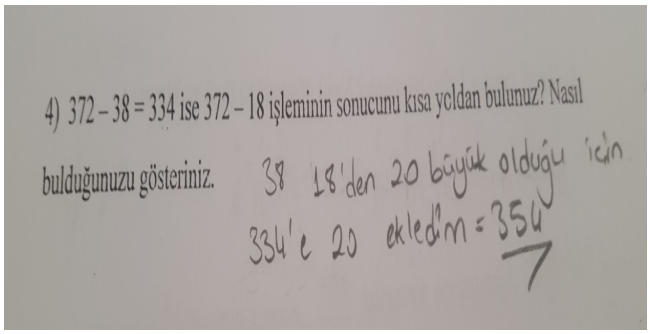
Sports	**Students**
Football	2/5
Basketball	7/12
Ping Pong	1/12
Volleyball	1/10

Explanation: Basketball $\frac{7}{12}$ = More than half likes it.

Figure 1 shows the answer of the student who reached the result by using number sense skill in solving the 17th problem. The student reached the result by using the half-whole relationship. The student answered: "Basketball $\frac{7}{12}$ = more than half likes it." In this question, students who did not use number sense equalized the denominator of fractions.

Figure 2 shows the answer of the student who reached the result by using number sense skill in solving this question.

Figure 2
Student Answer to Question 4 (Student Using Number Sense)



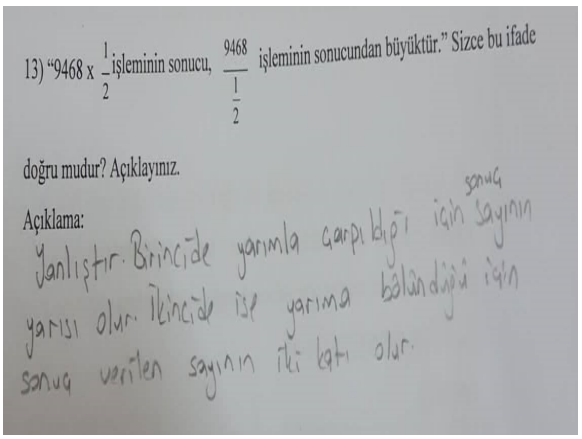
4) If 372-38=334, find the result of the 372-18 operation most shortly. Explain how you have found the result.

Explanation: Since 38 is 20 more than 18, I added 20 to 334 = 354

Figure 2 shows that the student makes mental calculations using the information in the question instead of performing the given operation. This student answered the question as: "Since 38 is 20 more than 18, I added 20 to 334 = 354". The student used the ability of flexibility in the calculation, which is one of the sub-dimensions of the applied test. In this question, students who did not use number sense performed the given subtraction operation directly.

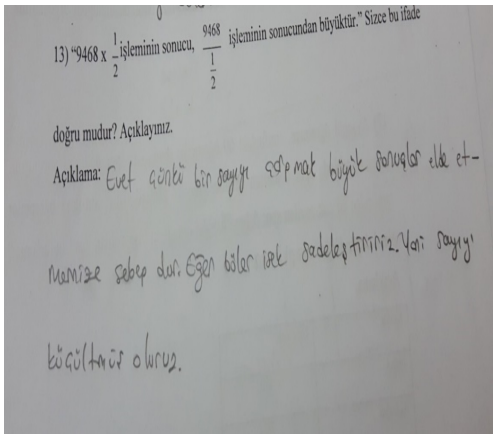
The answers of some students who used and did not use number sense in question 13 are given below.

Figure 3
Student Answer to Question 13 (Student Using Number Sense)

	**13)** "The result of the operation $9468\text{x}\frac{1}{2}$ is greater than the result of the operation $\frac{9468}{\frac{1}{2}}$." Is this statement correct? Explain. **Explanation:**" It is wrong. The first one is multiplied by half, so the result is half the number. The second is divided by half, so the result is twice the number.

It can be said that students who use number sense in this question correctly understand the effect of multiplication and division by half. A student with a developed number sense made the following explanation for this question: *"It is wrong. The first one is multiplied by half, so the result is half the number. The second is divided by half, so the result is twice the number (Figure 3)."*

Figure 4
Student Answers to Question 13 (Student Not Using Number Sense)

	**13)** "The result of the operation $9468\text{x}\frac{1}{2}$ is greater than the result of the operation $\frac{9468}{\frac{1}{2}}$." Is this statement correct? Explain. **Explanation:** Yes, because multiplying a number gives us bigger numbers. If we divide, we abbreviate it. It means, we minimize the number.

A student who didn't use the number sense explained it as*:* "*Yes, because multiplying a number gives us bigger numbers. If we divide, we abbreviate it. It means, we minimize the number.*"

Does Number Sense Show a Significant Difference According to Grade Level?

Within the scope of this question, the scores students received from the number sense test were examined according to their grade levels.

Table 3
Independent Sample t-Test Results About Students' Number Sense Scores Related to Grade Level

Grade Level	n	M	SD	df	t	p
7th grade	175	1.90	2.276	328	- .882	.379
8th grade	155	2.12	2.095			

When Table 3 was examined, it was seen that the average number sense score was higher in 8th grade. It was seen that the number sense increased as the grade increased. The independent sample t-test was used to determine whether the students' number sense scores show a statistically significant difference according to their grade level. At the end of the analysis, no significant difference was found between the test scores of students in grade 7 and 8 $(t_{328} = -.882,\ p > .05)$.

Does The Number Sense Differ Significantly by Gender?

The students' number sense was also examined considering gender. The independent sample t-test was used to determine whether this difference between students' number sense and gender was statistically significant.

Table 4
Independent Sample t-Test Results About Students' Number Sense Scores Related to Gender
Katılımcıların Cinsiyete Göre Dağılımı

Gender	n	M	SD	df	t	p
Male	161	1.71	2.078	328	-2.351	.019
Female	169	2.28	2.268			

According to Table 4, it was seen that the average score of male students from the number sense test was 1.71 and the average score of female students was 2.28. As a result of the analysis, a significant relationship was found between the test scores of male and female students in favor of female students $(t_{328} = -2.351,\ p < .05)$. It could be concluded that female students were more successful than male students in terms of number sense.

What is The Distribution of Students in Terms of Algebraic Thinking Levels?

According to the Algebraic Thinking Test, Level 1 is the level where questions such as finding the value of a letter as a result of arithmetic operations, concluding a problem by taking letters as objects, concluding an operation without giving any value to the letters are answered. Below are some student answers for different levels.

Figure 5
Student Answers to Question 6 (Level 0)

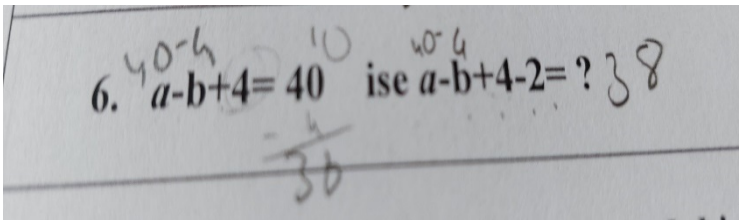
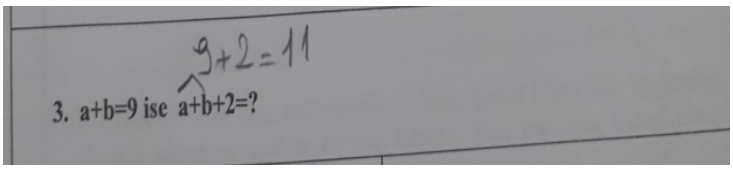
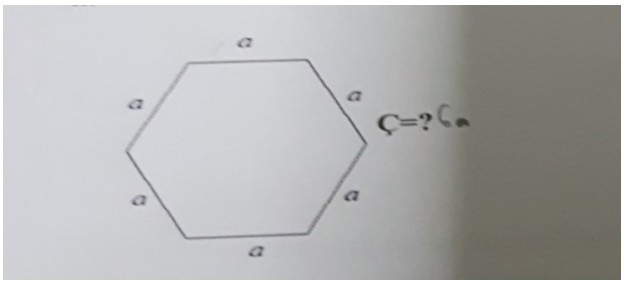


Figure 6
Student Answers to Question 3 (Level 1)



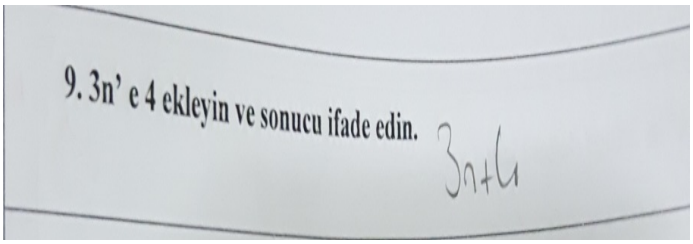
Level 2 is the same as the first level in terms of abstraction; the difference is that the questions are more complex.

Figure 7
Student Answers to Question 5 (Level 2)



Level 3 is the level where letters are perceived and used as unknowns.

Figure 8
Student Answers to Question 9 (Level 3)



9) To 3n add 4 and express the result.

Explanation:
3n+4

At Level 4, students are able to attach meanings and finalize operations to similar but more complex expressions to those at level 3.

Figure 9
Student Answers to Question 15 (Level 4)

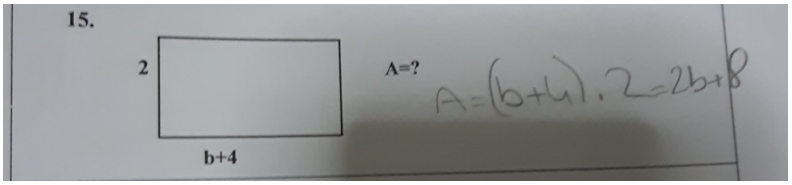


Table 5 presents the results of the algebraic thinking level test.

Table 5
Distribution of Students' Algebraic Thinking Levels (ATL) According to Grade Level

ATL		Level 0		Level 1		Level 2		Level 3		Level 4		Total
		f	%	f	%	f	%	f	%	f	%	f
Grade	7th	90	51.4	55	31.4	20	11.4	5	2.9	5	2.9	175
	8th	51	32.9	53	34.2	31	20.0	11	7.1	9	5.8	155
Total		141	42.7	108	32.7	51	15.5	16	4.8	14	4.2	330

Table 5 shows that 51.4% of 7th-grade students were at level 0, 31.4% were at level 1, 11.4% were at level 2, 2.9% were at level 3 and 2.9% were at level 4. 32.9% of 8th-grade students were at level 0, 34.2% were at level 1, 20% were at level 2, 7.1% were at level 3 and 5.8% were at level 4. Of all students, 42.7% were at level 0, 32.7% were at level 1, 15.5% were at level 2, 4.8% were at level 3, 4.2% were at level 4.

It was seen that both 7th and 8th grade students were accumulated at level 0 and level 1. There were more students from grade 7 at level 0 and level 1, while at levels 2, 3 and 4 there were more students from grade 8. Very few students from both two grades reached level 4.

Is There a Relationship Between Number Sense and Algebraic Thinking?

Correlations were calculated to compare the students' number sense and algebraic thinking.

Table 6
The Spearman's Rank Correlation Between Number Sense and Algebraic Thinking Level

Variables	n	Correlation Coefficient r	Significance (2 ways) p
Number Sense	330	.594	.000
Algebraic Thinking Level	330		

According to the results obtained, there was a moderate level, positive and significant relationship between the number sense and the algebraic thinking levels of the students $(r_{330} = .594,\ p < .05)$. This result shows that algebraic thinking levels increase with the increase in number sense and algebraic thinking levels decrease with the decrease in number sense.

DISCUSSION AND CONCLUSION

This research aims to determine the level of algebraic thinking and number sense of 7th and 8th-grade students and to examine whether there is a relationship between them. It is obvious that number sense is one of the basic skills in mathematics since numbers are in all parts of our lives. Therefore, number sense is one of the subjects which have been studied in recent years and its importance is frequently emphasized. Algebraic thinking is another way of thinking that includes the basic skills

necessary for mathematics. When we consider skills such as basic properties of numbers, patterns, problem solving, flexible representation, reasoning, and estimation, we can say that these skills are common to both number sense and algebraic thinking. For example, both number sense and algebraic thinking involve recognizing and analyzing patterns. Students with a well-developed number sense are better equipped to identify patterns in sequences, equations, and functions that are fundamental to algebra. Therefore, it was believed that there was a relationship between number sense and algebraic thinking, and it was examined. Also, number sense and algebraic thinking were examined in terms of grade level and gender.

Firstly, the study examined student performance on number sense test. As a result of the test scores, it was seen that students had a quite low number sense score average. It was seen that students solved the problems using rule-based methods while they were required to use number sense. This result is supported by the results of other studies carried out in this context (Altıntaş et al., 2023; Birgin & Peker, 2022; İymen, 2012; Kartal, 2016; Mohamed & Johnny, 2010; Singh, 2009; Şengül & Gülbağcı, 2013b; Takır, 2016; Yang, 2005; Yang & Sianturi, 2021). The reason for this was thought to be the methods used in mathematics teaching. It was seen that the students solved the questions by using methods based on rules and memorization and did not think of short and practical ways. The students preferred to do operations instead of predicting in the questions that assessed their ability to predict. This result is consistent with different studies (Kayhan Altay, 2010; Menon, 2004; Reys et al., 1999). Besides, in some questions, the students found very unreasonable results. Whereas number sense is an important way of thinking that represents logical thinking. However, the students answered without ever checking their results. Likewise, Işık and Kar (2011) stated that students could not interpret their results. Also, students had difficulty in understanding the relationship between numbers. They generalized a situation, which was valid only for a number, to different numbers. For example, some of them believed that the result of multiplication is always greater than the division with the same number. In this case, the student considered the division of integers and did not think that this would be invalid if the divisor number was rational.

It was also examined whether the scores of the students from the number sense test differed according to their grade levels and gender or not. It was seen that the average number sense score was higher in 8th grade. This result is supported by the results of other studies carried out in this context (Aunio et al., 2006; Işık & Kar, 2011; Singh, 2009; Şengül & Gülbağcı, 2013b; Takır, 2016; Yapıcı, 2013). Besides, there was no significant difference between the test scores of 7th and 8th grade students. As the grade level increased, the number sense increased but this increase was not significant. Also, there was a significant relationship between the test scores of female students and male students in favor of female students. It could be said that female students were more successful than male students related to number sense. This result contradicts the studies in the literature (Gülbağcı Dede & Şengül, 2016; Harç, 2010; Kayhan Altay, 2010; Menon, 2004; Singh, 2009; Takır; 2016). When the literature was examined, it was determined that number sense did not show a significant difference according to gender. Çetin and Çite (2022) determined that 4th grade students' fractional number sense did not show a significant difference according to gender. Birgin and Peker (2022) concluded in their study that there was no significant difference between the number sense performances of girls and boys. Yapıcı (2013) found that the students' number sense in percentages showed a significant difference in terms of gender but stated that this difference was higher in male students. The results presented in this study are in line with Yang et al., study (2008) results revealing that female students were more successful in recognizing the relative greatness of numbers, which was one of the number sense components, than male students. It is thought that whether there is a significant difference in terms of gender may vary depending on the study group. Because number sense is a skill that can be learned and taught (Griffin, 2004). The classroom environment, different teaching activities and methods provided by the teachers are important

for the development of number sense. This result might be because the sample of the study included female students from one school and male students from another school.

After the number sense test, the distribution of students in terms of algebraic thinking levels was examined. It was seen that both 7th and 8th grade students were accumulated at level 0 and level 1. Very few students from both two grades reached level 4. In the literature (Çağdaşer, 2008; Çakan Özbayar, 2017; Gülpek, 2006; Kaş, 2010; Kaya, 2017; Öner Sünkür et al.,2012) it is stated that the algebraic thinking levels of secondary school students develop up to the third level. As a result of the algebraic thinking level test, it was seen that the students could not perceive the letters as a variable or unknown and had difficulty in concluding the operations without assigning value to the letters. The students believe that they must find a numerical result while solving a math problem. Therefore, they tend to assign values to the letters. Similarly, in the study examining the misconceptions of students about algebra, Akkaya (2006) stated that the letters have the value of digits according to students and that they see the letters as numbers only. In another similar study (Dede et al., 2002), the researchers stated that students had difficulties in understanding the concept of variable, and the different uses of letters in the transition from arithmetic to algebra. Warren (2005) states that one of the main difficulties students experience in algebraic thinking is insufficient arithmetic knowledge. Lannin, Barker, and Townsend (2006) state that it is important for students to understand the meanings of algebraic symbols and applications in order to enable them to transition to algebraic thinking. Kieran (1992) emphasized that in order to be successful in learning algebra, it is necessary to understand what the symbols and basic concepts mean. Therefore, for the development of algebraic thinking, arithmetic knowledge, and the correct understanding of the meanings of basic algebraic symbols and concepts by students are of great importance.

In the study, most general problem examined was whether there was a relationship between number sense and algebraic thinking or not. As a result of the correlation analysis of the students' algebraic thinking levels and the scores obtained from the number sense test, it was found that there was a moderate level significant positive correlation between the number sense and the algebraic thinking levels of the students. Because when we consider skills such as basic properties of numbers, patterns, problem solving, flexible representation, reasoning, and estimation, we can say that these skills are common to both number sense, and algebraic thinking. This result shows that algebraic thinking and number sense are not independent of each other. For example, when the 6, 17, 18, 20 questions of the algebraic thinking test were examined, it was found that questions were requiring the use of number sense.

Suggestions

- ✓ As a result of the research, it was seen that the students' success in number sense was quite low. However, number sense is a skill that can be developed with certain learning environments. Therefore, strategies such as the classroom environment, rich educational activities, and different methods to be used by teachers are important for students to develop these skills. It is recommended that differently activities should be included in lessons.
- ✓ Teachers can use different activities that can improve the students' number sense such as using different methods to solve problems, making mental computations, providing students with the opportunity to estimate and use references, and allowing them to compare their predictions with real results. Also, it is thought that it would be beneficial for teachers to elicit short, practical, and effective methods rather than giving rule-based ways to solve problems.
- ✓ Teachers should encourage students to use their number sense in lessons. Rather than memorizing, students should understand what they use and why. Also, students should be

guided to question the reasonableness of the result in problem-solving.

- ✓ At the level of algebraic thinking, which is another variable of the research, it was seen that very few students were at the 4th level. Different teaching methods that can improve students' algebraic thinking levels can be examined, along with the number sense associated with this skill.
- ✓ As a result of the research, it was determined that there is a moderate level relationship between the number sense of the students and their algebraic thinking levels. By making use of this relationship, teaching that can develop both skills can be done.

In further studies,

- ✓ The relationship between number sense and different thinking ways and variables can be examined.
- ✓ Number sense is a very comprehensive subject. The components of number sense can be examined in detail on a subject basis.
- ✓ Learning environments that can improve the number sense can be examined.
- ✓ The study can be conducted with more students and different grade levels.
- ✓ The study can be improved using mixed methods.

Among the limitations of the research are that this research is carried out only with secondary school students, that the data is limited only to the data collection tools used, and that it is in the 2018-2019 academic year in terms of time. The limitations of the study include the attention of the participants while answering the questions and external factors such as light, sound and noise that may affect them.

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Ethical approval

It is declared that scientific and ethical principles have been followed while carrying out and writing this study and that all the sources used have been properly cited. Additionally, was conducted with the permission of “Necmettin Erbakan University Social and Humanistic Sciences Scientific Research Ethics Committee (Date: 12 November 2021, Decision No: 2021/548).

Conflict of Interest

The authors have no conflict of interest to declare.

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