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Error Analysis of Absolute Rate Coefficient Extrapolated under *Pseudo*-First Order Conditions

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Abstract: Computer based simulations for the reaction of nitrogen(II) oxide with the hydroxyl radical have been used to estimate the error associated with the *pseudo*-first order approximation under different simulated conditions. For the absolute rate coefficients, calculated by dividing the *pseudo*-first order constant by the concentration of the reactant in excess, the analysis of the relative error associated with working under *pseudo*-first order conditions shows that for a reactants’ ratio higher than 10, the relative error is less than 5%.

Keywords: *Pseudo*-first order conditions, absolute rate coefficient, computer simulations, error analysis.

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INTRODUCTION

The *pseudo*-first order approximation is an important technique used by kineticists to extrapolate absolute rate coefficients (k_{II}) of second order reactions with simple mathematical calculations (1,2). The *pseudo*-first order approximation relies on the fact that, in case of a substantial difference between the initial concentrations of the reactants in a second order reaction, the disappearance of one reactant over time would only marginally affect the concentration of the other, and therefore, effectively reduces the order of the reaction from two to one. When the difference in the concentration of the reactants is large enough, *pseudo*-first order conditions are established. On the other hand, when the ratio of the concentration of the reactants approaches each other, the error associated with the absolute rate coefficient extrapolated with first order formalism increases dramatically, and the *pseudo*-first order approximation breaks down. With the advent of more sophisticated mathematical algorithms (3), it has been argued that working under *pseudo*-first order conditions could be considered obsolete. In addition to that, as shown in Table 1, there is substantial confusion with regards to the term of *pseudo*-first order conditions. Table 1 lists the conditions required for *pseudo*-first order according to different authors. The statement *"to a great excess"* has been interpreted by more than ten authors (3), but, as shown in Table 1, there is currently no systematic theoretical treatment addressing this limitation that has been accepted by the scientific community.

Table 1: Borders of validity of the *pseudo*-first order conditions reported in selected literature.

	Reference	**Initial ratio between reagents' concentrations**
Specialized Literature	Sicilio and Peterson, 1961 (4)	55 to 1
	O'Ferral and Miller, 1963 (5)	10
	Lente, 2015 (6)	10
	Espenson, 1995 (7)	10
	Corbett, 1972 (8)	5
Teaching Literature	Rawn, 2008 (9)	*"…initial concentration (of A) much greater than the concentration of B."*
	Atkins and De Paula, 2006 (10)	*"…large excess…"*

Additional confusion is created by the first attempt of Sicilio and Peterson (4) to provide a theoretical analysis of the chemical reactions carried under *pseudo*-first order conditions. While their results were published in the Journal of Chemical Education, which is historically important for the reasons to be further elaborated in this study, validity of their approach may not be relevant to a kineticist. The Sicilio and Peterson’s paper (4) reads in part: “*The first part of a reaction is often stressed in the kinetic studies, and unfortunately, it is during this part that the ratio error is greatest. During the latter stages of a reaction, the error decreases rapidly and becomes zero at 100% completion*”. Sicilio and Peterson thus implied that looking at the decay in its entirety could possibly lead to better fits under *pseudo*-first order conditions. This is not the case for two main reasons: First, in a *pseudo*-first order reaction, the researchers usually build the kinetic profiles by plotting the concentration of the reactant in deficit over time. When the reaction gets close to completion, the reactant in deficit is almost completely exhausted and its presence is more difficult to detect. Therefore, the relative error becomes more important in comparison with the signal it generates. Second, as shown in Table 2, the propagation of errors in a semi-log plot, such as a first order plot, the relative error for lower values is substantially higher.

Table 2: Propagation of error in a semi logarithmic plot: As the value of the variable decreases, the experimental accuracy related to the instrument used for the detection of that variable remains the same. The semi-log scale magnifies the error bars by a factor of about 10 from 0.01 to 0.125.

Value	Error	Range	Natural Log	Difference
98	0.5	97.5	4.5900	0.0102
		98.5	4.5798	
8	0.5	8.5	2.1400	0.1251
		7.5	2.0149	

If both of these limitations of the 1961 Sicilio and Peterson’s (4) analysis are taken into account, the conclusions of their reasoning lose reliability. In 1963, O’Ferral and Miller (5) published a note, also in the same journal, refuting the conclusions of Sicilio and Peterson on the basis that most kineticists work under experimental conditions that are well within the range of functionality allowed for the *pseudo*-first order conditions. Corbett (8), published simulation results for a hypothetical reaction, again in the very same journal, showing that a 2% error could be achieved even if the initial ratio of concentrations of the two reagents was about 2. Other references (6, 7, 9, and 10) discuss the limits of the *pseudo*-first order conditions, but do not provide a theoretical framework to support their discussions. One of the controversies in the kinetic community is whether the *pseudo*-first order approximation is a reliable tool. Although the *pseudo*-first order approximation is used almost since the inception of chemical kinetics as a specific discipline (7), modern techniques such as non-linear least square fitting without any transformation of axis made possible by modern computer processing, make the this approximation look at least obsolete if not inadequate (6), anonymous reviewer to (1) and anonymous reviewer to the present work. The question is open for discussion in the kinetic community, and while the present work is not designed to take a stand in either direction, it is the intention of the author to produce a solid background for the estimate of the error introduced by working under *pseudo*-first order conditions. More recently, the robustness of the *pseudo*-first order conditions was reviewed by Schnell and Mendoza (11), and by Pedersen and Bersani (12). These more recent studies are focused on applications of the *pseudo*-first order approximation to the analysis of enzymatic reactions. Instead of helping to reach conclusions of broad validity, their application is limited to a very narrow field of the art.

MATERIALS AND METHODS

Computer simulations of chemical kinetics have been used since the 1970s (13) and currently are a well-established methodology to mimic chemical systems (14). Java-based chemical simulator Billite TENUA (15) was used to simulate the validity of the *pseudo*-first order approximation on the reaction of nitrogen(II) oxide (IUPAC name nitric oxide or systematic IUPAC name oxidonitrogen) with the hydroxyl radical. This recombination reaction is a well-known system (2,16) that follows a simple mechanism (17). The generally accepted mechanism for the reaction of the hydroxyl radical with nitrogen(II) oxide (18) consists of three elementary steps, including the collision of the reactant molecules to form an energized adduct, $[HONO]^*$:

$$NO + OH \rightarrow [HONO]^* \qquad \text{(Eq. 1)}$$

Followed by stabilization of the adduct:

$$[HNOO]^* + M \rightarrow \text{Products} \qquad \text{(Eq. 2)}$$

The decomposition of the energized adduct back into the original reactants:

$$[HNOO]^* \rightarrow NO + OH \qquad \text{(Eq. 3)}$$

Adduct $[HONO]^*$ is formed in the first step of the mechanism, and it can either back dissociate or stabilize into products. The internal structure, and the possible isomeric configurations of adduct $[HONO]^*$ have been discussed elsewhere (2,16) and it is most likely to follow the bond sequence HOHO, as other possible structures for this state are energetically unfavourable. M represents the third body; a third molecule that it is not consumed in the reaction, but that plays a significant role by stabilizing the adduct by removing from it the excess energy, in turn favouring the formation of the reaction products. In the system that was reproduced in this study M is helium gas. The *pseudo*-first order approximation assumes the following order of concentrations: $[M] >> [NO] >> [OH]_o$. The overall reaction can be written as follows:

$$NO + OH + M \rightarrow Products \qquad (Eq.\ 4)$$

To simulate experimental conditions, the initial concentration of NO can be varied in the range of 10^{12} - 10^{16} , in large excess with respect to the $[OH]_0$, typically kept at 10^{12} molecules cm^{-3}. Simulated OH temporal profiles are then analysed assuming simple first order exponential behaviour to determine k'.

$$[OH]_t = [OH]_0 x\ exp(-k\text{^}\ t) \qquad (Eq.\ 5)$$

With reference to equation 5, $k\text{^} = k_{II}\ x\ [NO] + k_d$, where k_{II} is the absolute rate coefficient for the reaction, and k_d is the loss of OH due to other factors, such as wall collisions, reactions with its precursor, self-reaction, and others. The reaction rate is derived using the following equation:

$$rate = k\text{^}\ x\ [OH] \qquad (Eq.\ 6)$$

The absolute rate coefficients are calculated by dividing k^ by the concentration of the reactant in excess. Alternatively, absolute rate coefficients can be extrapolated with the least square mean, from linear fits of the plot k^ against [NO] for each different pressure with the assumption that:

$$k_d << k_{II}\ x\ [NO] \qquad (Eq.\ 7)$$

This second technique, also referred as the second order plot or linearization method, minimizes potential experimental errors as it dilutes the effect of the relative concentrations on the determination of the absolute rate coefficient over a relatively wide range of concentrations. On the other hand, it is slightly more elaborated and not all researchers necessarily use it, particularly in solution. Since in the simulation k_d is set to be null, Equation 7 is then reduced to:

$$k_{II} = k\text{^}\ x\ [NO]^{-1} \qquad (Eq.\ 8)$$

The bimolecular rate coefficient for the reaction k_{calc} is obtained from simulated OH decays for different ratios of $r = [NO]_0/[OH]_0$. Ratios are then compared with the accepted values of k_{II} at room temperature under different pressures, including 50, 200, and 600 Torr. The percentage error is then derived as:

$$\varepsilon = 100X(k_{true} - k_{calc})/k_{true} \qquad (Eq.\ 9)$$

The different estimates for the percentage errors are tabulated and analyzed. Table 3 shows the absolute rate coefficient calculated with NASA-JPL expression (19) at 298.15 K for three different pressure values. All data treatment was performed with Microcal Origin (20).

Table 3: Absolute rate coefficient for the reaction of nitrogen(II) oxide with hydroxyl radical calculated with NASA-JPL expression (19) at 298.15 K under 50, 200, and 500 Torr of helium.

p (Torr)	**k_{II} [cm^3 $molecule^{-1}$ s^{-1}]**
50	1.4×10^{-12}
200	5.0×10^{-12}
500	1.0×10^{-11}

RESULTS AND DISCUSSION

Hydroxyl radical temporal decays were simulated for at least three orders of magnitude, i.e. from 10^{12} [molecules cm^{-3}] to 10^9 [molecules cm^{-3}] respectively at 50, 200, and 500 Torr at 298.15 K. Figure 1 represents five typical simulated decays at 298.15 K, under 50 Torr of total pressure, assuming helium to be the bath gas at different relative concentrations of the reactants ranging from 20 to 100 . The decays are linear for at least three orders of magnitude and the temporal resolution between points (ε) was adjusted to allow a sufficient number of points for each curve. The slope calculated by least-square fit of each curve represents the *pseudo*-first order constant k^ [s^{-1}].

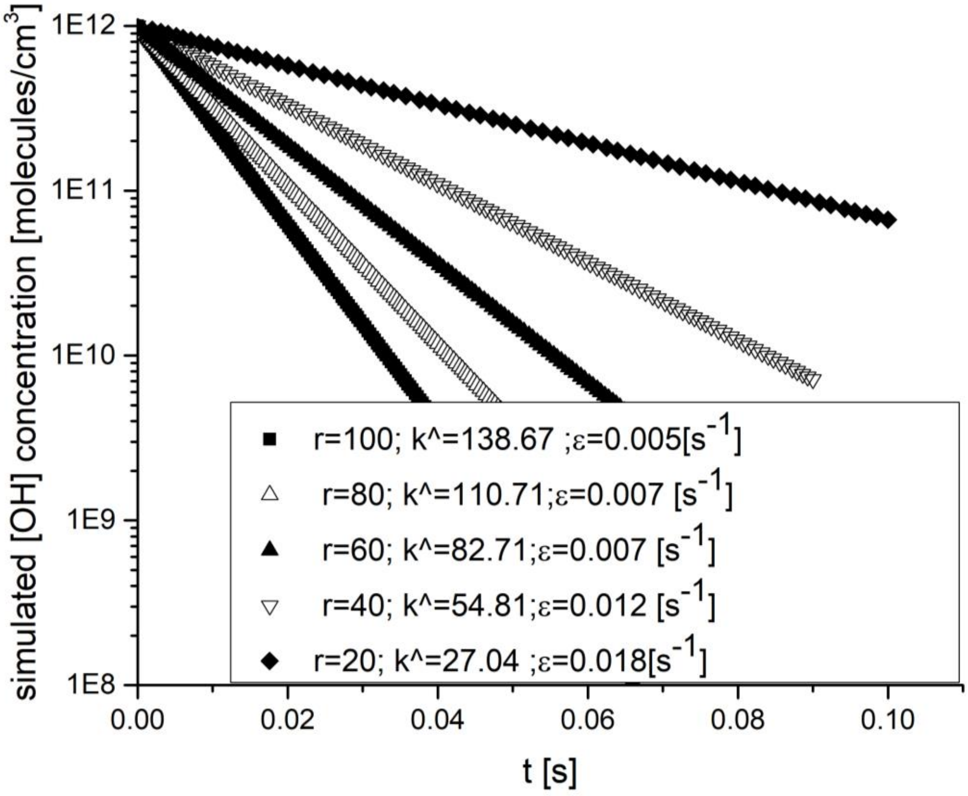

Figure 1. Typical simulated temporal profiles for different ratios of initial concentration of reactants between 20 and 100.

Figure 1 shows five typically simulated temporal profiles of the hydroxyl radical in the reaction with nitrogen(II) oxide at 298.15 K, under 50 Torr of total pressure. This is when assuming helium to be the bath gas, at different relative concentrations of the reactants ranging from r = 20 to r = 100.

The temporal profiles (decays) represented in Figure 1 are substantially linear over a three orders of magnitude drop of concentration of the reactant in defect, suggesting no systemic deviation from a first order kinetic decay. Shown in Figure 2, however, are simulated hydroxyl radical temporal decays under the same conditions of pressure and temperature of the ones represented in Figure 1. The temporal profiles represented in Figure 2 are the outcome of simulations run with relative reactant concentrations ranging from 1 to 5 showing a substantial loss of linearity in a semi logarithmic plot in the first part of the decay. The loss of linearity of the temporal profile is a first indication that the *pseudo*-first order conditions are only partially met in this range of relative concentrations, and that the least square fit of the data point may lead to unreliable conclusions.

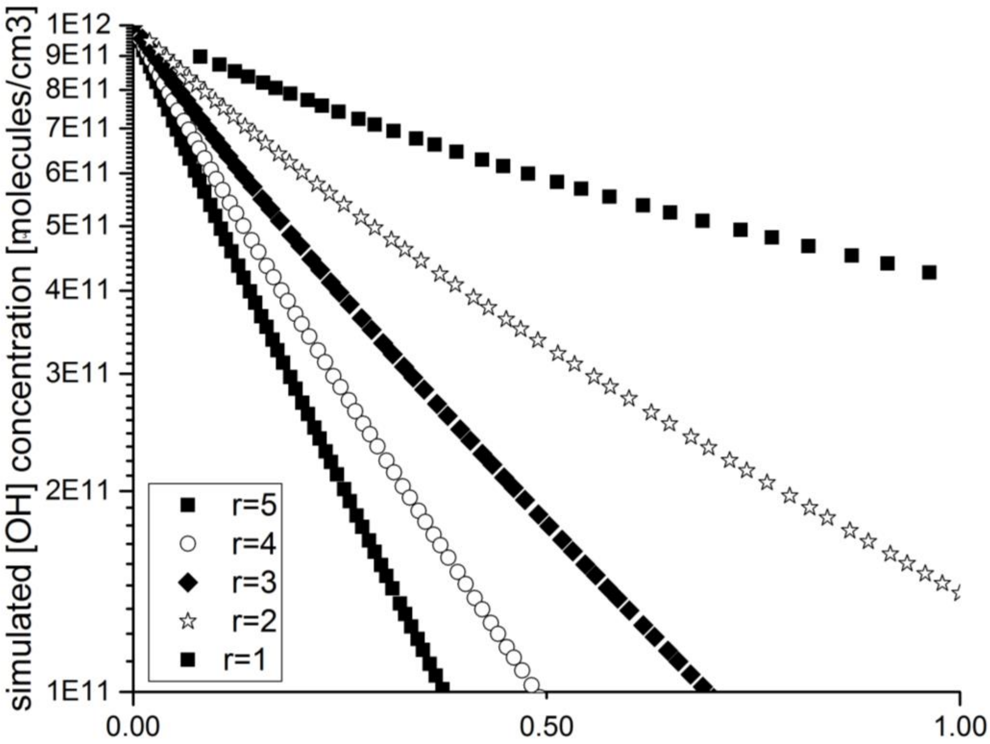

Figure 2: Typical simulated temporal profiles for different ratios of initial concentrations of reactants approaching to 1.

Figure 2 represents five typically simulated temporal profiles of the hydroxyl radical in the reaction with nitrogen(II) oxide at 298.15 K, under 50 Torr of total pressure, assuming helium to be the bath gas, at different relative concentrations of the reactants ranging from r = 1 to r = 5. Absolute rate coefficients k_{II} [cm^3 $molecule^{-1}$ s^{-1}] are then calculated by dividing the *pseudo*-first order constants k^ by the concentration of the reactant in excess. Alternatively, they are determined as the slope of the second order plot, where k^ is plotted against the different concentrations of the reactant in excess, in this case the nitrogen(II) oxide.

Figure 3 shows the simulated *pseudo*-first order constant plotted against the concentration of nitrogen(II) oxide. The plot of in this figure is constructed with simulated data. Therefore, k_{II} extrapolated from it is expected to be the same k_{II} fed into the model, unless the breakdown of the *pseudo*-first order approximation.

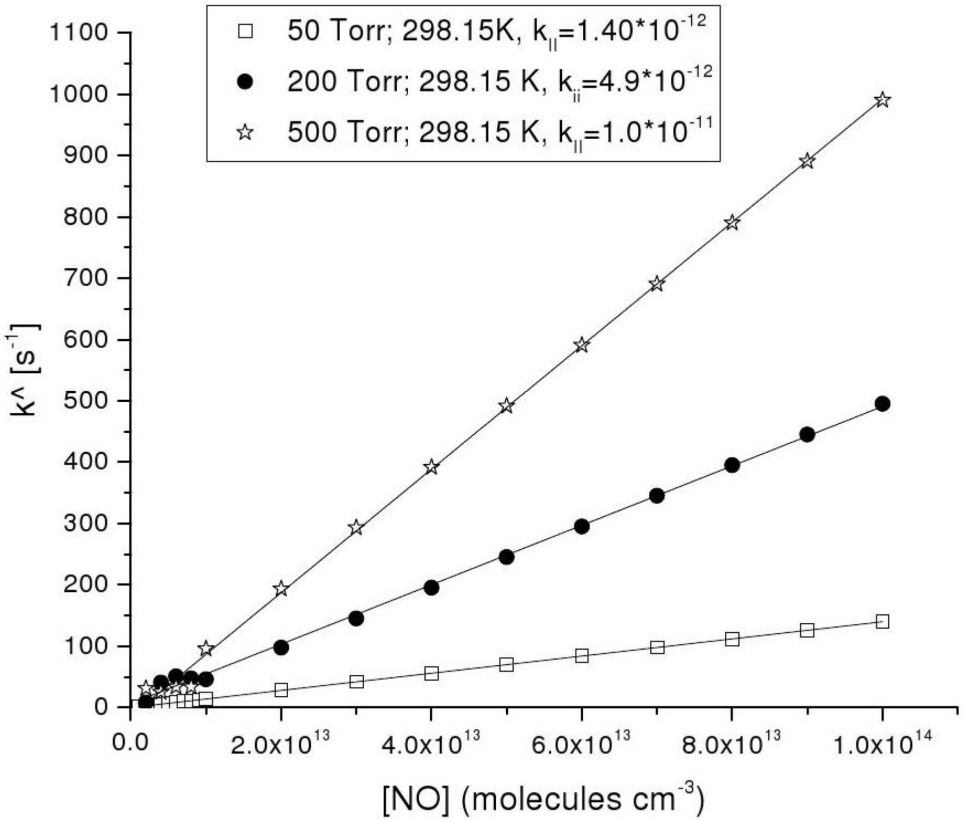

Figure 3: Second order plot of the simulated *pseudo*-first order rates versus the concentration of nitrogen(II) oxide at 298.15 K, under 50, 200, and 500 Torr of Helium. The absolute kinetic constant k_{II} [cm^3 $molecule^{-1}$ s^{-1}] is determined as the least square slope of the linear fit.

Once the absolute rate coefficient is extrapolated for each different simulated condition, its value is compared with the literature value used as input in the simulation mechanism. The relative error is then calculated with Equation 9. Table 4 illustrates the output of the relative error calculation on the absolute rate coefficient derived in the simulation done in the present study. The data tabulated in Table 4 show that for r greater than 10 the relative error introduced by using the *pseudo*-first order approximation is less than 5%. If the absolute rate coefficient instead of being calculated independently as the ratio of the *pseudo*-first order constant k^ by the concentration of nitrogen(II) oxide are calculated with the second order plot, a ratio as low as 5 can be attained with an error less than 5%. The data of Table 4 are reported in Figure 4 where the relative percentage error is plotted against the relative concentrations of the reactants. If a 5% relative ratio is considered to be acceptable, then the conclusions of O'Ferral and Miller (5) would set the tone for the proper determination of an acceptable level of uncertainty associated with *pseudo*-first order conditions.

Table 4: Relative error calculated with Equation 9.

r	**k'** **[s^{-1}]**	**[OH]** **[molecules cm^{-3}]**	**[NO]** **[molecules cm^{-3}]**	**$k_{II(calc)}$** **[cm^3 $molecule^{-1}$ s^{-1}]**	**$k_{II(true)}$** **[cm^3 $molecule^{-1}$ s^{-1}]**	**error**
1	0.0846	10^{12}	10^{12}	8.46×10^{-13}	1.4×10^{-12}	94%
2	1.944	10^{12}	2×10^{12}	9.72×10^{-13}	1.4×10^{-12}	31%
3	3.236	10^{12}	3×10^{12}	1.08×10^{-12}	1.4×10^{-12}	23%
4	4.616	10^{12}	4×10^{12}	1.15×10^{-12}	1.4×10^{-12}	18%
5	6.072	10^{12}	5×10^{12}	1.21×10^{-12}	1.4×10^{-12}	13%
6	7.387	10^{12}	6×10^{12}	1.23×10^{-12}	1.4×10^{-12}	12%
7	8.726	10^{12}	7×10^{12}	1.25×10^{-12}	1.4×10^{-12}	11%
8	10.57	10^{12}	8×10^{12}	1.32×10^{-12}	1.4×10^{-12}	6%
9	11.92	10^{12}	9×10^{12}	1.32×10^{-12}	1.4×10^{-12}	5%
10	13.35	10^{12}	1×10^{13}	1.33×10^{-12}	1.4×10^{-12}	5%
20	27.04	10^{12}	2×10^{13}	1.35×10^{-12}	1.4×10^{-12}	3%
30	40.91	10^{12}	3×10^{13}	1.36×10^{-12}	1.4×10^{-12}	3%
40	54.81	10^{12}	4×10^{13}	1.37×10^{-12}	1.4×10^{-12}	2%
50	68.75	10^{12}	5×10^{13}	1.38×10^{-12}	1.4×10^{-12}	2%
60	82.71	10^{12}	6×10^{13}	1.38×10^{-12}	1.4×10^{-12}	2%
70	96.73	10^{12}	7×10^{13}	1.38×10^{-12}	1.4×10^{-12}	1%
80	110.7	10^{12}	8×10^{13}	1.38×10^{-12}	1.4×10^{-12}	1%
90	124.68	10^{12}	9×10^{13}	1.39×10^{-12}	1.4×10^{-12}	1%
100	138.67	10^{12}	1×10^{14}	1.39×10^{-12}	1.4×10^{-12}	1%

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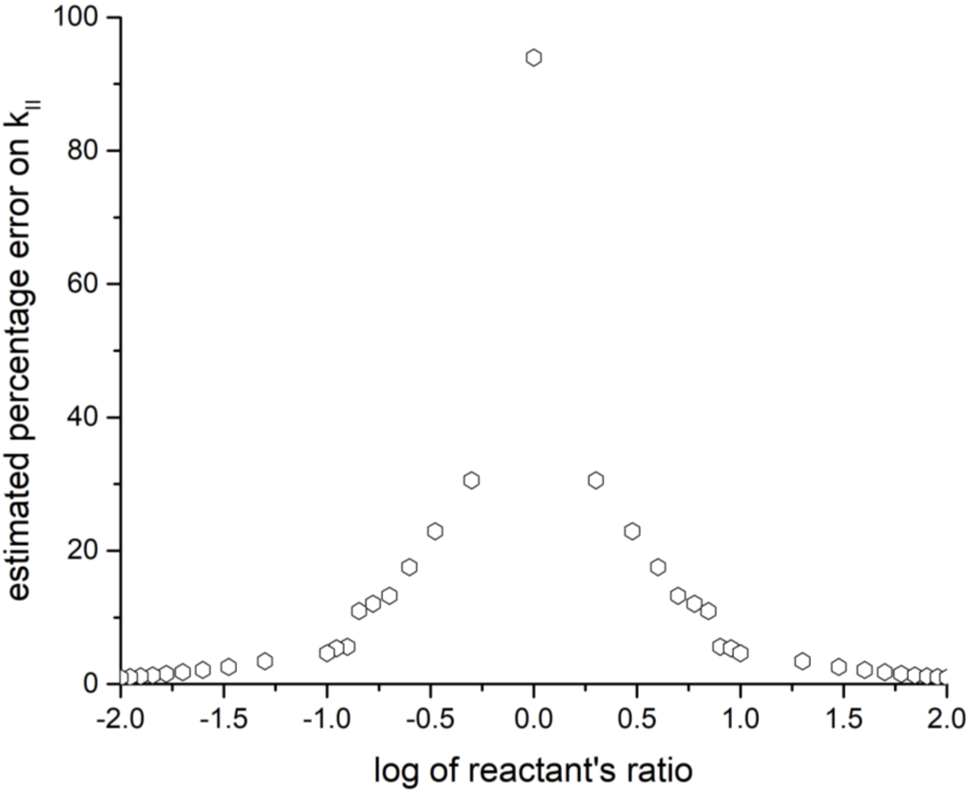

Figure 4: Error analysis plot: Relative error on the absolute rate coefficient for the reaction with nitrogen(II) oxide at 298.15 K under 50 Torr of helium.

CONCLUSIONS

Computer based simulations for the reaction of nitrogen(II) oxide with the hydroxyl radical have been used to estimate the error associated with the *pseudo*-first order approximation under different simulated conditions. The analysis of the relative error associated with the *pseudo*-first order conditions shows that for a reactants' ratio higher than 10, the relative error is less than 5%, thus supporting the view expressed by O'Ferral and Miller (5) and Kiss (3). Further and more comprehensive research may be needed to generalize these results, with specific reference to more complex systems like the reaction of nitrogen(IV) oxide with the hydroxyl radical (21). Other aspects of this problem suitable for further investigation are related to the transferability of the conclusion of the present study to the practicality of experiments, especially in the range of low signal to background ratio, where it is not always possible to observe a temporal profile for three orders of magnitude.

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