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Research Article

Evaluation of several geoid models for GNSS height transformation in Turkiye

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ABSTRACT

The main purpose of this paper is to present the results of 'external' quality tests for global, regional and local geoid models. In this context, available geoid models were evaluated for the height transformation in three different geographic locations in Turkiye on the basis of ground truth GNSS/Leveling data. The differences between observed and computed quantities were investigated. The comparison results of the global, regional and local geoid models for three test areas were presented. The tested geoids are the national geoid model Turkiye-Geoid2003 (TG-03) released by the General Command of Mapping in 2003 which was computed using land and sea gravity data, topographic heights from digital terrain model and GPS/Leveling data, including EGM96 global geopotential model for long wavelengths, the ultra-high resolution model EGM2008 that was released by the US National Geospatial Intelligence Agency, the refined versions of these two geoid models and the GNSS/Leveling derived geometric models produced by the metropolitan municipalities. The comparison of results show that TG03 and EGM2008 models provide about 2 times lower accuracy than the precise local geoid models. In addition, the RMS values of the other two methods (IMPTG03 and IMPEGM2008) compared in the study were obtained as 2.45cm (Istanbul), 5.09cm (Izmır) and 4.77cm (Bursa) for IMPTG03, 2.61cm (Istanbul), 5.45cm (Izmır) and 5cm (Bursa) for IMPEGM2008. It seems that the results are suffici t for many engineering applications in the local areas and EGM2008 can be reach the accuracy of regional or local geoid models after the improvement procedure.

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INTRODUCTION AND A BRIEF OVERVIEW OF THE GEOID DETERMINATION METHODS

GNSS derived ellipsoidal height is a geometrical height and it has no control over practical measurements and engineering and geophysical applications [1]. The height used in practice is the distance from the geoid surface also known as the orthometric height. The geopotential surface, which includes the underground levels and possesses a W0 value for the potential surface close to the seafl or, is the initial surface for the ground positions and is called a geoid [2, 3]. According to this defin tion and using the Helmert orthometric height system, the height of any point on earth is defi ed by the length of the plumb line from that point to the geoid. As the geoid is a function of density and mass distribution, data used in geoid determination represents the mass-density distribution of the Earth. The geoid surface can be defi ed using the data obtained via applying different measuring techniques to the Earth in general or to a specific region. The common sources of data and results of observations can be listed as below [2-9].

- Combined method (GNSS/Leveling, GNSS- Gravimetric)
- Gravity fi ld models
- Global models
- Astro-geodetic methods and
- Digital density models.

The Earth's gravity potential coeffici ts are being used in the global models and geoid height values are calculated with the equation (1) [4, 10, 11].

$$N = \frac{GM}{r\gamma_p}\sum_{n=2}^{n\max}\left(\left(\frac{R}{r}\right)^n \sum_{m=0}^{n}\bar{C}^S_{nm}\bar{Y}_{nm}(\theta,\lambda)\right)+\frac{\Delta g_b}{\gamma_p}H \qquad (1)$$

Where;

N: Geoid height,
γ_p: Normal gravity value,
Δg_b: Bouguer gravity anomoly,
H: Orthometric height,
GM: Multiplication of the mass of Earth's and gravitational constant in GRS-80 system
R: The mean radius of the earth,
r: Distance from the center of the earth to earth point
$P_{nm}\cos(\theta)$: Fully normalized Legendre function,
θ, λ: Centralized latitude (90-ϕ) and longitude
m: Order
n: Degree

$\bar{Y}_{nm}(\theta,\lambda)=\bar{P}_{nm}(\cos\theta)\cos m\lambda$ if $m\geq 0$,

$\bar{Y}_{nm}(\theta,\lambda)=\bar{P}_{nm}(\cos\theta)\sin m\lambda$ if $m>0$,

According to the gravimetric method, geoid is a function of the density and mass distribution and the changes in geoid are actually representing the distribution of Earth's mass density. The mass above and below the Earth's surface is in no homogeneous state causing the gravity vector to vary. Gravity is the best measure to describe the density distribution in question. Therefore, geoid surface can be determined via modeling the changes in the gravity fi ld values. Thus, the gravity measurements performed for the characteristic points selected depending on the topographic structure and the mass distribution of the region which is subject to geoid surface determination are reduced to a geoid surface via several reductions thereby can be defi ed as a model or grid data using varying evaluation techniques. Geoid height of a point in the region of the model is determined by the Stokes integral and the equation (2) [2, 6, 12, 13, 14].

$$N = \frac{R}{4\pi\gamma}\int\int_{\sigma}(\Delta g)S(\psi)d\sigma \qquad (2)$$

Where;

N: Geoid height,
R: The mean radius of the earth,
γ: Normal gravity,
Δg: Gravity anomaly,
S(ψ): Stokes' function on the sphere,
σ: The surface of integration.

Another technique for geoid determination is the plumb line deviation appointed with regard to the astrogeodetic measures. The gravity vectors of the plumb line deviation, true and normal gravity fi lds are different in direction. In other words, plumb line deviation is the angle between the normal of ellipsoid and the vertical straight (plumb line direction). Th s angle is symbolized as θ and equals to the total plumb line deviation. ξ, η and ε are the components of the total plumb line deviation. $\xi = \phi - \varphi$ is the fi st component and it is the difference between the astronomical latitude φ and the geodesic latitude ϕ. $\eta = (\Lambda - \lambda)\cos\varphi$ is the second component and it can be determined depending on the astronomical longitude (Λ) and geodesic longitude(λ). On the other hand, $\varepsilon = \eta \sin\alpha + \xi \cos\alpha$ is the component at the α azimuth of plumb line deviation. The geoid height difference between two points can be calculated by the equation $\Delta N = -\varepsilon\Delta s$; ΔN: difference in geoid undulation, Δs: distance nearby two points [2, 6, 15].

The other approach for the determination of short and ultra-short components involves the application of anomaly values obtained by subtracting the long wavelength components of geoid which is calculated by using global models from the point gravity anomaly value. The geoid determined with this approach comprises the long wavelength errors and it is on a different datum from GNSS/Leveling geoid [13, 15-17]. The inadequate frequency of the points, the non-cost effici cy and impracticality of the points selected for the purpose of refl cting the topographic properties and breaking effect reduce the accuracy of the geoid determined using common astrogeodetic measurements [3]. Geoid accuracy can be improved by using the existing astrogeodetic plumb line deviations and making use of the plumb line deviations obtained by increasing the frequency of the astronomic points. According to the geoid determination survey preformed based on the Finland astrogeodetic network, the average accuracy of the geoid detected by taking the observation points with a distance less than 60km is ±16 cm and the ΔN accuracy at 31.6 km's is around ±8.3cm [6]. Today, it is possible to detect the astronomical latitudes and longitudes at an average of 0.1" accuracy depending on the star catalogue with simply obtaining the geodetic coordinates with an adequate accuracy using GNSS and with the availability of the zenith and CCD (Charged Coupling Devices) cameras. Although, it is possible to detect "cm" level geoid accuracy using adequate frequencies of points, the applications need to become more practical.

There is a need for detecting the gravity anomalies to an accuracy of 1-2mgal in a span of a few km to obtain "cm" level geoid accuracy with gravity measurements. If there are suffici t point gravity anomalies in the area subject to geoid surface study, the boundary value problem is easily solved univocally. However, in the areas with irregular topographies, gravity measures involve meaningful frequencies even to the highest level. The point gravity values must have an adequate frequency to give the frequencies in question. The average gravity anomalies defi ed for an average topographical surface do not improve the results and they cause systematic errors. Th s case shows the diffi ulties in "cm" level geoid accuracy determination with short and ultra-short components using gravimetric methods [13]. In particular, it is difficult to predict the local effects of the geoid by the global model based on the spherical harmonics or the gravity values. GNSS/Leveling derived geometric geoid models have great signifi ance for more accurate height transformation of GNSS derived ellipsoidal heights for practical geodetic applications [5, 18-22].

Users need orthometric heights and ellipsoidal heights obtained from geometric levelling network and geodetic GNSS network respectively for the determination of the geoid surface with GNSS/Leveling. The common points of GNSS/Leveling networks called as control or basis points. The geoid heights of the control (basis) points are obtained by taking the differences, between orthometric and ellipsoidal heights. The geoid surface can be defi ed through a model which is a function of basis points. The basis points must be cover the entire of the working area and homogeneous distributed. Although geoid and geopotential surfaces are smooth surfaces, there are complex geometric structures. Such surfaces can be expressed mathematically is difficult, and these need analytical or an analogue model to precisely determine it. To identify these surfaces, hyperboloid paraboloid, or other high-level functions are used in practice.

On the other hand, recently developed global geoid model EGM2008 can be used directly in many studies to provide the absolute and relative accuracy requirement. However, several studies conducted by the General Command of Mapping for the development of national geoid model within the borders of Turkiye. In Turkiye, various regional geoid models have been computed with different methods, since 1970s. Th ough this effort, a summary of the developed geoid models, are given in next section.

EVALUATION OF GEOID MODELS USED IN STUDY

The geoid models are EGM2008, TG-03 and local geoid models which are used Izmir, Istanbul and Bursa. In the following section there will be descriptions about these models.

Global Geoid Models

Several national and international institutions are working on the determination of geoids. Some of them are General Directorate of Mapping (in Turkiye) [23], International Service for the Geoid (ISG) [24]. The most popular international global geoid models can be sorted in chronological order as OSU91A [25], EGM-96[26] and EGM2008 [27] global geoid models.

The OSU91A global model is a model developed by the Ohio State University which involves the application of global spherical harmonics complete to degree/order 360 and is calculated depending on the one-year GEOSAT altimeter data, the surface gravity observations and geopotential coeffici ts of GEM-T2 satellite. Th s model has 130682 coeffici ts and a few meters of absolute accuracy. Th s accuracy is in dm level relevantly for the short distances [6, 25].

On the other hand, the EGM-96 global model is developed by DMA (U.S Defense Mapping Agency), NIMA (National Imagery and Mapping Agency) and GSFC (NASA Goddard Space Flight Centre) and with the assistance of OSU (Ohio State University). Global spherical harmonics complete to degree/order 360 are used in this model. Besides the gravity data used in the other global models, data from the former eastern countries like Russia, China etc. used in the development process of this model. The model developed was subjected to comparison with the regional and local geoid models and GNSS/Leveling measures. As a result of the research, 0.5 m to 1 m of absolute accuracy and a dm level relative accuracy for shorter distances was yielded. It is noted that this model is the best global model developed up to the present [6, 26].

The Earth Gravitational Model 2008 (EGM2008) is the global geoid model published by the National Geospatial-Intelligence Agency (NGA). It replaced the EGM96 model which had been the default global geoid since its publication in 1996.The offi al Earth Gravitational Model EGM2008 has been publicly released by the National Geospatial-Intelligence Agency (NGA) EGM Development Team. Th s gravitational model is complete to spherical harmonic degree and order 2159, and contains additional coeffici ts extending to degree 2190 and order 2159. The model was computed from a global 5 arc-minute grid of gravity anomalies from land and satellite based sources. The model is provided complete to spherical harmonic degree and order 2159, which equates to a grid size of approximately 6.5 km. The global agreement to GNSS/Leveling is approximately 7cm. EGM2008 is available from the NGA website. It is provided in terms of spherical harmonic coeffici ts which generally need to be converted into a grid of geoid heights before they can be used [10, 27, 28].

Global models were obtained by combining the dispersed data groups throughout the earth, as the result of serious efforts. It can be mentioned about several factors that restrict the ability of global models.

- The accuracy, distribution and frequency of the regional and local terrestrial data that involved in the computation are the most important factor. If the region contains enough observations of terrestrial gravity, the global model can provide the accuracy required for engineering applications.
- Although, the satellite-based gravity data are homogeneously distributed to the whole earth, the terrestrial data used in the creation of global models are different both qualitatively and quantitatively. Therefore, the performance of the global model differs from region to region.
- It was available frequent gravity observations (almost 100 m interval) for the Nordic countries such as Sweden, Norway and Denmark. On the other hand, almost no gravity observation could be used in less developed countries such as Afghanistan, Pakistan and Bangladesh. Although suffici t data could be available for some regions and countries (such as Turkiye), they have not contributed to the solution of global models due to the military and strategic reasons.
- Several factors such as data errors, datum siftings, stochastic model and the others leads to errors called as commission error in estimation of coeffici ts of the global models.
- As known, the gravity fi ld of the earth consists of a combination of an infin te number of frequencies. However, even the numerical solution of the EGM2008 model was conducted yet 2190 degree. Therefore, an omission error has occurred due to negligence of the next order of frequencies. Th s important factor restricts the capabilities of the global models.
- Even the best combined global geoid model (Nmax = 2190, Nmax: Maximum degree) has about 8-9 km spatial resolution. In engineering applications, a higher resolution (such as 2 km to 3 km) geoid model which minimizes interpolation error is needed.
- It is possible to meet this expectation with a high-resolution regional model supplemented by gravity and GNSS/Leveling data.

National Geoid Models of Turkiye

A number of geoid solutions in regional as well as in local areas of the Turkiye have been presented over the last 35 years [29-36]. Turkiye has started in the 1970s to the present day determination of local and regional geoid models by using a variety of methods (Gravimetric, Astrogeodetic, GNSS/Leveling). The fi st studies conducted by [37-39], vertical defl ction components were used in 98 astronomic observation sites. Th s attempt could not meet the needs of the geoid due to the inadequate and inhomogeneous data used in this study. Later, South West Anatolia Doppler geoid was calculated by using Doppler measurements in levelling networks thanks to the development of satellite technologies [40].

Turkiye's fi st gravimetric geoid model (TG-91) was calculated in conjunction with gravity and terrain elevation data acquisition and the publication of a global geopotential models [41]. It is national gravimetric geoid model for Turkiye which determined by the General Command of Mapping 1991. GPM2-T1 global earth potential model, gravity data and the topographic heights from digital terrain model data was used. TG91 geoid computed based on the remove-restore and least squares collocation technique. In 1992, Turkiye Doppler geoid (TDG-92) was calculated by ellipsoid heights derived from satellite data and orthometric heights in levelling networks at 184 sites. Turkiye Astro-geodetic geoid (TAG-94) have been determined in 1994, using data obtained from astro-gravimetric levelling technique [42].

With the development of GNSS (Global Navigation Satellite System) technology, Turkish National Fundamental GPS Network (TUTGA99) was established in 1999 by the General Command of Mapping in order to form a control network that will be the reference geodetic studies. The construction of Turkiye National Fundamental GPS Network (TUTGA) made it possible to determination of consistent and homogeneous ellipsoidal heights. The orthometric heights of the selected 197 TUTGA point are determined via precise geometric leveling observation based on Turkiye's National Vertical Control Network (TUDKA). Thus the GNSS/Leveling derived geoid heights were obtained for these points. Turkiye Geoid-1999 (TG-99) and its updated version (TG-99A) was calculated [43] in the early 2000s, for the creation of a consistent geoid model with the GNSS datum by combination of the GNSS/Leveling data and the TG-91 Geoid model.

TG-99A geoid model has been computed at the 3×3 arcs of minutes through modeling of differences on GNSS/

Leveling points of long wavelength effects in the Turkish Gravimetric Geoid (TG-91) computed in 1991. GNSS coordinates (ϕ,λ,h) and orthometric heights(H) of 196 points and the 3×3 arcs of minutes used in TG-91 has been used. GNSS/Leveling geoid heights were obtained from the difference between the GNSS ellipsoidal heights and the orthometric height values. As a basic data, the study used the difference between gravimetric geoid height and GNSS/ Leveling geoid height. The inner accuracy of model is achieved in ±5cm from the difference between interpolated and measured geoid heights for 196 points. Furthermore, the accuracy of the model is tested in 122 independent points throughout Turkiye, and the outer geoid height accuracy has been found to be ±10 cm. It is possible to obtain values with relatively higher accuracy values (TUTGA-99A, 1999). TG99A geoid model is obtained by combining the TG-91with GNSS / leveling and includes of the short-wavelength topography effects [43].

In subsequent years, the geoid models namely (TG-03) [44] and THG-09 [45] were calculated by Remove-Restore method thanks to the improvement of the global geopotential model, the surface gravity data, GNSS / leveling geoid heights and with the development of digital terrain models. (TG-03) is a revised version of previous model was computed using land and sea gravity data, topographic heights from digital terrain model and GNSS/Leveling data, including EGM96 global geopotential model for long wavelengths and was released by the General Command of Mapping in 2003. The absolute accuracy of TG03 is given to be 8.8 cm in the national report of Turkish National Union of Geodesy and Geophysics.

Finally, (THG-09) is the latest version of Turkish geoid. EGM2008 geoid model for long wavelengths effect, land gravity anomalies, DNSC08 gravity anomalies from ERS1, ERS2 and TOPEX/POSEIDON altimetry data, topographic heights, GNSS/Leveling geoid heights were used together by Remove-restore technique, least squares collocation method and Fast Fourier Transform [45].

Local Geometric Geoid Models

Many local governments and municipalities in Turkiye have developed geodetic GNSS and leveling networks, mainly for digital photogrammetric map and orthophoto production, engineering surveys and cadastral purposes. The other important purpose of these projects was the determination of local geoid model for transformation of GNSS derived ellipsoidal heights to orthometric heights by using the common points of GNSS/Leveling networks. In this regard, Istanbul, Izmir and Bursa samples can be summarized as follows:

- To determine "cm" accuracy geoid by GNSS/Leveling data, within the borders of Istanbul municipality, 1005 reference points with ITRF96 derived ellipsoidal heights and the Helmert orthometric heights in TUDKA, covering the said region has been taken in an area of 65×160km. Istanbul local geoid model has been determined using the "multi-parameter polynomial regression" and "adaptive artific al neural network and fuzzy inference system" methods for practical use [46]. The accuracy of the model is tested via independent levelling and GNSS measurements different parts of Istanbul. As a result of studies made for the accuracy of the model, model consistency has been found to be about ±4.5 cm [47].
- Izmir's geoid model was created using the geostatistical method called as kriging. Th s study was performed within the borders of Izmir Municipality area, covering approximately 115km x 112km between 37.87° and 38.91° north latitudes and 26.47° and 27.76° east longitudes (Figure 2). It was a part of a national research project, namely "Izmir geodetic infrastructure for the production of 1/5000 scaled digital photogrammetric maps and orthophotos" carried out by co-operation of Izmir Metropolitan Municipality and Yildiz Technical University. The 857 points were obtained from GNSS/Leveling network. Besides, 301 previously established points from the Izjrs-2001 project were also used [48]. Finally, the 1148 reference points with ITRF96 derived ellipsoidal heights and the Helmert orthometric heights in TUDKA (data density is approximately 4.75km^2/ point) were used. The accuracy of the model was found to be ± 3.9 cm as the result of performed investigations [49].
- Local geoid model for the province of Bursa were calculated under the project called as "The production of 1/1000 scaled photogrammetric digital maps and orthophoto maps for Bursa Metropolitan Areas" similar to previous ones in a similar way. The 1280 reference points with ITRF96 derived ellipsoidal heights and the Helmert orthometric heights in TUDKA (data density is approximately 3.44km^2/ point) were used. The achieved model is covering approximately 65km x 82km between 39.86° and 40.49° north latitudes and 28.35° and 29.44° east longitudes. It is possible to say that the model accuracy is around 4 cm to 5cm in here [50].

Local Improvement/Fitting of EGM2008 and TG03 Geoid Models with GNSS/Leveling Data

As described in the introduction three models were evaluated in this study to estimate their qualities' for the transformation of GNSS derived heights. For the reasons discussed in previous sections, the global gravity fi ld models are not yet fully meet the precise geodetic application requirements. Global models still need improvement with local gravity observations and/or GNSS/Leveling data to achieve high accuracy both regional and local level. Turkish proprietary data were not used in EGM2008 computations. From the mean discrepancy between EGM2008 and

GNSS/Leveling, it is found that the local model (national vertical datum of Turkiye) for test area is off et from the global geoid by 60 cm [51]. On the other hand, the National regional geoid models are one of the most important components of Turkish national geodetic infrastructure. Some difficulties were occurred during the determination of the homogeneous National models, covering the whole country with a high precision. It is necessary to use a fitting model to minimize the long and medium wavelength geoid errors, the systematic datum discrepancies between the global geoid and the GNSS/Leveling data (for the improvement procedure) in order to make a meaningful validation. The following section describes in detail how the fitting were performed.

In practice, the various wavelength errors in the global and regional geoid solution may be approximated by different kinds of functions in order to adjust the local geoid to a set of GNSS levelling points through an integrated least squares (LS) adjustment. Several models can be used ranging from a simple linear regression to more complicated transformation model. These are summarized as below;

- Polynomial Expansion of various order
- Similarity Transformation Models
- Least Squares Collocation
- Interpolation methods
- The other methods (Differential similarity, Legendre polynomial, Fourier series and etc.)

The general issues related to the improvement of the regional geoid model is situated in Large Scale Map and Map Information Production Regulation (came into force in 2005). According to this regulation, static GNSS measurements are required in uniformly dispersed control points for the improvement of regional geoid model. In addition, these points must be connected to the nearby TUDKA-99 levelling points with precise geometric levelling. Thus, the orthometric heights and then geoid heights can be calculated. The differences between GNSS / Leveling geoid heights and the regional geoid model in these specifi d points are modeled with a suitable surface to improved regional geoid [52].

For this purposes, at least four appropriately scattered points up to 200 km2 and additionally one point for per 200 km2 were determined in the test area for the improvement EGM2008 and TG03 models. These points were positioned by hierarchical GNSS network densifi ation in 1-2cm accuracy. Then it has connected to the National Vertical Control Network of Turkiye by precise geometric leveling, thus their geographical coordinates, ellipsoidal and orthometric heights were determined. The geoid height differences between GNSS/Leveling and TG03 or EGM2008 and the geoid heights calculated by the equation DN were modeled separately with an appropriate function by using reference points with orthometric and ellipsoidal height. Th s procedure was applied to obtain the correction surface. It was used four parameters model in the studies dealing with the combined adjustment of GNSS, levelling and geoid data to remove the systematic errors introduced by the datum discrepancies between the data sets. It is the most commonly used in such adjustments and is given by the following equation;

$$
\begin{aligned}
&f(\varphi,\lambda)=\Delta N=h_i-H_i-N_i^{EGM2008}=N_i^{GNSS-LEV.}\\
&-N_i^{EGM2008}=a_i^T.x+v_i=a_0+a_1.(\lambda_i-\lambda_0)\\
&+a_2.(\varphi_i-\varphi_0)+a_3.(\lambda_i-\lambda_0).(\varphi_i-\varphi_0)
\end{aligned}
\tag{3}
$$

where, h_i and H_i are respectively ellipsoidal and orthometric heights, λ_i and ϕ_i are longitude and latitude of point i, ($N_i^{EGM2008}$) is EGM2008 geoid heights, ($N_i^{GNSS-LEV.}$) is the corresponding GNSS/Leveling-derived height difference, is the shift parameter between the GNSS/Leveling datum and the EGM2008 datum are the shift parameters between two parallel' datums and v_i denotes residual. λ_0 and ϕ_0 are appropriate selected arbitrary value for reduction of geographical ellipsoidal coordinates.

Thus, the improved versions of for TG03 and EGM2008 geoid models were determined by the help of fitted corrector surface. The evaluated models in this study can be sorted as;

- GNSSLEV (**A**): GNSS/Leveling derived geometric models that was produced by the metropolitan municipalities;
- EGM2008 (**B**): The ultra-high resolution model EGM2008 that was released by the US National Geospatial Intelligence Agency;
- IMPEGM2008 (**C**): Locally improved version of EGM2008;
- TG-03 (**D**): The regional height datum of Turkiye is expressed by is Turkiye-Geoid2003 (TG-03) released by the General Command of Mapping in 2003 which was computed using land and sea gravity data, topographic heights from digital terrain model and GNSS/Leveling data, including EGM96 global geopotential model for long wavelengths; and,
- IMPTG03 (**E**): Locally improved version of TG03.

Colored contour plots of all geoid models used in the study are shown in Figure 1. As expected, the EGM2008 and IMPEGM2008 models have quite soft contour lines and identify long-range variations in the geoid surface. Although TG03and IMPTG03 models have also soft contour lines, they show regional differences due to the terrestrial gravity data. GNSSLEV model have come to the fore in all cases based on the accuracy criteria and the ability to represent the local variations. In practice, there is no statistical meaning of these visual interpretations. The availability of GNSS/Leveling data in the whole of the test areas with a homogeneous distribution and suffici t density allow a more reliable assessment, at the local scale, of the quality of the geoid models.

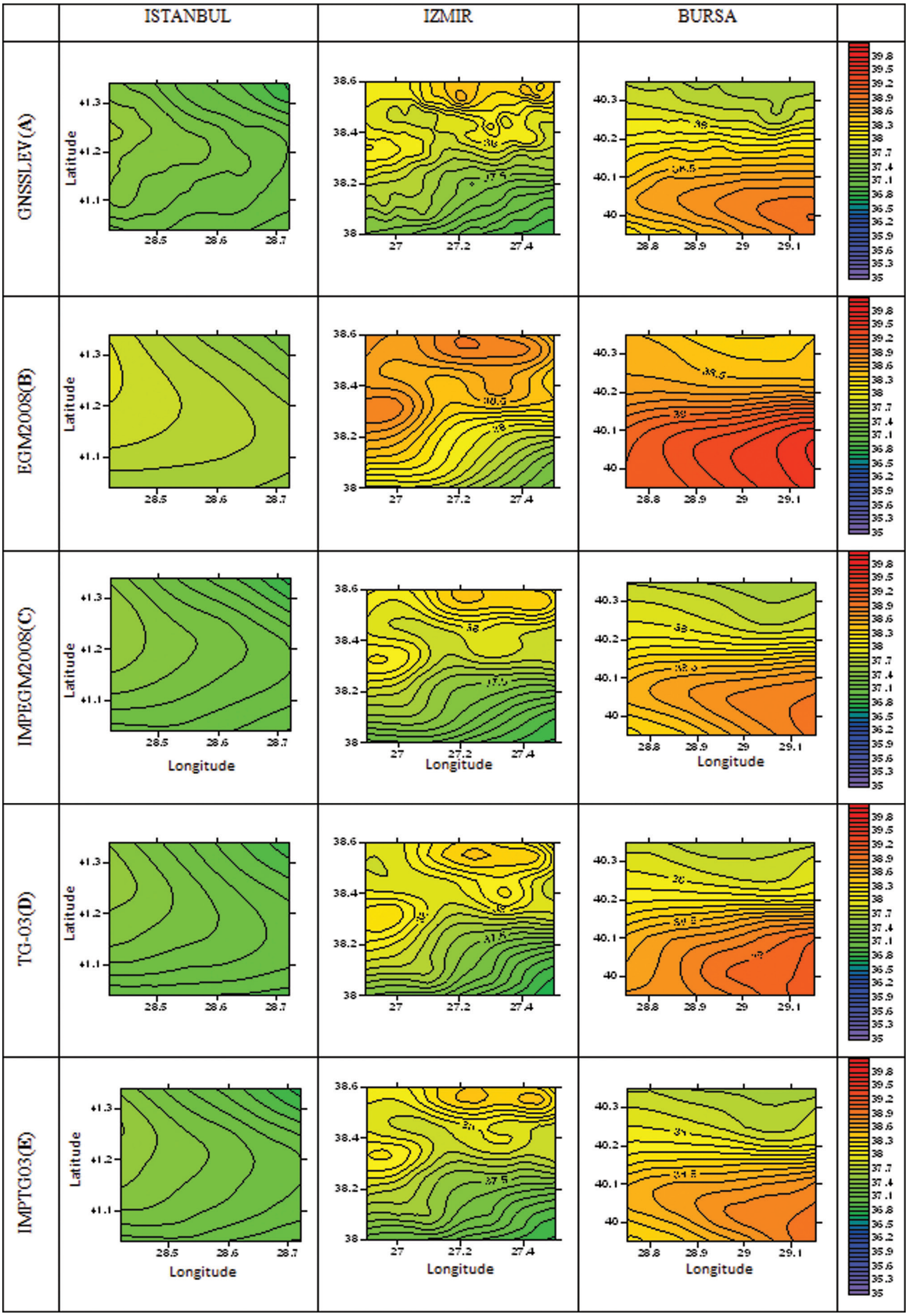

Figure 1. Contour plots of all geoid models used in the study (Contour units are in m, the axes units are in degrees).

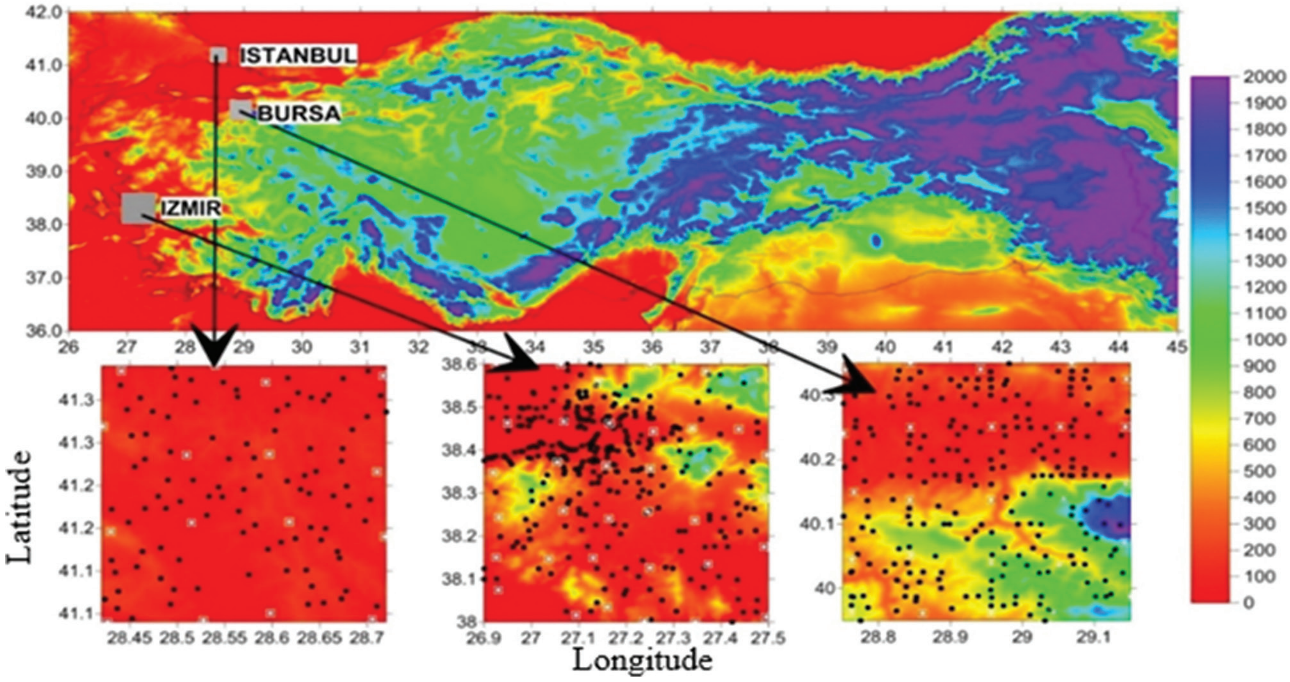

Figure 2. Test areas and used data (The units are degrees for the axes, m for the colour scale.

VALIDATION OF MODELS WITH GNSS/LEVELING DATA

As is well known, the geoid heights or height anomalies derived from GNSS/Leveling data are the highest quality and most reliable data in validating the global or regional geoids. It is also called as "GNSS/Leveling test" that is a comparison of the geoid height derived from GNSS/Leveling data and the geoid height calculated from another geoid model [12, 53-55]. Th s validation procedure gives a reasonable indication of the geoid model's accuracy. However, the accuracy of the validation points must be at least equivalent to the accuracy of geoid model for a significant control. In this regard, it is necessary to establish that an appropriate data set with suffici t number and frequency in a consistent reference system.

In Turkiye, GNSS/Leveling points are mainly established by General Command of Mapping and the General Directorate of Land Registry and Cadastre for geodetic and cadastral purposes. At the regional level, several GNSS/Leveling points were produced by the local municipalities, for the studies under the digital map productions. In addition, smaller areas at various universities and research institutes carried out the GNSS/Leveling work for scientific purposes.

There are several points with GNSS/Leveling data sets can be available for three different local regions for evaluation purposes. The following data sets have been used:

1. The fi st one is a part of Istanbul territories of about 1000 kilometers squared area covering approximately 0.3 x 0.3 arcs of degrees within 41.04° and 41.34° north latitudes and 28.42° and 28.72° east longitudes, this dataset consist of 97 co-located (the common points of GNSS/Leveling networks) benchmarks provided by Istanbul Metropolitan Municipality, which has good coverage over selected region of Istanbul (Figure 2).
2. The second set is derived from the control-surveying of "Izmir geodetic infrastructure for the production of 1 / 5000 scaled digital photogrammetric maps and orthophotos" project. It includes 308 benchmarks with an area approximately 0.6 x 0.6 arcs of degrees in the West part of Turkiye, between the 38o.00 to 38o.60 latitudes and the 26.9o to 27.50o longitudes on the Izmir.
3. As for the third set, it has been chosen in a part of Bursa city, it involves an area of approximately 0.4 x 0.4 arcs of degrees between the 39.95o to 40.35o latitudes and the 28.75o to 29.15o longitudes with 218 benchmarks.

Figure 2 shows that the benchmarks are uniformly distributed over the test regions. The lack of points in the some part of test regions corresponds to an area with high mountains in which the precise levelling would be impractical so there is no possibility to collect test data in mountainous regions. The white square dots in Figure 2 shows the

Table 1. Statistical information for used data and evaluation results (The units are in m for differences)

ISTANBUL	**Latitude**	**Longitude**	**N**	**Elev.**	**A**	**B**	**C**	**D**	**E**
Number of values	97	97	97	97	97	97	97	97	97
Minimum	28.4239	41.0446	36.8260	12.3726	-0.0893	-0.5453	-0.0851	-0.1251	-0.0884
Maximum	28.7198	41.3369	37.5490	214.2835	0.0901	-0.2823	0.1100	0.1898	0.0857
Mean	**28.5689**	**41.1967**	**37.2969**	**94.9932**	**0.0032**	**-0.4124**	**0.0106**	**0.0487**	**0.0052**
Median	28.5700	41.1953	37.3110	91.3069	0.0001	-0.4122	0.0068	0.0412	0.0083
Average deviation	0.0772	0.0719	0.1212	39.7654	0.0246	0.0505	0.0331	0.0529	0.0304
Standard deviation	**0.0875**	**0.0848**	**0.1522**	**46.2132**	**0.0329**	**0.0601**	**0.0415**	**0.0659**	**0.0371**
Root Mean Square	**28.5690**	**41.1968**	**37.2972**	**105.6379**	**0.0331**	**0.4168**	**0.0428**	**0.0819**	**0.0375**
IZMIR	**Latitude**	**Longitude**	**N**	**Elev.**	**A**	**B**	**C**	**D**	**E**
Number of values	308	308	308	308	308	308	308	308	308
Minimum	26.9002	38.0006	36.8070	0.0000	-0.1886	-0.8630	-0.2138	-0.3141	-0.2187
Maximum	27.4779	38.6000	38.5900	1276.7018	0.1468	-0.2336	0.2740	0.2740	0.2688
Mean	**27.1567**	**38.3639**	**37.8666**	**183.3889**	**-0.0018**	**-0.5974**	**0.0179**	**-0.0404**	**0.0127**
Median	27.1376	38.3950	37.9135	114.9997	-0.0023	-0.5877	0.0175	-0.0370	0.0165
Average deviation	0.1115	0.1086	0.2123	162.4864	0.0382	0.0768	0.0490	0.0623	0.0461
Standard deviation	**0.1401**	**0.1369**	**0.3017**	**229.4675**	**0.0514**	**0.0979**	**0.0637**	**0.0819**	**0.0606**
Root Mean Square	**27.1570**	**38.3641**	**37.8678**	**293.7462**	**0.0515**	**0.6053**	**0.0662**	**0.0913**	**0.0619**
BURSA	**Latitude**	**Longitude**	**N**	**Elev.**	**A**	**B**	**C**	**D**	**E**
Number of values	218	218	218	218	218	218	218	218	218
Minimum	28.7502	39.9500	37.6000	40.0175	-0.1579	-1.0449	-0.1768	-0.6060	-0.1764
Maximum	29.1456	40.3489	39.0030	1920.7618	0.1175	-0.4339	0.1836	0.0418	0.1955
Mean	**28.9496**	**40.1532**	**38.2994**	**445.7403**	**0.0033**	**-0.6177**	**0.0234**	**-0.1194**	**0.0228**
Median	28.9566	40.1668	38.3220	378.5153	0.0014	-0.6065	0.0258	-0.0986	0.0179
Average deviation	0.0962	0.1018	0.3587	289.0606	0.0224	0.0633	0.0496	0.0686	0.0451
Standard deviation	**0.1121**	**0.1164**	**0.4095**	**359.3724**	**0.0365**	**0.0800**	**0.0604**	**0.0906**	**0.0579**
Root Mean Square	**28.9498**	**40.1534**	**38.3016**	**572.5670**	**0.0366**	**0.6229**	**0.0648**	**0.1499**	**0.0623**

reference points used in the study. The black dots represent validation benchmarks. Statistical information for the distribution of test data can be seen in the fi st five columns of the Table 1.

The last five columns of Table 1 show that, the statistical information (mean, standard deviation and root mean square) related to the evaluation results for geoid models. As the result of the evaluation of statistical analysis, some information was achieved in Table 1. The most important indicator about quality of the model is standard deviation and, RMS (root mean square) errors. To evaluate the accuracy of the fitted model, the RMS of geoid height discrepancies were examined individually for each model. RMS indicates how closely model predicts the measured values. It is seen that the RMS values are varying between 0.4168m, 0.6053m 0.6229m, the means are -0.4124m,-0.5974m, -0.6177m, the standard deviations are 0.0601m, 0.0979m, 0.0800m respectively for Istanbul, Izmir and Bursa regions for EGM2008 before the bias and tilt fit. After the bias and tilt fit, the RMS values were decreasing to 0.0428m, 0.0662m 0.0648m, the means are 0.0106m, 0.0179m, 0.0234m, the standard deviations are 0.0415m, 0.0637m, 0.0604m respectively for Istanbul, Izmir and Bursa regions for EGM2008. According to Table 1 the standard deviation and RMS values of the differences show signifi ant improvements by fitting of EGM2008 Geoid models with GNSS/Leveling data.

Although at a lower level, similar improvement is observed for the TG03 model. IMPEGM2008 and IMPTG03 very close to GNSSLEV after fitting in terms of the standard deviation. The improvement is almost at the 65-70 % level for EGM2008, the 60-65 % for TG03. GNSSLEV, IMPEGM2008, IMPTG03 present very similar results. The best agreement with validation data is GNSSLEV at the 0.0331m, 0.0515m, 0.0366m respectively for Istanbul, Izmir and Bursa regions in terms of the RMS of the differences. For the IMPEGM2008, it is at the 0.0428m, 0.0662m, 0.0648m, for the IMPTG03 models, 0.0375m,

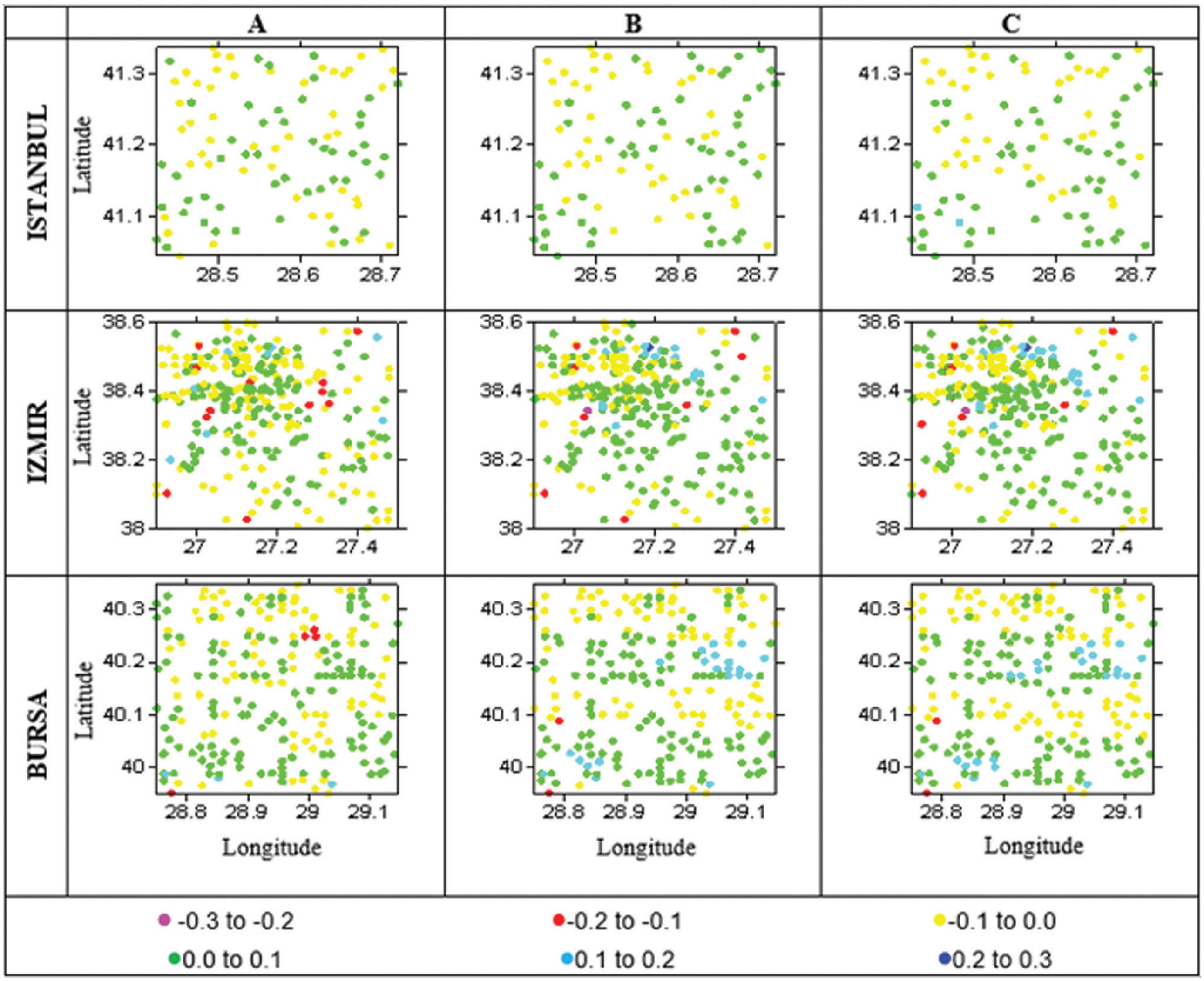

Figure 3. Geoid height discrepancies in validation points for each test areas (The axes' units are in degrees).

0.0619m, 0.0623m respectively for Istanbul, Izmir and Bursa regions.

Figure 3 represents the classed post map of the differences between the evaluated geoid models and corresponding ones from GNSS/Leveling validation data. The majority of the geoid heights computed from models show a good correspondence with the GNSS/Leveling validation data. According to the used validation data, the GNSSLEV is able to transform ellipsoidal heights over 100 %, 92.9 %, 96.3 %, IMPEGM2008 model is able to transform ellipsoidal heights over 97,9 %, 87.7 %, 88.6 %, IMPTG03 model is able to transform ellipsoidal heights over 100 %, 90.6 %, 89.0 % of the Istanbul, Izmir and Bursa territories respectively to within ±10cm (Figure 3). It is concluded from these results that the three models present the same behavior in the data sets used in this work.

COMPARISON OF MODELS WITH EACH OTHER

In this section, it was intended to compare the three geoid model in question with the each other geoid models established using different data sources and different methodology. For this purpose, the geoid models were converted to regular grid data format, then the geoid height differences in grid nodes was compared for the GNSS/Leveling geoid model (A), IMPTG03 (E) and IMPEGM2008 (C) geoid models. The models were compared grid by grid for A-C, E-C and A-E. The main purpose of the comparisons among the three geoid models is to show how they fit to each other, to identify areas of larger discrepancies, and to detect systematic and random differences. Such a comparison is signifi ant for the examination of the consistencies of the different geoid models. To make a meaningful comparison it was focused

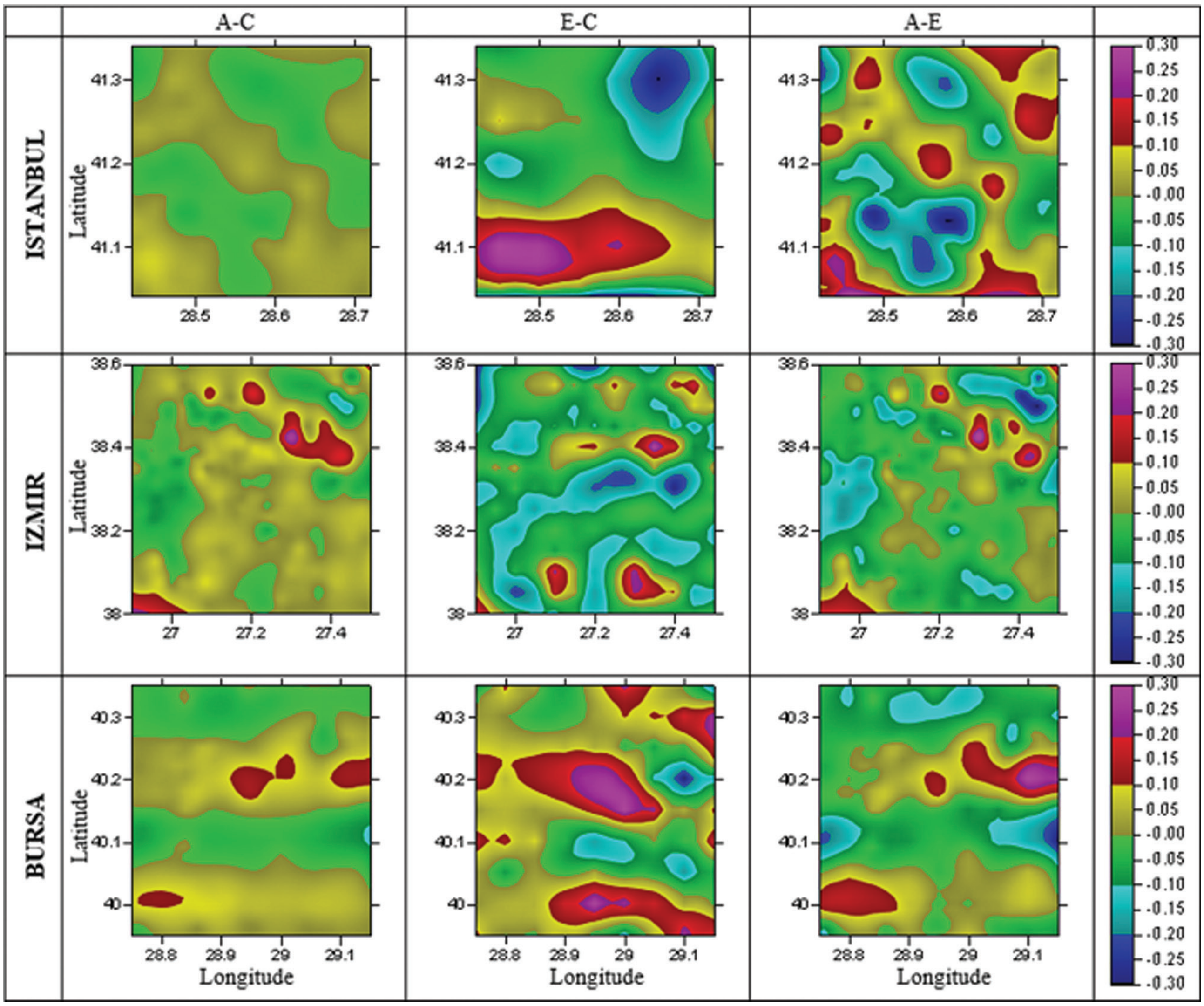

Figure 4. Geoid heights differences between compared models (difference' units are in m and the axes' units are in degrees).

on improved versions TG03 and EGM2008 and GNSSLEV model. There is no signifi ant visually differences between these models are (Figure 4).

Figure 4 shows the distribution of the differences between -30 cm and +30 cm. A visual interpretation of Figure 4 shows that the three geoid models agree well with the differences mostly between -30 cm and +20 cm (Table 2). It can be seen that the agreement between the models is within ± 10 cm for most parts of the test regions. In some areas the discrepancies are larger, but in most cases just a little. The differences are considerably larger in the rough mountains in the test regions. On the other hand, it can be recognized some localized oscillations in the differences, mainly due to systematic biases and unsystematic deviations in the differences. The achieved statistics for comparisons are given in Table 2.

According to Table 2, IMPTG03 and IMPEGM2008 gives the best agreement with 1.67-2.01cm RMS. The most interesting result is that IMPEGM2008 agree with the GNSS/Leveling data with RMS values between 2.61 and 5.45 cm (after fitting/improvement procedure). Th s is comparable to the corresponding RMS value for the IMPTG03 geoid model. Thus, IMPEGM2008 agrees well with the GNSS/Leveling data and the national geoid model in Turkiye. The difference between GNSSLEV and geoid heights computed using IMPEGM2008 model range between -0.0536 m and 0.0915 m with an average of 0.0053 m, standard deviation of about 0.0255 m and RMS 0.0261 m for Istanbul. Despite the fact that, RMS values in question increase two times in Izmir and Bursa areas, they are within acceptable limits (respectively 5.00 cm and 5.45 cm). The large discrepancies (with the maximum 27.65 cm) in local scale occurring in

Table 2. Statistical information for comparison results (difference' units are in m)

	ISTANBUL			IZMIR			BURSA		
	A-C	E-C	A-E	A-C	E-C	A-E	A-C	E-C	A-E
Number of values	961	961	961	3721	3721	3721	1681	1681	1681
Minimum	-0.0536	-0.0350	-0.0676	-0.1367	-0.0569	-0.1704	-0.1132	-0.0669	-0.1443
Maximum	0.0915	0.0471	0.0678	0.2765	0.0935	0.2425	0.1396	0.0516	0.1888
Mean	**0.0053**	**0.0052**	**0.0001**	**0.0217**	**0.0040**	**0.0177**	**0.0171**	**0.0014**	**0.0157**
Median	0.0052	0.0037	0.0027	0.0214	0.0014	0.0194	0.0130	0.0008	0.0118
Average deviation	0.0211	0.0122	0.0203	0.0363	0.0147	0.0353	0.0387	0.0139	0.0352
Standard deviation	**0.0255**	**0.0159**	**0.0245**	**0.0500**	**0.0197**	**0.0478**	**0.0470**	**0.0179**	**0.0451**
Root Mean Square	**0.0261**	**0.0167**	**0.0245**	**0.0545**	**0.0201**	**0.0509**	**0.0500**	**0.0179**	**0.0477**

Izmir and Bursa areas where no GNSS leveling data were available, due to the topography.

CONCLUSION

Although, GNSS/Leveling are the most popular and reliable integration for determination of local transformation surface for GNSS height transformation in small and flat areas it may not be rational to use GNSS/Leveling data for determination of the geoid, when the error budget of GNSS/Leveling observations are considered and the coverage and resolution of the data is taken into account for big and rough terrains. It is possible to obtain "cm" accuracy routinely for surface models by using appropriate surface estimation technique and regular reference points. Th s procedure called as geometric geoid modeling, it come to the fore in studies for the determination of short and ultra-short wavelength of geoid for years. Moreover, it is mostly recommended for the ellipsoidal to orthometric height transformation studies in Turkiye. Although, an adequate number of properly spread out reference points are used for surface modeling, accuracy of local models in test areas are about ±4 cm to 5 cm.

The regional height datum of Turkiye is expressed by national vertical control network created based on the Antalya tide gauge station. The accuracy of GNSS derived ellipsoidal heights of reference points in the network is 3cm approximately. On the other hand, the accuracy of orthometric heights, depending on the distance of the region to Antalya tide gauge station, it varies between 0.3 cm and 9.0 cm throughout Turkiye. The accuracy of the geometric geoid models with GNSS/Leveling data may access to the value of 4 cm to 5 cm at the best when considering distance from Antalya station of the test areas.

With the combination of GNSS/Leveling and gravimetric data, it is possible to construct cm accurately precise geoid model especially in regions with high mountains. Th s procedure has been globally accepted and adopted because of its accuracy and reliability. Due to the lack of reliable gravity and GNSS/Leveling data, there are only limited improvements was achieved on the precise geoid over Turkiye. However, the new gravity satellite missions provide new global solutions that allow modelling the long and medium wavelengths of the Earth's gravity field. The development of the Earth Gravitational Model 2008 (EGM2008) model is a signifi ant contribution for modelling the Earth's gravity and geoid. Recently, it can be confide tly used versus geometric or gravimetric models following a simple improvement procedure. The spherical harmonic models with an optimal degree of expansion are combined with terrestrial gravity and terrain data in order to express the gravity fi ld in every frequency in the spectrum. In this manner one can claim that EGM2008 model is exception because it is ultra-high-resolution model and with its 2190 degree/order of expansion it should be different from other earth geopotential models.

Direct use of TG03 and EGM2008 models provide about 2 times lower accuracy than the precise local geoid models. It is undeniable contribution of the local geoid solutions for engineering applications and practical geodetic works. Also, the global and regional geoid models currently available for Turkiye may be improved with GNSS/Leveling in local level as done in this study. As a result of the improvement procedure, the average RMS values of the improved versions of TG03 and EGM2008 models are achieved 4.10 cm and 4.35 cm respectively for the test areas. It seems to be suffici t for many engineering applications in the local areas. However, the successful use of this kind of models for GNSS height transformation depends on the minimization of systematic effects between GNSS/Leveling datum and related geoid.

Th s study shows that EGM2008 can be reach the accuracy of regional or local geoid models after the improvement procedure (modeling the differences between the GNSS/Leveling geoid heights and EGM2008 derived geoid heights at identifi d control points). Considering the topography of Turkiye and the difficulty to collect regular gravity data and determine the orthometric heights of the control points needed to establish a geoid surface in these regions using the geometric levelling, the EGM2008 global model becomes crucial.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confi m that the data that supports the fi dings of this study are available within the article. Raw data that support the fi ding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential confli ts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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