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THE STRUCTURE OF CERTAIN UNIQUE CLASSES OF SEMINEARRINGS

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Abstract. In this paper, we introduce the classes of α and strictly-α seminearrings and establishes some of their properties, mostly in relation to the possession of a mate function. Then we get the criterion for an α-seminearring to become a strictly-α seminearring. We also obtain a complete characterisations of α and strictly-α seminearrings and proved certain results for α and strictly-α seminearrings via certain unique classes of seminearrings.
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1. Introduction

In the literature, the notion of seminearrings was firstly introduced in [18] (in a German version "Fasthalbring"). However, only a very special type of seminearrings was considered in [18] and the question arisen whether a more general theory of seminearrings can be developed and the results in [9] make it clear that such a theory was developed by expanding the existing theories of nearrings and semirings. Indeed, they have studied some fundamental properties of seminearrings and introduced various notions of ideals. Later, Albert Hoogewijs studied embeddings and $\mathfrak{I}$-congruences of seminearrings [8]. Weinert has made some significant contributions to seminearrings [20,21,22] and also studied non-associate seminearrings towards the contributions to seminearfields and seminearrings as ordered algebras. The theory was further enhanced by S. A. Huq [10], Javed Ahsan [1] and Tim Boykett [6]. For example, applications and investigations we further refer to [9,20]. Semigroup sets $(\Gamma, +)$ and its mapping with absorbing zero respect to $\mathfrak{M}(\Gamma)$, addition is pointwise and composition of mapping is the obvious example of seminearring. The role of seminearring structure applied in many places of theoretical computer science, viz. algebra communicating processes, theory of automata and also seen in semigroup mapping and reversible computation models. Nowadays research in the group theory is quite intense. The theory of group is a meeting point of various extensions of algebraic theories. This work is an attempt to extend the classical results of group theory to seminearrings. We consider the following group theoretic facts.

Let $(S,.)$ be a group, H and K be any two subgroups of S. Then (i) HK is a subgroup of S iff $HK = KH$ and (ii) H is normal in S if and only if $xH = Hx$ for all x in S. Now it is quite natural for us to extend these concepts to seminearrings and we have named α-seminearring and strictly α-seminearring. In this paper, the notions of α and strictly-α seminearrings are introduced and we discuss some of their properties with respect to the possession of a mate function.

This paper comprises four sections. Section 2 review some basic results and definitions about seminearrings. In Section 3, we define α and strictly-α seminearrings and give some results concerning such seminearrings. In Section 4, we obtain a complete characterisations of α and strictly-α seminearrings. We also furnish certain results obtained through the special structures of seminearrings.

2. Preliminaries

We consider some basic terminologies and the results related to seminearrings are used in subsequent sections.

According to [8,17] a (right or left distributive) seminearring is defined to be an algebra $(R,+,.)$ such that $(R,+)$ and $(R,.)$ are semigroups and $(a+b)c = ac+bc$ holds for all $a,\ b,\ c$ in R. If for all $a \in R$, $a\cdot 0 = 0\cdot a = 0$ and $a+0 = 0+a = a$, then R is zero absorbing. All along this paper, R always denotes a right seminearring with zero absorbing.

Non-empty subset $\mathcal{I}$ of R is a right (respectively left) ideal of R if (i) $l+a \in \mathcal{I}\ \forall\ l,a \in \mathcal{I}$ and (ii) $l\cdot r \in \mathcal{I}$ $(r\cdot l \in \mathcal{I})$ $\forall\ l \in \mathcal{I}$, $r \in R$. An ideal is a both left as well as a right ideal of R [1].

Suppose seminearring R has non-empty subsets P and Q, then all finite sums set PQ is of the form $\sum p_k q_k$ with $p_k \in P$ and $q_k \in Q$. That is, $\sum pr_k(\sum r_k p)$ is a sum of all elements of $pR(Rp)$ for every $p \in R$. The fact that seminearring R is right distributive implies $Rp = \{rp : r \in R\}$. Therefore $pR(Rp)$ is a right(left) ideal of R. The ideal $pR(Rp)$ is called the principal right(left) ideal. In particular, if the non-empty set is singleton $\{p\}$, then $pR(Rp)$ is respectively the principal right(left) ideal generated by p [2]. An ideal $\mathcal{I}$ of a seminearring R is called a $\mathcal{K}$-ideal of R if for any $a,\ m \in R$, there exist $g,\ k \in \mathcal{I}$ such that $a+m+g = k+m+a$ [11].

An element x of a seminearring R is called distributive if for all $p,q \in R$, $x(p+q) = xp+xq$. The set of all distributive elements of R is denoted by R_d. We say that R is distributive if $R = R_d$. A seminearring R is called distributively generated, or d.g. seminearring for short, if R contains a multiplicative subsemigroup D of distributive elements which generates $(R, +)$, [19]. A subseminearring of the direct product $\prod_{k \in K} R_k$ of a family $\{R_k \mid k \in K\}$ (where K is an index set) of seminearrings is called a subdirect product of the R_k's if every projection map π_k (restricted to R) is onto. An element $a \in R$ is said to be idempotent if $a^2 = a$. (We denote by E, the set of all idempotents of R). A ring R is regular (von Neumann) if $\forall\ a \in R$, $\exists\ r \in R \ni\ a = ara$.

In the literature, regular rings were studied extensively. The concept has been naturally extended to fewer axioms of algebraic structures namely nearrings as well as seminearrings. The element z in R, $x = xzx$ is not necessary to be unique. The "mate function" concept arises as a result of this and introduced in [12]. A mate function is a self-map $f : R \to R$ such that $a = af(a)a\ \forall\ a \in R$. Then f is a mate function for R and $f(a)$ is called a mate of a. R admits mate functions iff it is regular. A mate function f of R is known as a mutual mate function if $f(a)$=$f(a)af(a)\ \forall\ a \in R$. We refer to each of a and $f(a)$ as a mutual mate of each other [12].

The following special seminearring structures are extended from nearrings to seminearrings. R is said to be a $P(r, m)$ seminearring if $x^rR = Rx^m\ \forall\ x \in R$, where r, m are positive integers [4,15]. A seminearring R is a P_k seminearring (P'_k seminearring) if $x^kR = xRx$ ($Rx^k = xRx$) $\forall\ x \in R$, where k is a positive integer. In particular R is said to be a P_2 seminearring (P'_2 seminearring) if $x^2R = xRx$ ($Rx^2 = xRx$) $\forall\ x \in R$ [5,16]. R is a left-duo seminearring if every left ideal of R is two-sided [3,13,14].

Remark 2.1. [11] If $\mathcal{I}$ is an ideal of R, then $R/\mathcal{I}$ denotes the set of all congruence classes, that is, $R/\mathcal{I} = \{s + \mathcal{I} : s \in R\}$.

Theorem 2.2. [11] *Consider a seminearring R with a $\mathcal{K}$-ideal $\mathcal{I}$. Then $R/\mathcal{I}$ is defined where $(h + \mathcal{I}) + (m + \mathcal{I}) = h + m + \mathcal{I}$ and $(h + \mathcal{I}).(m + \mathcal{I}) = hm + \mathcal{I}$.*

Remark 2.3. By expanding the definition of a ring, the notions of seminearring isomorphisms, epimorphisms, homomorphisms and so on are defined. The following outcomes have been obtained:

(i) $g : R \to R/\mathcal{I}$ is a seminearring epimorphism if $\mathcal{I}$ is a $\mathcal{K}$-ideal of R. As a result, the homomorphic image of R is $R/\mathcal{I}$.
(ii) If $g : R \to R/\mathcal{I}$ is the canonical epimorphism, then $(R/\mathcal{I})/(J/\mathcal{I}) = (R/J)$ for all $\mathcal{K}$-ideals J of R containing $\mathcal{I}$.

Remark 2.4. (i) If there exists $k \in K$ such that the projection map π_k is an isomorphism, a subdirect product of seminearrings $\{R_k \mid k \in K\}$ is considered trivial.

(ii) R is considered subdirectly irreducible if R is not isomorphic to a non-trivial subdirect product of seminearrings.

Remark 2.5. If seminearring is a subdirect product R_k's $(k \in K)$, then homomorphic images are R_k's of R (under the projection maps π_k), [7].

Theorem 2.6. (i) *Every seminearring is isomorphic to a subdirect product of a seminearrings which is subdirectly irreducible.*

(ii) *Every d.g. seminearring is a d.g subdirect product of d.g. subdirectly irreducible d.g. seminearrings.*

Proposition 2.7. *When R admits a mate function f, then $Rz = Rf(z)z$ and $zf(z)R = zR \ \forall \ z \in R$.*

Proof. We observe that $Rz = Rzf(z)z \subseteq Rf(z)z \subseteq Rz$ and hence $Rz = Rf(z)z$. Similarly, $zR = zf(z)R$. □

Proposition 2.8. *Let f be a mate function for R. Then every left ideal A of R is idempotent.*

Proof. Let A be a left ideal of R. Then $RA \subseteq A$. Therefore $A^2 = AA \subseteq RA \subseteq A$. Also for any a in A, $a = af(a)a = a(f(a)a) \in A(RA) \subseteq AA = A^2$ and hence A is idempotent. □

3. α and strictly-α seminearrings

In this section we give the precise definition of an α and strictly-α seminearring and illustrate this concept with suitable examples.

Definition 3.1. R is called an α-seminearring if all the left ideals of seminearring R commute with one another i.e., $(Rx)(Ry) = (Ry)(Rx) \ \forall \ x, y \in R$.

Definition 3.2. A seminearring R is called a strictly-α seminearring if every left ideals Rx of R commutes with every element of R i.e., $y(Rx) = (Rx)y \ \forall \ x, y \in R$.

Example 3.3. $R = \{0, v_1, v_2, v_3\}$ is given by

$+$	0	v_1	v_2	v_3
0	0	v_1	v_2	v_3
v_1	v_1	0	v_3	v_2
v_2	v_2	v_3	0	v_1
v_3	v_3	v_2	v_1	0

$.$	0	v_1	v_2	v_3
0	0	0	0	0
v_1	0	v_1	0	v_1
v_2	0	0	0	0
v_3	0	v_1	0	v_1

this seminearring is an α-seminearring as well as a strictly-α seminearring.

Example 3.4. $R = \{0, v_1, v_2, v_3, v_4\}$ is given by

$+$	0	v_1	v_2	v_3	v_4
0	0	v_1	v_2	v_3	v_4
v_1	v_1	v_1	v_2	v_4	v_4
v_2	v_2	v_2	v_2	v_4	v_4
v_3	v_3	v_4	v_4	v_3	v_4
v_4	v_4	v_4	v_4	v_4	v_4

$.$	0	v_1	v_2	v_3	v_4
0	0	0	0	0	0
v_1	0	v_1	v_1	v_1	v_1
v_2	0	v_1	v_2	v_2	v_2
v_3	0	v_1	v_2	v_2	v_2
v_4	0	v_1	v_4	v_4	v_4

then $(R, +, \cdot)$ is an α-seminearring but not a strictly-α seminearring.

Proposition 3.5. *The following hold in a strictly-α seminearring R.*

(i) *R is a P_2' seminearring.*

(ii) *If $y \in R_d$, then $yM = My$ for all ideals M of R.*

(iii) *If $R = R_d$, then R is left-duo.*

Proof. (i) As R is a strictly-α seminearring, $y(Rx) = (Rx)y\ \forall\ x, y \in R$. Taking $y = x$, we get $xRx = Rx^2$ and (i) follows.
(ii) We have $M = \sum_{x \in M} Rx$. If $y \in R_d$, then $yM = y(\sum_{x \in M} Rx) = \sum_{x \in M} y(Rx) = \sum_{x \in M} (Rx)y = (\sum_{x \in M} Rx)y = My$ and (ii) follows.
(iii) Suppose R has any ideal M. For any $y \in R(= R_d)$. (ii) implies $My = yM \subseteq RM \subseteq M$. This yields $MR \subseteq M$. Thus M is a right ideal of R. Hence R is left-duo. □

Proposition 3.6. *If R is a strictly-α seminearring, then it is an α-seminearring.*

Proof. As R is a strictly α-seminearring, we have $y(Rx) = (Rx)y\ \forall\ x, y \in R$. Let $a, b \in R$ and for $r \in R$,

$$(ra)Rb \subseteq RaRb \Rightarrow (Rb)ra \subseteq RaRb \Rightarrow RbRa \subseteq RaRb \tag{1}$$

In a similar fashion,

$$(rb)Ra \subseteq RbRa \Rightarrow (Ra)rb \subseteq RbRa \Rightarrow RaRb \subseteq RbRa. \tag{2}$$

From (1) and (2), we get $RaRb = RbRa$ and the desired result follows. □

The converse of the above Proposition 3.6 is not valid, i.e., an α-seminearring need not be a strictly-α seminearring. The seminearring in Example 3.4 comes in handy to justify this assertion. It is worth noting that this seminearring admits mate function.

Proposition 3.7. *If R is a strictly-α (an α) seminearring, then any homomorphic image of R is also a strictly-α (an α) seminearring.*

Proof. Let R and R' be two seminearrings and we define a map $g : R \to R'$ a seminearring epimorphism. Assumption of $a, b, r' \in R'$ where onto of g, $\exists\ x, y, r$ in R with $a = g(x)$, $b = g(y)$ and $r' = g(r)$.
$br'a = g(y)g(r)g(x)$
$= g(yrx)$
$= g(r_1xy)$ (since $yrx \in y(Rx) = (Rx)y \Rightarrow yrx = r_1xy$ for some $r_1 \in R$)
$= g(r_1)g(x)g(y)$
$= r'_1ab$ (where $r'_1 = g(r_1) \in R'$)
$\in (R'a)b$.
So $bR'a \subseteq (R'a)b$.
Similarly, $(R'a)b \subseteq b(R'a)$. Thus $(R'a)b = b(R'a)$.
Therefore R is strictly-α seminearring.
Next we assume that $a, b, s'_1, s'_2 \in R'$. Hence onto g, $\exists\ x, y, s_1, s_2 \in R$, $a = g(x)$, $b = g(y)$ and $s'_1 = g(s_1)$ and $s'_2 = g(s_2)$. Now $s'_1as'_2b = g(s_1)g(x)g(s_2)g(y)$
$= g(s_1xs_2y)$
$= g(s_2ys_1x)$ (since $s_1xs_2y \in RxRy = RyRx \Rightarrow s_1xs_2y = s_2ys_1x;\ s_1, s_2 \in R$)
$= g(s_2)g(y)g(s_1)g(x)$
$= s'_2bas'_1$ (where $s'_1 = g(s_1), s'_2 = g(s_2) \in R'$)
$\in R'aR'b$.
So $R'aR'b \subseteq R'bR'a$.
Similarly, $R'bR'a \subseteq R'aR'b$. Thus $R'bR'a = R'aR'b$.
Hence R is an α-seminearring. □

As a consequence of Proposition 3.7 we obtain the following:

Theorem 3.8. *If R is an α (a strictly-α) seminearring, then R is isomorphic to a subdirect product of an α (a strictly-α) seminearring which is subdirectly irreducible.*

Proof. By Theorem 2.6, we have R to be isomorphic to a subdirect product of a seminearrings say R_i and every R_i subdirectly irreducible therefore homomorphic image of seminearring R, π_i. The remaining proof follows by Proposition 3.7. □

Theorem 3.9. *If an α (a strictly-α) seminearring has an ideal $\mathcal{I}$, then $R/\mathcal{I}$ is also an α (strictly-α) seminearring.*

Proof. $R/\mathcal{I}$ is a homomorphic image of R by the canonical-homomorphism and the needed result is got from Proposition 3.7. □

4. Main results

Proposition 4.1. *Let R be a strictly-α seminearring with mate function. Then every ideal of R is also a strictly-α seminearring with mate function.*

Proof. Suppose f is a mate of R, and that A is a two-sided ideal. Let $a, b, c \in A$. Since R is a strictly-α seminearring, then

$$bca \in bRa = Rab = Raf(a)ab \subseteq Aab$$

since A is two-sided. Similarly,

$$cab \in Rab = bRa = bRaf(a)a \subseteq bAa.$$

Therefore $bAa = Aab$, and A is a strictly-α seminearring.

We observe that, as A is an ideal, the mutual mate function $g : A \to A$ defined by $g(a) = f(a)af(a)$, for $a \in A$, serves as a mate function for A and the result is achieved. □

We shall now give a complete characterisation of α-seminearrings.

Theorem 4.2. *Suppose f is a mate of R. Then the seminearring R is an α-seminearring if and only if $Rx \cap Ry = RxRy \ \forall \ x, y \in R$.*

Proof. Let R be an α-seminearring has a mate function. Given principal left ideals Rx and Ry, their intersection $Rx \cap Ry$ is also a left ideal, and so by Proposition 2.8, $Rx \cap Ry$ is idempotent. Hence

$$Rx \cap Ry = (Rx \cap Ry)^2 \subseteq RxRy.$$

Now, $RxRy \subseteq Ry$, and since R is an α-seminearring, $RxRy = RyRx \subseteq Rx$.
Hence $RxRy \subseteq Rx \cap Ry$, and therefore $RxRy = Rx \cap Ry$.
Conversely, if $RxRy = Rx \cap Ry$ for all x, y, then

$$RxRy = Rx \cap Ry = RyRx$$

and so R is an α-seminearring. □

Theorem 4.3. *Suppose f is a mate of a seminearring R. Then the seminearring R is left-duo iff it is a P_1 seminearring.*

Proof. We first observe that a left-duo seminearring for the "only if" part. Since a left ideal Rz of R, for each $z \in R$, it also becomes a right ideal of R which implies $(Rz)R \subseteq Rz$. For every $r \in R$, $\exists$ some $r' \in R \ni zr = (zf(z)z)r = z(f(z)zr) = zr'z$. Thus $zR \subseteq zRz$. Obviously, the reverse inclusion, namely $zRz \subseteq zR$ always holds. Subsequently, we got $zR = zRz \ \forall \ z \in R$. For the "if" part, let $\mathcal{I}$ be a left ideal of R. Thus by the definition $R\mathcal{I}$ is a subset of $\mathcal{I}$. For each $h \in \mathcal{I}$, we get $hR = hRh = (hR)h \subseteq R\mathcal{I} \subseteq \mathcal{I}$. As such $\mathcal{I}R \subseteq \mathcal{I}$ and the result is achieved. □

Theorem 4.4. *If a seminearring R is admitting a mate function f, then the following are equivalent:*

(i) *R is a left-duo seminearring.*
(ii) *R is a P_1 seminearring.*
(iii) *$\mathcal{I}_1 \cap \mathcal{I}_2 = \mathcal{I}_1\mathcal{I}_2$ $\forall$ left ideals ($\mathcal{I}_1$ & $\mathcal{I}_2$) of R.*

Proof. (i) $\Longrightarrow$ (ii) By the proof of Theorem 4.3, it follows.
(ii) $\Longrightarrow$ (iii) Suppose $z \in \mathcal{I}_1 \cap \mathcal{I}_2$. Then $z = zf(z)z = z(f(z)z) \in \mathcal{I}_1 \cap \mathcal{I}_2$. Thus $\mathcal{I}_1 \cap \mathcal{I}_2 \subseteq \mathcal{I}_1\mathcal{I}_2$.
Now let $r = hm \in \mathcal{I}_1\mathcal{I}_2$ with $h \in \mathcal{I}_1$ and $m \in \mathcal{I}_2$. We have $r = hm \in hR = hRh$ and therefore $r = hth$ (where t in R) $= h(th) \in h\mathcal{I}_1$. Since $h\mathcal{I}_1 \subseteq \mathcal{I}_1$, we have $r \in \mathcal{I}_1$. This ensures that $\mathcal{I}_1\mathcal{I}_2 \subseteq \mathcal{I}_1$.
Again as $r = hm \in R\mathcal{I}_2 \subseteq \mathcal{I}_2$, we see that $\mathcal{I}_1\mathcal{I}_2 \subseteq \mathcal{I}_2$. Collecting all these pieces, we get $\mathcal{I}_1\mathcal{I}_2 \subseteq \mathcal{I}_1 \cap \mathcal{I}_2$, then (iii) follows.
(iii) $\Longrightarrow$ (i) Let $\mathcal{I}_1$ be a left ideal of R and let $\mathcal{I}_2 = R$. So (iii) implies that $\mathcal{I}_1 R = \mathcal{I}_1 \cap R = \mathcal{I}_1$, hence $\mathcal{I}_1$ is a right ideal, this implies (i). □

Proposition 4.5. *If f is a mate function of an α-seminearring R, then R is a left-duo seminearring.*

Proof. Let $x \in R$ and for y in R, $xy = xf(x)xy = xf(x)xf(x)xy$ (since $xf(x) \in E$) $= x(f(x)x)(f(x)xy) \in x(RxRy) = x(RyRx)$ (since R is a α-seminearring)$\subseteq xRx \Rightarrow RyRx \subseteq R(Rx) \subseteq (Rx)$. Thus $xR \subseteq xRx$. But obviously $xRx \subseteq xR$. Hence $xR = xRx$ $\forall$ $x \in R$. Now the desired fact is proved from Theorem 4.3. □

Proposition 4.6. *Let R be a seminearring admitting mate functions. Then R is an α-seminearring if and only if it is a left-duo seminearring.*

Proof. "Only if" part follows from Proposition 4.5 and "if" part is a consequence of Theorem 4.2 and Theorem 4.4. □

We conclude this paper as we discuss one more situation under which an α-seminearring becomes a left-duo seminearring.

Proposition 4.7. *If an α-seminearring has a right identity e, then it is a left-duo seminearring.*

Proof. Suppose R has any left ideal A. Then $A = \sum_{x \in A} Rx$. Now $RxR = RxRe = ReRx = RRx \subseteq Rx$. Hence $AR = (\sum_{x \in A} Rx)R = (\sum_{x \in A} RxR) \subseteq \sum_{x \in A} Rx = A$. Hence $AR \subseteq A$ and the needed result is achieved. □

5. Conclusion and future work

Motivation for these classes of seminearrings stems from the elementary group-theoretic facts. This yields to develop more classes of seminearrings with the help of group-theoretic facts. In the future, the ideas that need further consideration will lead to a significant piece of seminearrings theory.

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