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Compartmental Models of the COVID-19 Pandemic: A Systematic Literature Review

Deniz YERİNDE[1], Merve ER[2*]

[1]*Marmara University, Institute of Pure and Applied Sciences, Istanbul, Turkey*
[2]*Marmara University, Industrial Engineering Department, Faculty of Engineering, Istanbul, Turkey*

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Abstract

As COVID-19 rapidly spread all around the world, different methods have been proposed to explore the dynamics of the pandemic, understand the transmission mechanism, and assess the preventive measures. Mathematical models are frequently used worldwide to predict various parameters and develop effective policies for disease control. Compartmental models are the most popular mathematical models in epidemiology. These models divide the population into distinct groups (compartments) based on their status and describe the movement of an individual from one compartment to another. Various compartmental models and their variations have been developed to model the pandemic dynamics and measure the efficiency and necessity of different initiatives such as lockdowns, face masks, and vaccination. This paper provides a systematic literature review of different compartmental models proposed to model the COVID-19 pandemic. These models are discussed in detail based on the compartmental structure in the model, the aim of the model, variables, and methodological approaches..

*e-posta: merve.er@marmara.edu.tr

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1 INTRODUCTION

In recent years, the world is facing serious and unprecedented global health problems due to emerging infectious diseases and increased vulnerability of the population. In the last 20 years, people have been encountered with important epidemics such as SARS, H5N1, and H1N1. COVID-19 which spreads among people through connection with the droplets of infected person is among the world's deadliest epidemics [1]. COVID-19 pandemic has affected the health, society, economy, and business in many areas across the globe. After the first COVID-19 case was detected in the world on December, 2019; the total number of cases exceeded 657 million people [2]. During this period, many practices and interventions had been applied to decrease virus transmission and prevent the epidemic, e.g. drugs, isolation, physical distancing, use of masks, closure of schools, restriction of trips, vaccination, and contact tracing. Although some of these applications had positive impact on the prevention of this disease, some of them led to negative consequences in mental and physical health. The impacts and effectiveness of both pharmaceutical and non-pharmaceutical methods are not clear, and variants of the virus have emerged like alfa, beta, gamma, delta, and omicron [3].

Undoubtedly, this epidemic, which has taken the whole world under its effect, has left humanity in a difficult situation in many areas. We are still experiencing social, economic and environmental effects such as home quarantine, being socially alienated, and new post-pandemic trends in education field and business world [4,5]. All of these situations placed pandemic modelling at the forefront of academic research and government decision making. Governments have tried many preventive strategies for overcoming COVID-19. Most of the research in this area focus on fundamental reproduction number of the epidemic and the efficiency of different treatments to decrease virus transmission [6-7].

Many different studies have been conducted to model and forecast the spread of Covid-19 and test the impact of different prevention strategies [8,9]. Compartmental models are commonly used mathematical models in epidemiology [10,11]. These models divide the population into distinct groups (compartments) based on their status and describe the movement of an individual from one compartment to another. Various compartmental models and their variations were developed to model the pandemic dynamics and measure the efficiency and necessity of different initiatives such as lockdowns and vaccination. The most used model is the basic SIR model that consists of three compartments including susceptible, infected, and recovered individuals [12]. Many variations and enhancements of these models have been proposed, e.g., SEIR [13], SEIRD [14], SEIARD [15]. Development of powerful vaccines also provided a critical additional tool for governments to prevent severe illness, hospitalization, and death due to COVID-19. Therefore, vaccine-induced protection was also incorporated to the proposed mathematical models too [16].

Most of the existing literature focus on forecasting the COVID-19 cases. Some researchers used time series analysis and forecasting to predict the number of new cases and deaths, e.g., ARIMA model and machine learning models such as Prophet [17,18]. These models try to predict future events based on past data. Grey prediction models are also employed to forecast new cases, deaths, and recovered individuals [19,20]. Grey models produce very satisfactory results in solving problems with poor data, hence, they are useful for forecasting new coronavirus infections with small data in the short-term. Artificial Intelligence, machine learning, and deep learning techniques are also used at different stages of pandemic management including fast discovery of the disease, clinical prognosis, predicting COVID-19 cases, foreseeing serious cases based on patient data, assessment of CT scans, and disease tracking [21,22].

Understanding the disease dynamics and their interaction is vital to control the pandemic. Especially, compartmental models are the most extensively used structures in mathematical modelling of infectious diseases due to their success in formulating disease transmission, adaptivity, and diversity in application. Kong et al. [23] presented a scoping review on compartmental models expanded based on SEIR model. There is a need for an up-to-date systematic review that examine different compartmental models with a comprehensive structure by considering the compartmental structure, target problem, model variables, and utilised methodological approaches. Compartmental models provide a very general approach and scheme that may be integrated into many other methods. Different solution methodologies may be based on compartmental structures. As an example, Duan and Nie [24] developed a grey prediction model based on the well-known SIR compartmental model. Zisad et al. [25] integrated Neural Network and SEIR compartmental model to predict the number of confirmed COVID-19 cases. Therefore, the scope of the study was limited to the use of epidemiological compartmental models. The aim of this paper is to provide a systematic literature review on the compartmental models that are proposed to model the COVID-19 pandemic. Definitions, properties, compartmental structures and differences of these models are explained in detail. The authors expect this comprehensive review will be a basis for building new models and facilitate further research in COVID-19 related literature.

The remainder of the paper is as follows. Section 2 provides a background on pandemic modelling. Section 3 summarizes the research methodology. Section 4 gives the results of the literature review and discusses the current status of the area. Finally, Section 5 concludes the paper.

2 BACKGROUND

The world has faced different infectious disease outbreaks, epidemics, and pandemics throughout the history. Although the academic literature includes various studies in the modelling of infectious diseases, there has been a substantial growth in the academic literature after the global COVID-19 pandemic [26-28]. Compartmental models are commonly used in mathematical modelling of infectious disease transmission [28,29]. These models divide the population into different groups and call it "compartments". These epidemiological models compute the theoretical number of people for each mutually exclusive compartments within a closed population over time. The most basic compartmental model in this area is the SIR (Susceptible – Infected – Recovered) model which forms the cornerstone in the study of infectious disease dynamics [30]. Kermack and McKedrick [31] modelled the plague disease for the first time in 1927 using the SIR model. “Susceptible” refers healthy individuals who have not yet been diagnosed with the disease and are open to catching it. “Infected” refers to infected/sick individuals. “Recovered” refers to individuals who recovered or died due to the disease.

The aim of compartmental mathematical models is to identify mathematical equations that describe disease transmission and flow of individuals between different epidemiological states over time. Each state is treated as a different compartment. These states are dependent on the disease under study. In the basic SIR model, population size is considered to be constant. Dynamics of the system is represented by a set of differential equations. In the SIR model, the population is sum of the susceptible, infected, and recovered people. The population in these three compartments change over time as the virus spreads. As more individuals become infected, S decreases and I increases. In the I compartment, either the people from the S group who get sick come and the people who recover leave. It is assumed that people who recover will become immune and will not be re-infected. Transitions between groups are determined using differential equations. The beginning and progress of the epidemic can be explained as in the following [32,33]:

- Before the start of the disease epidemic, S is equal to the whole population as no one is immune, and the number of people in groups I and R is 0 because other diseases are not considered.
- It is assumed that only 1 individual is infected at the beginning of the epidemic, and as soon as the first individual is infected, S decreases by 1 and I increases by 1.
- This first contagious individual infects other vulnerable people (before they recover or die).
- The epidemic continues dynamically as recently infected individuals infect other vulnerable people.
- As people recover from the disease or die, the number of people in the R group increases over time.

When modelling the dynamics of the epidemic, three different equations are needed to describe the rates of change in each group [32]:

$$S(t) = -\frac{\beta I(t) S(t)}{N} \tag{1}$$

$$I(t) = \frac{\beta I(t) S(t)}{N} - \gamma I(t) \tag{2}$$

$$R(t) = \gamma I(t) \tag{3}$$

where N is the total number of people in all groups (S + I + R = N), β is the infection rate controlling the transition between S and I, γ represents the rate of removal or recovery that controls the transition between I and R. R0, which is the basic reproduction number shows how many people an infected person will infect. R0 is calculated by β/γ. When R0 >1, virus will continue to spread among susceptible people, if there are no changes in the environment or external effects. When R0 <1, virus will disappear inevitably.

There are different variants and extensions of the SIR model. SI model assumes that patients who recover can become infected again. Dead factor is included in the SIRD model [34-38]. SEIR model considers the incubation period of the disease [39-41]. SEIR model emerged to separate those who were exposed to the virus but did not show symptoms [42-44]. Some people have innated or vaccine immunity against certain diseases, and there is a MSIR model that expresses them with M. Over time, new compartmental models have been developed by considering various disease dynamics and additional compartments such as exposed, dead cases, maternally derived immunity, and insusceptible. Other example models include SEI [45], SIRS [46,47], MSEIR [48], SEIRV [49], SEIJR [50], SEIHR [51], SEIRD [14,29], SEIARD [15], SEIAQRDT [52], SEIQRDP [53] and SPEIQRD [54,55].

Governments applied different strategies to fight with the COVID-19 pandemic including lockdowns, wearing mask, online education, etc. Developed compartmental models give the opportunity to simulate the effects of quarantine and other government strategies. Vaccines are very effective at preventing hospitalization, severe disease, and death from COVID-19. Hence, they are frequently included in mathematical models. For instance, Poonia et al. [56] developed three SEIR models considering three different scenarios; no social distancing, social distancing and social distancing with vaccination. Reproduction number is 2.16 without social distancing, and 1.3 with social distancing. Indeed, social distancing and vaccination has an important role to surpass COVID-19. In addition, Rajapaksha et al. [57] studied SEIRV model and simulated vaccine strategies. According to their results, vaccination rate should be higher than %45 to reduce infected people as soon as possible.

3 REVIEW OF LITERATURE ON COMPARTMENTAL MODELS USED IN MODELLING COVID-19

3.1 Search procedure

The aim of the study is to provide a systematic literature review on different compartmental models proposed to model the Covid-19 pandemic. Article search is performed in the Scopus database. First, articles that use official COVID-19 term and related terms (SARS-COV-2, Coronavirus) in article title, abstract and/or keyword list are screened. Totally 476,465 publications were found. Then, the publications that use the “compartmental model” terms or specific compartmental model names in article title, abstract and keyword were eliminated, which results in 2,689 publications. The following compartmental model terms were used in the elimination: “SEIR Model, SE(Is)(Ih)(Iicu)AR Epidemic Model, SEAIRD, SEAIRPD-qSEAIRPD-Q Model, SEI, SEIHR Model, SEIQHRD Model, SEIQRDP, SEIR, SEIR Epidemic Models, SEIR Models, SEIR Stochastic Model, SEIRD, SEIRD-model, SEIRV Model, SIDARTHE, SIDARTHE Model, SIER Model, SIR, SIR And Integro-differential Models, SIR And SEIQR Models, SIR Epidemic Model, SIR Epidemic Models, SIR Estimation, SIR Modeling, SIR Models, SIR-F, SIRD, SIRD Epidemiological Model, SIRDC Model, SIS Model, SQIR Model”.

Then, source type was selected as journal, document type was selected as article, and language was selected as English. This elimination resulted in 2,040 articles. Records that are irrelevant with the subject of this article were excluded and qualified publishers were selected for the review which are Elsevier, Springer, Institute of Electrical and Electronics Engineers (IEEE), John Wiley and Sons, Emerald, and Taylor and Francis. Finally, 150 articles were identified for full paper review. Distribution of the articles by subject area are as follows: 32% Mathematics, 25% Computer Science, 23% Engineering, 9% Decision Sciences, 6% Economics, Econometrics and Finance, and 5% Business, Management and Accounting.

3.2 Keywords, authors, and funding sponsors of reviewed papers

COVID-19 has affected all segments of the world population and national economies across the globe. Therefore, relevant literature includes many different publications studying the topic from different perspectives. The scope of this review was limited to the use of compartmental models for COVID-19. These models may be used to analyse different epidemiological and behavioural dynamics and patterns (e.g., transmission behaviour, lock-down policies). Compartmental modelling schemes may also be integrated to different methodologies such as Artificial Intelligence tools [58], Monte Carlo methods [59] etc. Therefore, the keyword map in Figure 1 is developed to visualize the keywords that are used in the reviewed papers with their frequency. Vosviewer software is used to prepare this keyword map.

Table 1 shows the most active countries in the reviewed articles based on the total number of publications and total count of citations. As it is seen from the table, United States, China, and India are the main countries contributing to this research field.

There were some sponsors in the reviewed papers. Figure 2 gives the funding sponsors based on the total number of supported studies.

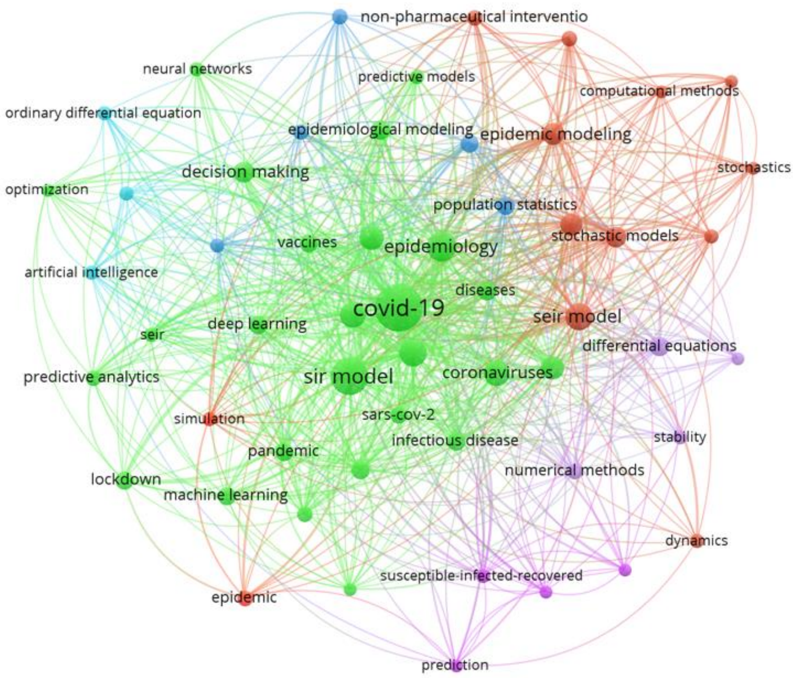

Figure 1. Keyword map of the reviewed articles

Table 1. Top ten countries based on total number of publications and citations

Countries	Number of publications	Count of citations
United States	36	468
China	26	384
India	18	203
Saudi Arabia	13	128
Canada	10	111
France	10	35
Taiwan	9	110
Brazil	8	44
United Kingdom	7	129
Germany	7	37

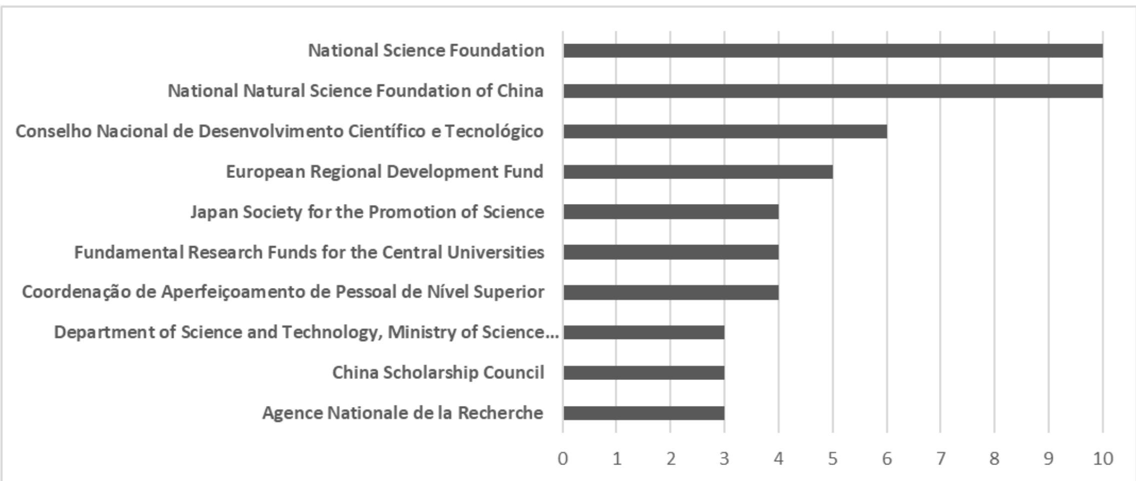

Figure 2. Funding sponsors based on the total number of supported studies

3.3 Different types of compartmental models

Compartmental models describe the system under study by decomposing the population into a number of interacting subsystems (components). Different kinds of models have been developed by varying the compartments, using different number of compartments, and developing different equations to describe the interaction between different compartments [60]. One of the most basic compartmental models is the SIR model that divides the population into three compartments; susceptible (S), infectious (I), and recovered (R) [61]. The structure and dynamics of the COVID-19 pandemic were changed as different variants of the virus emerged, symptoms changed, and new treatment protocols, medications, and vaccinations were emerged. This has resulted in a need to use new and different compartments in epidemiological models of COVID-19. Different compartmental models have been developed for COVID-19 to adapt to these changing environments and provide more accurate epidemiological models. Researchers included different compartments considering the status of the individual such as quarantined, hospitalized, asymptomatic, vaccinated, etc. Different compartments used in the reviewed papers to model the COVID-19 pandemic are summarised in Appendix. All of the models need to have sub-groups representing susceptible and infected individuals. The compartmental structure may change based on the goal of the analysis. As an example; "Quarantined" or "Vaccinated" compartments are used to measure the impacts of preventive policies [62]. Also, asymptomatic cases play a critical role in increasing the transmission rate of the virus. For this reason, compartmental models are further subdivided to "Asymptomatic" [63]. Some researchers also used different words to refer to similar cases: e.g., healed [7] & recovered [52].

Table 2. Categorization of the reviewed articles based on target problem type

Problem Category	Number of articles
Examining the effects of different factors (health system, immigration rate, contact distance, vaccination, etc.) on the COVID-19 disease	62
Prediction of the total number of COVID-19 cases / pandemic trend	61
Examining the effects of different isolation measures and identifying the best response strategy	9
Examining different control measure scenarios	5
Modelling public opinion	2
Analysis of information dissemination of social networks and understanding information propagation as an outbreak of disease	1
Analysing the worker and firm behaviour changes in the presence of virus	1
Assess the macroeconomic impact of the COVID-19	1
Classifying COVID-19 cases using demographic features: Sex, Patient Age, Temperature, Finding and Clinical Trials and SIR model	1
Developing an extended SIR model with agents optimally choosing outdoor activities	1
Formulating an efficient social distancing policy based on economic situations, transmission, and mobility control	1
Identifying policies that maximize social welfare considering economic activity during infectious disease epidemics	1
Making county- or city level estimates on the number of new infections, hospitality rates and demands on intensive care units	1
Predicting COVID-19 dynamics upon partial and total airport reopening with perfect and imperfect quarantine conditions and decision making for reopening of airport	1
Quantitative modelling of an economy hit by pandemic	1
Transmissibility estimation	1
TOTAL	150

3.4 Problems solved by compartmental models

Main aim of developing compartmental models for COVID-19 pandemic is to understand the dynamics and transmission patterns of the infectious disease. These models help us to understand how the disease spreads, predict regional peaks, estimate next waves, and understand the impact of various initiatives to control the pandemic. Studied problem type and aim of the reviewed articles are identified and their frequency is given in Table 2.

Most of the compartmental models in the literature focus on examining the effects of different factors (health system capacity, immigration, vaccination, social distancing, wearing mask, etc.) on the COVID-19 pandemic [64,65] Secondly, a high percentage of the articles aim to predict the total number of COVID-19 positive cases over time [66,67]. As it is seen from Table 2, there are also some other papers that focus on more specific problems such as classification of COVID-19 cases based on demographic features [68], modelling public opinion [69], assessment of the macroeconomic impact of COVID-19 [65], etc.

3.5 Different methods used in reviewed articles

Compartmental model is a very general modelling approach that is frequently used in mathematical modelling of infectious diseases. Publications combine and integrate compartmental models with other techniques to achieve their goals. The most frequently used technique is simulation [59,70]. Pandemic modelling includes many unknown parameters such as incubation period, transmission rate, etc. Simulation provides the opportunity to analyze different alternative scenarios. Different compartments are used in these simulation models to investigate the behavior of the entire population. Even, epidemic simulators were developed by researchers [71].

Some of the researchers strengthened their model by incorporating compartmental approach with machine learning and deep learning techniques [41,72], Artificial Neural Network [37], Genetic Algorithm [73], and regression [45]. They tried to obtain insights on key uncertainty drivers, predict the pandemic trend, and solve complex disease control optimization problems. Furthermore, artificial intelligence is used for diagnosis of the disease by using sensor data for temperature and thermographic temperature. AI based detector helps to reduce the mistaken alarm [58]. Other methods that are integrated with the compartmental modelling approach in the COVID-19 literature can be listed as follows: agent-based modelling [74], big data analytics [75], fuzzy set algorithm [76], Markov process [77], and system dynamics approaches [78]. Decision support systems were also developed for clinical decisions, allocation of resources, lock-down management, and optimization of response strategies [79].

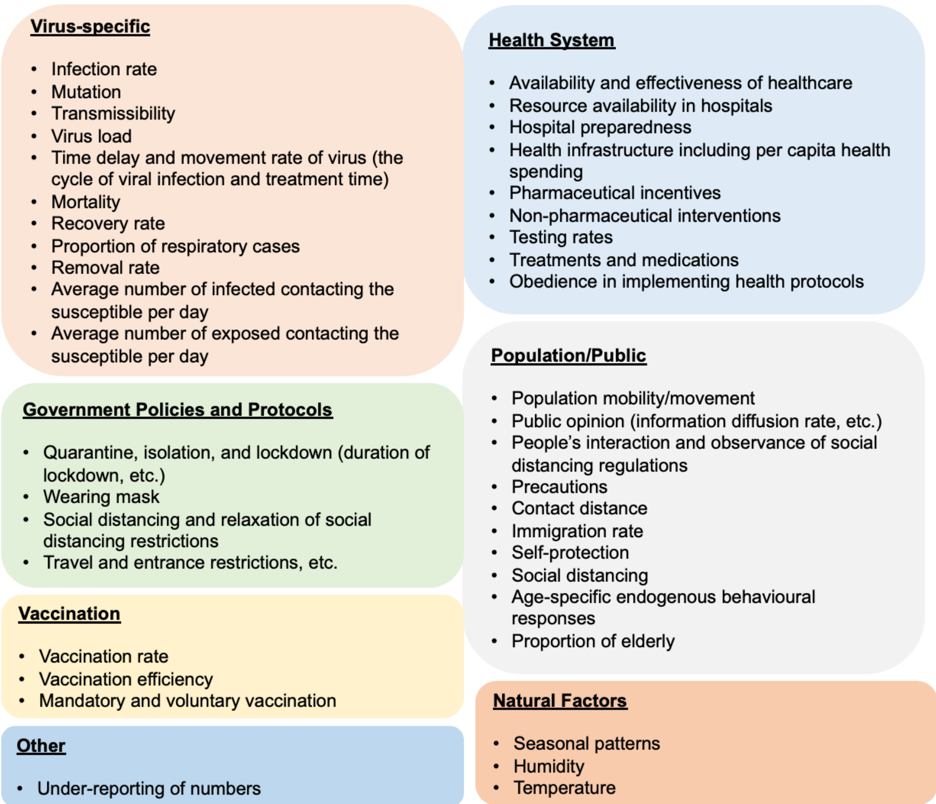

Figure 3. Covid-19 related factors considered in the reviewed articles

3.6 Different variables used in reviewed articles

Various mathematical models of COVID-19 have been proposed to understand the dynamics of the pandemic [80,81]. Various factors have been included in these models to predict transmission dynamics accurately and assess the combined effects on the overall system. Complex mathematical models were developed to provide insightful information by incorporating different variables and parameters that have an impact on the transmission, growth, and intensity of the pandemic. Estimating and studying the dynamics of the pandemic is critical to identify effective health policies to fight with this disease. Researchers included different aspects of the pandemic in their mathematical models to capture additional insights about the spread of the disease at different stages of the pandemic [82,83]. Kong et al. [23] emphasized the compartments based on virus characteristics and interventions, age groups and public health interventions. Figure 3 summarizes COVID-19 disease related factors and variables that are considered in the reviewed papers. These factors are categorized under seven main headings which are virus-specific [77], government policies and protocols [65], vaccination [84], health system [65], population/public [69], natural factors [77], and other (under-reporting of numbers [85]).

4 CONCLUSION

The rapid spread of the coronavirus pandemic led to a dramatic loss of human life, and seriously disrupted societies and economies across all countries. Many incentives were made by governments to fight against the virus, e.g., wearing mask, social distancing, quarantine, lock downs, school closures, and travel restrictions. National health systems, even in the most developed countries, suffered from this health crisis. Researchers worldwide published many papers on numerous aspects of the disease such as causes, symptoms, spread, consequences, preventive measures, etc. Compartmental epidemiological models such as SIR and SEIR have a critical importance in understanding the behaviour of the pandemic and studying different scenarios to identify the best pandemic policies. This paper reviewed and summarised different compartmental models used in COVID-19 modelling. The contribution of this work is to provide a detailed analysis on different compartmental structures used in the existing compartmental epidemiological models, pandemic dynamics used in mathematical equations, addressed problem types, and used methodologies in the reviewed papers.

Scientists warn us about the risk of future pandemic. Hence, lessons learned from COVID-19 pandemic are of vital importance. Especially insights gathered from compartmental mathematical models are very helpful in making decisions about forecasting progress of the pandemic, resource allocation, pandemic planning, effects of government and health policies. However, some difficulties and limitations exist for compartmental epidemiological models. Available epidemiological data may not be enough to provide an accurate estimation of model parameters. Automated and dynamic parameter estimation approaches (instead of fixed value parameter) utilizing advanced statistical methods help to overcome this limitation. Secondly, degree of public awareness, public panic, and government attention may be difficult to model. Thirdly, some compartments such as asymptomatic and unreported cases may be more difficult to quantify. Despite the difficulties, compartmental models provide valuable, acceptable and reliable results to describe pandemics, estimate the impacts of interventions, and assist policy making. Especially, previous researches show that integration of different methods (e.g., Artificial Neural Network) with compartmental models strengthen the quality of the results significantly. Compartmental models are also highly adaptive and can easily be modified for a future pandemic similar to COVID-19.

This study provides an informative summary of the compartmental structures used in modelling the COVID-19 pandemic. A systematic review was performed to examine the articles that has been released over a short time period in the rapidly growing subject area. Existing compartmental models were consolidated to provide a useful guidance for future research addressing the design of compartmental epidemic models. However, the study does not cover the entire field as the literature search is restricted with some qualified publisher. Therefore, although basis compartmental model types are covered in the review, there may be some other modified compartmental models which are not included this article.

For future research, this study may be enhanced by investigating different approaches and methodologies integrated with compartmental models in the literature. Another interesting possible future research direction may be applying a compartmental model of COVID-19 to compare the variability of the progress of epidemic in term of shape and peak at various countries with different population structure and national health system. Model parameters may be adjusted based on local parameters and population age structure may be included in the model.

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Author Contributions
Deniz YERİNDE: Conceptualization, Investigation, Resources, Writing-Original Draft, Visualization
Merve ER: Conceptualization, Methodology, Writing-Original Draft, Writing – Review & Editing, Supervision

All authors read and approved the final manuscript.

Conflict of Interest
No conflict of interest was declared by the authors.

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Appendix. *Different Compartmental Models Used for Modelling COVID-19*

Compartmental Model	Reference	Susceptible	Exposed	Infected	Removed	Asymptomatic	Death (Extinct)	Quarantined	Insusceptible	Recovered	Hospitalized	Vaccinated	Diagnosed	Ailing	Recognized	Threatened	Healed	Resolving
SEAIRD	[15]	X	X	X	X	X	X											
SEI	[45]	X	X	X														
SEIAQRDT	[52]	X	X	X		X	X	X	X	X								
SEIHR	[51]	X	X	X	X						X							
SEIQRDP	[53]	X	X	X			X	X	X	X								
SEIR	[86]	X	X	X						X								
SEIRD	[29]	X	X	X			X			X								
SEIRV	[49]	X	X	X	X							X						
SIDARTHE	[7]	X		X			X						X	X	X	X	X	
SIR	[33]	X		X						X								
SIRD	[36]	X		X			X			X								
SIRDC	[87]	X		X			X			X								X
SIRS	[37]	X		X	X													