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The Source of Γ-Primeness on Γ-Rings

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Abstract
The source of the primeness texture is a skeleton that generalizes traditional prime rings. In this context, our primary aim in this study is to describe the source of Γ-primeness in Γ-rings not included in the literature. This work's purpose is to generalize the concept of the source of primeness to a Γ-ring. In this study, the characteristics provided by the defined concept are also discussed, and the results achieved are exemplified.

Keywords: *Prime Γ-ideal, Source of Γ-primeness, Γ-ring*

AMS Subject Classification (2020): *16N60; 16Y80*

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1. Introduction

The structure of the Γ-ring was first proposed by Nobusawa in 1964 as a generalization of the ring [1]. The author determined the notion of the Γ-ring under certain conditions and obtained some significant results. Afterward, Barnes [2], inspired by Nabusawa, introduced and analyzed some concepts for Γ-rings. Many studies have extended important results on the structure of rings to Γ-rings [3–8].

Prime and semiprime ideals contribute extremely to important results in ring theory. Some properties of prime and semiprime ideals are studied in ring theory and generalized to Γ-rings. Recently, Aydın et al. [9] and Camcı [11] suggested the concept of the source of semiprimeness for a ring and described three ring types that were not previously included in the literature. Next, Arslan and Düzkaya [10] generalized the set of the source of semiprimeness defined for a ring to the Γ-ring and inquired about the properties of the set. Motivated by the set of the source of semiprimeness, Yeşil and Camcı [12] characterized the concept of the source of primeness for a ring. The authors regarded the relation between a ring's idempotent, nilpotent, and zero divisor elements and the set of the source of primeness and described new ring types.

This study set one's sights on generalizing the set of the source of primeness of a ring to the Γ-ring. Moreover, in this paper, the characteristics of the concept of the source of Γ-primeness of a Γ-ring and the different results created by idempotent, strongly nilpotent, nilpotent, and zero divisor elements in the set of the source of Γ-primeness are mentioned. The relationship between the source of semiprimeness and the source of Γ-primeness in the Γ-ring was also observed.

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2. Preliminaries

In this section, basic definitions previously lay one's laid in the literature are presented [1, 2, 8, 10, 13–16].

Definition 2.1. Let R and Γ be two additive abelian groups. If there exists a mapping $(a, \gamma, b) \to a\gamma b$ of $R \times \Gamma \times R \to R$ satisfies the following conditions:

1. $a\gamma b \in R$
2. $(a+b)\gamma c = a\gamma c + b\gamma c$, $a\gamma(b+c) = a\gamma b + a\gamma c$, and $a(\beta+\gamma)b = a\beta b + a\gamma b$
3. $(a\gamma b)\beta c = a\gamma(b\beta c) = a\gamma b\beta c$

for all $a, b, c \in R$ and $\beta, \gamma \in \Gamma$, then R is called a Γ-ring.

Definition 2.2. Let A be an additive subgroup of a Γ-ring R. If $a\gamma b \in A$, for all $a, b \in A$ and $\gamma \in \Gamma$, then A is called a Γ-subring of R.

Equivalently; if $A\Gamma A \subseteq A$, then A is called a Γ-subring of R.

Definition 2.3. Let A be an additive subgroup of a Γ-ring R. If $r\gamma a \in A$ (left ideal), $a\gamma r \in A$(right ideal), for all $r \in R$, $\gamma \in \Gamma$, and $a \in A$, then A is called a Γ-ideal of R.

Equivalently; if $A\Gamma R \subseteq A$ and $R\Gamma A \subseteq A$, then A is called a Γ-ideal of R.

Definition 2.4. Let P be a proper Γ-ideal of R. If $A\Gamma B \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$, for Γ-ideals A and B of R, then P is called a prime Γ-ideal of R.

Definition 2.5. For $a, b \in R$, if $a\Gamma R\Gamma b = (0)$ implies that $a = 0$ or $b = 0$, then R is called a prime Γ-ring.

Definition 2.6. Let R be a Γ-ring and $e \in R$. If $\gamma \in \Gamma$ exists such that $e\gamma e = e$, then the element $e \in R$ is called an idempotent element.

Definition 2.7. Let R be a Γ-ring. R is called a Boolean Γ-ring if $m\gamma m = m$, for all $m \in R$ and $\gamma \in \Gamma$.

Definition 2.8. An element x of a Γ-ring R is called nilpotent element if for some $\gamma \in \Gamma$, there exists a positive integer n such that $(x\gamma)^n x = 0$.

Definition 2.9. An element x of a Γ-ring R is called strongly nilpotent if there exist a positive integer n such that $(x\Gamma)^n x = 0$.

Definition 2.10. If there exist $1 \in R$ and $\gamma \in \Gamma$ such that $1\gamma r = r\gamma 1 = r$, for all $r \in R$, then R is called a Γ-ring with unit.

Definition 2.11. An element $0 \neq a \in R$ is called a zero divisor if there exists $b \neq 0$ such that $a\gamma b = b\gamma a = 0$.

Definition 2.12. Let R and S be Γ_1-ring and Γ_2-ring respectively. An ordered (ϕ, ψ) is called a Γ-homomorphism if the following conditions are satisfied:

1. $\phi : R \to S$ is a group homomorphism
2. $\psi : \Gamma_1 \to \Gamma_2$ is a group homomorphism
3. $\phi(x\gamma y) = \phi(x)\psi(\gamma)\phi(y)$

for all $x, y \in R$ and $\gamma \in \Gamma$.

Remark 2.1. Let R and S be Γ_1-ring and Γ_2-ring respectively. The product $R \times S$ is a $\Gamma_1 \times \Gamma_2$-ring with the following operation:

$$(a, b) + (c, d) = (a + c, b + d)$$
$$(\alpha, \delta) + (\beta, \gamma) = (\alpha + \beta, \delta + \gamma)$$
$$(a, b)(\beta, \gamma)(c, d) = (a\beta c, b\gamma d)$$

for all $(a, b), (c, d) \in R \times S$ and $(\beta, \gamma), (\alpha, \delta) \in \Gamma_1 \times \Gamma_2$.

Definition 2.13. Let A be a subset of a Γ-ring R. The source of semiprimeness of A is defined as $S_R(A) = \{b \in R : b\Gamma A\Gamma b = (0)\}$. When $A = R$, S_R is adopting instead of $S_R(R)$.

3. Results

In this section, the concept of the source of Γ-primeness is characterized for the Γ-ring. To understand the concept better, the basic characteristics of the set are first inspected. Furthermore, the relationship between a ring with unit, zero divisor, idempotent, and nilpotent elements, and the set of the source of Γ-primeness is discoursed.

Definition 3.1. Let A be a non-empty subset of the Γ-ring R and $a \in R$. The set described as

$$\{b \in R : a\Gamma A\Gamma b = (0)\}$$

is denoted by $S^a_{R_\Gamma}(A)$. The intersection of sets $S^a_{R_\Gamma}(A)$ is demonstrated by $P_{R_\Gamma}(A)$, and $P_{R_\Gamma}(A)$ is called the source of Γ-primeness of A in R. When $A = R$, the $S^a_{R_\Gamma}$ notation will be operated instead of $S^a_{R_\Gamma}(R)$. Therefore, the source of Γ-primeness of the R is

$$P_{R_\Gamma} = \bigcap_{a \in R} S^a_{R_\Gamma}$$

The primary and necessary features are stated below to comprehend the concept of Γ-primeness's source.

1. Let R be a Γ-ring. $P_{R_\Gamma}(A) = \bigcap_{a\in R} S^a_{R_\Gamma}(A) \neq \emptyset$ because of $a\Gamma A\Gamma 0 = (0)$, for all $a \in R$.
2. $S^0_{R_\Gamma}(A) = R$.
3. Let A be a Γ-subring of R. If $x \in S^a_{A_\Gamma}$, then $x \in A$ and $a\Gamma A\Gamma x = (0)$. Since $A \subseteq R$, $x \in S^a_{R_\Gamma}(A)$. Therefore, $S^a_{A_\Gamma} \subseteq S^a_{R_\Gamma}(A)$.

Remark 3.1. Let $K = \{b \in R : a\Gamma A\Gamma b = (0), \forall a \in R\}$, for a non-empty subset A of a Γ-ring R. If $x \in P_{R_\Gamma}(A)$, then $a\Gamma A\Gamma x = (0)$, for all $a \in R$. Hence, $P_{R_\Gamma}(A) \subseteq K$. Similarly, $K \subseteq P_{R_\Gamma}(A)$. In line with this explanation, the source of Γ-primeness of A in R is expressed as

$$P_{R_\Gamma}(A) = \{b \in R : R\Gamma A\Gamma b = (0)\}$$

Proposition 3.1. *Let A and B be two non-empty subsets of a Γ-ring R. Then,*

$$P_{(R\times R)_\Gamma}(A \times B) = P_{R_\Gamma}(A) \times P_{R_\Gamma}(B)$$

Proof. Let $(x, y) \in P_{(R\times R)_\Gamma}(A \times B)$. Then, $(R \times R)(\Gamma \times \Gamma)(A \times B)(\Gamma \times \Gamma)(x, y) = (0, 0)$. Thus, $(R\Gamma A\Gamma x, R\Gamma B\Gamma y) = (0, 0)$. From here, $R\Gamma A\Gamma x = (0)$ and $R\Gamma B\Gamma y = (0)$. This means that $x \in P_{R_\Gamma}(A)$ and $y \in P_{R_\Gamma}(B)$. Hence, $(x, y) \in P_{R_\Gamma}(A) \times P_{R_\Gamma}(B)$. The converse is similar. Therefore, $P_{(R\times R)_\Gamma}(A \times B) = P_{R_\Gamma}(A) \times P_{R_\Gamma}(B)$. □

Example 3.1. Let $R = \mathbf{Z_4}$ and $S = \mathbf{Z_6}$ be $\mathbf{Z_4}$-ring and $\mathbf{Z_6}$-ring respectively, and $A = \{\overline{0}, \overline{2}\} \subseteq R$ and $B = \{\overline{0}, \overline{3}\} \subseteq S$. Then, $R \times S$ is a $\mathbf{Z_4} \times \mathbf{Z_6}$-ring and $A \times B \subseteq R \times S$.

$$P_{(R\times S)_{\mathbf{Z_4}\times\mathbf{Z_6}}}(A \times B) = \{(\overline{c}, \overline{d}) \in R \times S : (R \times S)(\mathbf{Z_4} \times \mathbf{Z_6})(A \times B)(\mathbf{Z_4} \times \mathbf{Z_6})(\overline{c}, \overline{d}) = (\overline{0}, \overline{0})\}.$$

$$\begin{aligned}
(\overline{c}, \overline{d}) \in P_{(R\times S)_{\mathbf{Z_4}\times\mathbf{Z_6}}}(A \times B) \quad &\Rightarrow \quad (R \times S)(\mathbf{Z_4} \times \mathbf{Z_6})(A \times B)(\mathbf{Z_4} \times \mathbf{Z_6})(\overline{c}, \overline{d}) = (\overline{0}, \overline{0}) \\
&\Rightarrow \quad (R\mathbf{Z_4}A\mathbf{Z_4}\overline{c}, S\mathbf{Z_6}B\mathbf{Z_6}\overline{d}) = (\overline{0}, \overline{0}) \\
&\Rightarrow \quad R\mathbf{Z_4}A\mathbf{Z_4}\overline{c} = (\overline{0}) \text{ and } S\mathbf{Z_6}B\mathbf{Z_6}\overline{d} = (\overline{0}) \\
&\Rightarrow \quad \overline{c} \in \{\overline{0}, \overline{2}\} \text{ and } \overline{d} \in \{\overline{0}, \overline{2}\}
\end{aligned}$$

Therefore,

$$P_{(R\times S)_{\mathbf{Z_4}\times\mathbf{Z_6}}}(A \times B) = \{(\overline{0}, \overline{0}), (\overline{0}, \overline{2}), (\overline{2}, \overline{0}), (\overline{2}, \overline{2})\}$$

Let $(a, b) \in P_{R_{\mathbf{Z_4}}}(A) \times P_{S_{\mathbf{Z_6}}}(B)$. Then, $R\mathbf{Z_4}A\mathbf{Z_4}\overline{a} = (\overline{0})$ and $S\mathbf{Z_6}B\mathbf{Z_6}\overline{b} = (\overline{0})$. From here, $\overline{a} \in \{\overline{0}, \overline{2}\}$ and $\overline{b} \in \{\overline{0}, \overline{2}\}$. Thus,

$$P_{R_{\mathbf{Z_4}}}(A) \times P_{S_{\mathbf{Z_6}}}(B) = \{(\overline{0}, \overline{0}), (\overline{0}, \overline{2}), (\overline{2}, \overline{0}), (\overline{2}, \overline{2})\}$$

Proposition 3.2. *Let R be a Γ-ring with unit. Then, $P_{R_\Gamma} \subseteq \{x \in R : x\Gamma x = (0)\}$.*

Proof. Let $M = \{x \in R : x\Gamma x = (0)\}$. If $x \in P_{R_\Gamma}$, then $R\Gamma R\Gamma x = (0)$. Since R is a Γ-ring with unit, $(0) = x\Gamma 1\Gamma x = x\Gamma x$. Hence, $x \in M$. □

Proposition 3.3. *Let A and B be two non-empty subsets of a Γ-ring R. Then, the following holds.*

1. *If $A \subseteq B$, then $P_{R_\Gamma}(B) \subseteq P_{R_\Gamma}(A)$. In particular, $P_{R_\Gamma} \subseteq P_{R_\Gamma}(A)$ is provided.*
2. *If A is a Γ-subring of R, then $A \cap P_{R_\Gamma}(A) \subseteq P_{A_\Gamma}$.*

Proof.
1. Let $A \subseteq B$. If $x \in P_{R_\Gamma}(B)$, then $R\Gamma B\Gamma x = (0)$. Since $A \subseteq B$, $R\Gamma A\Gamma x = (0)$. Therefore, $x \in P_{R_\Gamma}(A)$.
2. Let $x \in A \cap P_R^\Gamma(A)$. Since $x \in A$ and $R\Gamma A\Gamma x = (0)$, $x \in P_{A_\Gamma}$.

□

Proposition 3.4. *Let A be a nonempty subset of a Γ-ring R. Then, $P_{R_\Gamma}(A) \subset S_R(A)$.*

Proof. If $x \in P_{R_\Gamma}(A)$, then $R\Gamma A\Gamma x = (0)$. Thus, $x\Gamma A\Gamma x = (0)$. Hence, $x \in S_R(A)$. □

Lemma 3.1. *Let R be a Γ-ring and $\emptyset \neq I \subseteq R$. Then,*

1. *$S^a_{R_\Gamma}(I)$ is a right Γ-ideal of R.*
2. *If I is a right Γ-ideal, then $S^a_{R_\Gamma}(I)$ is a Γ-ideal of R. In addition, $S^a_{R_\Gamma}$ is a Γ-ideal of R.*

Proof.
1. If $x, y \in S^a_{R_\Gamma}(I)$, then $a\Gamma I\Gamma x = (0)$ and $a\Gamma I\Gamma y = (0)$, for all $a \in R$. Thus, $x\Gamma R \subseteq S^a_{R_\Gamma}(I)$ and $x - y \in S^a_{R_\Gamma}(I)$ because of
$$a\Gamma I\Gamma(x - y) = a\Gamma I\Gamma x - a\Gamma I\Gamma y = (0)$$
and
$$a\Gamma I\Gamma(x\Gamma R) = (a\Gamma I\Gamma x)\Gamma R = 0\Gamma R = (0)$$
Accordingly, $S^a_{R_\Gamma}(I)$ is a right Γ-ideal of R.
2. From 3.1, $S^a_{R_\Gamma}(I)$ is a right Γ-ideal of R. In addition
$$a\Gamma I\Gamma(R\Gamma x) = (a\Gamma I\Gamma R)\Gamma x \subseteq a\Gamma I\Gamma x = (0).$$
Thus, $R\Gamma x \subseteq S^a_{R_\Gamma}(I)$. Consequently, $S^a_{R_\Gamma}(I)$ is a Γ-ideal of R. Moreover, since R is its ideal, $S^a_{R_\Gamma}$ is a Γ-ideal of R.

□

Theorem 3.1. *Let R be a Γ-ring and $\emptyset \neq I \subseteq R$. Then,*

1. *$P_{R_\Gamma}(I)$ is a right Γ-ideal of R.*
2. *If I is a right Γ-ideal of Γ-ring R, then $P_{R_\Gamma}(I)$ is a Γ-ideal of R. Specially, P_{R_Γ} is a Γ-ideal of R.*

Proof.
1. If $x, y \in P_{R_\Gamma}(I) = \bigcap_{a\in R} S^a_{R_\Gamma}(I)$, then $x, y \in S^a_{R_\Gamma}(I)$, for all $a \in R$. From Lemma 3.1, $S^a_{R_\Gamma}(I)$ is a right Γ-ideal of R. As a result, $x\Gamma R \subseteq \bigcap_{a\in R} S^a_{R_\Gamma}(I) = P_{R_\Gamma}(I)$ and $x - y \in S^a_{R_\Gamma}(I) = P_{R_\Gamma}(I)$, for all $x \in S^a_{R_\Gamma}(I)$. Therefore, $P_{R_\Gamma}(I)$ is a right Γ-ideal of R.
2. From 1, $P_{R_\Gamma}(I)$ is a right Γ-ideal of R. Moreover, if I is a right Γ-ideal, then by Lemma 3.1, $S^a_{R_\Gamma}(I)$ is a Γ-ideal of R. Thence, $R\Gamma x \subseteq \bigcap_{a\in R} S^a_{R_\Gamma}(I) = P_R^\Gamma(I)$, for all $x \in S^a_{R_\Gamma}(I)$. Therefore, $P_R^\Gamma(I)$ is Γ-ideal of R. Furthermore, since R is its ideal, P_R^Γ is a Γ-ideal of R.

□

Example 3.2. Let $R = M_{2\times 2}(\mathbb{R}) = \left\{\begin{pmatrix} a & x \\ b & y \end{pmatrix} : a, b, x, y \in \mathbb{R}\right\}$ and $\Gamma = M_{2\times 2}(\mathbb{Z}) = \left\{\begin{pmatrix} k & 0 \\ 0 & h \end{pmatrix} : k, h \in \mathbb{R}\right\}$. Then, R is a Γ-ring according to the addition and multiplication operations in matrices. Let $I = \left\{\begin{pmatrix} 0 & t \\ 0 & t \end{pmatrix} : t \in \mathbb{R}\right\}$. Here, I is a subset of R but is not a right or left Γ-ideal. Hence, when the set $P_{R_\Gamma}(I)$ is observed, it is concluded that $P_{R_\Gamma}(I) = \left\{\begin{pmatrix} e & f \\ 0 & 0 \end{pmatrix} : e, f \in \mathbb{R}\right\}$. Evidently, $P_{R_\Gamma}(I)$ is a right Γ- ideal but not a left Γ-ideal of R.

Theorem 3.2. *Let R be a Γ-ring. The following are provided.*

1. *If R is a prime Γ-ring, then $P_{R_\Gamma} = \{0\}$.*
2. *The source of Γ-primeness P_{R_Γ} is contained by every prime Γ-ideal of the R.*

Proof. 1. Let R be a prime Γ-ring and $x \in P_{R_\Gamma} = \bigcap_{a \in R} S^a_{R_\Gamma}$. Then, $b\Gamma R\Gamma x = (0)$, for all $0 \neq b \in R$. From the hypothesis, $x = 0$. Therefore, $P_{R_\Gamma} = \{0\}$.

2. Let I be a prime Γ-ideal of R and $x \in P_{R_\Gamma}$. Then, $R\Gamma R\Gamma x = (0) \subseteq I$. Since I is a prime Γ-ideal, $R \subseteq I$ or $x \in I$. Accordingly, $P_{R_\Gamma} \subseteq I$.

□

Example 3.3. Let $R = M_{1\times 2}(\mathbb{R}) = \left\{\begin{pmatrix} a & a \end{pmatrix} : a \in \mathbb{R}\right\}$ and $\Gamma = M_{2\times 1}(\mathbb{Z}) = \left\{\begin{pmatrix} k \\ 0 \end{pmatrix} : k \in \mathbb{Z}\right\}$. Then, R is a Γ-ring. It is straightforward to verify that R is a prime Γ-ring. Further, it can be examined that $P_{R_\Gamma} = \left\{\begin{pmatrix} 0 & 0 \end{pmatrix}\right\}$.

The following example can be donated to signalize that the reverse does not work.

Example 3.4. Let $R = \mathbf{Z_4}$ and $\Gamma = \mathbb{Z}$. Then, R is a Γ-ring. Precisely, $P_{R_\Gamma} = \{0\}$. However, since $\overline{x}\Gamma R\Gamma\overline{y} = \overline{0}$, for $\overline{x} = \overline{y} = \overline{2}$, R is not a prime Γ-ring.

Proposition 3.5. *Let R be a Γ-ring. The followings are satisfied.*

1. *If R is a Boolean Γ-ring, then $P_{R_\Gamma} = \{0\}$.*
2. *If $a \in P_{R_\Gamma}$, then a is a zero divisor element of R.*
3. *If R is a Γ-ring with unit, then $P_{R_\Gamma} = \{0\}$.*

Proof. 1. If $x \in P_{R_\Gamma}$, then $R\Gamma R\Gamma x = (0)$. Thus, $(0) = x\Gamma x\Gamma x = x$. Hence, $P_{R_\Gamma} = \{0\}$.

2. If $0 \neq a \in P_{R_\Gamma}$, then $R\Gamma R\Gamma a = (0)$. Thus, $a\Gamma a\Gamma a = (0)$. If it is stated that this equality is $a\Gamma(a\Gamma a) = (0)$ or $(a\Gamma a)\Gamma a = (0)$, then $a\Gamma a = (0)$ or $a\Gamma a \neq (0)$ since $a \neq 0$. If $a\Gamma a = (0)$, then a is a zero divisor element. If $a\Gamma a \neq (0)$, a is a zero divisor element because of $a\Gamma(a\Gamma a) = (0)$.

3. If $a \in P_{R_\Gamma}$, then $R\Gamma R\Gamma a = (0)$. Thus, $(0) = 1\Gamma 1\Gamma a = a$. Consequently, $P_{R_\Gamma} = \{0\}$.

□

As a result of the above proposition, the following corollary is acquired.

Corollary 3.1. *Let R be a Γ-ring. Then,*

1. *There is no idempotent element other than zero in P_{R_Γ}.*
2. *If $x \in P_{R_\Gamma}$, then x is a strongly nilpotent element of R.*
3. *Every element in P_{R_Γ} is a nilpotent element.*

Proof. 1. Let $x \in P_{R_\Gamma}$ be an idempotent element. Then, $R\Gamma R\Gamma x = (0)$. Thus, $x\Gamma x\Gamma x = 0$. Since x is an idempotent element, $x = 0$.

2. If $x \in P_{R_\Gamma}$, then $R\Gamma R\Gamma x = (0)$. Therefore, $(0) = x\Gamma x\Gamma x = (x\Gamma)^2 x$.

3. Since every strongly nilpotent element is nilpotent, every element of P_{R_Γ} is a nilpotent.

□

Theorem 3.3. *Let R and S be Γ_1-ring and Γ_2-ring, respectively. If ordered pair (f, ψ) is a Γ-homomorphism, then $f(P_{R_\Gamma}) \subseteq P_{f(R)_\Gamma}$. If f is an injective, then $f(P_{R_\Gamma}) = P_{f(R)_\Gamma}$.*

Proof. Since (f, ψ) is a Γ-ring homomorphism, $f(R)$ is a $\psi(\Gamma_1)$-ring with multiplication

$$f(a)\psi(\gamma)f(b) = f(a\gamma b).$$

Let $x \in f(P_{R_\Gamma})$. Then, there exists $a \in P_{R_\Gamma}$ such that $x = f(a)$. Since $a \in P_{R_\Gamma}$, $R\Gamma R\Gamma a = (0)$. From here,

$$(0) = f(R\Gamma R\Gamma a) = f(R)\psi(\Gamma)f(R)\psi(\Gamma)f(a)$$

Thence, $x = f(a) \in P_{f(R)_\Gamma}$.

Let f be an injective function and $a \in P_{f(R)_\Gamma}$. Then, $f(R)\psi(\Gamma)f(R)\psi(\Gamma)a = (0)$. Since

$$f(R\Gamma R\Gamma x) = f(R)\psi(\Gamma)f(R)\psi(\Gamma)a = (0)$$

$R\Gamma R\Gamma x = (0)$ is obtained. This means $x \in P_{R_\Gamma}$. Accordingly, $a = f(x) \in f(P_{R_\Gamma})$. □

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