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A NEW HYBRID GENETIC ALGORITHM FOR VERTEX COVER PROBLEM

ABSTRACT

The minimum vertex cover problem belongs to the class of NP-complete graph theoretical problems. This paper presents a hybrid genetic algorithm to solve minimum vertex cover problem. In this paper, it has been shown that when local optimization technique is added to genetic algorithm to form hybrid genetic algorithm, it gives more quality solution than simple genetic algorithm. Also, a new mutation operator has been developed especially for minimum vertex cover problem, which converges faster to the global optimal solution. The new hybrid genetic algorithm has been compared with the previous works. The experimental results have shown that the proposed algorithm can yield quality solutions in reasonable times.

Keywords: Vertex cover problem, Hybrid algorithms, Genetic algorithms, Local optimization techniques

TEPE ÖRTÜSÜ PROBLEMİ İÇİN YENİ BİR HİBRİD GENETİK ALGORİTMA

ÖZ

Minimum tepe örtüsü problemi, NP-Tam sınıfına ait teorik bir graf problemidir. Bu makale minimum tepe örtüsü problemini çözmek için yeni bir hibrid algoritma sunar. Bu makalede genetik algoritmaya yerel optimizasyon tekniğini eklenince basit genetik algoritmadan daha kaliteli sonuçlar elde edildiği gösterilmiştir. Ayrıca özellikle minimum tepe örtüsü problemi için, genel en iyi sonuca daha hızlı yakınsayan yeni bir mutasyon operatörü geliştirilmiştir. Yeni hibrid algoritma önceki çalışmalarla karşılaştırılmıştır. Hesaplama sonuçları algoritmanın makul sürelerde kaliteli sonuçlar verebileceğini göstermektedir

Anahtar Kelimeler: Tepe örtüsü problemi, Hibrid algoritmalar, Genetik algoritmalar, Yerel optimizasyon teknikleri

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1.INTRODUCTION

A vertex cover of an undirected graph G=(V,E) is a subset $V^* \subseteq V$ such that if $(u,v) \in E$, then $u \in V^*$ or $v \in V^*$ (or both). That is, each vertex "covers" its adjacency edges, and a vertex cover for G is a set of vertices that covers all the edges in E. The size of a vertex cover is the number of vertices in it (Cormen et al, 2001). According to the NP-completeness theory, this problem cannot be solved in polynomial time unless P=NP. Examples of the areas where the minimum vertex-cover problem occurs in the real world applications are communications, civil and electrical engineering, and bioinformatics. In this study, a genetic algorithm is used for solving minimum vertex cover with a local optimization technique.

In solving the minimum vertex cover problem, we also have a solution for two other graph problems: The maximum independent problem and the maximal clique problem (Garey and Johnson, 1979).

An independent set in a graph G=(V,E) is a subset $V' \subseteq V$ such that, for all u, v $\in V'$, the edge (u,v) is not in E (Cormen et al, 2001). The maximum independent set problem calls for finding the independent set of maximum cardinality.

The clique in an undirected graph G=(V,E) and subset is $V^* \subseteq V$, a subset of vertices, each pair of which is connected by an edge in E. In other words, a clique is a complete subgraph of G (Cormen et al, 2001).

The following relationship independent sets, cliques, and vertex covers are easy to verify (Garey and Johnson, 1979).
Lemma: For any graph G=(V,E) and subset $V^* \subseteq V$, the following statements are equivalent:

V* is a vertex cover for G.
V-V* is an independent set for G.
V-V* is a clique in the complement G^C of G, where $G^C = (V,E^C)$ with $E^C = \{(u,v): u, v \in V \text{ and } (u,v) \in E\}$.

Consequently, one can obtain a solution of the minimum vertex cover problem by taking the complement of the solution to the maximum independent set problem. A solution to the maximum clique problem is obtained by applying the maximum independent set heuristic to $G^C=(V,E^C)$ (Garey and Johnson, 1979).

As the importance of the minimum vertex cover problem and its complexity, many researchers have instead focused their attention on the development of heuristic, metaheuristic and approximation for delivering quality solutions in a reasonable time. One of the first approximation algorithms in the literature for the Unweighted Vertex Cover problem belongs to Gavril (1976). Clarkson has modified the greedy algorithm for the vertex cover problem (1983). Garey and Johnson (1979) have presented a simple approximation algorithm based on maximal matching that give an approximation ratio of 2 for the general graphs. Recently, Ashay Dharwadker (2011) has presented a new polynomial-time algorithm to find minimal vertex covers in graphs. Khuri and Back (1994) have presented an evolutionary heuristic for the minimum vertex cover problem. For a comprehensive survey on the algorithms for MVC, the reader is referred to Monien and Speckenmeyer (1985) and Pullan (2009).

In this paper, a new hybrid genetic algorithm called HGA is presented to find the minimum vertex cover of the graph. The proposed algorithm has been tested on the DIMACS benchmark instances and compared with other existing algorithms. The paper is organized as follows: Section II briefly describes the genetic algorithms. Section III outlines the proposed algorithm. In Section VI, computational efficiency of the proposed algorithm is discussed, and the proposed algorithm is compared with two other algorithms on the DIMACS benchmark instances. Section V summarizes and concludes the paper.

2. GENETIC ALGORITHMS

GA is an optimization technique based on the natural evolution. It maintains a population of strings, called chromosomes that encode candidate solutions to a problem. The algorithm selects some parent chromosomes from the population set according to their fitness value (Goldberg, 1989), which are calculated using fitness function. The fittest chromosomes have more chances of selection for genetic operations in the next generation. Different types of genetic operators are applied to the selected parent chromosomes; possibly according to the probability of operator, and next generation population set is produced. In every generation, a new set of artificial creatures is created using bits and pieces of the fittest chromosomes of the old population (Mitcell, 2002).

Although GA is probabilistic, in most cases it produces better population compared to their parent population because selected parents are the fittest among the whole population set, and the worse chromosomes die off in successive generations. This procedure is continued until some user defined termination criteria are satisfied. The basic steps of a genetic algorithm as follows:

```
Algorithm GA:
begin
   Initialize Population;
   Generation = 0;
   repeat
      Generation = generation+1;
      Selection(Population);
      Crossover(Population)
      Mutate(Population);
   until Termination_Criterion;
end.
```

Reproduction, crossover (recombination), mutation are widely used genetic operators in GA. To achieve quality solutions, these operators must adjust according to the problem. The important weakness of the genetic algorithm is may has a tendency to converge towards local optima. Mutation is the crucial part of genetic algorithm to avoid local optima. In this study, a heuristic mutation method is designed for the minimum vertex cover problem.

3. THE NEW HYBRID GENETIC ALGORITHM

Hybrid genetic algorithms have received significant interest in recent years and are being increasingly used to solve real-world problems. A genetic algorithm is able to incorporate other techniques within its framework to produce a hybrid that reaps the best from the combination (El-Mihoub et al., 2006). Recently, Kotecha and Gambhava (2003) have proposed a Hybrid Genetic Algorithm for minimum vertex cover problem which uses *Procedure1* as a local optimization technique, and they also have proposed a new crossover operator (heuristic vertex crossover called HVX) especially for minimum vertex cover problem.

In the proposed algorithm, we have combined a new local optimization with genetic algorithm to achieve global optimal solution consistently and also we have developed a new heuristic mutation operator.

3.1. Local Optimization Technique

The idea of combining genetic algorithm and local optimization technique has been investigated extensively during the past decade. Kotecha and Gambhava (2003) have used the *Procedure1*to remove the unnecessary vertex from the solution. But when the solution converges towards the local optima, it is almost imposibble to find an unnecessary vertex. To avoid the local optima, we have used the *Procedure2. Procedure2* provides the population diversity by making small moves arond the solution neighbours. For minimum vertex cover problem, we have proposed the following procedures.

Procedure1: If a vertex *v* and its all neighbor in the solution, remove the vertex *v* from the solution. Repeat this procedure till the solution has no removable vertices.

Procedure2: If a vertex *v* is in the solution and its all neighbor in the solution except only one vertex *w*, then define *v* as can be slide vertex and remove the vertex *v* from solution and add the vertex *w* to the solution. Perform the *Procedure1*.

3.2. The New Heuristic Mutation

After the crossover operation, local optimization technique carries out to all solution. Because of the *Procedure1,* if any vertex remove from a solution with mutation, then the solution cannot be feasible. Thus, the proposed mutation only adds vertex or vertices to solution. Also, mutate number of vertices changing in the program if the best solution cannot progress during a certain amount of generation. For each vertex, the number of how many times it is in the best solutions which come from the previous generation with elitism is calculated. Then, all vertices calculated values ascending sort. The fewer vertex has been used, the more the vertex has change for mutation. At first sight, it seems a destructive effect on solution, local optimization technique will make the solution better. Thus, instead of being stuck in a certain set of vertices, almost all vertices will be tested in solutions by generations, and the risk of stuck local minima is minimized.

4. EXPERIMENTAL RESULTS

This section presents results of computational experiments for the proposed hybrid algorithm. All the procedures of HGA have been coded in C++ language. The experiments have been carried out an Intel Pentium Core2 Duo 2.6 GHz CPU and 2GB of RAM. The algorithm has been tested on the complement graphs of DIMACS clique instances. The heuristic has been compared with The CliqMerge Method (C-Merge) (Balas and Niehaus, 1996) and Optimized Crossover (OCH) (Aggarwal and Orlin 1997). The results for C-Merge and OCH was taken from (Aggarwal and Orlin 1997). The comparison of the algorithm with previous work is summarized in table I.

For experiment, we have used population size $\mu \leq 100$ and have iterated for only 10 generations, because by means of the local optimization technique and the new heuristic mutation, the proposed algorithm converges faster to global optimal solution. The elitizm rate have been setted to %25 and the remaining %75 of the population have been reproduced in every generation by one point crossover. After the crossover, the new heuristic mutation operator have been applied to all solutions of the population. The local optimization techiques have been applied after the mutation operator. For each of these problem, a total of 10 runs of the algorithm is performed. The results are summarized in table I for the best results that were encountered during the 10 runs for each instance.

Balas and Niehaus 1996 have used the method of optimal merging (optimized crossover) in order to come up with a heuristic for the independent set problem. This heuristic enumerates many more sets of parents than normallyand accordingly has a much greater running time. The OCH has been proposed by Charu et al. This is a genetic algorithm for the maximum independent set problem. The centerpiece of the method is the so-called optimized crossover, which generates one optimal offspring based on the same merging procedure of Cliqmerge and one other having a random character. This method is faster than C-Merge.

In table I, the first three columns represent the name of the instance, number of its vertices and cardinality of vertex cover, respectively. This information is available from the DIMACS web site. Following three columns denote C-Merge, OCH and the proposed algorithm HGA. Finally, the last three columns report the computational times for C-Merge, OCH and HGA in seconds, correspondingly.

Table 1. Computational Experiments

Problem Name	N	Opt	C-Merge	OCH	HGA	$T_{C\text{-}Merge}$	T_{OCH}	T_{HGA}
brock200-2	200	188	189	189	188	9	0,96	1,42
brock200-4	200	183	184	184	184	17	0,80	1,32
brock400-2	400	371	375	376	375	115	8,25	5,71
brock400-4	400	367	375	376	375	110	5,11	5,31
brock800-2	800	776	779	781	779	286	15,31	25,79
brock800-4	800	774	779	781	779	282	33,01	26,28
keller4	171	160	160	160	160	9	0,59	1,14
keller5	776	749	749	752	749	625	38,40	37,25
MANN-a27	378	252	252	252	252	2422	8,19	16,46
MANN-a45	1035	690	691	692	693	4119	110,36	73,64

As one can see, HGA and C-Merge performed better than OCH nearly for all instances. The proposed algorithm HGA found the optimal solution 4 times whereas C-Merge found 3 times and OCH found only 2 times. Even though HGA's computational times were calculated in more advanced computer, the computational times can give idea about the effectiveness of the algorithms. OCH is much faster than both C-Merge and HGA but it did not find quality solution as much as HGA and C-Merge. HGA and C-Merge found nearly similar solution but HGA can find the solution much faster than C-Merge. Consequently, table I shows than HGA is more effective than C-Merge and OCH.

5. CONCLUSIONS

Minimum vertex cover problem is a classic graph optimization problem that is NP-complete even to approximate well. In this paper, a new hybrid genetic algorithm for minimum vertex cover problem has been proposed. We have used a new local optimization technique and a new heuristic mutation operator in order to improve the performance of the algorithm. The algorithm has been implemented in C++ language. The performance of algorithm tests on DIMACS clique instances by comparing with the previous works. The experimental results have shown that although OCH is much faster than our Hybrid Genetic Algorithm (HGA) and C-Merge, it did not find quality solution as much as HGA and C-Merge. C-Merge found quality solution similar to HGA, but it's computational times is not reasonable. The results have shown that HGA is more effective than C-Merge and OCH.

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