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Advanced Fractional Mathematics, Fractional Calculus, Algorithms and Artificial Intelligence with Applications in Complex Chaotic Systems

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ABSTRACT Chaos, comprehended characteristically, is the mathematical property of a dynamical system which is a deterministic mathematical model in which time can be either continuous or discrete as a variable. These respective models are investigated as mathematical objects or can be employed for describing a target system. As a long-term aperiodic and random-like behavior manifested by many nonlinear complex dynamic systems, chaos induces that the system itself is inherently unstable and disordered, which requires the revealing of representative and accessible paths towards affluence of complexity and experimental processes so that novelty, diversity and robustness can be generated. Hence, complexity theory focuses on non-deterministic systems, whereas chaos theory rests on deterministic systems. These entailments demonstrate that chaos and complexity theory provide a synthesis of emerging wholes of individual components rather than the orientation of analyzing systems in isolation. Therefore, mathematical modeling and scientific computing are among the chief tools to solve the challenges and problems related to complex and chaotic systems through innovative ways ascribed to data science with a precisely tailored approach which can examine the data applied. The complexity definitions need to be weighed over different data offering a highly extensive applicability spectrum with more practicality and convenience owing to the fact that the respective processes lie in the concrete mathematical foundations, which all may as well indicate that the methods are required to be examined thoroughly regarding their mathematical foundation along with the related methods to be applied. Furthermore, making use of chaos theory can be considered to be a way to better understand the internal machinations of neural networks, and the amalgamation of chaos theory as well as Artificial Intelligence (AI) can open up stimulating possibilities acting instrumental to tackle diverse challenges, with AI algorithms providing improvements in the predictive capabilities via the introduction of adaptability, enabling chaos theory to respond to even slight changes in the input data, which results in a higher level of predictive accuracy. Therefore, chaos-based algorithms are employed for the optimization of neural network architectures and training processes. Fractional mathematics, with the application of fractional calculus techniques geared towards the problems' solutions, describes the existence characteristics of complex natural, applied sciences, scientific, engineering related and medical systems more accurately to reflect the actual state properties co-evolving entities and patterns of the systems concerning nonlinear dynamic systems and modeling complexity evolution with fractional chaotic and complex systems. Complexity entails holistic understanding of various processes through multi-stage integrative models across spanning scales for expounding complex systems while following actuality across evolutionary path. Moreover, Fractional Calculus (FC), related to the dynamics of complicated real-world problems, ensures emerging processes adopting fractional dynamics rather than the ordinary integer-ordered ones, which means the related differential equations feature non-integer valued derivatives. Given that slight perturbation leads to a significantly divergent future concatenation of events, pinning down the state of different systems precisely can enable one to unveil uncertainty to some extent. Predicting the future evolution of chaotic systems can screen the direction towards distant horizons with extensive applications in order to understand the internal machinations of neural and chaotic complex systems. Even though many problems are solvable and have been solved, they remain to be open constantly under transient circumstances. Thus, fields with a broad range of spectrum range from mathematics, physics, biology, fluid mechanics, medicine, engineering, image analysis, based on differing perspectives in our special issue which presents a compilation of recent research elaborating on the related advances in foundations, theory, methodology and topic-based implementations regarding fractals, fractal methodology, fractal spline, non-differentiable fractal functions, fractional calculus, fractional mathematics, fractional differential equations, differential equations (PDEs, ODEs), chaos, bifurcation, Lie symmetry, stability, sensitivity, deep learning approaches, machine learning, and so forth through advanced fractional mathematics, fractional calculus, data-intensive schemes, algorithms and machine learning applications surrounding complex chaotic systems.

KEYWORDS
Fractional mathematics
Complexity
Fractional calculus
Deep learning
Computational complexity
Fractal methodology
Fractalization
Complex versus chaotic systems and chaos
Bifurcation
Control and optimization
Strange attractors
Approximation theory
Lie symmetry
Complex chaotic systems
Complex systems
Data-intensive computational application processes
Real data interpolation and applications
Differential equations
Machine learning
Deep neural network.

INTRODUCTION, PRELIMINARY REMARKS AND OVERVIEW

Theory of chaos, as having been referred to the qualitative exploration of unstable aperiodic behaviors in deterministically nonlinear dynamical complex systems, bears a plenus of definitions where unstability means the system does not settle into a form of behavior resisting small disturbances, while aperiodic behavior signifies the variables' description of a state of a system that does not go through an iteration of values, which comes to mean that the system in question does not repeat itself at all continuing to manifest the impacts of any slight perturbation. Notwithstanding, these conditions render exact predictions impossible, yielding a series of measurements that are apparent randomly on small disturbances, which is a situation more commonly known as the 'butterfly effect' referring to the fact that even a very minor and remote factor can produce disruptions with a large-scale magnitude; and thus, sensitive dependence on initial conditions marks

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the chaotic systems' distinctive features among which being topologically mixing and having dense periodic orbit happen to be the other ones. The indication of time-chaos, on the other hand, referring to sensitivity to initial conditions means that when one has you have two sets of initial conditions or as another option two points in phase space, extremely in proximity with each other, the two ensuing trajectories, are close to each other at the beginning, will show eventual and exponential divergence away from each other. The other principle chaos theory lies on is uncertainty that interdicts accuracy while the third principle belongs to strange attractors showing that complex systems are inclined to settle in one specific situation. When the situation is dynamic, it is known as "strange attractor", whereas it is referred to as attractor when it is static. Given all these, small perturbations bring about chaos in a chaotic system, and chaos theory is involved with the way order irrupts into chaos, whereas complexity theory, suggesting the conception that there is order within chaos, emphasizes self-organization related to chaos into order. With a plethora of diverse independent variables and constituents in nonlinear interaction with one another, complex systems exhibiting a unique characteristic known as emergence, with interactions among subcomponents producing novel properties surpassing individual capabilities can be stated to provide balance in order and chaos (Farsi 2017).

Key Constructs of Complexity Theory, Complex versus Chaotic Systems, and Chaos

Complex systems are said to be more coherent compared to chaotic systems, with uncertainty arising differently in both systems. Complexity theory, providing the implications of analysis and explanation of complex systems, addresses the emergence of order in complex systems at the edge of chaos which signifies a point across the boundary oscillating between randomness and determinism. Thus, complexity theory focuses on non-deterministic systems, whereas chaos theory rests on deterministic systems (Karaca 2022b). Furthermore, uncertainty in chaotic systems results from the inability of knowing the initial condition of the system, whereas uncertainty arises from the notion of emergence in a complex system (Lartey *et al.* 2020). Concerning the uncertainty quantification, which is the quantitative characterization and estimation of uncertainties in both computational and real-world applications, attempts to determine the degree of likelihood regarding certain outcomes if certain aspects of the system are not know in an exact sense. Aleatoric uncertainty refers to a sort of uncertainty that is peculiar to a problem or to an experimental setup in which it is not possible to do reduction to additional experimental knowledge or physical lineage (Barbano *et al.* 2022). As Pierre Simon Laplace put forth, the theory regarding probabilities lies at the bottom of common sense that is reduced to calculus, which enables one to appreciate the exactness an accurate mind can feel based on a kind of instinctive hunch that cannot often be accounted for (Pierre-Simon 1986). Chaos-based applications in science, engineering and other relevant domains require the understanding that some chaotic systems display a unique feature by having two or more coexisting attractors with every attractor being achieved due to the same range of parameters which depend on the initial condition at stake. In these respects, multistable chaotic systems are equipped with the potential applications correspondingly with several parameters of multistable dynamical systems that have sensitivity to initial conditions, noise as week as system parameters. The appearance of hidden attractors, associated with multistability, demonstrates the existence of self-excited attractors in multistable systems with the employment of computational processes. Yet, it is not possible to predict the hidden attractors by typical computational approaches, and thus, the growing level of complexity in physical problems requires more complex and advanced mathematical differential operators. Fractal-fractional operator provides the combination of fractional differentiation with fractal derivative for performing a single differentiation. All these physical processes exhibit attributes characterized by a fractal nature (Khan *et al.* 2023). Both complexity and chaos, being deeply rooted in physics, display the endeavor through an attempt to observe similar systematics across an extensive varying range of phenomena so that a more profound and precise understanding thereof can be achieved. Comprised of a set of mathematical concepts, chaos and complexity theory provides the description of the way systems change over time. Mathematical modeling, oriented towards describing multiple and diverse facets of the real world, reciprocal interactions and dynamics of them from the lenses of mathematics, needs to tackle universal concepts efficiently, promptly and accurately. From this point of view, mathematical models are unique in that they enable the control, mechanization and automation of intellectual activities as well as processes. Mathematical models depending on specialized knowledge are those which with inherent mathematical nature encompass the process of determining the properties of a model with rigor elucidating the different multiple components being identified, revised, designed, organized, formulated and arranged in harmony. Given all these, chaos and complexity theory provide a synthesis of emerging wholes of individual components unlike some of the traditional scientific approaches that analyze systems in isolation.

Both mathematical modeling and scientific computing are considered to be amongst the chief tools for the purpose of solving the challenges and problems related to complex systems by means of innovative ways attributable to data science with a precisely-tailored approach so that sense can be derived from chunks of big data. This kind of tailor-made customized approach can only realize the opportunity of examining data applied, which heavily relies on the capacity of the computer at work as different capacities of computers can have impact on the computational outputs, and thus, the application of the method in question is based on the code by step to be taken into account. Therefore, the complexity definitions needs to be weighed over different data offering a highly extensive applicability spectrum endowed with more practicality, convenience and availability due to the fact that the respective processes lie in the concrete mathematical foundations, which all may as well indicate that the methods are required to be examined thoroughly regarding their mathematical foundation in conjunction the methods to be applied. This is the sole manner which can make foreseeability possible as regards what level of complexity will emerge concerning any data chosen to be employed.

Key Constructs of Nonlinearity, Complex Dynamics Systems, Chaos and Order

Nonlinearity, being a required condition for chaos, with almost all nonlinear systems whose phase space having three or more dimensions, display chaotic features in at least part of the phase space. Exhibiting complex dynamics, complex systems which span across several scales, display order and chaos in a simultaneous way, operating at the critical "edge of chaos", which provides maximization of emergence, spanning from micro-level to macro-level to illustrate the propagation of critical decisions ranging from lower to higher levels, adaptability, creativity and evolvability. While complex systems may have several scales, chaos may reign upon scale n, with the coarser scale above it (scale n-1) which might be self-organizing, which indicates that it is the opposite of chaos in some sense. When there is the case of the edge of chaos, the precise value of the control manifests a switching dynamics, which happens to be a critical point in phase transitions where the long-range correlations are significant. At this point, adaptability with memory, the capability of modifying the environment to be able to operate appropriately at the edge of chaos, becomes evident and it is this place where self-organization becomes likely to occur. Consequently, the interplay between chaos that producing new possibilities and order coupled with them ensures self-organization and open-ended evolution (Baranger 2000). As a dynamical system dependent on diverse parameters, complex systems manifest themselves in a constant sort of evolution formed by a huge number of unities and distances along trajectories increase or decrease in a polynomial way not exponential way. Furthermore, fractal structures are to be seen in many complex systems (Palis 2002).

Sharing fundamental features with chaos theory, complexity theory encompasses nonlinearity, dynamism, feedback, loops, and so forth. Both being sensitive to initial conditions result in unpredictable outcomes, and self-organization, in this regard, is emphasized with global patterns emerging from local reciprocal interactions. As a compelling challenge, chaotic systems are ones belonging to the unknown unknowns with chaotic motion being almost impossible or very challenging to forecast. Thus, the chaotic behaviors of correlations in chaotic systems prove the hardships concerning prediction of the chaotic systems, while the identified state transitions of correlations can lend a quantitative rule for the selection of appropriate methods. Systems that are deterministic, made up of simple differential equations, are not attributed to reference points to implicit chance mechanisms. Complex systems ofttimes display self-organization which arises when systems spontaneously order themselves optimally or in a more stable way without the external adjustment of any control parameters, which is a feature not found in chaotic systems. This situation is often referred to as anti-chaos in chaotic systems that are inclined to be out of equilibrium, meaning that the system does not settle into a steady state of behavior, which refers to the notion of openness. Most of the real-world systems are open, which poses problems in terms of modeling and experimentation. One other feature related to complex systems is the notion of feedback where the output of a process in the system is exposed to being recycled, as a result of which the output becomes the new input of the system. Feedback occurrences in complex systems are seen to be across the levels of organization, which are micro levels and macro levels. Between the subunits of micro level interactions, some patterns are generated, reacting back again which is a global o local positive feedback known as coevolution which is a concept originating from evolutionary biology for the description of how organisms create their environments and how they are in return molded by the environment they exist in (Rickles *et al.* 2007), (Ruhl 1995). Chaotic systems do not depend on their history unlike the complex systems which rely on their history. Across this line, chaotic behaviors push a system acting in equilibrium into chaotic order out of order. Complex systems, on the other hand, evolve distantly from the equilibrium at the edge of chaos.

Chaos theory posits that even the most seemingly random processes can be described and predicted through the use of a set of complex mathematical equations. Concerning nonlinearity and complex dynamics with chaos, it was noticed by French mathematician Henri Poincaré that nonlinear deterministic systems could behave in an apparently chaotic and unpredictable way. Despite this important contribution, the significance of chaos was accredited with full appreciation after the extensive availability and exponential growth of computational processes through digitalization employed for numerical simulations as well as for the demonstration of chaos in various physical systems. Figure 1 depicts the Poincaré section in z = 0 along with the return maps having three associated elements and the scaled axis system, demonstrating the sensitivity to changes in initial conditions, which is an important characteristic of chaotic systems.

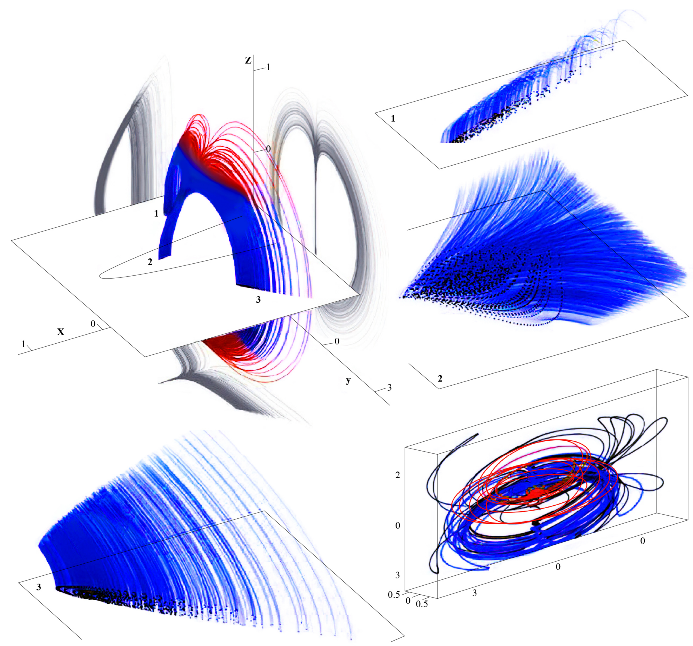

Figure 1 3D perspective segment of a typical chaotic attractor system having a hyperbolic equilibrium.

Given these notions, facts and considerations, the role of mathematical modeling and scientific computation comes to the foreground in processes, including analyses, decision-making, solution of real-world problems, prediction and simulation. These processes entail the definition of which level of detail needs to be introduced in different parts of a mode along with which simplifications are to be conducted to achieve its integration into different models emulating highly complex problems while considering uncertainty as well.

Key Constructs of Bifurcation Theory, Control, Strange Attractors in Complex Chaotic Systems

Bifurcation theory is concerned with the examination of changes in topological or qualitative structure of a given family of curves including the integral curves of vector fields as well as the solutions concerning differential equations. Bifurcation theory is generally applied to the mathematical study of dynamical systems with bifurcation introduced by Henri Poincaré in 1885 occurring both in continuous, characterized by ordinary, delay or partial differential equations, and in discrete systems, described by maps, when a slight smooth change made to the parameter values of a given system brings about an abrupt change in its behavior (Poincaré 1885), (Blanchard *et al.* 2006). Local bifurcations can be analyzed by changes in local stability of equilibrium, periodic orbits and other sets which are invariant as parameters across critical thresholds, whereas global bifurcations often occur as larger invariant system sets collide with one another, which cannot be detected by fixed points. Causing sudden changes in the system's direction, outcomes and characteristics, bifurcation points are those in the system being unstable, leading the system to change structure, character or direction in a dramatic way (Lartey *et al.* 2020). If these unstable points are known, prediction of a bifurcation can be ensured, however, the outcome of the bifurcation or the next state of the system remains unpredictable.

Control theory, in these regards, are concerned with the questions of how the behavior of a system is influenced through the inputs appropriately chosen in order that the output of the system adopts a desired final state or trajectory. Feedback process happens to be the key notion of control theory, with the difference between actual and desired output is implemented as feedback into the input of the system so that the output of the system is completed to converge into the output desired. Correspondingly, bifurcation is used to describe significant qualitative changes occurring in the trajectories of a generally nonlinear dynamical system, considering that the key system parameters are varied. Since a control input and feedback are involved in a nonlinear control system, its nature is very complex as a dynamical system (Chen and Moiola 1994). Even though control input is given and fixed, the controlled system is a non-autonomous dynamical system. Therefore, control input is necessary to be determined to achieve a certain performance considering the combination of the design and dynamics of controllers, which is challenging. All in all, bifurcation equilibrium, oscillation and therefore chaos is found in many systems and new kinds of attractors representing a new sort of behavior entails the understanding that nonrandom chaotic behavior enables the handling of the system. Demonstrating the repetitive abilities, chaotic systems may enable the identification of strange patterns, and although dynamic systems are unpredictable, they still keep the boundaries where they operate their transformations. This creates patterns which are referred to as strange attractors which have different shapes and forms characterizing chaotic systems (Murphy 1996). It is these strange attractors which define the dynamic systems' boundaries since such systems show progress towards chaos owing to their constant growth, and they are identifiable as well as measurable through the use of fractals.

Key Constructs of Fractal Methodology, Real Data Interpolation and Applications in Complex Chaotic Systems

In mathematics, fractal, signifying any of a class of complex geometric shapes with a common attribute of fractional dimension was first introduced as a concept by mathematician Felix Hausdorff in 1918. Distinctive from the simple figures pertaining to Euclidean or classical geometry, fractals are endowed with the capability of describing diverse irregularly shaped objects or spatially nonuniform phenomena in nature from cliffs to seashores, coastlines to mountain ranges. The term *fractal*, as derived from *fractus* meaning fragmented or broken in Latin was coined by Benoit B. Mandelbrot. A fractal system, as a complex, nonlinear interactive system, has the ability of adapting to a changing environment, and it is marked by the self-organization potential existent within a nonequilibrium setting. Fractal theory, on the other hand, has sought to comprehend seeks to complexity in order to ensure an innovative way for the identification of irregularity and complex dynamical systems. The applications that deal with fractal geometry concern various subject matters from turbulence to errors, word frequencies to aggregation and fragmentation-related processes. The growth of the use of fractals in application areas has spawned not only new directions but also new methodological issues. Furthermore, multifractals, arising as a more complex form of fractals, have paved the way of multifractal analysis with the assignment of fractal spectrum to an object, while fractal analysis provides the assignment to a single fractal value. Consequently, multifractal algorithms have been proposed to be employed for practical applications to characterize the signals in medicine, clinical research, biology, and so forth (Karaca 2022a), (Karaca *et al.* 2022).

These developments challenge the prescriptions of reductionism, which assumes that the resultant component behavior and dynamics provide the representation of the entire system behavior by synthesizing approaches and showing that in most complex systems, there is a high level of interconnectivity, dynamic aspects and reasons attributed to nonlinear behaviors (Gowrisankar and Banerjee 2021). While complexity theory explores the way individual components generate simple outcomes by nonlinear and intense interactions, chaos theory explores the possible ways simple systems generate complex outcomes which cannot be described through the components per se (Watt and Willey 2005). Fractals, as very complex, having symmetry of scale and being infinitely detailed geometric shapes show the direction of a procedure that describes the way of constructing and defining a small section in which their small sections resemble the large ones. For a function, one can consider fractal as follows: $f(x)$ to x, $g(x)$, $g(g(x))$, $g(g(g(x)))$, $g(g(g(g(x))))$, $g(g(g(g(g(x)))))$, etc. Given all these aspects, fractals are related to chaos as they both are complex systems with similar properties (Meta 2016). By assessing the fractal characteristics of data, fractal analysis is made up of different methods for assigning a fractal dimension to a dataset, whether it be pattern or signal, which makes it helpful in understanding the functions, structures as well as *spatial and temporal complexity* of various systems, and thus, facilitation is provided quantifying patterns in nature and identifying deviations from such natural sequences.

As the process of using known data values for the estimation of the unknown data values or a missing value, data interpolation is used as a method to predict the future based on the past trends and data, which improves the way to collect data and work on it (Karaca and Cattani 2018). Among some of the elements interpolated are dense evenly space points, extreme changes in terrains, obstacles, and increase or decrease amount of sample points which influence cell values. The values of non-sampled data from a set of discrete sensory data are measured by interpolation which is required in different fields as sensors cannot constantly cover the region under study. Natural systems include complex dynamics which extend across multiple spatiotemporal scales, and efforts to understand and forecast the dynamics of these systems have brought about advances in large-scale simulations along with the dimensionality reduction techniques and a multitude of complementing forecasting methods. High dimensionality and chaotic behavior of the systems reveals a convergence of different approaches as a result of the advances in innovations in algorithms, computing power and ample data accessibility.

Key Constructs of Fractional Mathematics, Fractional Calculus and Data-intensive Computational Application Processes in Complex Chaotic Systems

Fractional mathematics along with the application of fractional calculus techniques oriented towards the solution of problems can describe the existence characteristics of complex natural, scientific and engineering-related as well as medical systems in a more accurate way to reflect the actual state properties, besides the co-evolving entities observations and patterns of such systems truly concerning the nonlinear dynamic systems and modeling complexity evolution in combination with order of fractional chaotic as well as complex systems (Karaca 2022a). Notwithstanding, Fractional calculus (FC), deeply related to the dynamics of complicated real-world problems, allows emerging processes in various fields adopting fractional dynamics rather than the ordinary integer-ordered ones, which means the respective differential equations feature non-integer valued derivatives (Jacob *et al.* 2020), (Karaca 2023).

Fractal patterns, albeit in an array of scales rather than in an infinite manner, having been modeled extensively due to the time and space-related limits concerning practice-wise elements (Karaca and Cattani 2017), (Karaca *et al.* 2020). It is possible that the models might simulate theoretical fractals or natural phenomena with fractal features, and the results derived from modeling processes can be employed as benchmarks for fractal analysis purposes. Fractional calculus, which emerged as a formulation extending ordinary calculus, procures a constructive and algorithmic approach towards the smooth differentiable-structured modeling of natural processes through fractals. Fractal calculus procures a constructive approach towards the smooth differentiable-structured modeling of natural processes through fractals which are perplexing to solve, while differential equations concerning fractals congregate a profound understanding of analysis along with different constructions. The models constructed accordingly can be applied to processes that occur in fractal time and spaces, which propounds the dimensionality aspects as well as the endless patterns at temporal and spatial scales. To put differently, the application of calculus concepts as well as techniques can be beneficial for analyzing and describing the behavior of not fractal objects only but also systems.

With the inherent feature of fractional derivatives in terms of spatiotemporal memory as well as the capability of expressing phenomena occurring in a naturally complex way, machine learning, as a powerful tool, has also come to the foreground in an integrated way owing to its learning behavior and patterns based on historical data lending upper hand in analyzing data, solving problems, modeling, prediction, and so forth by providing new genesis and points of view. The potential of the combination of these approaches facilitates the description process of complex dynamics based on the schemes relying on fractional derivatives and machine learning with novel and innovative corresponding techniques. Furthermore, with its differentiation and integration of non-integer order, FC provides the representation of the generalization of classical differential and integral calculus, providing an amalgam of computational methods concerning various complex systems in tandem with fractional derivatives, fractional differential equations, fractional wavelet, fractional entropy, fractional neural networks, fractional fuzzy, and so on to open the frontiers towards systematic optimized solutions, tackling the systemic properties holistically by seeing through the spontaneous processes (Karaca and Baleanu 2022b), (Karaca and Baleanu 2022a).

Data, being at the center of many compelling challenges in system design, modeling and other related processes, require the need of figuring out reliability, efficiency, consistence, maintainability, scalability. The real-life applications of data-intensive systems and applications make an intensive use of data in all their heterogeneous forms, and computational problems can be solved in this sort of a nested network with concurrent or distributed systems paying attention to operational processes, memory, communication between nodes, machine instructions, among many other processes and elements. Based on the voluminous amounts of data produced by experiments as well as high-throughput technologies disseminated by cyberinfrastructures, data-intensive research comprises a rich variety of scientific methodology that shares the common feature of relying on the accumulation and sharing of evidences across an extensive scale and research contexts, ranging from automated data analysis and automated reasoning to extraction of significant patterns in exact sciences based on data through computational means with human intervention as minimized as possible. With these amenities, applications of data-driven methods have demonstrated that computational methods are empowered with transforming research substantially in terms of how it is performed and the ways by which experiments are set up, conducted and verified. Considering that slight perturbation leads to a significantly divergent future concatenation of events, pinning down the state of different systems in a precise way can to some extent unveil uncertainty. Predicting the future evolution of chaotic systems can show the direction to distant horizons with extensive applications to understand the internal machinations of neural and chaotic complex systems.

Key Constructs of Machine Learning, Algorithmic and Artificial Intelligence-related Application Processes in Complex Chaotic Systems

Chaos theory evolved from a niche mathematical field into a transformative force, demonstrated that quite simple mathematical equations were able to model systems with each bit as violent as a waterfall (Gleick 2008) Rooted in the exploration and investigation of dynamic systems with extreme sensitivity to initial conditions, the world of chaos theory has projected an exponential impact on the realm of Artificial Intelligence (AI) by empowering it in terms of tackling complex problems and providing enhancement in adaptability and learning capabilities related to the AI algorithms. Besides sensitivity to initial conditions, chaos also arises in nonlinear systems with relationships across variables may not be proportional, which is known as nonlinearity that presents intricate and unpredictable behaviors. Another characteristic is the strange attractors as chaotic systems are known to exhibit randomly appearing but deterministic and self-similar complex patterns in a related system's behavior, known as strange attractors. Randomness abruptly becomes an orderly disorder, both in existential terms and in the real-world scenarios where there is a hidden order to chaos. The incorporation of chaos theory and AI provides other improvements in the predictive capabilities of AI algorithms through the introduction of adaptability, which makes chaos theory respond to even slight changes in the input data bringing about a higher level of predictive accuracy. Furthermore, chaos-based algorithms are employed for the optimization of neural network architectures and training processes. That being said, chaos theory also provides facilitation in feature selection, namely the identification of significant attributes in complex and big datasets, leading to more efficient AI models and more streamlined in the meantime. Sensitivity to initial conditions make chaos theory a significant one in the detection of anomalies, which allows the AI systems to identify critical deviations from normal behaviors or those which are unexpected. Besides these, chaos-based data augmentation techniques host controlled perturbations, which improves the generalization capabilities concerning the AI models. Last but not least, reinforcement learning attribute in chaos theory is applied to enhance the AI agents to discover the related environments in a more effective way, which also results in coming up with optimal policies.

Referring to models employed to find patterns within data, a wide variety of advanced machine learning methods, including supervised, unsupervised and reinforcement learning, can be utilized effectively for prediction and classification with a nested hierarchy of features. As for deep learning, it is utilized for solving the same kind of problems as in conventional machine learning means, yet the difference lies in the models' architecture to comprehend the way the decisions are made. Machine learning means are run through the estimation of parameters bringing about the optimal outcome possible, with parameters being existent for each input feature in simple linear cases of models. Deep learning models also exhibit corresponding parallelism, and yet, they integrate more features compared to conventional means oriented towards making predictions. Generating new features from the input features as integral to the training process, deep learning does not use the combination of input features for direct prediction. Machine learning, as the prediction state evolution of chaotic systems, is considered to be an emerging paradigm along with reservoir computing that has a core with dynamical network made up of artificial neurons, which can provide facilitation in predicting unexpected situations like system collapse and chaotic transients linked with crisis situations as well as bifurcation points and asymptotic behaviors (Kong *et al.* 2021). Managing uncertainties and changes in processes can happen on machine learning level with pattern recognition in addition to algorithmic processes which lends applicable processes to solve mathematical problems in a finite set of steps involving recurrence broadly. Artificial Neural Networks (ANNs) and AI systems, in that regard, have their applications with overlapping fields concerning process modeling, adaptive control issues and tool condition monitoring with a focus on learning abilities with the recognition that it is not possible to treat learning separate from other points like signal processing, fusion abilities, critical decision making and self-calibration, among others (Monostori 2003). Thus, different machine learning techniques have significant impacts on building effective models in various application terrains based on the learning capabilities, the particular nature of data as well as the targeted outcome.

The rest of the Editorial for our special issue is organized as follows: Section 2 presents the Work in Progress providing the overview information and inputs of the accepted papers compiled and published. Finally, Section 3 is comprised of Concluding Remarks, Challenges and Future Directions.

WORK IN PROGRESS

Comprising of a set of mathematical concepts, chaos and complexity theory propounds the description of the way particular systems evolve over time, and in this context, chaos-based applications in engineering, science, applied sciences, mathematics, physics, medicine, biology, and other related realms require the reflective, holistic and accurate comprehension, which unveils a rigorous attempt to observe similar systematics spanning across a broad varying range of phenomena. Mathematical modeling and scientific computing also serve these purposes while describing, analyzing and interpreting multiple aspects of the real-world problems blended with the dynamics, complexities and reciprocal interactions in addressing universal concepts effectively. Thus, the integration of mathematical modeling and computational methods empower solution-oriented approaches related to chaotic and complex systems based on innovative ways that can be ascribed to data science from a precisely customized perspective while dealing with large chunks of big data. With reference to the content of accepted papers, the aim of our special issue has been to provide novel directions based on advanced mathematical modeling and computational practicalities in conjunction with chaos-driven model training as well as optimization methods.

Across these strands of thought and aspects, deep learning approaches, deep neural networks, fractional calculus, approximation theory, medical imaging, image denoising, machine learning methods, learning algorithms, complexity, wave propagation, Newtonian mechanics on fractals subset, bifurcation, PDEs, ODEs, wave equations with different models as Nonlinear Coupled Konno-Oono model, Jaulent–Miodek, Korteweg–de Vries (KdV) equation, peak signal-to-noise ratio, Cantor sets, n-Term Klein-Gordon equations, local fractional Laplace equation related to complexity and chaos in electromagnetic fields, fractal methodology, fractal spline, non-differentiable fractal functions and linear fractal function have been addressed, explained and exemplified through the schemes of different areas including physics, mathematics, fluid dynamics, medicine engineering, science, control, optimization geared towards applicable solutions. The theoretical and applied dimensions of nonlinear dynamics and complex systems, merging mathematical analysis, advanced methods and computational technologies have been presented for exhibiting the implications of applicable approaches in real systems and other related domains. Accordingly, the main contributions, novelties and contents of the seven papers accepted for our special issue are provided herein.

Deep learning and machine learning have had a pivotal impact in healthcare systems owing to their capability of handling large complex data with as minimal human intervention as possible, and thus, the applications of deep learning and machine learning are geared towards the achievement of a higher level of service quality besides the quality of health concerning patients, doctors, researchers, practitioners and healthcare professionals. Among the critical tasks deep learning and machine learning have proven to be effective are acute disease detection, disease diagnosis, classification, image analysis, signal analysis, drug discovery and delivery as well as smart health monitoring, among others. Accordingly, The first manuscript in our special issue entitled "Unveiling the Complexity of Medical Imaging through Deep Learning Approaches" presents a comprehensive review of deep learning methodologies which are applied to different healthcare aspects with a focus on various tasks among which disease segmentation, classification and detection are included (Rasool and Iqbal Bhat 2023). The study provides contributions in terms of the intricate nature of medical imaging, revealing the hidden patterns by the application of deep learning-related approaches. Furthermore, the authors of the manuscript provide the discussion of the key features and characteristics of deep learning approaches and significant contributions made by different deep learning techniques in the field of medicine, highlighting the classification approaches and advancements in medical imaging, with a specific emphasis placed on the Convolutional Neural Network (CNN) as a popular method in computer vision tasks. The merits and demerits of various deep learning methods are also depicted through an evaluation in tabular format. The findings indicated through the study reveal the immense potential and benefits belonging to deep learning technology in healthcare, which can empower researchers and practitioners while navigating through the complexities of medical imaging with enhanced diagnostics and interpretation.

Wave propagation is one of the cornerstones in the study of linear and nonlinear Partial Differential Equations (PDEs) where a wave is referred to as a recognizable signal transferred from one part of the medium to another part of it at an identifiable speed of propagation. In this regard, the transfer of energy occurs as the wave propagates, yet, for the matter, it may not be the case. A travelling wave, advancing in a particular direction with the addition of retaining a fixed shape is associated with a constant velocity throughout its related propagation course. It is possible to observe these kinds of waves in various scientific areas such as in combustion occurring after a chemical reaction. In addition, PDEs are one result of the mathematical modeling of dynamical systems, and phenomena such as conservation, reaction and diffusion, to name some can be expressed by means of PDEs which owing to their quintessence are examined profusely in science and engineering. In this regard, Lie symmetry analysis is known to be a robust tool to mathematically analyze PDEs, and it can be employed to secure analytic solutions or to converge PDEs into solvable ordinary differential equations (ODEs). Correspondingly, Nonlinear Coupled Konno-Oono model (NCKOM) represents a current-field string interaction with an external magnetic field, whereas Jaulent–Miodek (JM) equation is a kind of evolution equation possible to be identified in physics, remarkably fluid dynamics, matter physics as well as optics to describe these aspects. The next research paper with the title "Novel Traveling Wave Solutions of Jaulent-Miodek Equations and Coupled Konno-Oono Systems and Their Dynamics" provides the contributions with regard to deriving of some novel variety of solutions for Jaulent-Miodek equations (JMEs) and coupled Konno-Oono equations (CKOEs) (Kumar *et al.* 2023a). (1+1) coupled Jaulent-Miodek system of equations is associated with the energy-dependent Schrödinger potential, while the coupled Konno-Oono system related to complexity and chaos in electromagnetic fields are solved analytically in the research in question. Similarity reductions via Lie-symmetry analysis is carried out for the systems to derive their analytical solutions. The authors supplement the analytical solutions graphically to shed light on the dynamical behavior of the solutions. The research paper, which has dealt with the Lie-symmetry analysis as explored, provides the obtaining of seven analytic solutions for the CKOEs and two analytic solutions for the JMEs. Similarity reductions are conducted by the authors via Lie-symmetry analysis so that it can be possible to derive the related analytical solutions. As another contribution, traveling wave profiles are obtained and solution for CKOEs are shown to different from the one obtained by an earlier research.

As a prototypical example of an exactly solvable nonlinear system, the Korteweg–de Vries (KdV) equation aims at describing shallow water waves which are in nonlinear and weak interactions, concerning long internal waves in a density-stratified fluid, ion acoustic waves in a plasma as well as acoustic waves on a crystal lattice. As a model for many physical phenomena including the propagation of small-amplitude large-wavelength waves in plasma physics and shallow waters, the Korteweg–de Vries (KdV) equation is considered to be an extensively-employed model. On the other hand, bifurcation in dynamical system happens in the case a slight smooth change exerted to the parameter values, namely bifurcation parameters, of a system leads to an abrupt topological or qualitative change in its behavior. Within this regard, the authors of the subsequent work "Study of Fixed Points and Chaos in Wave Propagation for the Generalized Damped Forced KdV (GDFKdV) Equation using Bifurcation Analysis" consider the Generalized Damped Forced KdV (GDFKdV) equation given by $U_t + PU^n U_x + QU_{xxx} + SU = \gamma F(U, x, t, v_i)$ with P, Q and S denoting non-linear, dispersion, damping coefficients, respectively (Chadha and Tomar 2023). The authors also investigate the behavior of the fixed points evaluated for the corresponding dynamical system of their model problem. In addition, the effects of significant parameters involved in the model, which are the free parameters v_1 and v_2, the nonlinear, dispersion and damping coefficients denoted by P, Q and S respectively, are analyzed using the bifurcation tools. Another input to note is the obtaining of the plots for the critical values of the nonlinear and dispersion coefficients for which the system becomes unstable and exhibit chaotic behavior. The chaos in the related dynamical system under various conditions is confirmed with the help of the Lyapunov exponents.

Approximation theory having a significant role in machine learning regarding its tasks like classification or regression plays a key role with its techniques in terms of learning from the data. Via a learning algorithm, many machine learning methods approximate a function or a mapping between the inputs and outputs, and a typical example of models approximating functions in classification tasks is one that belongs to neural networks which are as a whole assumed to be able to approximate a true function mapping the inputs to the class labels. Deep neural networks, on the other hand, own the same order of computational complexity as deep convolutional neural networks. Across these lines, another paper entitled "Different variants of Bernstein Kantorovich operators and their applications in Sciences and Engineering field" aims to highlight the different variants of Bernstein-Kantorovich operators which are used extensively for the approximation of functions in L^p spaces (Bhardwaj and Bawa 2023). The authors put forth the benefit of employing Kantorovich variants over discrete operators that are not suitable for approximating functions which are not continuous. Thus, the operators are generalized into operators of integral type, Kantorovich being one technique which helps to approximate integral functions. The study provides the other inputs addressing the discussion of the important applications of Kantorovich operators that depict the pragmatic and theoretical aspects of approximation theory which concerned with the approximation of complicated quantities by simpler functions.

Having become more significant over the recent times in different fields including but not limited to medical imaging, defect detection, machine vision, image processing provides practical benefits like making the digital image available in any wanted format which improves the images for human interpretation and enables the processing and extracting of information for machine interpretation. Likewise, the process of denoising aims at enhancing the quality of the image through noise reduction while preserving the significant structures and details. Image denoising removes noise from a noisy image so that the true image can be restored, yet, due to factors such as edge, texture, noise, sharp structures and texture pose difficulties in these processes. It is possible that denoised images cause to lose some details, so when an image is being denoised, it is of importance to keep the visual details and components mentioned above. The peak signal-to-noise ratio (PSNR) is the one of the frequently emplpyed objective measure to assess perceptual image quality in tasks related to images and video compression. In the next paper entitled "Weighted and well-balanced non linear TV based time-dependent model for image denoising", the authors address image denoising and deblurring issues which require the adoption of a time-dependent model as a fundamental idea (Kumar *et al.* 2023b). The aim of the research is to enhance the image formation process, and weighted well-balanced flow as a total variation-based time-dependent model is utilized by the authors for the purpose of removing additive noise while preserving the edges successfully. As another contribution, the authors apply the new variation of the flow in the TV-based time-dependent model. The weighted model is said to improve the quality of the restored images and preserve the edges better. The numerical results, which are expressed as a static known as the peak signal-to-noise ratio (PSNR), demonstrate that the scheme proposed yields better results compared to the previous model.

As a method used to solve for a broad range of problems that have mathematical models yielding equations or systems thereof, the differential transform scheme is effective. The Cantor set, created by repeatedly deleting the open middle thirds of a set of line segment, is a closed set that entirely consists of boundary points, which is a noteworthy counterexample in the fields of set theory and general topology. The local fractional calculus is applied for modeling and processing non-differentiable phenomena in different fractal physical phenomena, with some local fractional models being wave equations on the Cantor sets, local fractional mechanics of elastic materials, Newtonian mechanics on fractals subset of real-line, local fractional Laplace equation, and so forth. The subsequent paper named "Analysis of the *n*-Term Klein-Gordon Equations in Cantor Sets" aims at demonstrating the effectiveness of the local fractional reduced differential transformation method (LFRDTM) in approximating the solution of the extended n-term local fractional Klein-Gordon equation (Goswami *et al.* 2023). For this aim, the authors use the fractional complex transform and the local fractional derivative, in combination, to analyze the n-term Klein-Gordon equations and in cantor sets. The method proposed by the paper is said to provide a powerful mathematical instrument for solving fractional linear differential equations. As the other contributions, the authors address the existence of the solution followed by some examples. Ultimately, the study provides an effective and accurate method for modeling complex physical systems displaying fractal or self-similar behavior at various length scales. The authors conclude that the fractional complex transform with the local fractional differential transform method proves to be a powerful and flexible approach for obtaining effective approximate solutions of local fractional partial differential equations. By demonstrating the effectiveness of the LFRDTM in approximating the solution of the local fractional Klein-Gordon equation of term n, the authors also expect to encourage its use in an extensive range of applications in fields like physics and engineering.

Providing a general setting and context to understand real-world phenomena, fractal methodology provides the generalization of real-data interpolation by means of fractal techniques. Numerous mathematical models developed and which can generate free-form shapes show two varieties which are known to be deterministic and stochastic. With deterministic qualities, spline models have established themselves to be powerful and convenient to model smooth shapes. On the other hand, fractal models are used to recreate different shapes which are found in nature, and most fractal models are endowed with stochastic components, which render them appropriate to generate irregular, nonsmooth shapes. In this regard, a fractal spline is a function which is made of spline functions having different sclaes maintaining the self-similarity attribute. Consequently, the last study in our special issue "Fractalization of Fractional Integral and Composition of Fractal Splines" is concerned with the perturbation of fractional integral of a continuous function f defined on a real compact interval, namely $(I^v f)$ by means of a family of fractal functions $(I^v f)^\alpha$ reliant upon the scaling parameter α (Apulprakash 2023). The authors of the study propose a fractal operator within the space of continuous functions, an analogue to the existing fractal interpolation operator perturbing f, which results with α-fractal function f^α to elicit the phenomenon. The composition of differentiable fractal function $h^{(k)}$ with a non-differentiable fractal function g yields a non-differentiable fractal function $g(h^{(k)})$, which satisfies the end point conditions that are necessary. Furthermore, the study provides the discussion regarding the composition of α-fractal function with the linear fractal function besides the extension of the composition operation on the fractal interpolation functions to the case of differentiable fractal functions.

CONCLUDING REMARKS, CHALLENGES AND FUTURE DIRECTIONS

Chaos theory is capable of offering an alternative that describes and explains the particular behavior of some nonlinear systems, fundamentally almost in all naturally occurring physical, biological, chemical or social systems or structures. This qualitative exploration of unstable aperiodic behaviors in deterministically nonlinear dynamical complex systems also holds a plethora of definitions in which unstability means the system resists small disturbances and does not settle into a form of behavior whereas aperiodic behavior denotes the variables in a state of a system which does not go through an iteration of values. These particular conditions can make exact predictions not possible; and thus, generates a series of measurements appearing randomly on small disturbances. Butterfly effect, uncertainty and strange attractors are some of the most notable features chaotic systems, whereas more coherence is attributed to complex systems where complexity theory addresses the emergence of order there at the edge of chaos, signifying a boundary point between randomness and determinism. Chaos-based applications in engineering, science and other related trajectories entail the profound and precise comprehension revealing a rigorous attempt to observe similar systematics over an extensive varying range of phenomena. Made up of a set of mathematical concepts, chaos and complexity theory grants the description how particular systems evolve over time. Mathematical modeling geared towards the description of diverse multiple aspects of the real world in addition to the dynamics and reciprocal interactions tackles universal concepts in a prompt, accurate and efficient way. As they are unique in enabling the mechanization, automation and control of intellectual activities and processes, mathematical models are acknowledged to be unique. The integration of mathematical modeling and scientific computing are the principles that empower means to solve challenges pertaining to complex systems through innovative ways attributable to data science with a precisely customized approach plausible sense can be derived from large chunks of big data.

Fractional mathematics encompassing the application of fractional calculus techniques can be used to solve problems that describe the existence characteristics of complex natural, scientific, engineering-related and medical systems accurately. FC is profoundly concerned with the dynamics of real-world problems, which allows emerging processes in diverse trajectories by adopting fractional dynamics rather than the ordinary integer-ordered ones. Fractal patterns, in an array of scales rather than in an infinite manner, have been modeled extensively owing to the time and space-related limits concerning practice-wise elements. It is possible that the models might simulate theoretical fractals or natural phenomena with fractal features, and the outcomes derived from modeling processes can be employed as benchmarks for fractal analysis purposes. As a result, the application of calculus concepts as well as techniques are highly beneficial to describe and analyze both behavior of fractal objects and that of systems. Owing to the inherent feature of fractional derivatives in terms of spatiotemporal memory and the capability of expressing phenomena that occur in a naturally complex way, machine learning is an integrated way through its learning behavior and patterns based on historical data, which provides benefits in analyzing data, solving problems, modeling, forecasting, prediction, and so forth by providing new genesis and perspectives. The coherent combination of these approaches facilitates the description process of complex dynamics based on the schemes relying on fractional derivatives and machine learning with novel corresponding techniques. Furthermore, with its differentiation and integration of non-integer order, FC provides the representation of the generalization of classical differential and integral calculus, providing an amalgam of computational methods concerning various complex systems in tandem with fractional derivatives, fractional differential equations, fractional wavelet, fractional entropy, fractional neural networks, fractional fuzzy, and so on to open the frontiers towards systematic optimized solutions, tackling the systemic properties holistically by seeing through the spontaneous processes. Data, while causing compelling challenges in system design, modeling and other related processes, also require the need of sorting out efficiency, reliability, consistence, maintainability and scalability. The real-life applications of data-intensive systems and applications make an intensive use of data in all their heterogeneous forms, and computational problems can be solved in this sort of a nested network with concurrent or distributed systems paying attention to operational processes, memory, communication between nodes, machine instructions, among many other processes and elements.

Challenges also become evident considering the asymptomatic, chaotic, complex, dynamic and nonlinear systems. If one has the aim of managing chaos and complex systems, it is important to identify the correct level of the system and consider it within its particular setting adopting vigilance by interpreting and analyzing the system. Therefore, the related challenge and wrongdoing is identifying individual agents as the agents of the system. Another compelling issue is the dependent components regarding the system's complexity as the result is coupled systems in a tight way if multiple components depend on each other, while the other one is managing work in progress with common delays in a certain workflow concerning feedback and making use of the related information. This causes somehow overload leading to problems and challenges due to highly chaotic and complex issues. To solve this challenge, tasks need to be managed constantly on track with no delays if possible. One more challenge worthy of mentioning has to do with predicting changes that are possible to occur. This challenge can be sorted out by managing chaos and complex systems by using advanced technologies and probing the trends as well as establishing forecasting models so that the evolutions of complex systems can be predicted with a relatively strong precision.

Based on these aspects, trends and challenges, the following points can be provided as future directions to unlock new frontiers in research and application terrains: novel and solution-oriented avenues can be explored for the ultimate the medical, clinical impacts of machine learning in imaging and signal processing. In addition to deep learning methods and parallel training implementation techniques, having become dominant in computer vision-related tasks, Convolutional Neural Networks (CNNs), made up of multiple building blocks, can be oriented for automatic and adaptive learning in spatial hierarchies. Another direction is related to large medical datasets and enhancing of the potential in minimizing overfitting and providing generalizability through better pre-trained sets of unis so that deep learning research can be fostered. Moreover, experiments with high dimensional or multimodal data to represent and analyze them through the selection of powerful tools. All these challenges and directions show that further research may be carried out so that it can be figured out how it could be possible to control and manage the chaotic behavior of different systems for the purpose of expanding validity, coherence and reliability concerning future plans, schemes and models. Chaos theory, in this regard, provides an alternative to explain and describe the behavior of nonlinear systems, and the rationale behind the use of chaos theory is to better understand the internal machinations of neural networks. As a matter of fact, being profoundly rooted in physics, complexity and chaos attempt to observe comparable similar systematics over a broad range of phenomena. Taken together, chaos and complexity theory provide a version synthesis comprising emerging wholes of individual components unlike some traditional scientific approaches which handle the analysis of systems in isolation. Unpredictability, being at the pedestal of some challenges, this approach is one way which can render foreseeability possible concerning what level of complexity will emerge related to the data chosen to be employed. As a last resort, what needs to be endowed with is the ability to see deep relationships and how they can fit in a whole coherently, which can be put differently as the simplicity on the other side of complexity.

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Authors' contributions

The Editorial for the special issue entitled *Advanced Fractional Mathematics, Fractional Calculus, Algorithms and Artificial Intelligence with Applications in Complex Chaotic Systems* has been written by Yeliz Karaca. Having conducted the related editorial duties and assignments, both Editors, Yeliz Karaca and Dumitru Baleanu, have approved the submission of the Editorial work pertaining to the special issue.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Availability of data and material

Not applicable.

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