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Introduction of the Runge-Kutta method in GPS orbit computation

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Abstract

In all Global Navigation Satellite Systems (GNSS) applications, the determination of the satellite orbits is an important task. In this study, we present the equations given in the Interface Specification Document of GPS and the Runge-Kutta method in the computation of the position P, velocity V, and acceleration A of the GPS satellites using the broadcast ephemeris. The definition of the differential equation describing the GPS satellite's motion has enabled us to introduce the Runge-Kutta method in the GPS orbit computation; this method uses the initial conditions determined in this study from the Keplerian elements provided in the broadcast ephemeris files. The Lagrange interpolation method is used for comparison of the results, where the vectors P, V, and A are estimated using the precise ephemeris. The difference not exceeding 2.4 m was obtained in the X, Y, and Z axes during seven days on the position of the GPS satellite number 9 tested in this study. In velocity and acceleration, the difference is about a few mm/s and mm/s^2, respectively.

1. Introduction

GNSS stands for Global Navigation Satellite System; it is a general term describing any satellite constellation. The most well-known and widely used GNSS is the Global Positioning System (GPS), developed and operated by the United States. The objectives of GPS are the instantaneous determination of position, velocity, and precise time in the World Geodetic System 84 (WGS84) [1, 2].

GLONASS is the Russian system, which stands for Globalnaya Navigazionnaya Sputnikovaya Sistema, or Global Navigation Satellite System, it is one of the GNSS systems [3]. GLONASS uses the PZ-90 (Parametry Zemli 1990 or Parameters of the Earth 1990) as a reference coordinate system [4].

There are also other global and regional GNSS systems, such as BeiDou China's system, the Galileo system of the European Union, the Navic (Navigation with Indian Constellation) system of India, and the QZSS (Quasi-Zenith Satellite System) system of Japan.

The computation of satellite positions is a fundamental task in all GPS positioning software [5]. The determination of a GPS position begins with the determination of the GPS satellite's coordinates in orbit; these coordinates are used for a combined use of the GPS and GLONASS observations [6, 7].

In the GPS positioning technique, the data used for the determination of the PVA (Position, Velocity, Acceleration) of satellites can come in the form of a broadcast or a precise ephemeris [5].

In the ICD-GLONASS (Interface Control Document-GLONASS) [4], the authors presented the Runge-Kutta method for the determination of the PVA using the initial conditions provided in the GLONASS broadcast ephemerides file. In [3, 6, 7] the authors used the Runge Kutta method in the computation of the GLONASS satellite orbits. In these papers, several Runge Kutta orders were used.

In the ISD (Interface Specification Documents-GPS) [2], the authors presented the equations of the PVA computation from the GPS broadcast ephemerides file. The ISD-GPS are technical documents that define the specifications for signals, messages, and interfaces within GPS systems [8]. In [5,6], the authors used the equations given in the IS-GPS document in the computation of the position; in Remondi [9], the author presents the derivation of the position equations (velocity); and in Thompson et al. [10], the authors present the acceleration equation of the satellite's motion.

The main objective of this manuscript is the introduction of the Runge-Kutta method in the computation of the orbits (Position, Velocity, and Acceleration) using the Keplerian elements transmitted in the broadcast files.

The methodology used in the estimation and comparison between the positions (P), velocities (V), and accelerations (A) of the GPS satellites computed from the broadcast and precise ephemeris is as follows:

1-The equations given in the ISD noted IS-GPS Algorithm. This algorithm is applied when the Toe (Time of ephemerides) is equal to Tc (Time of Computation).

2-The Runge-Kutta integration method; this method is used when the Toe and Tc are different. The Runge Kutta method requires the initial conditions to compute the PVA vectors of the GPS satellites in orbit. In this study, these initial conditions (P0; V0; A0) were computed from the Keplerian elements provided in the broadcast ephemeris file using the IS-GPS Algorithm.

3-The Lagrange interpolation method to compute the polynomial of position using the X, Y, and Z coordinates of the GPS satellites provided in the precise ephemerides file. The first and second derivations of the position polynomial give the polynomials of velocity and acceleration, respectively.

The broadcast and precise ephemerides of the GPS satellite number 9, registered between January 8 and 14, 2023, are used to get the PVA of this satellite using the IS-GPS Algorithm, the Runge Kutta integration method, and the Lagrange interpolation method. These ephemerides are produced by the International GNSS Service (IGS) and downloaded from the Crustal Dynamics Data Information System (CDDIS) database. The application date corresponds to the GPS week number 2244.

This paper is organized according to the following sections: Section 2 describes the PVA computation by the IS-GPS Algorithm and the Runge-Kutta method using the Keplerian elements provided in the broadcast ephemerides file. Section 3 describes the PVA estimation by the Lagrange interpolation method using the coordinates of the GPS satellites provided in the precise ephemerides file. In Section 4, a comparison and discussion of the results application are given. Finally, a summary of the conclusions is given.

2. PVA computation from broadcast ephemeris data

2.1. Broadcast ephemeris data

The broadcast ephemeris is the Keplerian elements transmitted by the GPS satellite to the user every two hours and referenced to the time of ephemeris (Toe).

Figure 1 shows an example of the GPS broadcast ephemeris file of GPS SV 9. This figure is taken from the navigation message and has been modified.

According to [11-14], the parameters given in Figure 1 and needed in the computation of PVA are:

- Toe: Reference time of ephemeris (sec of GPS week).
- $\sqrt{a}$: Square root of the semi-major axis (sqrt(m)).
- M_0: Mean anomaly at the reference time (radians).
- Δn: Mean motion difference from the computed value (radians/sec).
- e: Eccentricity.
- w (Omega): Argument of perigee (radians).
- i_0: Inclination angle at reference time Toe (radians).
- i (IDOT): Rate of inclination angle (radians/sec).
- Cuc: Amplitude of the cosine harmonic correction term to the argument of latitude (radians).
- Cus: Amplitude of the sine harmonic correction term to the argument of latitude (radians).
- Crc : Amplitude of the cosine harmonic correction term to the orbit radius (meters).
- Crs : Amplitude of the sine harmonic correction term to the orbit radius (meters).
- Cic : Amplitude of the cosine harmonic correction term to the angle of inclination (radians).
- Cis : Amplitude of the sine harmonic correction term to the angle of inclination (radians).
- Ω_0 OMEGA0: Longitude of the ascension node of the orbit plane at the weekly epoch (radians).
- $\dot{\Omega}$ OMEGA DOT: Rate of right ascension (radians/sec).

G09	2023	01	13	09	59	44
SV	Y	M	D	H	M	S
	4.00E+00		-6.03E+01		4.60E-09	1.80E+00
	IODE		Crs		Delta n	M0
	-3.22E-06		2.58E-03		8.26E-06	5.15E+03
	Cuc		e		Cus	sqrt(a)
	4.67E+05		-2.60E-08		5.44E-01	-4.84E-08
	Toe		Cic		OMEGA0	Cis
	9.55E-01		2.13E+02		1.95E+00	-8.08E-09
	i0		Crc		Omega	OMEGA DOT
	-1.75E-10		1.000E+00		2.24E+03	0.00E+00
	IDOT		Codes L2		Crs	L2 P data
	2.00E+00		0.00E+00		1.39E-09	4.00E+00
	SV accury		SV health		TGD	IODC
	4.66E+05		4.00E+00			
	Tran time		Fit int			

Figure 1. Broadcast ephemeris data of GPS SV 9.

2.2. IS-GPS algorithm

The position, velocity, and acceleration at time (Tc) are computed as given in the following sections:

2.2.1. Positions of the GPS satellites (P)

The positions are computed by using the equations given in [2, 14-16];

-Compute mean motion (Equation 1):

$$n_0 = \sqrt{\mu/a^3} \tag{1}$$

Where; $\mu = 3.986005 \times 10^{14} m^3/s^2$ is the gravitational constant

-Time from ephemeris reference epoch (Equation 2):

$$t_k = t - T_{oe} \quad (2)$$

-Corrected mean motion (Equation 3):

$$n = n_0 + \Delta n \quad (3)$$

-Mean anomaly (Equation 4):

$$M_k = M_0 + nt_k \quad (4)$$

-Eccentric anomaly: Kepler's equation of eccentric anomaly solved by iteration [13, 16, 17] (Equation 5):

$$E_k = M_k + e\sin E_k \quad (5)$$

-True anomaly (Equation 6):

$$\upsilon_k = \arctan(\sin\upsilon_k/\cos\upsilon_k) \quad (6)$$

Where: $\sin\upsilon_k = \frac{(\sqrt{1-e^2\sin E_k})}{(1-e\cos E_k)}$ and $\cos\upsilon_k = \frac{(1-e\cos E_k)}{(\cos E_k - e)}$

-Argument of latitude (Equation 7):

$$\varphi_k = \upsilon_k + w \quad (7)$$

-Argument of latitude correction (Equation 8):

$$\delta u_k = C_{us}\sin 2\varphi_k + C_{uc}\cos 2\varphi_k \quad (8)$$

-Radius correction (Equation 9):

$$\delta r_k = C_{rs}\sin 2\varphi_k + C_{rc}\cos 2\varphi_k \quad (9)$$

-Inclination correction (Equation 10):

$$\delta i_k = C_{is}\sin 2\varphi_k + C_{ic}\cos 2\varphi_k \quad (10)$$

-Corrected argument of latitude (Equation 11):

$$u_k = \varphi_k + \delta u_k \quad (11)$$

-Corrected radius (Equation 12):

$$r_k = a(1 - e\cos E_k) + \delta r_k \quad (12)$$

-Corrected inclination (Equation 13):

$$i_k = i_0 + \delta i_k + i(IDOT)t_k \quad (13)$$

-Position in the orbital plane (Equation 14):

$$\begin{cases} x'_k = r_k\cos u_k \\ y'_k = r_k\sin u_k \end{cases} \quad (14)$$

-Corrected longitude of the ascending node (Equation 15):

$$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)t_k - \dot{\Omega}_e T_{oe} \quad (15)$$

Where: $\dot{\Omega}_e = 7.2921151467 \times 10^{-5}\, rad/s$ is the Earth's rotation rate.

-Earth-fixed geocentric satellite coordinates (Equation 16):

$$\vec{P} = \begin{cases} X_k = x'_k\cos\Omega_k - y'_k\cos i_k\sin\Omega_k \\ Y_k = x'_k\sin\Omega_k + y'_k\cos i_k\sin\Omega_k \\ Z_k = y'_k\sin i_k \end{cases} \quad (16)$$

2.2.2. Velocities of the GPS satellites (V)

The velocities are computed by taking the time derivative of the position equations [2,9]:

-Eccentric anomaly rate (Equation 17):

$$\dot{E}_k = n/1 - e\cos E_k \quad (17)$$

-True anomaly rate (Equation 18):

$$\dot{\upsilon}_k = \dot{E}_k\sqrt{1-e^2}/1 - e\cos E_k \quad (18)$$

-Corrected Inclination angle rate (Equation 19):

$$\left(\frac{di_k}{dt}\right) = i(IDOT) + 2\dot{\upsilon}_k(C_{is}\cos 2\varphi_k - C_{ic}\sin 2\varphi_k) \quad (19)$$

-Corrected argument of latitude rate (Equation 20):

$$\dot{u}_k = \dot{\upsilon}_k + 2\dot{\upsilon}_k(C_{us}\cos 2\varphi_k - C_{uc}\sin 2\varphi_k) \quad (20)$$

-Corrected radius rate (Equation 21):

$$\dot{r}_k = e \times a\sin\dot{E}_k + 2\dot{\upsilon}_k(C_{rs}\cos 2\varphi_k - C_{rc}\sin 2\varphi_k) \quad (21)$$

-Longitude of ascending node rate (Equation 22):

$$\dot{\Omega}_k = \dot{\Omega} - \dot{\Omega}_e \quad (22)$$

-Velocity in the orbital plane (Equation 23):

$$\begin{cases} \dot{x}'_k = \dot{r}_k\cos u_k - r_k\dot{u}_k\sin u_k \\ \dot{y}'_k = \dot{r}_k\sin u_k + r_k\dot{u}_k\cos u_k \end{cases} \quad (23)$$

-Earth's fixed geocentric velocity satellite (Equation 24):

$$\vec{V} = \begin{cases} \dot{X}_k = -x'_k\dot{\Omega}_k\sin\Omega_k + \dot{x}'_k\cos\Omega_k - \dot{y}'_k\sin\Omega_k\cos i_k \\ \quad -y'_k(\dot{\Omega}_k\cos\Omega_k\cos i_k - (di_k/dt)\sin\Omega_k\sin i_k) \\ \dot{Y}_k = x'_k\dot{\Omega}_k\cos\Omega_k + \dot{x}'_k\sin\Omega_k + \dot{y}'_k\cos\Omega_k\cos i_k \\ \quad -y'_k(\dot{\Omega}_k\sin\Omega_k\cos i_k + (di_k/dt)\cos\Omega_k\sin i_k) \\ \dot{Z}_k = \dot{y}'_k\sin i_k + y'_k(di_k/dt)\cos i_k \end{cases} \quad (24)$$

2.2.3. Accelerations of the GPS satellites (A)

The accelerations are computed by taking the time derivative of the velocity [2,10] (Equation 25):

$$\vec{A} = \begin{cases} \ddot{X}_k = -\mu \frac{X_k}{r_k^3} + F\left[(1 - 5\left(\frac{Z_k}{r_k}\right)^2)\left(\frac{X_k}{r_k}\right)\right] \\ \qquad +2\dot{Y}_k\dot{\Omega}_e + X_k\dot{\Omega}_e^2 \\ \ddot{Y}_k = -\mu \frac{Y_k}{r_k^3} + F\left[(1 - 5\left(\frac{Z_k}{r_k}\right)^2)\left(\frac{Y_k}{r_k}\right)\right] \\ \qquad -2\dot{X}_k\dot{\Omega}_e + Y_k\dot{\Omega}_e^2 \\ \ddot{Z}_k = -\mu \frac{Z_k}{r_k^3} + F\left[(3 - 5\left(\frac{Z_k}{r_k}\right)^2)\left(\frac{Z_k}{r_k}\right)\right] \end{cases} \tag{25}$$

Where:$F = (3/2)J_2(\mu/r_k^2)(Re/r_k)^2$;

$J_2 = 0.0010826262$ is the second-order harmonic coefficient; $Re = 6378137.0m$ is the WGS 84 Earth equatorial radius.

2.3. Runge-Kutta (R-K) method

The high accuracy that is nowadays required in the computation of satellite orbits can only be achieved by using numerical methods for the solution of the equation of motion [16]. Among these methods; the Runge-Kutta integration method, originally presented by Carl Runge (1856–1927) in 1895 and Wilhelm Kutta (1867–1944) in 1901 [17]. Runge-Kutta method is particularly easy to use and may be applied to a wide range of different problems [14].

In this section, we present the Runge-Kutta method as an alternative solution for the PVA computation of the GPS satellites using the broadcast ephemerides.

Equation 25 is a second-order differential equation describing the motion of the GPS satellites in orbit. This equation is transformed into a first-order differential Equation (26) and can be resolved numerically by the Runge-Kutta method [3, 4, 16, 18].

$$\begin{cases} \dot{X} = V_x \\ \dot{Y} = V_y \\ \dot{Z} = V_z \\ \dot{V_x} = -\frac{\mu}{r^3}X + F\left(1 - \frac{5Z^2}{r^2}\right)X + \dot{\Omega}_e^2 X + 2\dot{\Omega}_e V_y \\ \dot{V_y} = -\frac{\mu}{r^3}Y + F\left(1 - \frac{5Z^2}{r^2}\right)Y + \dot{\Omega}_e^2 Y - 2\dot{\Omega}_e V_x \\ \dot{V_z} = -\frac{\mu}{r^3}Z + F\left(3 - \frac{5Z^2}{r^2}\right)Z \end{cases} \tag{26}$$

Equation 26 has the general form: $\dot{Y} = F(t, Y)$ and the resolution by the fourth-order R-K method is given in [3, 13, 16] by (Equation 27):

$$Y_{i+1} = Y_i + (K_1 + 2K_2 + 2K_3 + K_4)/6 \tag{27}$$

The coefficients K_i are:

$$\begin{cases} K_1 = f(t_n, Y_n) \\ K_2 = f(t_n + h/2, Y_n + K_1/2) \\ K_3 = f(t_n + h/2, Y_n + K_2/2) \\ K_4 = f(t_n, Y_n) \end{cases} \quad h \text{ is the integration step.}$$

The initial conditions required by the Runge-Kutta method are the position (P_0), the velocity (V_0), and the acceleration (A_0) determined using the IS-GPS Algorithm at time T_{oe} when ($T_{oe} \neq T_c$).

For the computation of the GPS satellite PVA vectors by using the R-K integration method, the Equation 27 becomes (Equation 28):

$$Y = \begin{cases} X_{i+1} = X_i + (K_1 + 2K_2 + 2K_3 + K_4)/6 \\ Y_{i+1} = Y_i + (K_1 + 2K_2 + 2K_3 + K_4)/6 \\ Z_{i+1} = Z_i + (K_1 + 2K_2 + 2K_3 + K_4)/6 \\ Vx_{i+1} = Vx_i + (K_1 + 2K_2 + 2K_3 + K_4)/6 \\ Vy_{i+1} = Vy_i + (K_1 + 2K_2 + 2K_3 + K_4)/6 \\ Vz_{i+1} = Vz_i + (K_1 + 2K_2 + 2K_3 + K_4)/6 \\ Ax_{i+1} = Ax_i = Ax_0 \\ Ay_{i+1} = Ay_i = Ay_0 \\ Az_{i+1} = Az_i = Az_0 \end{cases} \tag{28}$$

More information about the application of this method for GLONASS satellites is given in [3, 4, 18].

3. PVA computation from precise ephemeris data

3.1. Precise Ephemeris Data

The coordinates of all GPS satellites are given in precise ephemeris data (predicted, rapid, and final) produced by various agencies [5, 14, 19].

Figure 2 shows an example of a GPS precise ephemeris file corresponding to the GPS week number 2244. This figure is taken from the precise file and has been modified.

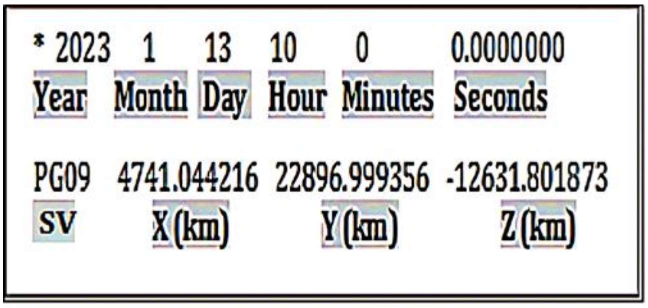

Figure 2. Precise ephemeris data of GPS SV 9.

According to [20], the parameters given in Figure 2 are the X, Y, and Z coordinates of the GPS satellites in. The format of the precise ephemeris is SP3 (Standard Product 3), proposed in 1989 by Remondi [20].

3.2. Lagrange Interpolation

In this section, we present the Lagrange interpolation method used in the PVA estimation of the GPS satellites using precise ephemeris, and thereafter compare our results from the R-K method.

Lagrange interpolation is the most commonly used because of its ease of implementation and accuracy in the interpolation of precise orbits.

The polynomial $P_m(x)$ of order $(n-1)$ that passes through the (n) points is given by [1, 12, 21]:

$$P_m(x) = \sum_{i=0}^{n} f(a_i).L_i(X) \ \text{ where } \ L_i(x) = \prod_{j=0,j\neq i}^{n}\left(\frac{X-a_i}{a_i-a_j}\right)$$

To interpolate the X-coordinates of a GPS satellite at a given time (t), the Lagrange polynomial formula becomes (Equation 29):

$$P_m(x) = X_1 \frac{(t-t_2)(t-t_3)\dots(t-t_n)}{(t_1-t_2)(t_1-t_3)\dots(t_1-t_n)} + \cdots \quad (29)$$

The P_m (y) and $P_m(z)$ Lagrange polynomials of the Y and Z coordinates are formed in the same way.

The Lagrange polynomials of P, V and A can be written as (Equation 30-32):

3.2.1. Lagrange polynomial of P:

$$P_m(P) = \begin{cases} P_m(X) = A_1 t^{13} + \ldots A_{14} \\ P_m(Y) = B_1 t^{13} + \ldots B_{14} \\ P_m(Z) = C_1 t^{13} + \ldots C_{14} \end{cases} \quad (30)$$

3.2.2. Lagrange polynomial of V (derivation of $P_m(\boldsymbol{P})$):

$$P_m(V) = \begin{cases} P_m(Vx) = 13A_1 t^{12} + \ldots A_{13} \\ P_m(Vy) = 13B_1 t^{12} + \ldots B_{13} \\ P_m(Vz) = 13C_1 t^{12} + \ldots C_{13} \end{cases} \quad (31)$$

3.2.3. Lagrange polynomial of A (derivation of $P_m(\boldsymbol{V})$):

$$P_m(A) = \begin{cases} P_m(A_x) = 156A_1 t^{11} + \cdots A_{12} \\ P_m(A_y) = 156B_1 t^{11} + \cdots B_{12} \\ P_m(A_z) = 156C_1 t^{11} + \cdots C_{12} \end{cases} \quad (32)$$

4. Application: Broadcast and precise PVA of GPS satellites computation

4.1. Data sources and PVA estimation steps

In this application, the vectors of positions P, velocities V, and accelerations A of the GPS satellite number 9 are computed and compared using the broadcast and precise files.

The data used are the broadcast and precise ephemerides of the GPS satellite number 9 between January 8 and 14, 2023. This data, produced by IGS and downloaded from (https://cddis.nasa.g/) corresponds to the GPS week number 2244.

In the Figure 3 and 4, we present the broadcast and precise orbits of the GPS satellite number 9 between January 8 and 14, 2023.

The computational steps from the broadcast and precise ephemeris of the P, V, and A are given in the Figure 5.

5. Results and discussion

The positions of the GPS satellite number 9 in the orbital plane (Equation 14) computed by the IS-GPS Algorithm during the GPS week number 2244 are shown in Figure 6.

The Lagrange polynomial functions $P_m(X), P_m(Y), P_m(Z)$ of January 14, 2023, computed between 09:15 and 12:30 are given in Figure 7.

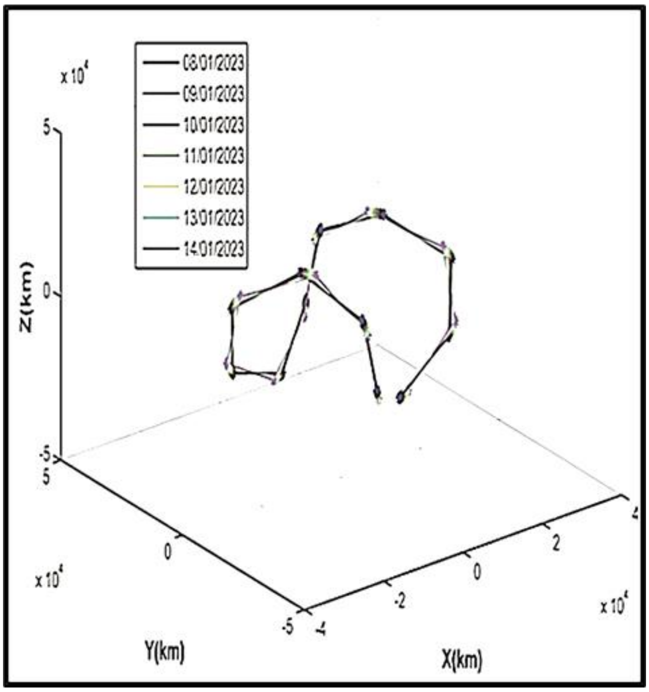

Figure 3. Broadcast orbits of SV 09 (7 days).

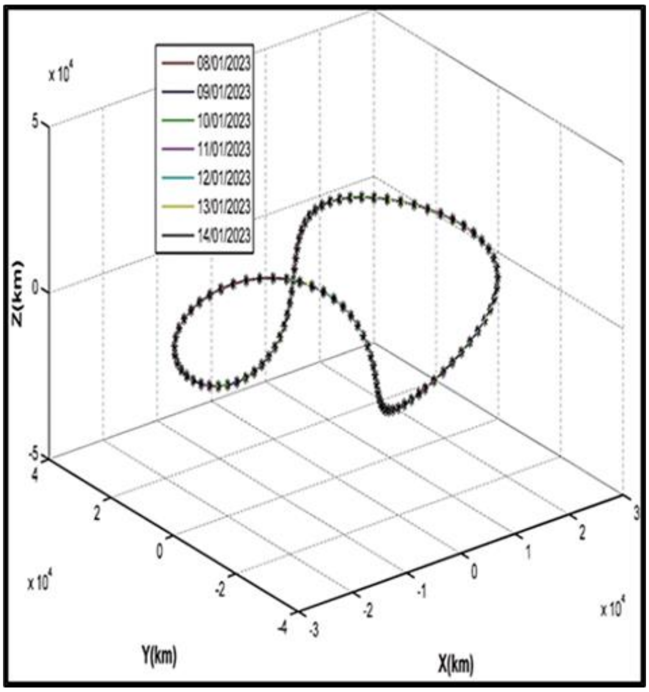

Figure 4. Precises orbits of SV 09 (7 days).

Table 1 gives an example of the differences between the R-K method and the Lagrange interpolation at 10h00m00s on January 13, 2023.

In the Table 1, the computation is carried out at time (Toe = 09h59m44s) using the IS-GPS algorithm and the ephemerides given in Figure 1. The results are used as initial conditions to determine the P, V, and A of the GPS satellite number 9 at time Tc = 10h00m00s by the Runge-Kutta method. The Lagrange interpolation was applied at time (Tc) for the computation of V and A using P given in the precise ephemerides file (Figure 2). In this application, the differences between the Toe and Tc equal 16 sec.

Table 1. 3D Differences (Broadcast vs. Precise).

Tc	P cm	V cm/s	A cm/s²	Methods of computation
10h00m00s	183	0.016	0.0003	IS-GPS algorithm R-K method Lagrange interpolation

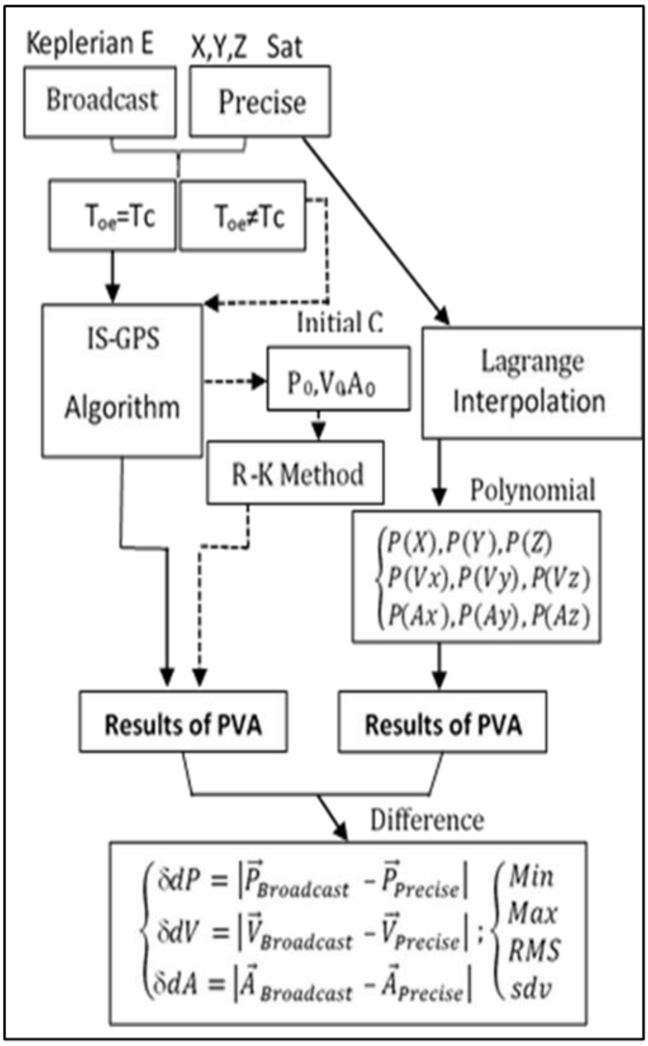

Figure 5. Computational steps of PVA.

During the GPS week number 2244, this process of combined use (IS-GPS Algorithm and the R-K method) is repeated 14 times (twice a day) for the GPS satellite number 9. The differences obtained between the R-K method and the Lagrange interpolation are illustrated in the Figure 8.

The 3D-differences between the broadcast and the precise PVA computation in position varies between 1.38 m and 2.54 m. In velocities the difference varies between 0.14 mm/s and 0.24 mm/s, and in acceleration, the difference varies between 0.003 mm/s^2 and 1.300 mm/s^2.

Figure 9 shows the differences every two hours in position (dx, dy, and dz) between the broadcast and precise ephemerides results during the GPS week number 2244 of the satellite GPS number 9.

The differences obtained from the broadcast and precise data seemed to be within a meter or so in position (dx,dy, dz) and do not exceed 2.4 m in the X, Y, and Z axes; the RMS is 1.66 m and the standard deviation is 0.19 m.

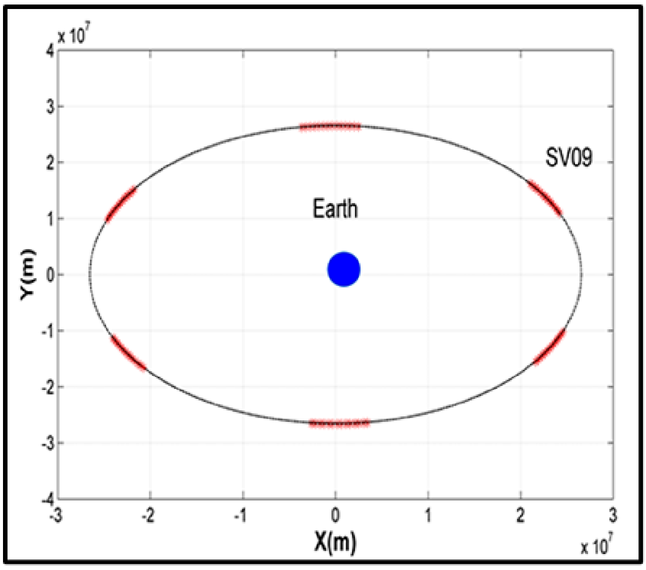

Figure 6. Position of SV 9 in the orbital plane.

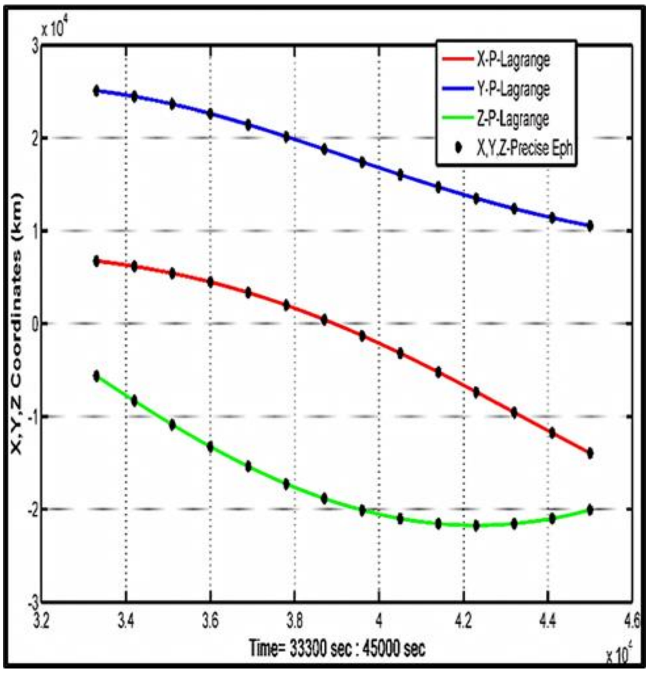

Figure 7. Lagrange polynomial's function $P_m(P)$ of SV 9.

These differences are due to a number of factors, such as:

- The accuracy of each type of ephemeris (broadcast and precise) affects the PVA result.

-The accuracy of the methods used in computation:

1- IS-GPS Algorithm (resolution of the Kepler equation).

2- R-K method (order = 4 and integration step size = 1 sec).

3- Lagrange integration order (order of P(m) = 13), Figure 7 and Equations 30-32.

-The difference between the reference of the broadcast ephemeris (Antenna Phase Center) and the reference of the precise ephemeris (Center of Mass). The determination of this offset using ANTEX files (ANTenna Exchange format) reduces these differences [13].

- In Equations 25 and 26, the acceleration of the satellites due to the luni-solar perturbation is assumed to be null. This assumption gives less accurate position, velocity, and acceleration results. In the GLONASS system, these perturbations are transmitted in the broadcast ephemeris files and added as constant values in Equations 25 and 26.

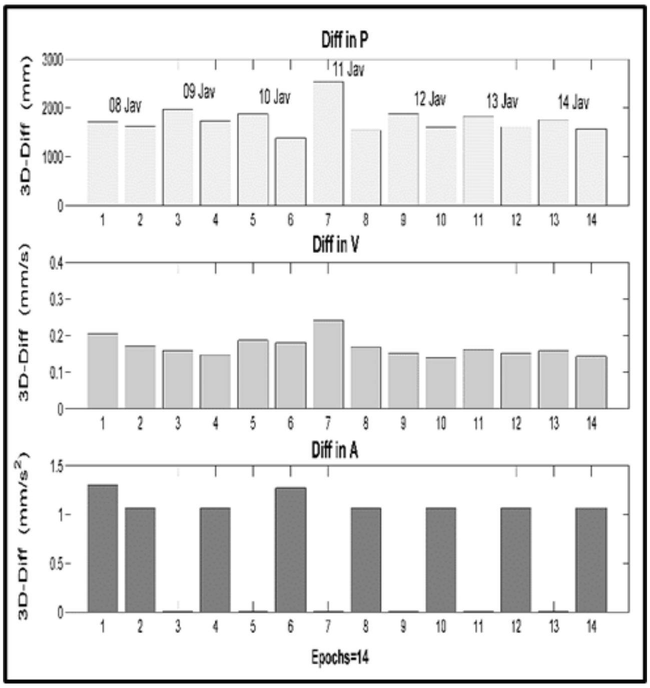

Figure 8. Differences (Broadcast vs. Precise) at 14 epoch (R-K Method and Lagrange interpolation).

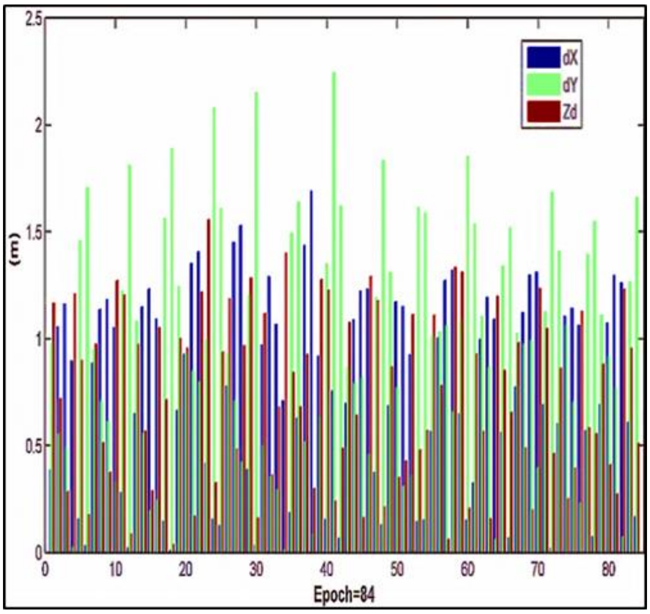

Figure 9. Differences (Broadcast vs. Precise) at 84 epoch (IS-GPS Algorithm, R-K Method and Lagrange interpolation).

6. Conclusion

In GNSS theory, the determination of the receiver position begins with the determination of the satellite position.

This paper presents the IS-GPS Algorithm and the Runge-Kutta integration method for computing the position, velocity, and acceleration of the GPS satellites using the broadcast ephemerides data. In addition, we compared our results with the Lagrange interpolation method used in the estimation of the GPS satellite vectors P, V, and A using precise ephemeris data.

The velocity and acceleration of satellites are important measures in several GPS applications. In this study, the velocity and acceleration of the GPS satellites are obtained by the derivation of the position equations (IS-GPS Algorithm), the integration of the differential equation of GPS satellite motion by the Runge-Kutta method, and derivation of the Lagrange polynomial.

The introduction of the Runge-Kutta method in the computation of the position, velocity, and acceleration (PVA) of the GPS satellites as an alternative solution requires the determination of initial conditions from the Keplerian elements by the IS-GPS Algorithm.

The difference obtained between the broadcast and the precise PVA computation does not exceed 2.4 m (X, Y, and Z axes) in the position of the GPS satellite number 9 and a few millimeters in velocity and acceleration.

Conflicts of interest

The authors declare no conflicts of interest.

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