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Research Article

Effect of soil coefficients and Poisson's ratio on the behavior of modified Euler-Bernoulli beam lying on Winkler foundation[#]

B. Gültekin SINIR[1]*, Kabil ÇETİN[1], Leyla USTA[2]

[1]Manisa Celal Bayar Üniversity, Engineering Faculty, Civil Engineering Department, 45140, Manisa-Turkey
[2]Manisa Celal Bayar Üniversity, Science and Arts Faculty, Mathematics Department, 45140, Manisa-Turkey

* Corresponding Author : gultekin.sinir@cbu.edu.tr

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Abstract: Beside the geometric acceptations on deriving the mathematical model of the structural elements the material specifications can affect their different fundamental responses. Respectively, in this study standard Euler-Bernoulli beam theory is adjusted and modified to comprise the effect of Poisson's ratio on the mechanical model of the beam. The interaction between beams and the soil medium below the beams alter the actual behavior of the beams. In this study, the equation of motion of the beam lying on Winkler foundation is derived via adopting the extended Hamilton principle. This new formulation is used to investigate the soil-structure interaction features of the modified beam with uniform cross sectional area. Three different soil types is considered. The non-dimensional mathematical model of the proposed beam is also obtained. Finally, the effect of the Poisson's ratio and foundation spring coefficient on the behavior of the proposed beam is discussed and demonstrated through given diagrams.

1. Introduction

Beams are among the most commonly used structural members that are quite important for many researchers. Therefore engineering beam theories are commonly studied subjects and there are some different beam theories. This study presents mathematical model of the modified Euler Bernoulli beam. The Euler–Bernoulli beam (EBB) theory based on the assumptions of straight lines normal to the central axis.

The interaction between beams and the soil medium below the beams alter the actual behavior of the beams. It means that the interaction has considerably role on the behavior of the beams. Thus, a reasonably accurate model for the soil–foundation–structure interaction system is needed in improved design of structures. Winkler foundation is a well-known soil idealization model [1]. According to Winkler's idealization, deformation of foundation is confined to loaded regions only. Thus, in this idealization, the soil medium is represented as a system of identical but mutually independent, discrete, elastic springs. The equation of motion of the beam lying on Winkler foundation is derived via adopting the extended Hamilton principle [2]. This new formulation is used to investigate the soil-structure interaction features of the modified beam with uniform cross sectional area. In this study, we investigate dynamic behavior of the beam lying on three different soil models. These models are the first is homogenous Winkler foundation [3], the second is the Winkler foundation which has variable properties along beam length [4] and the last is partly Winkler foundation [5].

The non-dimensional mathematical model of the proposed beam is also obtained. The effects of the Poisson's ratio and foundation spring coefficients on the natural frequencies of the beam are discussed and demonstrated through given diagrams.

2. Derivation of equation of motion

We assume that the beam deflects only due to bending moment. And the effect of shear force is negligible. Here w is the vertical displacements. Deformations (u_1, u_2, u_3) in the coordinates x_1, x_2, x_3 are written using the geometry of the deformed beam, w. If we substitute these deformations into Green Lagrange strain relations. The strains is found.

$$u_1 = -x_3\left(\frac{\partial w}{\partial x_1}\right),\ u_2 = 0,\ u_3 = w(x_1,t) \quad (1)$$

$$\varepsilon_{11} = -x_3\left(\frac{\partial^2 w}{\partial x_1^2}\right) + \frac{1}{2}\left(-x_3\frac{\partial^2 w}{\partial x_1^2}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x_1}\right)^2$$
(2.a)

$$\varepsilon_{13} = -x_3\left(\frac{\partial^2 w}{\partial x_1^2}\right).\left(-\frac{\partial w}{\partial x_1}\right)$$
(2.b)

$$\varepsilon_{33} = \frac{1}{2}\left(\frac{\partial w}{\partial x_1}\right)^2$$
(2.c)

To obtain the simple version of the strain, w and x_3 has been compared. ε is very small number. Positions derivatives will be smaller. If $O(\varepsilon^5)$ and its over is eliminated, terms of the new strain is obtained.

$$x_3 \gg w \quad x_3 \to O(1)$$
(3.a)

$$w \ \to \quad w \to O(\varepsilon)$$
(3.b)

$$\frac{\partial w}{\partial x_1} \to \quad O(\varepsilon^2)$$
(3.c)

$$\varepsilon_{11} = -x_3\frac{\partial^2 w}{\partial x_1^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x_1}\right)^2$$
(4.a)

$$\varepsilon_{13} = 0$$
(4.b)

$$\varepsilon_{33} = \frac{1}{2}\left(\frac{\partial w}{\partial x_1}\right)^2$$
(4.c)

σ_{11} which is the normal stress in x_1 direction is enough to obtain the unit strain ε_{33}. This relation $\varepsilon_{33} = -\upsilon\varepsilon_{11}$ is used for homogeneous isotropic elastic beam. New ε_{11} is obtained by using this relation.

$$\varepsilon_{11} = \frac{1}{1+\upsilon}\left(-x_3\frac{\partial^2 w}{\partial x_1^2}\right)$$
(5)

We benefit from the principle of virtual displacements for the equation of motion. This is carried out by using extended Hamilton's principle. Where δK denotes the virtual kinetic energy, δU is the virtual potential energy, δV denotes the virtual work done by external forces. The virtual kinetic energy, the virtual potential energy, the virtual work and Hamilton's principle are written by

$$\int_{t_2}^{t_1}(-\delta K + \delta U + \delta V)dt = 0 \quad (6)$$

$$\delta K = \int_0^l \int_A \ \rho \dot{u}_3 \delta \dot{u}_3 \, dA dx_1$$
(7.a)

$$\delta K = \int_0^l \int_A \ \sigma_{11}\delta\varepsilon_{11}\, dA dx_1$$
(7.b)

$$\delta V = -\int_0^l(-kw\delta w)dx_1$$
(7.c)

Where ρ and l are the mass density and the length of the beam, respectively. Where k denotes the foundation stiffness. Using the fundamental lemma of calculus variations, we obtain the equation of motion. At this point, the beam will be assumed to consist of homogeneous, isotropic and elastic material. If we substitute these obtaining expressions into the equations of motion and boundary conditions, equations of motion is obtained in terms of displacement. The small transverse vibration of a beam and lying on an elastic foundation of the Winkler type is governed by the linear fourth order partial differential equation.

$$\rho A\ddot{w} + \frac{EI}{(1+\upsilon)^2}\frac{\partial^2}{\partial x_1^2}\left(\frac{\partial^2 w}{\partial x_1^2}\right) + k(x_1)w = 0$$
(8)

Where the boundary conditions are

$$\delta w: -\frac{EI}{(1+\upsilon)^2}\frac{\partial}{\partial x_1}\left(\frac{\partial^2 w}{\partial x_1^2}\right)$$
(9.a)

$$\frac{\partial \delta w}{\partial x}: \frac{EI}{(1+\upsilon)^2}\left(\frac{\partial^2 w}{\partial x_1^2}\right)$$
(9.b)

Where A and I are the cross-sectional area and the moment of inertia, respectively. E is Young's modulus of the beam material. υ is

Poisson's ratio. t is the time. The dot denotes the derivative according to time.

The soil medium is represented as a system of identical but mutually independent, discrete, elastic springs. The solution is made for three different soil conditions that lying on the beam. Firstly homogenous Winkler foundations stiffness constant throughout the soil. Where k_0 is constant foundation stiffness. Secondly variable Winkler foundation stiffness varies along the soil. γ is coefficient depends on the soil property. The natural frequencies are calculated for three values of γ, namely $\gamma = 0{,}1$. Finally partly Winkler foundation is given. H is the Heaviside step function. A portion of the beam is free, x_0 and the other portion is lying on the soil. The beam is gradually modelled with a spring for different values of x_0. For the homogenous foundation, variable foundation, partly foundation the coefficient are can written as $k = k_0$, $k = k_0\left(4(1-\gamma)\left((x_1^2 - x_1) + 1\right)\right)$, $k = k_0 H(x_1 - x_0)$, respectively.

3. Galerkin Solution Procedure

The equation is solved for the hinged-hinged boundary condition.

$$X = \sin \pi x_1 \quad (10)$$

The change with the time is harmonic due to linear equation of motions. Substituting the approximate solution into the equation of motion, the residual function, $R = (x_1, t)$ is obtained due to the fact that the approximate solution does not exactly provide the equation.

$$w = (x_1, t) \cong \sum_{j=1}^{N} X_j(x_1)\, q_{j e^{i\omega t}} \quad (11)$$

$$R = \frac{1}{1+\upsilon} \sum \rho A X_i^{IV} + (k(x_1) - \omega^2) X_i \quad (12)$$

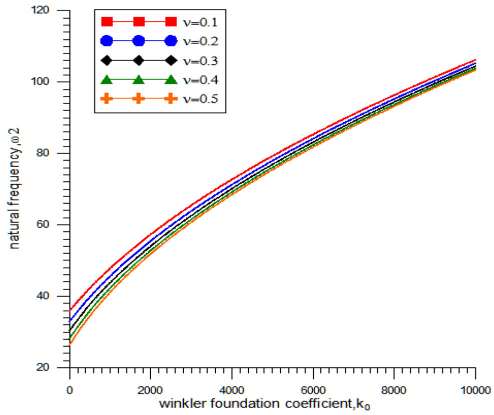
The arbitrary constant q_j in the solution are to be determined after the operation:

$$\int_0^1 R(x_1, t) X_j(x_1) dx_1 = 0 \qquad i = 1,2, \dots, N \quad (13)$$

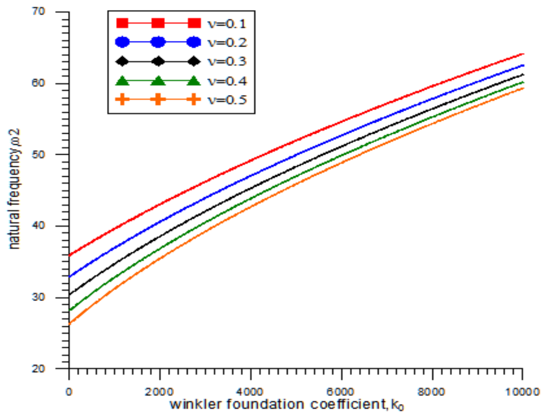
The first and second natural frequency values of the beam is found as results.

4. Numerical Result

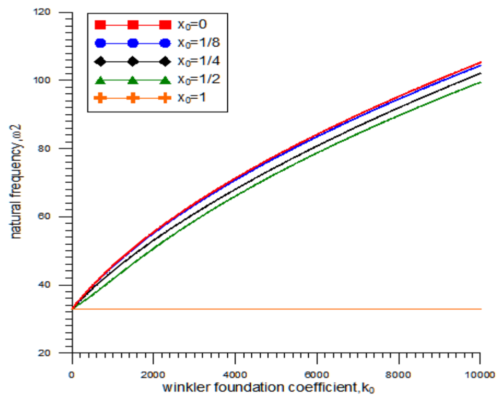
As the foundation stiffness is increase, natural frequency of the beam also increase. This behavior is taken place since the global stiffness of the system is raised. Furthermore, the relation between foundation stiffness and beam frequency seems to be nonlinear at different Poisson's ratio value as depicted in Figure 1. As the Poisson's ratio decrease, natural frequency increase.



$(a)\ \gamma = 1$



$(b)\ \gamma = 0$



(c)

***Figure 1**. The natural frequencies of the beam lying on (a) homogenous (b) variable and (c) partly foundation*

It is remarkable, that for $\gamma = 1$ the homogeneous form of Winkler foundation is obtained. The effect of foundation stiffness on the natural frequency can be more clearly at second natural frequency. So the second natural frequency graphs are given.

5. Conclusion

The standard Euler-Bernoulli beam theory is adjusted and modified to comprise the effect of Poisson's ratio on the mechanical model of the beam. In this study, we investigate the dynamic behavior of the beam lying on the Winkler foundation. The interaction between the soil medium and the beams alter the actual behavior of the beams. In this study, the equation of motion of the beam is derived via adopting the extended Hamilton principle. Three different foundation types which are homogenous, variable and partly founded are considered.

The natural frequency of a beam depends on the foundation types and stiffness. The natural frequencies of the beam is higher than the natural frequency of the same beam without foundation.

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