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Comparison of Small Scale Effect Theories for Buckling Analysis of Nanobeams

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Abstract

Theories which consider small scale effect have a great importance on analysis in micro and nano scale. In present paper, three kind of nanotubes (Carbon Nanotube (CNT), Boron Nitride Nanotube (BNNT), and Silicon Carbide Nanotube (SiCNT)) are analyzed in case of buckling on two parameters elastic foundation. Three different small scale theories (Nonlocal Elasticity Theory (NET), Surface Elasticity Theory (SET), and Nonlocal Surface Elasticity Theory (NET&SET)) are applied to calculate the buckling loads. Also Classical Euler-Bernoulli Beam Theory (CT) is used to see the effect of small scale effective theories. Comparative results are given for simply supported nanotubes in figures.

Keywords: CNT, BNNT, SiCNT, Nonlocal Elasticity, Surface Elasticity, Nonlocal Surface Elasticity.

1. Introduction

Nanotubes have a gigantic using area in nanotechnology based devices. Due to their superior mechanical properties, different kind of nanotubes are being used in many area such as space technology, nanosensors, nanoactuators, biotechnology etc . Carbon nanotube is the most used nanotube type since its discovery in 1991 by Iijima [1]. Carbon nanotubes consist of six Carbon (C) atoms bonded to each other in hexagonal shape. Carbon nanotubes attracted much attention due to its very high mechanical properties such as Young's modulus which is equal to 1 TPa [2-5]. As technology always needs better material with better properties, scientists have developed a new type of nanotube which is much stronger than CNT called BNNT. BNNT's material properties look better than CNT on paper with Young's modulus equal to 1.8 TPa while the cost of BNNT is much higher than CNT [5-10]. Another kind of nanotube is boron nitride nanotube. Due to its superior mechanical strength, BNNT have been researched and used widely [8, 11, 12]. Their limited thermal resistance pushed researchers to develop a new nanomaterial. Silicon carbide nanotube can stay stable until 1000^{o}C where Carbon nanotube and Boron Nitride nanotube can stay stable only until 600^{o}C [13].

2. Nanotubes

In present paper nanotubes are modeled as resting on double parameter foundation. To model double parameter foundation Winkler and Pasternak foundation models are used. Foundations parameter of Winkler and Pasternak foundations are k_w and k_p respectively. As it can be seen from Fig. 1, nanotubes are obtained, basically, by rolling over the long side of graphene or silicene sheets.

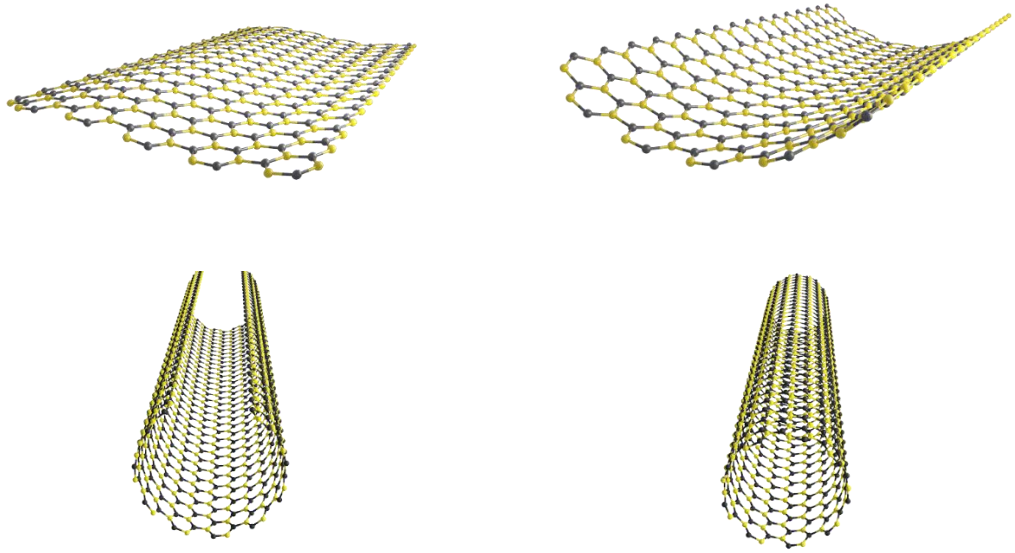

Fig. 1. Obtaining nanotubes

Nanotubes can be obtained in three form. These forms are determined by the rolling angle of graphene or silicene sheet. To illustrate, silicene sheet and three forms of nanotubes are demonstrated in Fig. 2. The red marked lines in silicene structure shows clearly the difference between rolling angles. As it can be seen from the top marked line in red at silicene, the *armchair* structure can be obtained by rolling the silicene sheet with zero degree. Likewise, *zigzag* nanotube structure can be obtained by a rolling of silicene sheet with 45^o. Furthermore, any rolling with any other angle will be called as *chiral*. For example, SiCNT can be obtained in armchair, zigzag, and chiral form [14]. Each of these three types of SiCNT have different Young's modulus. Three types of SiCNT's are demonstrated in Fig. 2. Furthermore, as it can be seen, three types of nanotube structures look different from each other.

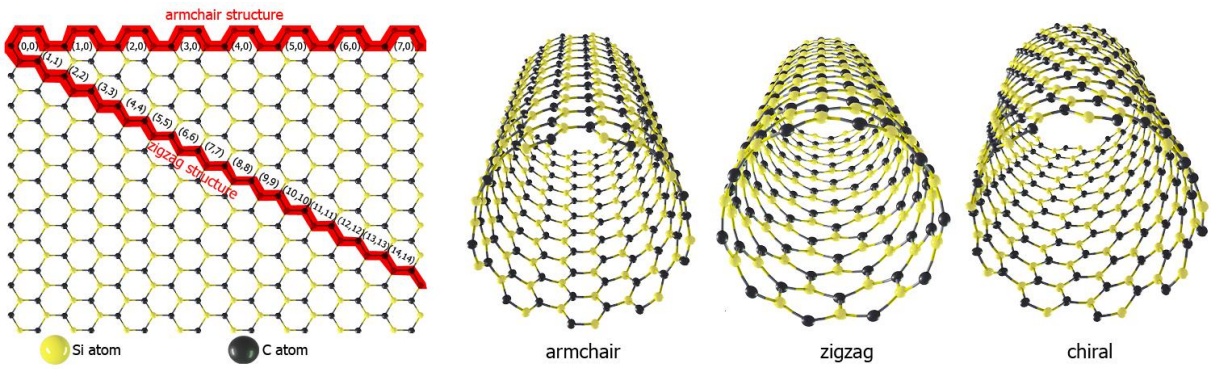

Fig. 2. Armchair, zigzag, and chiral structure

Graphene is one of the most futuristic material founded in present century. Within its discovery in 2004 (13 years after the foundation of carbon nanotube) graphene sheets attracted huge attention with its superior material properties [4, 15-19]. Graphene sheets consist of six carbon atoms. These six atoms are bonded to each other in honeycomb (hexagonal) shape. Graphene sheets can be produced in many methods. To illustrate two of most used methods are layer separation and chemical separation methods. Graphene sheets and graphene based nano-sized materials (carbon nanotubes, carbon nanowires) have limited thermal stability. Carbon based nanomaterials can stay stable only until 600ºC in air [20, 21]. Due to this thermal limitation, these materials can only be used in limited applications area where these nano-sized materials will not be effected by an environment higher than 600 ºC. On the other hand, scientists need to use strong nano-sized materials in environment much higher than 600ºC like aerospace. For example NASA will launch a spacecraft named *Parker Solar Probe* to the sun where the spacecraft will be in an environment around 1400ºC in 2018 summer [22]. To overcome this issue, scientists have produced a new nanomaterial, silicene, which can stay stable until 1200ºC without any damage [13]. Silicene is a layer of silicon atoms which are hexagonally arranged similarly to carbon atoms in graphene sheets. However, the mechanical properties of silicene are weaker than graphene. To explain, Si-Si bond length in silicene is 2.29 Å where the C-C bond length is 1.42 Å in graphene, so that silicene performs higher chemical reactivity than graphene. This bond length makes silicene weaker than graphene. To address this issue silicene and graphene are composed and a new more powerful, thermal resistant nanomaterial has been developed 'silicon carbide sheet'.

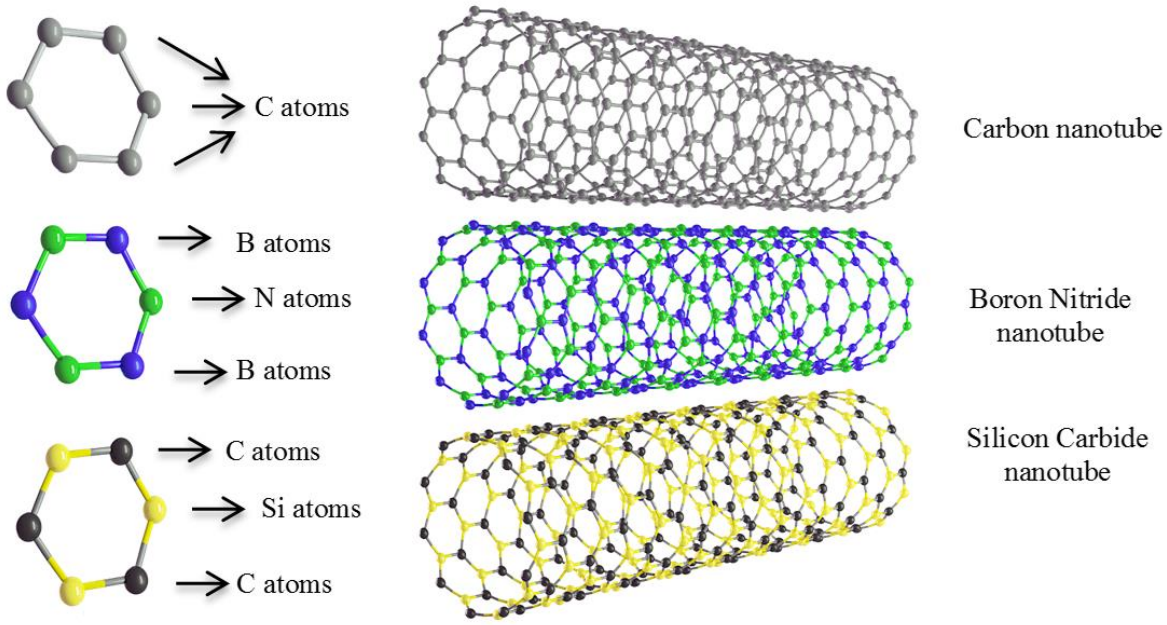

Fig. 3. The structure of CNT, BNNT, and SiCNT

NASA Glenn Research Center has collaborated with Rensselaer Polytechnic Institute in order to obtain silicon carbide sheets. Researches from the collaboration have developed several methods to obtain silicon carbide sheet. Some of these methods are chemical conversion of carbon nanotubes (nanotubes which have been obtained by rolling graphene sheet) to silicon carbide nanotube (nanotube which have been obtained by rolling silicon carbide sheet), direct

SiCNT growth on catalyst, and template-derived SiCNTs. Finally, scientist have produced the material, silicon carbide sheet, which is capable to stay stable under 1000^{o}C and stronger than silicene in case of mechanical stability. On the other hand, boron nitride nanotubes are another kind of nanotube which consist of boron "*B*" and nitrogen "*N*" atoms bonded to each other. The bond length between *B* and *N* atoms is 2.503 Å [11]. Although the bond length in BNNT is longer than CNT and SiCNT, BNNT is the strongest material in case of mechanical stability with Young's modulus equal to 1.8 TPa where CNT and SiCNT have Young's modules equal to 1 TPa and 0.62 TPa respectively [10, 13]. Three types of nanotubes are demonstrated in Fig. 3. Carbon atoms, boron atoms, nitrogen atoms, and silicon atoms are demonstrated with red, blue, green, and yellow balls in Fig. 3 (carbon atoms are demonstrated in darker color in SiCNT structure).

Nanotubes have been modeled as plate, shell, rod, and beam in literature [23-30] to make bending [31, 32], buckling [33] , vibration [34] analyses possible theoretically. In this paper, nanotubes are modeled as cylindrical beam by using Euler-Bernoulli beam model. In the model, *L* is the length, *r* is the average radius, *D* is the average diameter, *t* is the thickness of nanotube.

3. Formulation for Buckling Problem of Nanotubes

Buckling equation with surface effect can be obtained by using the following nonlocal Euler-Bernoulli beam model. Nonlocal Euler-Bernoulli model of nanobeams can be expressed as follows [35, 36]

$$\frac{d^2}{dx^2}\left(-EI\frac{d^2w}{dx^2}\right)+\mu\frac{d^2}{dx^2}\left(\frac{d}{dx}\left(P\frac{dw}{dx}\right)-q\right)+q-\frac{d}{dx}\left(P\frac{dw}{dx}\right)=0 \tag{1}$$

In Eq. (1), *w(x)* is the deflection at any *x* point, *P* and *q* are the axial compressive load and the transverse distributed force respectively. The moment of inertia of a nanotube can be calculated by using classical moment of inertia for circular cross sections as follows

$$I=\pi r^3 t \tag{2}$$

Where '*r'* is the radius of nanotube and '*t*' is the thickness. In order to take the surface elasticity effect into consideration, the classical flexural rigidity '*EI'* need to be replaced with '$\overline{EI}$ '. $\overline{EI}$ is the effective flexural rigidity, for a nanotube with a circular section $\overline{EI}$ can be calculated as 68]

$$\overline{EI}=\frac{E\pi D^4}{64}+\frac{E^s\pi D^3}{8} \tag{3}$$

As it can be seen in Eq. (3), there are two different Young's modulus parameter for one material. E^s is the surface Young's modulus and *E* the material's Young's modulus. "*D*" is the average

diameter which can be calculated by calculating the difference between outer and inner diameter of nanotube. The residual surface stress doesn't have any effect on the bulk in case of the deflection equal to zero. On the other hand, in case of any deflection, the residual surface tension generates a distributed transverse loading *q(x)* along the longitudinal direction. The Laplace-Young equation predicts that

$$q(x) = H\frac{d^2w}{dx^2} - k_w w + k_p \frac{d^2w}{dx^2} \tag{4}$$

Where *H* is a constant which depends on the residual surface tension and the cross-sectional shape and can be calculated with the following equation [37, 38]

$$H = 2\tau^0 D \tag{5}$$

In Eq. (5) 'τ^0' is the residual surface tension. By substituting Eqs. (3-5) in Eq. (1) we can obtain the equilibrium equation of a nanowire embedded in Winkler and Pasternak foundation as [39]

$$(-\overline{EI} + P\mu - H\mu - k_p\mu)\frac{d^4w}{dx^4} + (H + k_w\mu - P + k_p)\frac{d^2w}{dx^2} - k_w w = 0 \tag{6}$$

Where μ is the nonlocal parameter and equal to $(e_0 a)^2$. k_w and k_p are the Winkler and Pasternak foundation parameters, respectively. In case of choosing k_w and k_p equal to zero, the equation will be the nanowire equilibrium without any foundation effect. Hence by choosing *H* and μ equal to zero, the equation will be the nanowire equilibrium embedded in double parameter foundation without nonlocal and surface effect. To calculate the buckling loads of nanowire, boundary conditions must be used in Eq. (6). In case of simply supported nanobeams, boundary conditions can be expresses as [40, 41]

$$w=0 \ \ and \ \ M=0 \quad \text{at} \quad x=0 \ and \ x=L \tag{7}$$

To use Navier's Solution Procedure, generalized displacement series needs to be employed as follows

$$w(x) = \sum_{n=1}^{\infty} W_n \sin\left(\frac{n\pi x}{L}\right) \tag{8}$$

In Eq. (56), W_n is the undetermined Fourier coefficient. This means that Eq. (8) must satisfy the associated boundary conditions. Use of Eq. (8) in Eq. (6) yields the following relation for buckling loads of a nanowire which are embedded in double parameter foundation including size effect as;

For nonlocal elasticity theory (NET)

$$P(n)=\frac{(\overline{EI}+k_p\mu)\left(\frac{n\pi}{L}\right)^4+(k_w\mu+k_p)\left(\frac{n\pi}{L}\right)^2+k_w}{\mu\left(\frac{n\pi}{L}\right)^4+\left(\frac{n\pi}{L}\right)^2} \tag{9}$$

For surface elasticity theory (SET)

$$P(n)=\frac{\overline{EI}\left(\frac{n\pi}{L}\right)^4+(H+k_p)\left(\frac{n\pi}{L}\right)^2+k_w}{\left(\frac{n\pi}{L}\right)^2} \tag{10}$$

For nonlocal surface elasticity theory (NET & SET) (NSET)

$$P(n)=\frac{(\overline{EI}+H\mu+k_p\mu)\left(\frac{n\pi}{L}\right)^4+(H+k_w\mu+k_p)\left(\frac{n\pi}{L}\right)^2+k_w}{\mu\left(\frac{n\pi}{L}\right)^4+\left(\frac{n\pi}{L}\right)^2} \tag{11}$$

4. Numerical Examples

In present paper the buckling analyzes of three types of nanotubes have been made by using two different size effective theories. The length of each nanotube have been chosen equal to 50 nm. Furthermore, the diameter for each nanotube are also have been chosen equal to 1 nm to plot Fig. 5. To model the double parameter foundation, Winkler and Pasternak foundation models have been used. Foundation parameters have been chosen as $k_w=1500$ and $k_p=50$. To take the size effect into consideration, nonlocal elasticity theory, surface elasticity theory, and nonlocal surface elasticity theory is used. In Fig. 4, the buckling loads of CNT, BNNT, SiCNT have been plotted for various radius for first three modes. Buckling loads of BNNT, CNT, and SiCNT have been plotted in green, red, and blue colored lines respectively.

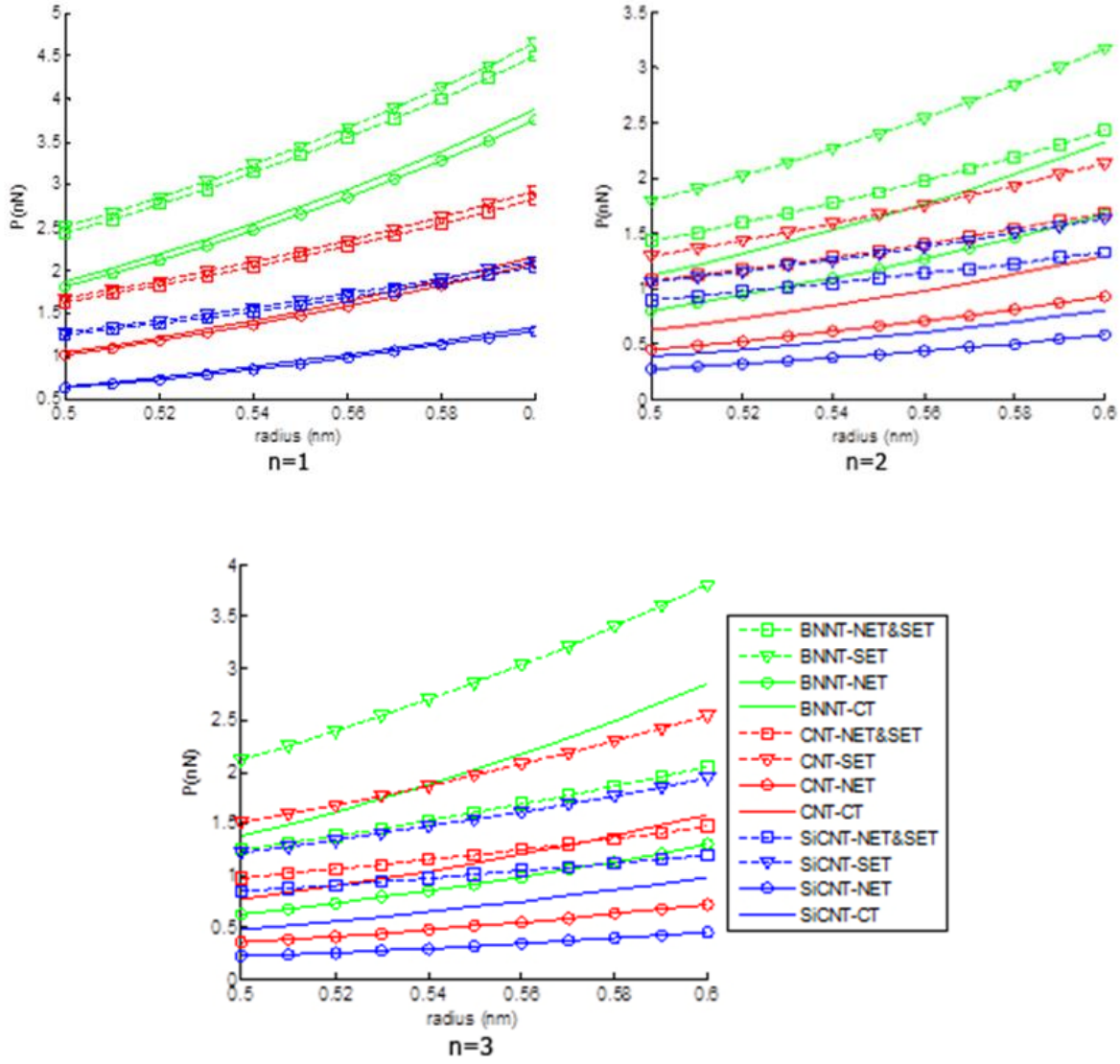

Fig. 4. Buckling loads of nanotubes for various radius

As it can be clearly seen from Fig. 4 with the increase in radius, as expected, the buckling load of CNT, BNNT, SiCNT follows an increasing trend. To compare size effective theories, surface elasticity theory always gives higher results than classical theory while nonlocal elasticity theory always gives lower results. On the other hand, nonlocal surface elasticity theory gives higher results than classical theory in first modes, but lower results in higher modes for BNNT. Furthermore, the effect of nonlocal surface elasticity theory has been observed to perform alike trend for CNT and SiCNT. As expected the buckling loads of BNNT is the highest while SiCNT is the lowest.

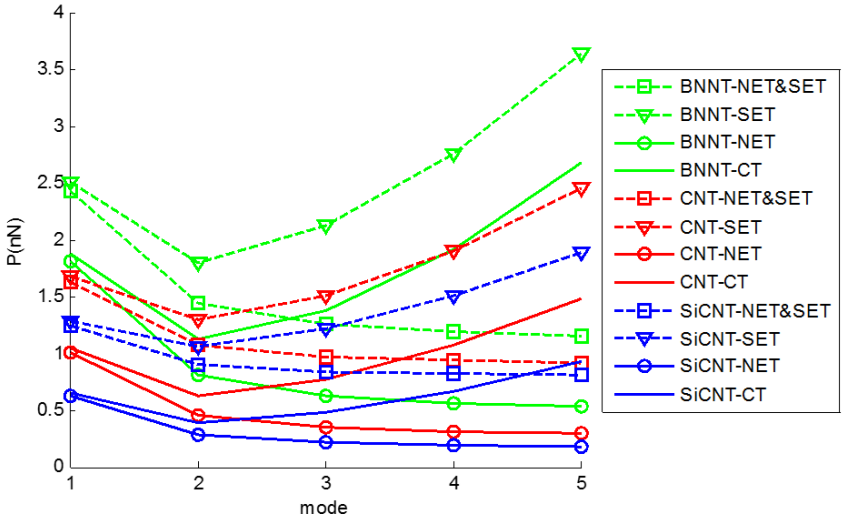

Fig. 5. Buckling loads for different mode numbers

In Fig. 5, the buckling load of BNNT, CNT, and SiCNT have been plotted for different mode numbers and size effective theories. To see the differences between results, the length of nanotubes is chosen equal to 100 nm. As it can be seen from Fig. 5, as expected, BNNT is the strongest nanotube in case of stability where SiCNT is the weakest. On the other hand, with the effect of double parameter foundation, the buckling loads doesn't always give higher buckling loads for higher mode numbers. To illustrate, for all nanotubes, when surface elasticity theory and classical theory is employed the buckling loads perform increasing trend after second mode number. However, when nonlocal elasticity theory and nonlocal surface elasticity theory is used the buckling loads perform decreasing trend with the increase in mode numbers.

4. Concluding Remarks

As it can be seen from Fig. 3 and Fig. 4 the buckling loads of all nanotubes increase with the increase in radius and mode numbers. As expected, boron nitride nanotubes have the strongest resistance to buckling where silicon carbide nanotubes have the lowest. Furthermore, surface elasticity theory gives highest buckling loads while nonlocal elasticity theory gives lowest and nonlocal surface elasticity theory's results are between two results.

Acknowledgments

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