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An appraisal of statistical and probabilistic models in highway pavements

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Abstract

Accurate performance prediction is crucial for safe and efficient travel on highway pavements. Within pavement engineering, statistical models play a pivotal role in understanding pavement behavior and durability. This comprehensive study critically evaluates a spectrum of statistical models utilized in pavement engineering, encompassing mechanistic-empirical, Weibull distribution, Markov chain, regression, Bayesian networks, Monte Carlo simulation, artificial neural networks, support vector machines, random forest, decision tree, fuzzy logic, time series analysis, stochastic differential equations, copula, hidden semi-Markov, generalized linear, survival analysis, response surface methodology and extreme value theory models. The assessment meticulously examines equations, parameters, data prerequisites, advantages, limitations, and applicability of each model. Detailed discussions delve into the significance of equations and parameters, evaluating model performance in predicting pavement distress, performance assessment, design optimization, and life-cycle cost analysis. Key findings emphasize the critical aspects of accurate input parameters, calibration, validation, data availability, and model complexity. Strengths, limitations, and applicability across various pavement types, materials, and climate conditions are meticulously highlighted for each model. Recommendations are outlined to enhance the effectiveness of statistical models in pavement engineering. These suggestions encompass further research and development, standardized data collection, calibration and validation protocols, model integration, decision-making frameworks, collaborative efforts, and ongoing model evaluation. Implementing these recommendations is anticipated to enhance prediction accuracy and enable informed decision-making throughout highway pavement design, construction, maintenance, and management. This study is anticipated to serve as a valuable resource, providing guidance and insights for researchers, practitioners, and stakeholders engaged in asphalt engineering, facilitating the effective utilization of statistical models in real-world pavement projects.

1. Introduction

Highway pavements are crucial elements of transportation infrastructure, serving as the backbone for safe and efficient travel. These surfaces endure continuous strain from heavy loads, weather fluctuations, and environmental factors, often leading to degradation and diminished serviceability over time [1]. To confront these challenges, the field of highway pavement engineering increasingly relies on statistical and probabilistic models. These models serve as robust tools to comprehend pavement behavior and aid in strategic decision-making regarding design, construction, and maintenance approaches [2].

Statistical and probabilistic models offer a quantitative framework for predicting pavement performance, integrating uncertainties, and optimizing resource allocation [3]. They surpass conventional deterministic methods by accommodating variability in materials, construction techniques, and environmental conditions. This adaptability enables engineers and decision-makers to better evaluate the reliability, durability, and life-cycle costs of highway pavements [4].

Performance modeling and prediction stand out as primary applications of these models in highway pavement engineering. They facilitate the development of mathematical relationships between pavement performance indicators and influential factors like traffic volume, climate conditions, design specifics, and material properties. Leveraging historical data and statistical techniques, these models forecast future performance, pinpoint critical factors affecting pavement deterioration, and guide maintenance strategies to prolong pavement service life [5].

Reliability analysis constitutes another crucial area where statistical and probabilistic models make significant contributions. The principles of reliability-based design, integrating probabilistic models, empower engineers to design pavements meeting precise performance criteria with a desired level of certainty. By quantifying uncertainties related to input variables such as traffic loads, material attributes, and environmental circumstances, reliability analysis presents a more realistic evaluation of pavement performance, reducing the risk of premature failure [6].

Life-cycle cost analysis (LCCA) forms an integral part of pavement management, aiming to optimize long-term expenses while upholding desired performance levels. Statistical and probabilistic models enrich LCCA by considering input variations, encompassing construction costs, maintenance outlays, and pavement performance data [6,7]. Integrating these models into decision-making processes allows engineers to evaluate the cost-effectiveness of diverse pavement designs, materials, and maintenance strategies throughout the pavement's life cycle [7,8].

The criticality of uncertainty quantification and sensitivity analysis emerges in understanding the impact of input parameters and variability on pavement performance. Statistical methods serve as tools to gauge uncertainties and evaluate their effect on prediction reliability. Sensitivity analysis aids in pinpointing the most influential factors affecting pavement performance, enabling engineers to allocate resources strategically and focus on areas with the most potential for improvement [8-10].

Table 1. Comprehensive overview of models in highway pavement engineering: from mechanistic-Empirical to data-driven approaches.

Model Type	Application	Advantages	Contribution	Key Considerations	References
Mechanistic-Empirical Models	Rutting Prediction	Estimation of rut depth in pavements - - Calibration with field data	Equations for rutting prediction - - Regression coefficients	Dependence on accurate field data	[1-2]
Mechanistic-Empirical Models	Fatigue Cracking Prediction	Estimation of fatigue cracks in pavements	Mathematical relationship between factors - - Regression coefficients	Influence of traffic load, asphalt properties, temperature	[3-4]
Mechanistic-Empirical Models	Thermal Cracking Probability Prediction	Estimation of thermal cracking probability	Relationship involving asphalt binder properties, temperature gradient	Consideration of temperature differences	[5-6]
Statistical and Probabilistic Models	Performance Modeling and Prediction	Quantitative framework for predicting pavement performance - - Integration of uncertainties	Forecasting future performance - - Pinpointing critical factors	Adaptability to variable conditions	[7-8]
Statistical and Probabilistic Models	Reliability Analysis	Reliability-based design for pavement meeting performance criteria	Integration of probabilistic models - - Quantifying uncertainties	Reduction of risk of premature failure	[9-10]
Statistical and Probabilistic Models	Life-Cycle Cost Analysis	Optimization of long-term expenses in pavement management	Consideration of input variations - - Enrichment of LCCA	Evaluation of cost-effectiveness	[6, 11]
Statistical and Probabilistic Models	Sensitivity Analysis	Understanding impact of input parameters and variability	Gauging uncertainties - - Identifying influential factors	Strategic resource allocation	[8-10]
Data-Driven Approaches	Advanced Analytics	Fusion of statistical models with machine learning	Enhanced performance prediction and decision-making	Utilization of big data and analytics	[12]
Data-Driven Approaches	Machine Learning-Based Models	Insights from vast datasets - - Enhanced accuracy and efficiency	Improved pavement analysis - - Proactive and evidence-based pavement management	Integration with traditional models	[11-12]
Overall Study	Comprehensive Assessment	Examination of various models in pavement engineering	Scrutiny of equations, parameters, and applicability	Identification of factors impacting model performance	[13-14]
Recommendations	Actionable Guidance	Enhancing model efficacy	Informed decision-making for pavement design, construction, and management	Focus on precise input parameters, validation, and data availability	[15-16]

The emergence of big data and advanced analytics has propelled data-driven approaches in pavement engineering. Statistical and probabilistic models, in conjunction with machine learning techniques, facilitate insights from vast datasets, leading to enhanced performance prediction, condition assessment, and decision-making [11]. The fusion of data-driven approaches with traditional models enhances the accuracy and efficiency of pavement analysis, fostering proactive and evidence-based pavement management practices [12]. Table 1 provides a comprehensive overview of various pavement engineering models, categorizing them based on type, application, advantages, contribution, key considerations, and references, offering valuable insights for informed decision-making in pavement design, construction, and management.

2. Overview of statistical and probabilistic models

Mechanistic-Empirical (M-E) models in pavement engineering represent a pivotal integration of mechanistic principles and empirical data to forecast pavement performance [13]. These models rely on a series of equations that encapsulate the structural response of pavements to diverse factors such as traffic loads, climate variations, and material attributes [14]. A thorough assessment of M-E models involves scrutinizing equations, delineating parameters, evaluating performance, and considering the associated advantages, limitations, and applicability [15].

In Mechanistic-Empirical (M-E) models, a foundational equation addresses rutting prediction, estimating pavement segment rut depth. The equation is expressed as Rutting = $\beta 0 + \beta 1 \times$ Load + $\beta 2 \times$ Thickness + $\beta 3 \times$ Asphalt Properties + $\beta 4 \times$ Subgrade Strength + ϵ, where Load represents applied traffic load, Thickness indicates layer thickness, Asphalt Properties encompass asphalt mixture parameters, and Subgrade Strength denotes subgrade characteristics. The coefficients β0, β1, β2, β3, and β4 are derived through calibration processes using field data.

Another vital equation in M-E models deals with fatigue cracking prediction, estimating pavement fatigue crack count: Number of Cracks = $\beta 0 + \beta 1 \times$ Load + $\beta 2 \times$ Asphalt Properties + $\beta 3 \times$ Temperature + ϵ. In this equation, Load signifies traffic load, Asphalt Properties include asphalt material parameters, Temperature represents pavement temperature, and β0, β1, β2, and β3 are regression coefficients.

A third notable equation in M-E models focuses on predicting thermal cracking probability: Probability of Cracking = $\beta 0 + \beta 1 \times$ Asphalt Binder Properties + $\beta 2 \times$ Temperature Gradient + ϵ. Here, Asphalt Binder Properties denote asphalt binder attributes, Temperature Gradient signifies temperature differences across pavement layers, and β0, β1, and β2 are regression coefficients.

M-E models present advantages by capturing fundamental pavement behavior, enhancing accuracy compared to purely empirical models. Their adaptability allows calibration and customization for specific pavement types, materials, and climate conditions. These models facilitate design optimization by simulating diverse design alternatives and considering factors like material properties, layer thicknesses, and traffic loads. Additionally, they support the evaluation of existing pavements under varying conditions, aiding in asset management decisions and maintenance strategies. Furthermore, M-E models assist in life-cycle cost analysis by considering long-term performance and associated maintenance expenses [17].

However, M-E models entail certain limitations. The acquisition of accurate and representative input parameters, including traffic loads, material properties, and climate data, is vital for reliable predictions [18]. Calibration for these models can be intricate and time-consuming, demanding extensive field data, laboratory testing, and validation efforts. Detailed and specific data requirements may pose challenges, particularly for existing pavements with limited data availability. Sensitivity to assumptions concerning material behavior, layer interfaces, and boundary conditions represents another limitation, potentially affecting prediction accuracy. The complexity of M-E models may necessitate specialized expertise and computational resources for proper implementation and understanding [19].

The applicability of M-E models depends on factors such as pavement type, climate conditions, traffic characteristics, and the availability of reliable calibration datasets. Accurate material characterization and validation against field performance are essential for ensuring precise predictions. Incorporating M-E models within a comprehensive decision-making framework, considering engineering judgment, validation against field data, and local conditions, is crucial [19-20].

In conclusion, Mechanistic-Empirical (M-E) models stand as a transformative advancement in pavement engineering by amalgamating mechanistic principles with empirical data. The meticulous appraisal of these models involves dissecting equations, defining parameters, and evaluating performance. While offering advantages like realistic behavior, adaptability, design optimization, and life-cycle cost analysis, M-E models face challenges related to input parameter variability, calibration, data requirements, sensitivity to assumptions, and model complexity. Employing M-E models warrants a comprehensive approach within a broader decision-making framework [20-21]. Table 2 summarizes the advantages and limitations of various equations within Mechanistic-Empirical Pavement Models, highlighting their benefits and challenges, offering a concise reference for their utilization and potential pitfalls.

2.1. Weibull Distribution Models

The Weibull distribution is commonly used in pavement engineering to model the time-to-failure of pavement distresses such as cracking, rutting, and potholes [22]. The distribution provides a probabilistic framework for estimating the probability of failure at different times [23]. Critically appraising Weibull distribution models involves examining the equations, defining parameters, analyzing their performance, and

Table 2. Critical insights and considerations on advantages and limitations of mechanistic-Empirical pavement models.

Model Type	Application	Equation and Description	Advantages	Limitations	Applicability	References
Mechanistic-Empirical Models	Rutting Prediction	Rutting = β0 + β1 × Load + β2 × Thickness + β3 × Asphalt Properties + β4 × Subgrade Strength + ϵ	Estimation of rut depth in pavements	Dependence on accurate field data	Pavement types, materials, climate conditions	[13-15]
Mechanistic-Empirical Models	Fatigue Cracking Prediction	Number of Cracks = β0 + β1 × Load + β2 × Asphalt Properties + β3 × Temperature + ϵ	Estimation of fatigue cracks in pavements	Influence of traffic load, asphalt properties, temperature	Pavement types, materials, climate conditions, traffic characteristics	[13-15]
Mechanistic-Empirical Models	Thermal Cracking Probability Prediction	Probability of Cracking = β0 + β1 × Asphalt Binder Properties + β2 × Temperature Gradient + ϵ	Estimation of thermal cracking probability	- Consideration of asphalt binder properties, temperature gradient	Pavement types, asphalt binder properties, temperature differences across layers	[13-15]
Mechanistic-Empirical Models	Advantages	Captures fundamental pavement behavior - Enhances accuracy compared to purely empirical models	Adaptability for specific pavement types, materials, climate conditions	Acquisition of accurate input parameters, calibration complexity, data requirements	Design optimization, simulation of diverse alternatives, evaluation of existing pavements	[17-19]
Mechanistic-Empirical Models	Limitations	Input parameter variability - Calibration complexity and time-consuming efforts - Specific data requirements - Sensitivity to assumptions	Reliable predictions contingent on accurate and representative input parameters	Pavement types, climate conditions, traffic characteristics, reliable calibration datasets	-	[17-20]
Mechanistic-Empirical Models	Applicability Considerations	Dependence on accurate material characterization and validation against field performance - Comprehensive decision-making framework	- Comprehensive approach required, considering engineering judgment, validation, local conditions	Pavement type, climate conditions, traffic characteristics, reliable calibration datasets	-	[19-21]
Mechanistic-Empirical Models	Conclusion	Transformative advancement in pavement engineering - Amalgamates mechanistic principles with empirical data	Meticulous appraisal involving equation dissection, parameter definition, and performance evaluation	Challenges related to input parameter variability, calibration, data requirements, and model complexity	Comprehensive approach within a broader decision-making framework	[20-21]

evaluating their advantages, limitations, applicability, and other important factors.

The Weibull distribution function for pavement distresses can be expressed as (Equation 1):

$$F(t) = 1 - \exp(-(t/\beta)^{\alpha}) \qquad (1)$$

In Equation 1, F(t) represents the cumulative probability of failure at time t, β is the scale parameter that determines the location of the distribution, and α is the shape parameter that influences the steepness of the distribution curve.

To estimate the parameters β and α, statistical techniques such as maximum likelihood estimation or least squares fitting can be employed. Once the parameters are determined, the Weibull distribution model can be used to estimate the probability of failure at specific time intervals.

Advantages of using Weibull distribution models in pavement engineering include their flexibility and ability to capture a wide range of failure behaviors [24]. The shape parameter α allows for modeling both early-life failures ($\alpha < 1$) and wear-out failures ($\alpha > 1$), making it suitable for representing different types of distresses [25]. The scale parameter β provides a measure of the time scale at which failure occurs. Weibull distribution models also allow for probabilistic predictions, enabling engineers to assess the probability of pavement distresses occurring within a given time frame [26].

Additionally, Weibull distribution models can be beneficial for analyzing the performance of pavement sections and making informed decisions regarding maintenance and rehabilitation strategies [27]. By estimating the probability of failure over time, engineers can prioritize repairs based on the expected life remaining for different distresses [28]. This probabilistic approach helps optimize resource allocation and reduce the risk of premature or delayed repairs.

However, there are limitations to consider when using Weibull distribution models. One limitation is the assumption of statistical independence, which may not always hold true for pavement distresses. For instance, the occurrence of one distress, such as cracking, may influence the development of other distresses, such as rutting. Deviations from independence can impact the accuracy of predictions [29].

Another limitation is the requirement of a sufficient amount of failure data for accurate parameter estimation. Obtaining a comprehensive dataset of failure times can be challenging, especially for rare or extreme distresses. Limited data can lead to uncertainties in parameter estimation and affect the reliability of predictions [30].

Furthermore, Weibull distribution models assume that the failure process follows a specific pattern. However, actual pavement distresses may exhibit complex behavior influenced by various factors, such as traffic loads, climate conditions, and material properties. The simplicity of the Weibull distribution may not fully capture the intricacies of the failure process, and additional factors may need to be considered [30-31].

The applicability of Weibull distribution models in pavement engineering depends on the specific distress being analyzed, the availability of failure data, and the objectives of the analysis [32]. These models are particularly suitable for time-to-failure analysis and can provide valuable insights into the reliability and performance of pavement sections [33, 34].

Weibull distribution models offer a probabilistic framework for analyzing the time-to-failure of pavement distresses [35]. They provide flexibility, probabilistic predictions, and aid in decision-making regarding maintenance and rehabilitation strategies [36]. However, limitations related to independence assumptions, data availability, and the simplicity of the model should be considered [37]. Weibull distribution models are applicable in pavement engineering for time-to-failure analysis, but they should be used in conjunction with other tools and engineering judgment to obtain a comprehensive understanding of pavement performance [38,39].

2.2. Markov Chain Models

Markov chain models have been widely used in pavement engineering to analyze the deterioration process of pavement infrastructure [40]. These models provide a stochastic framework for understanding the transition of pavement condition states over time. Critically appraising Markov chain models involves examining the equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [41].

The fundamental equation in Markov chain models is the transition probability matrix, which represents the probabilities of transitioning from one pavement condition state to another. Let's consider a simplified example where we have three condition states: Good (G), Fair (F), and Poor (P). The transition probability matrix can be represented as (Equation 2):

$$P = | p(GG)\ p(GF)\ p(GP) |\ |\ p(FG)\ p(FF)\ p(FP) |\ |\ p(PG)\ p(PF)\ p(PP) | \tag{2}$$

In this matrix, p(GG) represents the probability of transitioning from a good condition to Good condition, p(GF) represents the probability of transitioning from Good to Fair, and so on. The sum of probabilities in each row of the matrix is equal to 1, ensuring that a transition will occur at each time step [42].

The transition probability matrix can be estimated using historical data, expert opinions, or statistical techniques. Once the matrix is determined, it can be used to simulate the deterioration process of pavements over time and estimate the probabilities of being in different condition states at specific time intervals [43].

Advantages of using Markov chain models in pavement engineering include their ability to capture the stochastic nature of pavement deterioration and provide insights into the evolution of condition states over time. These models can assist in long-term planning, budgeting, and decision-making related to pavement maintenance and rehabilitation strategies. Markov chain models also offer a systematic approach to analyzing and predicting the future condition of pavement networks, allowing engineers to allocate resources efficiently [44-45].

Moreover, Markov chain models can accommodate various factors that influence pavement deterioration, such as traffic loads, climate conditions, and maintenance activities. By incorporating these factors into the transition probabilities, the models can provide a more realistic representation of the deterioration process [45].

However, Markov chain models have limitations that should be considered. One limitation is the assumption of stationarity, which assumes that the transition probabilities remain constant over time [46]. In reality, the transition probabilities may change due to external factors or changes in pavement management practices. Deviations from stationarity can affect the accuracy of predictions.

Another limitation is the requirement of reliable and representative data for estimating the transition probabilities. Obtaining comprehensive data on pavement condition states and their transitions can be challenging, especially for large pavement networks. Insufficient or biased data can lead to uncertainties in the estimated transition probabilities and impact the reliability of model predictions [47].

Additionally, Markov chain models assume a discrete set of condition states, which may not fully capture the continuous nature of pavement deterioration. The discrete nature of the model may result in limited resolution when analyzing pavement condition changes [48-49].

The applicability of Markov chain models in pavement engineering depends on the availability of data, the desired level of analysis (e.g., network-level or project-level), and the specific objectives of the analysis. These models are particularly useful for long-term asset management and can provide valuable insights into the evolution of pavement condition states [49]. Table 3 presents an overview and critical analysis of Distribution Models, focusing on Weibull Distribution, and Markov Chain Models in the context of pavement engineering.

Table 3. Distribution Models and Markov Chain models in pavement engineering.

Models	Equations	Advantages	Limitations	Applicability	References
Weibull Distribution Models	F(t) = 1 - exp(-(t/β)^α)	Flexibility, Probabilistic predictions, Maintenance decisions based on expected life	Independence assumptions, Data availability, Simplistic pattern assumption	Time-to-failure analysis, Reliability insights	[22-34]
Markov Chain Models	P = | p(GG) p(GF) p(GP) | | p(FG) p(FF) p(FP) | | p(PG) p(PF) p(PP) |	Stochastic representation, Long-term planning, Resource allocation	Assumption of stationarity, Data requirement, Discrete states' limitation	Long-term asset management, Pavement condition evolution insights	[40-49]

2.3. Regression Models

Regression models are extensively used in pavement engineering to establish relationships between various input variables and pavement performance indicators [50]. These models allow for the prediction of pavement behavior based on observed data and provide valuable insights for design, analysis, and decision-making processes. Critically appraising regression models involves examining the equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [51].

A common form of regression equation used in pavement engineering is the multiple linear regression model, which relates a dependent variable (e.g., pavement distress or performance indicator) to multiple independent variables (e.g., traffic load, climate conditions, material properties). The equation can be represented as (Equation 3):

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p + \varepsilon \qquad (3)$$

In Equation 3, Y represents the dependent variable, β_0 represents the intercept, β_1, β_2, ..., β_p are the coefficients corresponding to the independent variables X_1, X_2, ..., X_p, and ε represents the random error term.

To estimate the regression coefficients, statistical techniques such as ordinary least squares (OLS) estimation or maximum likelihood estimation are commonly employed. The coefficients reflect the magnitude and direction of the relationship between the independent variables and the dependent variable. By analyzing the coefficients, engineers can gain insights into the significance and contribution of each independent variable to the pavement performance [52].

Advantages of using regression models in pavement engineering include their simplicity, interpretability, and ability to incorporate various input variables. Regression models provide a quantitative framework for understanding the relationships between input variables and pavement performance [53]. They can help identify the most influential factors affecting pavement behavior and guide decision-making processes, such as selecting appropriate materials, optimizing design parameters, and estimating future performance.

Moreover, regression models can facilitate the development of empirical design guidelines and performance prediction models. By analyzing historical data and observing the relationship between independent variables and pavement performance, engineers can establish empirical equations that simplify the design process and improve the accuracy of performance predictions [54].

However, regression models have limitations that should be considered. One limitation is the assumption of linearity in the relationship between the dependent variable and independent variables. In reality, the relationships may be nonlinear, and the use of linear regression may not capture the full complexity of the underlying behavior. Nonlinear regression techniques or alternative modeling approaches may be required to address this limitation [55]. Another limitation is the reliance on available data for model development. Insufficient or biased data can lead to unreliable regression models and inaccurate predictions. Data quality, representativeness, and sample size are crucial factors that can affect the performance of the regression models [55].

Additionally, regression models assume that the relationship between the independent variables and the dependent variable remains constant over time and across different conditions. Changes in environmental factors, traffic patterns, or material properties may violate this assumption and impact the accuracy of model predictions [56]. The applicability of regression models in pavement engineering depends on the availability of relevant data, the suitability of the selected independent variables, and the objectives of the analysis [57]. These models are particularly useful for analyzing and predicting pavement performance based on observed data. However, caution should be exercised when extrapolating the results outside the range of observed data or when applying the models to significantly different conditions [58]. Table 4 provides an overview of regression models in pavement engineering.

Table 4. Overview of Regression Models in pavement engineering.

S/No	Aspect	Description	References
1	Introduction	Regression models in pavement engineering	[50-51]
2	Regression Equation	Multiple linear regression for pavement distress [51].	
3	Variables	Y, $0\beta0$, $\beta1$,$\beta2$,...,βp, εε	
4	Coefficient Estimation	Statistical techniques for coefficient estimation	[52]
5	Advantages	Simplicity, interpretability, and versatility	[53]
6	Empirical Design Guidelines	Facilitating design guidelines and predictions	[54]
7	Limitations	Linearity assumption, data reliance, and stability concerns	[55]
8	Applicability	Applicable based on data, variables, and analysis goals [57].	[57-58]

2.4. Bayesian Networks

Bayesian networks have emerged as a powerful modeling technique in pavement engineering, allowing for probabilistic reasoning and analysis of complex relationships between variables [59]. These networks provide a graphical representation of variables and their dependencies, incorporating both observed data and prior knowledge to make informed predictions and decisions. Critically appraising Bayesian networks involves examining the equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [60].

In a Bayesian network, variables are represented as nodes, and their relationships are depicted as directed edges or arcs between the nodes. The structure of the network captures the conditional dependencies among variables, while probability distributions represent the strength of these dependencies. The network is constructed based on prior knowledge, expert opinions, and available data. Once the network structure is established, Bayesian inference techniques can be used to update and refine the probabilities based on observed data [61].

The key equation in Bayesian networks is Bayes' theorem, which allows for the calculation of posterior probabilities given prior probabilities and observed evidence. In a simplified form, Bayes' theorem can be expressed as (Equation 4):

$$P(H \mid E) = P(H) \times P(E \mid H) / P(E) \quad (4)$$

In Equation 4, P(H | E) represents the posterior probability of hypothesis H given evidence E, P(H) is the prior probability of H, P (E | H) is the probability of observing evidence E given H, and P(E) is the probability of observing evidence E.

Bayesian networks offer several advantages in pavement engineering [62]. First, they allow for explicit representation of uncertainties and their propagation throughout the network. This enables engineers to make probabilistic predictions and assess the reliability of their estimates. Bayesian networks can also handle incomplete or uncertain data by incorporating prior knowledge, making them robust in situations where data availability is limited [63]. Table 5 presents Bayesian networks in pavement engineering.

Furthermore, Bayesian networks provide a framework for incorporating multiple sources of information, including both quantitative data and qualitative expert opinions. This integration of diverse knowledge sources enhances the decision-making process and improves the accuracy of predictions. Additionally, Bayesian networks enable sensitivity analysis, which allows engineers to assess the impact of changes in input variables on the output predictions. This sensitivity analysis aids in identifying the most influential factors and understanding their relative importance in pavement performance [64].

Despite their advantages, Bayesian networks have limitations that should be considered. One limitation is the complexity of model development, which requires expert knowledge in selecting appropriate variables, defining their dependencies, and estimating the necessary probabilities. Constructing and updating the network structure can be challenging, particularly for large and complex systems [65]. Another limitation is the requirement of sufficient and representative data to estimate the probabilities accurately. In situations where data scarcity exists, the network may rely heavily on prior knowledge, which can introduce uncertainties and biases into the model [66].

Moreover, Bayesian networks assume that the network structure and parameter values remain constant over time and across different conditions.

Changes in the system or underlying relationships may require updating the model, which can be labor-intensive and time-consuming [67].

The applicability of Bayesian networks in pavement engineering depends on the availability of data, the complexity of the problem, and the objectives of the analysis [68]. These models are particularly useful for decision support systems, risk assessment, and performance prediction under uncertainty [69]. Bayesian networks can provide valuable insights into pavement behavior and support informed decision-making by considering both quantitative and qualitative information [70].

Table 5. Bayesian Networks in pavement engineering: aspects and applicability.

S/No	Aspect	Description	References
1	Methodology	Bayesian networks offer probabilistic reasoning and complex relationship analysis in pavement engineering [59].	[59-60]
2	Structure	Graphical representation with nodes for variables and edges for dependencies; constructed based on prior knowledge and data [60].	[60-61]
3	Equation	Bayes' theorem for calculating posterior probabilities based on prior probabilities and observed evidence [61].	[61]
4	Uncertainty	Explicit representation of uncertainties, enabling probabilistic predictions and reliability assessment [62].	[62-63]
5	Data Handling	Robust handling of incomplete or uncertain data through incorporation of prior knowledge [63].	[63]
6	Knowledge Integration	Integration of quantitative data and qualitative expert opinions for improved decision-making and prediction accuracy [62].	[62]
7	Sensitivity Analysis	Enables sensitivity analysis to assess the impact of input variable changes on output predictions [64].	[64]
8	Model Complexity	Model development complexity due to the need for expert knowledge and challenges in constructing and updating large and complex systems [65].	[65]
9	Data Requirement	Requires sufficient and representative data for accurate probability estimation; reliance on prior knowledge in data-scarce situations [66].	[66]
10	Model Dynamics	Assumes constant network structure and parameter values, requiring model updates for changes in the system or underlying relationships [67].	[67]
11	Applicability	Useful in decision support, risk assessment, and performance prediction under uncertainty in pavement engineering [68].	[68-70]

2.5. Monte Carlo Simulation Models

Monte Carlo simulation models have become a popular technique in pavement engineering for analyzing the uncertainty and variability associated with pavement performance [71]. These models utilize random sampling and repeated simulations to estimate the range of possible outcomes and their probabilities. Critically appraising Monte Carlo simulation models involves examining the equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [72].

The fundamental principle behind Monte Carlo simulation is to generate random samples for input variables based on their probability distributions. These input variables can include traffic loads, material properties, climate conditions, and other parameters that influence pavement performance. The simulations are then performed repeatedly using the sampled input values, and the results are recorded for analysis [73].

Monte Carlo simulation models offer several advantages in pavement engineering. Firstly, they provide a comprehensive assessment of uncertainty by considering the entire range of possible outcomes instead of relying on single-point estimates [73-74]. This allows engineers to understand the probabilistic nature of pavement performance and make informed decisions considering the associated risks [75].

Additionally, Monte Carlo simulation models can handle complex systems and interactions between multiple variables. By incorporating various input distributions and their correlations, these models capture the interdependencies and provide a more realistic representation of pavement behavior [76].

Furthermore, Monte Carlo simulation models are flexible and adaptable to different scenarios. They can be used in various stages of the pavement life cycle, from design to maintenance and rehabilitation, to assess the potential risks and guide decision-making processes [77].

However, Monte Carlo simulation models also have limitations that should be considered. One limitation is the need for a sufficient number of simulations to obtain reliable results. As the number of simulations increases, the accuracy and precision of the estimates improve. However, this can be computationally expensive and time-consuming, particularly for complex pavement models [77].

Another limitation is the requirement for accurate probability distributions for the input variables. The quality of the simulations heavily relies on the accuracy and representativeness of the chosen distributions. In cases where limited data or expert judgment is available, there may be uncertainties associated with the input distributions, which can impact the reliability of the simulation results [78].

Moreover, Monte Carlo simulation models assume that the input variables are independent and that the underlying system is stationary [78]. These assumptions may not hold true in all pavement engineering scenarios, and deviations from these assumptions can affect the accuracy and validity of the results [79].

The applicability of Monte Carlo simulation models in pavement engineering is wide-ranging. They can be used for probabilistic design, risk assessment, sensitivity analysis, and optimization of pavement performance. These models provide insights into the variability and uncertainty associated with pavement behavior, supporting decision-making processes and aiding in the development of robust pavement designs [80].

Table 6 presents a summary of Monte Carlo simulation models in pavement engineering. In conclusion, Monte Carlo simulation models offer a powerful approach to analyze uncertainty and variability in pavement engineering [81-82]. They provide a probabilistic framework for estimating the range of possible outcomes and assessing risks. However, limitations related to the number of simulations, accuracy of input distributions, and assumptions should be considered [82-83]. Monte Carlo simulation models are applicable in various stages of pavement engineering and can provide valuable insights when combined with other modeling techniques and engineering judgment [83-84].

Table 6. Monte Carlo Simulation Models in pavement engineering.

S/No	Aspect	Description	References
1	Application	Analyzing uncertainty and variability in pavement performance	[71, 82]
2	Methodology	Utilizes random sampling and repeated simulations	[72, 81]
3	Advantages	Comprehensive assessment of uncertainty, handles complex systems	[73,74, 76]
4	Flexibility	Adaptable to different scenarios and stages of pavement life cycle	[77, 80]
5	Limitations	Requires sufficient simulations, accurate probability distributions	[77-79]
6	Assumptions	Assumes independence of input variables and stationary system	[78]
7	Applicability	Wide-ranging: probabilistic design, risk assessment, sensitivity analysis	[80, 83]
8	Conclusion	Powerful approach for uncertainty analysis in pavement engineering	[81-84]

2.6. Artificial Neural Networks (ANN)

Artificial Neural Networks (ANN) models have gained significant attention in pavement engineering for their ability to capture complex relationships and make predictions based on historical data [85]. These models mimic the structure and functioning of the human brain, allowing for non-linear modeling and learning from data. Critically appraising ANN models involves examining the equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [85].

The equations used in ANN models are based on the concept of neurons and their connections within a network. The basic equation for a neuron in an ANN is the weighted sum of inputs, followed by an activation function. It can be expressed as (Equation 5):

$$y = f\left(\sum (w * x) + b\right) \quad (5)$$

In Equation 5, y represents the output of the neuron, f is the activation function, w and x are the weights and inputs, respectively, and b is the bias term.

ANN models consist of an input layer, one or more hidden layers, and an output layer. The weights and biases in the network are adjusted during a training process, where the model iteratively learns from the input-output data pairs [86]. The training algorithm, such as backpropagation, adjusts the weights and biases to minimize the error between the predicted outputs and the actual outputs.

One advantage of ANN models in pavement engineering is their ability to capture complex non-linear relationships that may exist between pavement performance indicators and various input variables. ANN models can learn from large amounts of data and adapt to changing conditions, making them suitable for analyzing the behavior of complex pavement systems [87].

Furthermore, ANN models can handle missing or noisy data and are capable of generalizing patterns from the available data to make predictions for unseen situations. This flexibility allows for the integration of diverse input variables, such as traffic loads, material properties, and environmental factors, in predicting pavement performance [87,88].

Another advantage of ANN models is their ability to provide real-time predictions. Once trained, ANN models can quickly process input data and generate predictions, making them suitable for applications that require timely decision-making, such as real-time pavement monitoring or management systems [89].

However, there are limitations to consider when using ANN models in pavement engineering. One limitation is the "black box" nature of these models, meaning they lack interpretability. It can be challenging to understand the underlying reasons for the model's predictions or to extract explicit relationships between input variables and output predictions [90].

Additionally, ANN models require a significant amount of training data to accurately learn the underlying patterns and make reliable predictions. In situations where data availability is limited, the performance of the model may be compromised. Furthermore, ANN models are sensitive to the selection of model architecture, activation functions, and training algorithms. The performance of the model can vary based on these choices, and finding the optimal configuration can be a trial-and-error process [91]. The applicability of ANN models in pavement engineering is vast. They can be used for various applications, including pavement performance prediction, optimization of pavement design, and decision support systems. ANN models are particularly useful when there are complex interactions between input variables and when non-linear relationships need to be captured [92].

Table 7 provides a summary of artificial neural networks (ANN) in pavement engineering. In conclusion, Artificial Neural Networks (ANN) models offer a powerful approach in pavement engineering to capture complex relationships and make predictions based on historical data. They excel in handling non-linear relationships, adapting to changing conditions, and providing real-time predictions [93]. However, their "black box" nature, the need for extensive training data, and sensitivity to model configuration should be considered. ANN models are applicable in a wide range of pavement engineering tasks, but care should be taken in model development, interpretation of results, and consideration of domain-specific knowledge [94-95].

Table 7. A summary of Artificial Neural Networks (ANN) in pavement engineering.

S/No	Aspect	Description	References
1	Model Type	Artificial Neural Networks (ANN)	[85]
2	Equation	$y=f(\sum(w*x)+b)$	
3	Model Structure	Consists of an input layer, one or more hidden layers, and an output layer. Weights and biases are adjusted during a training process using algorithms like backpropagation	[86]
4	Advantages	- Captures complex non-linear relationships. - Learns from large datasets and adapts to changing conditions. - Handles missing or noisy data. - Provides real-time predictions	[87-89]
5	Limitations	- Lack of interpretability (black box nature). - Requires a significant amount of training data. - Sensitivity to model architecture, activation functions, and training algorithms	[90,91]
6	Applicability	Suitable for various applications, including pavement performance prediction, pavement design optimization, and decision support systems. Particularly useful in capturing complex interactions and non-linear relationships	[92-95]

2.7. Support Vector Machines (SVM)

Support Vector Machines (SVM) models have gained prominence in pavement engineering for their ability to handle complex classification and regression tasks. SVM models aim to find an optimal hyperplane that separates data points into different classes or predicts a continuous output based on input features. Critically appraising SVM models involves examining the equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [96].

The basic equation for an SVM model can be represented as follows (Equation 6):

$$f(x) = \text{sign}\left(\sum \left(\alpha_i * y_i * K(x_i, x) + b\right)\right) \quad (6)$$

In Equation 6, f(x) represents the predicted output, α_i is the Lagrange multiplier associated with each support vector, y_i is the corresponding class label (-1 or 1), $K(x_i, x)$ is the kernel function that measures the similarity between input vectors x_i and x, and b is the bias term.

SVM models aim to maximize the margin between the hyperplane and the nearest data points of different classes, known as support vectors. The choice of the kernel function, such as linear, polynomial, or radial basis function (RBF), determines the shape of the decision boundary and the ability to handle non-linear relationships [97].

One advantage of SVM models in pavement engineering is their ability to handle high-dimensional datasets and capture non-linear relationships between input variables and pavement performance indicators. By utilizing appropriate kernel functions, SVM models can effectively map input data into higher-dimensional feature spaces, where patterns and separability become more apparent [98].

Furthermore, SVM models have a solid theoretical foundation, characterized by the structural risk minimization principle. This principle allows SVM models to generalize well from the training data to unseen instances, thus reducing the risk of overfitting and improving prediction accuracy. SVM models also offer robustness against outliers and noise in the dataset. The optimization process aims to maximize the margin between classes, effectively ignoring data points that lie far from the decision boundary. This property makes SVM models particularly useful in pavement engineering, where outliers and noise can be present due to variability in material properties, traffic conditions, or environmental factors [99].

However, there are limitations to consider when using SVM models in pavement engineering. One limitation is the computational complexity, particularly when dealing with large datasets or complex kernel functions. Training an SVM model with a high number of data points and features can be time-consuming and memory-intensive [100].

Furthermore, SVM models can be sensitive to the choice of hyperparameters, such as the kernel function, regularization parameter (C), and kernel-specific parameters. Selecting appropriate values for these hyperparameters requires careful tuning and cross-validation to achieve optimal model performance [101].

The applicability of SVM models in pavement engineering is broad. They can be used for classification tasks, such as identifying different pavement distresses or pavement condition assessment, as well as regression tasks, including predicting performance indicators such as pavement roughness or fatigue life [102]. Table 9 provides an overview of support vector machines (SVM) in pavement engineering.

Table 9. Support Vector Machines (SVM) in pavement engineering.

S/No	Aspect	Description	References
1	Model Equation	$f(x)$=sign(∑($\alpha iyiK(xi,x)+b$))	[96]
2	Kernel Functions	Linear, Polynomial, Radial Basis Function (RBF)	[97]
3	Advantages	Effective in handling high-dimensional datasets and non-linear relationships. - Solid theoretical foundation based on structural risk minimization. - - Robust against outliers and noise.	[98, 99]
4	Limitations	Computational complexity, especially with large datasets or complex kernel functions. - - Sensitivity to hyperparameters, requiring careful tuning.	[100, 101]
5	Applicability	Classification tasks for identifying pavement distresses. - - Regression tasks for predicting performance indicators like pavement roughness or fatigue life.	[102]

2.8. Gaussian Process Regression (GPR)

Support Vector Machines (SVM) models have gained prominence in pavement engineering for their ability to handle complex classification and regression tasks. SVM models aim to find an optimal hyperplane that separates data points into different classes or predicts a continuous output based on input features. Critically appraising SVM models involves examining the equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors.

The basic equation for an SVM model can be represented as follows (Equation 7):

$$f(x) = \text{sign}\left(\sum(\alpha_i * y_i * K(x_i, x) + b)\right) \quad (7)$$

In Equation 7, f(x) represents the predicted output, α_i is the Lagrange multiplier associated with each support vector, y_i is the corresponding class label (-1 or 1), $K(x_i, x)$ is the kernel function that measures the similarity between input vectors x_i and x, and b is the bias term.

SVM models aim to maximize the margin between the hyperplane and the nearest data points of different classes, known as support vectors. The choice of the kernel function, such as linear, polynomial, or radial basis function (RBF), determines the shape of the decision boundary and the ability to handle non-linear relationships [103].

One advantage of SVM models in pavement engineering is their ability to handle high-dimensional datasets and capture non-linear relationships between input variables and pavement performance indicators [104]. By utilizing appropriate kernel functions, SVM models can effectively map input data into higher-dimensional feature spaces, where patterns and separability become more apparent [105].

Furthermore, SVM models have a solid theoretical foundation, characterized by the structural risk minimization principle. This principle allows SVM models to generalize well from the training data to unseen instances, thus reducing the risk of overfitting and improving prediction accuracy [106].

SVM models also offer robustness against outliers and noise in the dataset. The optimization process aims to maximize the margin between classes, effectively ignoring data points that lie far from the decision boundary. This property makes SVM models particularly useful in pavement engineering, where outliers and noise can be present due to variability in material properties, traffic conditions, or environmental factors [107].

However, there are limitations to consider when using SVM models in pavement engineering. One limitation is the computational complexity, particularly when dealing with large datasets or complex kernel functions. Training an SVM model with a high number of data points and features can be time-consuming and memory-intensive [108].

Furthermore, SVM models can be sensitive to the choice of hyperparameters, such as the kernel function, regularization parameter (C), and kernel-specific parameters. Selecting appropriate values for these hyperparameters requires careful tuning and cross-validation to achieve optimal model performance [109].

The applicability of SVM models in pavement engineering is broad. They can be used for classification tasks, such as identifying different pavement distresses or pavement condition assessment, as well as regression tasks, including predicting performance indicators such as pavement roughness or fatigue life [110].

2.9. Random Forest Models

Random Forest models have gained popularity in pavement engineering as a powerful machine learning technique for classification and regression tasks. Random Forest models are an ensemble of decision trees that make predictions by averaging the outputs of multiple individual trees. Critically appraising Random Forest models involves examining their equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicabilities, and other important factors [111].

The basic equation for a Random Forest model can be summarized as follows (Equation 8):

$$\hat{y} = RF(x) \quad (8)$$

In Equation 8, $\hat{y}$ represents the predicted output, and RF(x) denotes the Random Forest model's prediction based on the input features x.

Random Forest models consist of multiple decision trees, each trained on different subsets of the data using a technique called bootstrap aggregating (or "bagging"). Bagging helps reduce the variance and overfitting commonly associated with individual decision trees. The predictions of the individual trees are combined using averaging (for regression) or voting (for classification) to obtain the final prediction.

One advantage of Random Forest models in pavement engineering is their ability to handle high-dimensional datasets with a large number of input features. These models can effectively capture complex relationships between pavement performance indicators and a variety of input variables, such as traffic characteristics, climate conditions, and material properties [112].

Random Forest models also offer robustness against overfitting and noise in the data. By aggregating predictions from multiple trees, Random Forest models can reduce the impact of outliers and the influence of individual noisy data points, leading to more reliable predictions.

Furthermore, Random Forest models provide a measure of variable importance, which can be valuable in pavement engineering for identifying the most influential factors affecting pavement performance. This information can aid engineers in prioritizing interventions and optimizing maintenance strategies [113].

However, Random Forest models are not without limitations. One limitation is the lack of interpretability compared to simpler models like linear regression. Random Forest models are often considered black-box models, making it challenging to understand the exact relationships and mechanisms underlying the predictions [114].

Another limitation is the potential for overfitting if not properly tuned. Although Random Forest models are designed to mitigate overfitting, they can still exhibit overfitting behavior if the number of trees in the ensemble is too large or if the individual trees are allowed to grow too deep. Applicability-wise, Random Forest models find widespread use in pavement engineering for various tasks, including pavement distress identification, condition assessment, and performance prediction. They are suitable for both classification tasks, such as identifying different types of distresses, and regression tasks, including predicting performance indicators like pavement roughness or cracking [115]. Table 10 provides a comprehensive overview of random forest models in pavement engineering.

Table 10. A Comprehensive overview of Random Forest Models in pavement engineering.

Model	Equation	Advantages	Limitations	Applicability in Pavement Engineering
Random Forest Models	$y\hat{}=RF(x)$	Handles high-dimensional datasets	Lack of interpretability	Classification: Distress identification - Regression: Pavement condition assessment, performance prediction
Ensemble of Decision Trees	Multiple trees with bagging	Robust against overfitting and noise	Potential for overfitting if not tuned properly	
Variable Importance	Provides a measure of influential factors	Lack of interpretability compared to simpler models		

2.10. Decision Trees

Decision tree models have been widely used in pavement engineering for their ability to handle both classification and regression tasks. They provide a transparent and intuitive representation of decision rules based on input features [116]. Critically appraising decision tree models involves examining their equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors.

The basic equation for a decision tree model can be represented as follows (Equation 9):

$$\hat{y} = \mathrm{DT}(x) \quad (9)$$

In Equation 9, $\hat{y}$ represents the predicted output, and DT(x) denotes the decision tree model's prediction based on the input features x.

Decision trees partition the feature space into regions based on the values of input variables and their associated thresholds. Each internal node represents a decision based on a feature and its threshold, while each leaf node represents a prediction or class label. The decision process follows a series of binary splits, leading to a hierarchical tree structure [117].

One advantage of decision tree models in pavement engineering is their interpretability. The decision rules generated by decision trees can be easily understood and visualized, allowing engineers to gain insights into the factors that contribute to pavement performance. This transparency makes decision trees valuable in explaining the decision-making process and building trust with stakeholders [118].

Decision trees can handle both categorical and numerical input features and are capable of capturing non-linear relationships. They are relatively insensitive to outliers and can handle missing data by considering surrogate splits. Decision trees also have low computational complexity during both training and prediction phases [119].

However, decision trees have certain limitations. One limitation is their tendency to overfit the training data, leading to poor generalization on unseen data. Decision trees have high variance, which means they can be sensitive to small changes in the training set and produce different trees.

To address the overfitting issue, ensemble methods such as Random Forest and Gradient Boosting are often employed, which combine multiple decision trees to improve prediction accuracy and reduce variance [119].

Another limitation is their instability to small changes in the input data. Decision trees can create different splits or structures when trained on slightly perturbed versions of the same dataset. This instability can lead to different outcomes and limits the robustness of decision tree models. In pavement engineering, decision tree models find applicability in various tasks such as pavement distress identification, classification of pavement types, and predicting performance indicators like pavement condition or remaining life [120]. Table 11 below provides an overview of application of decision trees in pavements.

Table 11. Application of Decision Trees.

Model	Equation	Advantages	Limitations	Applicability	References
Decision Trees	ŷ = DT(x)	Interpretability. - Transparent decision rules. - Handles both categorical and numerical features. - Low computational complexity.	Tendency to overfit training data. - High variance. - Instability to small changes in input data.	Pavement distress identification. - Classification of pavement types. - Prediction of performance indicators (e.g., pavement condition).	[116, 118-120]

2.11. Fuzzy Logic Models

Fuzzy logic models have gained popularity in pavement engineering for their ability to handle uncertainty and imprecision in decision-making processes. Fuzzy logic extends traditional binary logic by allowing degrees of truth between 0 and 1, which is particularly useful in situations where crisp boundaries are difficult to define [121]. Critically appraising fuzzy logic models involves examining their equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors.

Fuzzy logic models utilize linguistic variables and fuzzy sets to represent and manipulate imprecise information. The basic equation for a fuzzy logic model can be represented as follows (Equation 10):

$$\hat{y} = F(x) \tag{10}$$

In Equation 10, ŷ represents the output, and F(x) denotes the fuzzy logic model's mapping from the input variables x to the output.

Fuzzy logic models consist of three main components: fuzzyfication, fuzzy rule base, and defuzzyfication. Fuzzyfication converts crisp input variables into fuzzy sets by assigning membership degrees to different linguistic terms. The fuzzy rule base consists of a set of IF-THEN rules that define the relationship between input fuzzy sets and output fuzzy sets. Defuzzyfication combines the outputs of the fuzzy rules to produce a crisp output [122].

One advantage of fuzzy logic models in pavement engineering is their ability to handle imprecise and uncertain information. This is particularly useful when dealing with subjective criteria or expert knowledge that may not be easily quantifiable. Fuzzy logic models can capture complex relationships between input variables and output predictions, allowing for more flexible and adaptive decision-making [123].

Fuzzy logic models also provide interpretability by using linguistic terms to describe input and output variables. This makes the models more accessible and understandable to engineers and stakeholders, enhancing the trust and acceptance of the model's predictions. Fuzzy logic models find applicability in pavement engineering tasks such as pavement condition assessment, risk analysis, and decision support systems. They can incorporate multiple input variables and expert knowledge to generate meaningful and actionable recommendations. Fuzzy logic models are particularly useful in situations where precise mathematical relationships are difficult to establish or when dealing with limited or uncertain data [124].

However, fuzzy logic models also have limitations. The design and tuning of fuzzy logic models require expert knowledge to define linguistic terms, membership functions, and fuzzy rules. This expertise may be time-consuming and subjective, and the performance of the model can be sensitive to these choices. Additionally, fuzzy logic models can suffer from the "curse of dimensionality" when dealing with a large number of input variables, leading to increased computational complexity [125].

Furthermore, fuzzy logic models may not capture complex non-linear relationships as effectively as other machine learning techniques such as neural networks or support vector machines. The interpretability of fuzzy logic models can sometimes come at the expense of predictive accuracy, as the simplicity of the fuzzy rules may not capture all nuances and interactions within the data [126].

2.12. Time Series Analysis Models

Time series analysis models play a crucial role in pavement engineering for understanding and predicting the behavior of pavement performance over time. These models analyze historical data collected at regular intervals to identify patterns, trends, and seasonal variations in the data. Critically appraising time series analysis models involves examining their equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [127].

One of the commonly used time series analysis models is the autoregressive integrated moving average (ARIMA) model. The ARIMA model is represented by the Equation 11:

$$Y(t) = \mu + \phi_1 Y(t-1) + \phi_2 Y(t-2) + \dots + \phi_p Y(t-p) + \varepsilon(t) + \theta_1 \varepsilon(t-1) + \theta_2 \varepsilon(t-2) + \dots + \theta_e \varepsilon(t-q) \tag{11}$$

In this Equation 11, Y(t) represents the observed value at time t, μ is the mean, ϕ and θ are the autoregressive and moving average coefficients, p and q are the order of the autoregressive and moving average components, and ε(t) represents the random error term.

Time series analysis models offer several advantages in pavement engineering. They can capture long-term trends, seasonality, and cyclic patterns in the data, allowing for accurate predictions of pavement performance over time. These models can be used to forecast future pavement conditions and estimate the remaining service life, which is essential for effective asset management and maintenance planning [128].

Time series analysis models also provide insights into the effects of external factors on pavement performance. For example, they can help identify the impact of traffic patterns, weather conditions, and maintenance interventions on pavement deterioration. By understanding these relationships, engineers can make informed decisions regarding pavement design, materials selection, and maintenance strategies [129].

Furthermore, time series analysis models are particularly useful when historical data is available but information about specific influencing factors or mechanisms is limited. They can extract valuable information from the data and provide a basis for decision-making even in the absence of detailed knowledge about underlying physical processes [130].

However, time series analysis models have certain limitations. They assume that the observed data follows a specific pattern and may not perform well if the data deviates significantly from this pattern. The accuracy of the models depends on the quality and representativeness of the historical data, and any biases or outliers in the data can affect the model's predictions.

Additionally, time series analysis models may not capture complex interactions between different variables or account for structural changes over time. In pavement engineering, where multiple factors can influence pavement performance, other modeling techniques such as regression analysis or machine learning may be more appropriate for capturing these complex relationships. It is important to note that the selection and performance of time series analysis models depend on various factors, including the availability of historical data, the nature of the data patterns, and the specific objectives of the analysis. Careful consideration should be given to the appropriate model selection and parameter estimation to ensure accurate predictions and reliable decision-making in pavement engineering [131]. Table 13 shows a summary on application of time series analysis models in pavements while Table 14 presents an overview of methods and applications in modeling techniques in pavement engineering.

Table 13. A summary on Application of Time Series Analysis Models in pavements.

Model	Equation	Advantages	Limitations	Applicability
ARIMA	$Y(t)=\mu+\phi 1Y(t-1)+\phi 2$ $Y(t-2)+...+\phi pY(t-p)+\varepsilon(t)+\theta 1$ $\varepsilon(t-1)+\theta 2\varepsilon(t-2)+...+\theta e\varepsilon(t-q)$	- Captures long-term trends, seasonality, and cyclic patterns. - Useful for forecasting future pavement conditions and estimating remaining service life. - Provides insights into the effects of external factors on pavement performance.	- Assumes a specific pattern in the data. - Performance may be affected if data deviates significantly from the assumed pattern. - May not capture complex interactions between different variables or account for structural changes over time.	- Forecasting pavement conditions. - Estimating remaining service life. - Identifying the impact of external factors on pavement performance. - Limited detailed knowledge about underlying physical processes.

2.13. Stochastic Differential Equations (SDE)

Stochastic Differential Equations (SDEs) have gained attention in pavement engineering for modeling the behavior of pavement systems under uncertain and random conditions. SDEs combine ordinary differential equations with stochastic processes, allowing for the inclusion of random variables and capturing the probabilistic nature of pavement performance. Critically appraising SDEs involves examining their equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [132].

A general form of a stochastic differential equation used in pavement engineering can be expressed as (Equation 12):

$$dY(t) = a(t, Y(t))\, dt + b(t, Y(t))\, dW(t) \qquad (12)$$

In Equation 12, Y(t) represents the pavement response variable at time t, a(t, Y(t)) is the drift term that describes the deterministic part of the equation, b(t, Y(t)) is the diffusion term that accounts for the random component, dt is an infinitesimal time step, and dW(t) represents a Wiener process or Brownian motion.

SDEs offer several advantages in pavement engineering. Firstly, they can capture the inherent uncertainties and randomness associated with pavement behavior, such as variations in traffic loads, material properties, and environmental conditions [133]. By incorporating stochastic processes, SDEs provide a more realistic representation of the complex and dynamic nature of pavement performance.

Moreover, SDEs allow for the estimation of probabilistic distributions of pavement responses, enabling the quantification of uncertainties in pavement design, analysis, and decision-making processes. This probabilistic framework facilitates risk assessment, reliability analysis, and the development of robust pavement designs that consider the uncertainties involved [134].

Table 14. Overview of methods and applications in modeling techniques in pavement engineering.

Modelling Technique	Description	Advantages	Limitations	Applicability in Pavement Engineering	References
Regression Models	Relates input variables to pavement performance indicators for prediction.	Simplicity and interpretability	Linearity assumption, sensitivity to data quality	Analyzing pavement performance based on data.	[50-56]
Bayesian Networks	Graphical representation of variable dependencies, allowing probabilistic reasoning.	Handling uncertainty, integrating knowledge	Model complexity, data requirements	Decision support, risk assessment in pavement.	[59-65]
Monte Carlo Simulation	Analyzes uncertainty by generating random samples for inputs.	Comprehensive uncertainty assessment	Computationally expensive, dependent on data quality	Probabilistic design, risk assessment in pavement.	[71-77]
Artificial Neural Networks	Captures complex relationships in pavement data with adaptability.	Capturing non-linear relationships	Lack of interpretability, data dependency	Pavement performance prediction and optimization.	[85-91]
SVM	Finds optimal hyperplanes for classification or regression tasks.	Handling high-dimensional datasets	Computational complexity, hyperparameter sensitivity	Distress identification, pavement condition assessment.	[96-102]
GPR	Handles complex pavement relationships, accounting for uncertainties.	Flexibility in capturing patterns, uncertainties	Computational complexity, sensitivity to parameters	Classification and regression tasks in pavement.	[103-110]
Random Forest	Ensembles decision trees for high-dimensional data, robust against overfitting.	Robustness, variable importance	Lack of interpretability, potential overfitting	Pavement classification, performance prediction.	[111-115]
Decision Trees	Provides transparent decision rules based on input features.	Interpretability, handles various features	Tendency to overfitting, sensitivity to input changes	Pavement distress identification, condition prediction.	[116-120]
Fuzzy Logic Models	Handles imprecise data using linguistic variables, allowing adaptive decisions.	Handling uncertainty, linguistic representation	Expertise for tuning, "Curse of dimensionality"	Pavement condition assessment, risk analysis.	[121-126]
Time Series Analysis	Analyzes historical data for pavement trends and patterns.	Captures trends, external factors' insights	Assumption limitations, sensitivity to data quality	Forecasting pavement conditions, understanding trends.	[127-131]

Another advantage of SDEs is their ability to model time-varying and nonlinear pavement behavior. They can capture dynamic processes and nonlinearity in pavement response, which are often observed in real-world conditions. This makes SDEs suitable for analyzing the performance of pavements under changing traffic conditions, environmental factors, and deteriorating mechanisms [135].

However, SDEs also have certain limitations and challenges in pavement engineering. Firstly, the estimation and calibration of parameters in SDEs can be complex and computationally intensive. Accurate estimation of drift and diffusion coefficients from limited data can be challenging, and their values may vary with different pavement types, conditions, and materials.

Table 15 presents a summary of Stochastic Differential Equations (SDE) in pavements. Furthermore, SDEs typically require a large amount of data to accurately capture the random component and determine the underlying stochastic processes. Obtaining reliable and extensive data for parameter estimation can be expensive and time-consuming, particularly for long-term pavement performance analysis. Applicability of SDEs in pavement engineering depends on the specific problem being addressed. They are particularly useful in studying pavement deterioration, life-cycle analysis, and reliability-based design, where uncertainties play a significant role. SDEs can assist in evaluating the long-term performance and reliability of pavements, facilitating informed decision-making for maintenance, rehabilitation, and asset management strategies [136].

2.14. Copula Models

Copula models have gained prominence in pavement engineering for capturing the dependence structure between random variables and analyzing their joint behavior. These models provide a flexible and powerful framework for modeling multivariate distributions, allowing for the incorporation of complex dependence patterns that cannot be adequately captured by traditional statistical approaches. Critically appraising copula models involves examining their equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [137].

Table 15. A summary of Stochastic Differential Equations (SDE) in pavements.

Model	Equation	Advantages	Limitations	Applicability
SDE	$dY(t)=a(t,Y(t))dt+b(t,Y(t))dW(t)$	Captures inherent uncertainties and randomness in pavement behaviour. Provides a realistic representation of complex and dynamic pavement performance. Enables estimation of probabilistic distributions for risk assessment and reliability analysis. Models time-varying and nonlinear pavement behaviour.	Complex and computationally intensive parameter estimation and calibration. Requires a large amount of data for accurate capture of random components. Challenges in obtaining reliable and extensive data for long-term analysis.	Pavement deterioration studies. Life-cycle analysis. Reliability-based pavement design. Evaluation of long-term performance and reliability.

The general form of a copula model used in pavement engineering can be expressed as (Equation 13):

$$C(F_1(x_1), F_2(x_2), ..., F_n(x_n)) \tag{13}$$

In Equation 13, C represents the copula function, $F_1(x_1)$, $F_2(x_2)$, ..., $F_n(x_n)$ are the marginal distribution functions of the individual random variables x_1, x_2, ..., x_n, respectively. The copula function captures the dependence structure between the variables and allows for modeling both linear and non-linear relationships.

One commonly used copula model in pavement engineering is the Gaussian copula, which assumes a multivariate normal distribution for the marginals. The correlation structure is then modeled using the copula function, typically the Gaussian copula. The equations and parameters for the Gaussian copula are as follows (Equation 14):

$$C(u_1, u_2, ..., u_n; \rho) = \Phi_n(\Phi^{-1}(u_1), \Phi^{-1}(u_2), ..., \Phi^{-1}(u_n); \rho) \tag{14}$$

Here, u_1, u_2, ..., u_n are the standardized values of the variables, Φ_n represents the joint cumulative distribution function for the multivariate normal distribution, Φ^{-1} denotes the inverse of the standard normal cumulative distribution function, and ρ is the correlation parameter that determines the strength and direction of the dependence.

Copula models offer several advantages in pavement engineering [138]. Firstly, they allow for the modeling of complex dependence structures, including asymmetry, tail dependence, and non-linear relationships. This flexibility enables a more accurate representation of the joint behavior of pavement variables, such as load and temperature, which are crucial in understanding pavement performance.

Additionally, copula models enable the estimation of joint probabilities and quantiles, which are useful in risk assessment and reliability analysis. By capturing the dependence structure, copula models can provide insights into the likelihood of extreme events, such as simultaneous high loads and high temperatures, that can lead to critical pavement failures [139].

However, copula models also have limitations and considerations in pavement engineering. Firstly, they require appropriate choice and calibration of copula functions. The selection of an appropriate copula function depends on the characteristics of the data and the underlying dependence structure. Choosing an incorrect copula can lead to inaccurate modeling results and unreliable predictions. Another challenge is the estimation of copula parameters, especially when dealing with limited data. The estimation process may require large sample sizes to achieve reliable parameter estimates, and the estimation can be computationally intensive for high-dimensional problems [140].

Table 16 presents a summary of the Copula Models in pavement engineering. Applicability of copula models in pavement engineering depends on the specific problem and the availability of relevant data. They are particularly useful when modeling the joint behavior of multiple variables that affect pavement performance. Copula models can be applied in various areas of pavement engineering, such as reliability analysis, performance prediction, and optimization of pavement designs [141].

2.15. Hidden Semi-Markov Models (HSMM)

Hidden Semi-Markov Models (HSMM) have shown promise in pavement engineering as they can capture the temporal dynamics of pavement conditions and predict their future states. HSMMs extend the traditional Hidden Markov Models (HMM) by allowing variable duration in each state, which is particularly useful for modeling the duration of pavement distresses. Critically appraising HSMMs involves examining their equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [142].

Table 16. A summary of the Copula Models in pavement Engineering.

Model	Equation	Advantages	Limitations	Applicability	References
Copula Models	$C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))$	Captures complex dependence structures.	Requires appropriate choice and calibration of copula functions.	Risk assessment and reliability analysis. Modelling joint behaviour of multiple pavement variables. Optimization of pavement designs.	[137-141]

In an HSMM, the pavement condition is represented by a sequence of hidden states, and the observed data correspond to these states. The key equations in an HSMM include the state transition probabilities, the duration probabilities, and the emission probabilities. HSMMs offer several advantages in pavement engineering. Firstly, they can capture the varying durations of pavement distresses, which is crucial for accurately predicting their evolution over time. Traditional HMMs assume fixed state durations, which may not accurately represent the real-world behavior of pavement conditions. HSMMs provide a more flexible framework for modeling the time spent in each state, leading to more accurate predictions [143].

Secondly, HSMMs allow for modeling multiple distinct states and their transitions, enabling the representation of different pavement distresses and their interrelationships. This capability is particularly valuable in capturing the complex nature of pavement deterioration processes, where multiple distresses can coexist and influence each other [143-144].

Furthermore, HSMMs provide a probabilistic framework for uncertainty quantification. They can estimate the uncertainty associated with the predicted pavement conditions, allowing for risk assessment and informed decision-making in pavement management and maintenance strategies [145].

However, HSMMs also have limitations and considerations in pavement engineering. One challenge is the estimation of model parameters, including the transition probabilities, duration probabilities, and emission probabilities. Adequate data for parameter estimation is crucial, and the accuracy of the model predictions heavily relies on the quality and representativeness of the training data. Another limitation is the computational complexity associated with the analysis and prediction using HSMMs, especially for large-scale pavement systems. The computational demands may increase with the complexity of the model, the number of states, and the length of the time series data [146]. Table 17 presents a summary of Hidden Semi-Markov Models (HSMM).

Applicability of HSMMs in pavement engineering depends on the specific problem and the availability of relevant data. They can be applied in various areas, such as pavement deterioration modeling, remaining service life prediction, and maintenance decision-making. HSMMs are particularly suitable when there is a need to capture the temporal dynamics and varying durations of pavement distresses [147].

Table 17. Hidden Semi-Markov Models (HSMM).

Model	Equations	Advantages	Limitations	Applicability	References
Hidden Semi-Markov Models (HSMM)	- State transition probabilities - Duration probabilities - Emission probabilities	- Captures varying durations of pavement distresses. - Models multiple distinct states and their transitions. - Provides a probabilistic framework for uncertainty quantification.	- Estimation of model parameters require adequate data. - Computational complexity, especially for large-scale pavement systems.	- Pavement deterioration modeling. - Remaining service life prediction. - Maintenance decision-making. - Suitable for capturing temporal dynamics and varying durations of pavement distresses.	[142-147]

2.16. Generalized Linear Models (GLM)

Generalized Linear Models (GLM) have been widely used in pavement engineering to analyze the relationships between predictor variables and pavement response variables. GLMs extend the traditional linear regression models by allowing for non-normal response variables and incorporating link functions to model the relationship between the predictors and the response. Critically appraising GLMs involves examining their equations, defining parameters, analyzing their performance, and evaluating their advantages, limitations, applicability, and other important factors [148].

The equation of a GLM consists of three main components: the linear predictor, the link function, and the probability distribution function.

1. Linear Predictor: The linear predictor in a GLM is defined as the sum of the predictor variables multiplied by their corresponding coefficients. It represents the systematic part of the model that captures the relationship between the predictors and the response variable. The equation of the linear predictor in a GLM can be written as follows (Equation 15):

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p \quad (15)$$

where η is the linear predictor, $\beta_0, \beta_1, \beta_2, \ldots, \beta_p$ are the coefficients, and $x_1, x_2, \ldots, x_p$ are the predictor variables.

2. Link Function: The link function in a GLM describes the relationship between the linear predictor and the expected value of the response variable. It transforms the linear predictor to ensure that the predicted values are within a valid range for the chosen probability distribution. Commonly used link functions include the identity, log, logit, and inverse functions.

3. Probability Distribution Function: The probability distribution function determines the probability distribution of the response variable given the linear predictor. The choice of the distribution depends on the nature of the response variable. Commonly used distributions in pavement engineering include the Gaussian (normal), Poisson, negative binomial, and gamma distributions.

GLMs offer several advantages in pavement engineering. Firstly, they can handle a wide range of response variables, including continuous, count, and binary variables. This flexibility allows for the analysis of various pavement performance indicators, such as roughness, cracking severity, and distress counts.

Secondly, GLMs provide a probabilistic framework that allows for the quantification of uncertainty in the model predictions. The probability distribution function provides information about the variability of the response variable, enabling the estimation of prediction intervals and confidence intervals [149].

Furthermore, GLMs can incorporate both categorical and continuous predictor variables, allowing for the inclusion of factors such as traffic volume, climate conditions, and material properties in the analysis. This capability enables the identification of significant factors influencing pavement performance and the estimation of their effects. However, GLMs also have limitations and considerations in pavement engineering. One limitation is the assumption of linearity between the predictor variables and the linear predictor. If the relationship is highly nonlinear, alternative modeling techniques such as generalized additive models (GAMs) may be more appropriate [150].

Another consideration is the choice of the link function and probability distribution. The selection should be based on the characteristics of the response variable and the assumptions made about its distribution. Mis-specifying these components can lead to biased or inefficient estimates. Applicability of GLMs in pavement engineering depends on the research question and the nature of the data. They can be used for various tasks, including predicting pavement performance, analyzing the effects of factors on pavement deterioration, and identifying significant variables for pavement design and maintenance [151].

Table 18 presents a summary of the generalized linear models (GLM) in pavements while Table 19 summarizes a comparison of key aspects of each modeling technique in pavement engineering, including their descriptions, advantages, limitations, and applicability, along with the corresponding references for further details.

Table 18. A summary of the Generalized Linear Models (GLM) in pavements.

Model	Equations	Advantages	Limitations	Applicability	References
Generalized Linear Models (GLM)	- Linear Predictor: $\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p$ - Link Function - Probability Distribution Function	- Handles various response variables (continuous, count, binary). - Probabilistic framework for uncertainty quantification. - Incorporates both categorical and continuous predictors. - Provides insights into significant factors influencing pavement performance.	- Assumes linearity between predictors and the linear predictor. - Choice of link function and distribution is critical.	- Predicting pavement performance. - Analyzing effects of factors on pavement deterioration. - Identifying significant variables for pavement design and maintenance.	[148-151]

2.17. Survival Analysis Models

Survival analysis models, also known as time-to-event analysis or reliability analysis, have been increasingly used in pavement engineering to analyze the time until the occurrence of specific events or failures, such as pavement distresses, rutting, or fatigue cracking. These models are particularly useful when dealing with censored data, where the event of interest may not have occurred for all observed samples [152].

The main equation used in survival analysis is the survival function, which represents the probability of an event not occurring before a certain time t. The survival function is typically estimated using nonparametric methods such as the Kaplan-Meier estimator or parametric models such as the exponential, Weibull, or log-logistic distributions. Here are the equations for the survival function in the exponential and Weibull models:

Table 19. A comparison of some Modeling Techniques in Pavement Engineering: Methods, Advantages, Limitations, and Applicability.

Modeling Technique	Description	Advantages	Limitations	Applicability	References
Stochastic Differential Equations (SDE)	Incorporates stochastic processes into ordinary differential equations, modeling pavement behavior under uncertainty.	Captures uncertainties in pavement behavior, enables probabilistic predictions, models non-linear pavement responses.	Complex parameter estimation, data-intensive, calibration challenges.	Long-term pavement performance analysis, risk assessment, reliability-based design.	[132-136]
Copula Models	Captures complex dependence structures between random variables, allowing for flexible multivariate modeling.	Models asymmetry, tail dependence, non-linear relationships, aids in risk assessment, provides joint probabilities and quantiles.	Challenges in copula selection, parameter estimation, and computational complexity.	Modeling joint behavior of multiple variables, reliability analysis, risk assessment.	[137-141]
Hidden Semi-Markov Models (HSMM)	Extends Hidden Markov Models to model variable state durations in pavement conditions.	Captures varying distress durations, represents multiple distresses, aids in uncertainty quantification.	Complex parameter estimation, computational complexity, dependency on quality data.	Pavement deterioration modeling, remaining service life prediction, maintenance decisions.	[142-147]
Generalized Linear Models (GLM)	Extends linear regression, handles various response types using link functions and diverse probability distributions.	Analyzes different response variables, provides probabilistic framework, handles categorical and continuous predictors.	Assumes linearity, choice dependency on link and distribution, might lack flexibility in highly non-linear relationships.	Pavement performance prediction, factors influencing deterioration, maintenance analysis.	[148-151]

1. Exponential Survival Function: The exponential survival function assumes a constant hazard rate, meaning that the probability of failure remains constant over time. The equation for the exponential survival function is given by (Equation 16):

$$S(t) = \exp(-\lambda t) \quad (16)$$

where S(t) is the survival probability at time t, λ is the hazard rate, and exp() is the exponential function.

2. Weibull Survival Function: The Weibull survival function allows for the hazard rate to change over time. It is defined as (Equation 17):

$$S(t) = \exp(-(t/\beta)^\alpha) \quad (17)$$

where S(t) is the survival probability at time t, β is the scale parameter, α is the shape parameter, and exp() is the exponential function.

Survival analysis models offer several advantages in pavement engineering. Firstly, they account for censored data, which is common in pavement performance studies. By considering the time until failure or distress, these models provide a more comprehensive analysis of pavement deterioration.

Secondly, survival analysis models allow for the estimation of key parameters such as the hazard rate, which represents the instantaneous probability of failure at a given time. This information is valuable for understanding the deterioration patterns and predicting the remaining useful life of pavements [153].

Table 19 presents a summary of the Survival Analysis Models. Furthermore, survival analysis models can incorporate covariates or predictors to analyze the effects of various factors on pavement survival. This enables the identification of significant variables influencing pavement performance and the estimation of their effects on the failure probability.

However, there are certain limitations and considerations to be aware of when using survival analysis models in pavement engineering. One limitation is the assumption of independent and identically distributed (IID) observations, which may not always hold true in practice. Correlation among observations due to spatial or temporal dependencies should be carefully addressed [154].

Another consideration is the choice of the appropriate parametric distribution for the survival function. The selection should be based on the characteristics of the data and the underlying assumptions about the hazard rate. Mis-specifying the distribution can lead to biased or inefficient estimates.

Applicability of survival analysis models in pavement engineering lies in their ability to analyze time-to-event data, predict failure probabilities, and identify significant factors affecting pavement deterioration. They can be used for assessing pavement performance, estimating remaining service life, and informing maintenance and rehabilitation decisions [155].

Table 19. A Summary of the Survival Analysis Models.

Model	Equations	Advantages	Limitations	Applicability	References
Survival Analysis Models	- Exponential Survival Function: $S(t)=\exp(-\lambda t)$	- Accounts for censored data. - Estimation of key parameters like hazard rate. - Incorporates covariates for analyzing the effects of various factors.	- Assumption of independent and identically distributed (IID) observations. - Choice of appropriate parametric distribution is critical.	- Assessing pavement performance. - Estimating remaining service life. - Informing maintenance and rehabilitation decisions.	[152-155]

2.18. Extreme Value Theory (EVT)

Extreme Value Theory (EVT) is a statistical approach that focuses on modeling the extreme values of a random variable. In the context of pavement engineering, EVT has been used to analyze extreme events such as peak traffic loads, extreme temperatures, or exceptional pavement distresses. Critically appraising EVT involves examining its equations, defining parameters, analyzing their performance, and evaluating the advantages, limitations, applicability, and other important factors.

The main equation used in EVT is the Generalized Extreme Value (GEV) distribution, which characterizes the distribution of extreme values. The GEV distribution is defined by three parameters: location (μ), scale (σ), and shape (ξ). The cumulative distribution function (CDF) of the GEV distribution is given by (Equation 18):

$$F(x) = \exp\{-[1 + \xi(x - \mu)/\sigma]^{-1/\xi}\} \quad (18)$$

where F(x) is the CDF at a given value x, exp() is the exponential function, and ξ is the shape parameter.

EVT offers several advantages in pavement engineering. Firstly, it provides a robust framework for analyzing extreme events and their probabilities, which is essential for designing pavements to withstand extreme conditions. By focusing on the tail behavior of the distribution, EVT enables engineers to assess the risks associated with rare events that have significant implications for pavement performance [156].

Secondly, EVT allows for the estimation of extreme quantiles, such as the design traffic load or design temperature, with associated return periods. This information helps in designing pavements to meet specified reliability or risk targets.

Furthermore, EVT provides a flexible modeling approach by accommodating different types of extreme value distributions, including the Gumbel, Fréchet, and Weibull distributions. This allows for tailoring the modeling to the specific characteristics of the data under consideration [157].

However, there are limitations and considerations when using EVT in pavement engineering. One limitation is the assumption of the underlying distribution of extreme values. The choice of the distribution should be guided by statistical techniques and domain knowledge, but selecting an inappropriate distribution can lead to unreliable results.

Another consideration is the limited availability of extreme data in pavement engineering. Extreme events are by definition rare, and reliable data on extreme values may be scarce. This can lead to challenges in parameter estimation and uncertainty quantification. Applicability of EVT in pavement engineering lies in its ability to analyze extreme events, estimate extreme quantiles, and assess the risks associated with rare events. It is particularly useful for designing pavements to withstand extreme conditions and ensuring their resilience and longevity [158]. Table 20 presents a summary of the Extreme Value Theory.

Table 20. A summary of the Extreme Value Theory.

Model	Equations	Advantages	Limitations	Applicabilities	References
Extreme Value Theory	Generalized Extreme Value (GEV) distribution: $F(x)=\exp\{-[1+\xi(x-\mu)/\sigma]-1/\xi\}$	- Provides a robust framework for analyzing extreme events. - Allows estimation of extreme quantiles and return periods. - Flexible modeling with different extreme value distributions.	- Assumes the underlying distribution of extreme values. - Limited availability of extreme data.	- Designing pavements to withstand extreme conditions. - Estimating extreme quantiles for design criteria. - Assessing risks associated with rare events.	[156-158]

3. Application of response surface methods (RSM) in pavement engineering

Response Surface Methods (RSM) serve as invaluable tools in pavement engineering, offering a systematic approach to model and optimize the intricate relationships between multiple factors influencing pavement performance [159]. These statistical techniques facilitate the exploration of optimal pavement materials mix design and provide insights into the complex interplay of variables [160]. The general equation for a response surface model is $Y=f(x1,x2,...,xk)+\varepsilon$, where Y is the response variable (e.g., pavement strength, durability),$x1,x2,...,xk$ are the independent variables (e.g., mix proportions, curing time), f is the response function, and ε is the error term. In quadratic response surface models, the equation becomes more specific, incorporating terms like βi, βii, and βij to represent linear, quadratic, and interaction effects.

RSM offers several advantages in pavement engineering. Firstly, it enables efficient optimization by identifying the optimal combination of factors that lead to desired pavement performance [161]. Secondly, RSM provides insights into the interactions between various factors, aiding engineers in making informed decisions. Lastly, it contributes to resource savings by reducing the need for extensive experimental trials through the development of predictive mathematical models [162]. However, RSM has limitations. It assumes linearity in relationships, which may not hold for highly nonlinear pavement behaviors. The method is also confined to polynomial models, potentially limiting its accuracy in capturing complex pavement material interactions. Additionally, the presence of outliers in the data can impact the precision of the response surface model [163].

RSM finds practical applications in pavement optimization and materials mix design. It plays a crucial role in optimizing aggregate gradation to enhance pavement mechanical properties, including stability and rut resistance [164]. Additionally, RSM aids in determining the optimal binder content, achieving the desired balance between pavement stiffness and flexibility. Moreover, RSM optimizes factors such as curing time and temperature for concrete pavements, contributing to improved compressive strength and durability [165]. In the realm of additives and fillers, RSM assists in optimizing types and quantities, enhancing overall pavement performance [166]. In conclusion, Response Surface Methods offer a systematic and efficient approach to pavement optimization and materials mix design. While acknowledging their limitations, their advantages make them indispensable for engineers seeking to achieve optimal pavement performance through informed decision-making and resource-efficient experimentation. The diverse types of response surface models provide flexibility in addressing specific challenges encountered in pavement engineering.

4. Research gap and contribution to knowledge

The study on statistical and probabilistic models in highway pavement engineering stands as a pivotal consolidation of diverse methodologies crucial for understanding pavement behavior and facilitating informed decision-making. Its comprehensive review encompasses an array of models, from mechanistic-empirical models to artificial neural networks, offering a detailed evaluation of each model's equations, parameters, strengths, limitations, and applicability. This meticulous analysis serves as a critical bridge between historical perspectives and modern advancements, anchoring the state-of-the-art discussion in a rich foundation of past research while highlighting the context of current developments.

Notably, the study's contribution extends beyond a mere enumeration of models; it undertakes a rigorous examination of their performance in predicting pavement distress, evaluating performance, optimizing design, and conducting life-cycle cost analysis. By acknowledging both strengths and limitations, this research provides invaluable insights into the necessity of accurate input parameters, calibration, validation procedures, and the impact of data availability and model complexity. It doesn't just stop at identification; it delves into actionable recommendations for enhancing the efficacy of these models in pavement engineering, guiding future research and practice towards overcoming these identified challenges.

What truly distinguishes this study is its anticipation of serving as a crucial resource for various stakeholders in asphalt engineering. By offering insights and guidance for the practical application of statistical models in real-world pavement projects, it directly benefits researchers, practitioners, and stakeholders involved in the design, construction, maintenance, and management of highway pavements. This anticipated impact underscores the study's significance in serving as a benchmark for understanding, evaluating, and improving the application of statistical and probabilistic models in the realm of highway pavement engineering, making it an invaluable contribution to the field.

5. Recommendations

Based on the appraisal of statistical models in pavement engineering, the following recommendations can be made:

1. Further Research and Development: Continued research and development efforts should focus on refining and enhancing the existing statistical models. This includes improving the accuracy of predictions, addressing limitations, and incorporating new advancements in the field of pavement engineering.
2. Data Collection and Standardization: Efforts should be made to collect comprehensive and standardized data for input parameters of statistical models. This includes traffic loads, material properties, climate data, and pavement performance data. Standardization of data will improve the accuracy and reliability of model predictions.
3. Calibration and Validation: Proper calibration and validation of statistical models are essential to ensure accurate representation of pavement behavior. More research should be conducted to develop standardized calibration and validation procedures

specific to each model, taking into account different pavement types, materials, and climate conditions.

4. Integration of Multiple Models: Instead of relying on a single statistical model, the integration of multiple models can provide a more comprehensive and robust approach to pavement engineering. The combination of different models, such as mechanistic-empirical models with machine learning techniques, can enhance the accuracy and reliability of predictions.

5. Decision-Making Framework: Statistical models should be used as part of a broader decision-making framework that considers engineering judgment, local conditions, and validation against field data. Pavement engineers should exercise caution in interpreting and applying model results, using them as a tool to inform decisions rather than relying solely on the model predictions.

6. Collaboration and Knowledge Sharing: Collaboration between researchers, practitioners, and stakeholders in the pavement engineering field is crucial for sharing knowledge, best practices, and data. This collaboration can lead to advancements in statistical modeling techniques and ensure their practical applicability in real-world pavement projects.

7. Continuous Model Evaluation: Statistical models should be continuously evaluated and updated based on new data and advancements in the field. Ongoing monitoring and evaluation of model performance will help identify areas for improvement and ensure the models remain relevant and accurate over time.

By implementing these recommendations, the field of pavement engineering can harness the full potential of statistical models, improving the design, maintenance, and management of highway pavements, and ultimately contributing to safer and more efficient transportation infrastructure.

6. Conclusion

In conclusion, this study critically appraised several statistical models commonly used in pavement engineering, including Mechanistic-Empirical (M-E) models, Weibull distribution models, Markov chain models, regression models, Bayesian networks, Monte Carlo simulation models, Artificial Neural Networks (ANN) models, Support Vector Machines (SVM) models, Random Forest models, Decision Trees models, Fuzzy Logic models, Time Series Analysis models, Stochastic Differential Equations (SDE), Copula models, Hidden Semi-Markov Models (HSMM), Generalized Linear Models (GLM), Survival Analysis models, and Extreme Value Theory (EVT).

Each of these models has its own set of equations, parameters, advantages, limitations, and applicabilities in the field of pavement engineering. The equations presented for each model demonstrated their mathematical foundations and how they capture the behavior of pavement structures, predict distresses, estimate performance, or model uncertainties.

The appraisal highlighted the advantages of these models, such as their ability to incorporate mechanistic understanding, reflect realistic behavior, adapt to specific conditions, optimize designs, evaluate performance, and conduct cost analysis. However, the limitations and challenges associated with these models were also identified, including the need for accurate input parameters, calibration efforts, data requirements, sensitivity to assumptions, model complexity, and the need for expert knowledge.

The applicability of these models varied depending on the specific pavement engineering context, including pavement type, climate conditions, traffic characteristics, available data, and the availability of calibration and validation datasets. It was emphasized that these models should be used as part of a broader decision-making framework that includes engineering judgment, validation against field data, and consideration of local conditions.

Overall, the critical appraisal of these statistical models in pavement engineering revealed their potential to enhance pavement design, evaluation, and decision-making processes. However, it is important to carefully consider their limitations and ensure their appropriate and accurate application. Further research and advancements in these models, as well as their integration with other approaches, can continue to improve the effectiveness and reliability of pavement engineering practices.

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Author contributions

Jonah Chukwuemeka Agunwamba: Conceptualization, Methodology, Study design, Result interpretation. **Michael Toryila Tiza:** Conceptualization, Writing-Original draft preparation, Methodology, Result interpretation **Fidelis Okafor:** Methodology, Result interpretation, insights into statistical and probabilistic models, enriching understanding in the field

Conflicts of interest

The authors declare no conflicts of interest.

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