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Investigation of prospective mathematics teachers’ noticing of student thinking related to probability

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ABSTRACT The aim of this study is to reveal and evaluate the attending and interpreting skills of student thinking of prospective teachers, as well as their instructional suggestions as responding skills. The current study was conducted with 29 prospective mathematics teachers (PMTs) within a qualitative design in the context of probability. First, three probability problems were asked to sixty-two 8th graders (13-14 years old) in a middle school, and their solutions were used to create tasks for PMTs. PMTs answered the tasks in a written report. Then, a class discussion was held, and PMTs were given the opportunity to revise the initial reports. Content analysis was used for data analysis. PMTs demonstrated partial or robust evidence for attending to and interpreting students' thinking. However, they struggled to respond to students' reasoning. In the revised reports, the PMTs' evidence for noticing skills was better with the support of the class discussion. This study provides an example of an approach that can be used for teaching in method courses, allowing PMTs' noticing skills for student thinking to be revealed and improved.

Keywords: *Noticing skills, Probability, Prospective mathematics teachers, Student’s thinking*

Matematik öğretmen adaylarının olasılık öğrenme alanına ilişkin öğrenci düşünüşünü fark etme becerilerinin incelenmesi

ÖZ Bu çalışmanın amacı, matematik öğretmen adaylarının öğrenci düşünüşünü tanımlama ve yorumlama becerileri ile birlikte öğrencilerin düşünüşüne ilişkin öğretimsel önerilerini (karşılık verme becerisi) ortaya çıkarmak ve değerlendirmektir. Bu çalışma 29 matematik öğretmeni adayının katılımıyla olasılık bağlamında nitel bir araştırma ile yürütülmüştür. İlk olarak, altmış iki ortaokul 8.sınıf öğrencisine üç olasılık problemi sorulmuş ve öğrencilerin çözümleri adaylara görev oluşturmak için kullanılmıştır. Matematik öğretmen adayları görevlere ilişkin yanıtlarını yazılı bir rapor halinde vermişlerdir. Daha sonra, bir sınıf tartışması yapılmış ve adaylara ilk raporlarını gözden geçirme ve düzeltme fırsatı verilmiştir. Veri analizi için içerik analizi kullanılmıştır. Matematik öğretmen adayları, öğrencilerin stratejilerini tanımlama ve yorumlama konusunda kısmi ya da güçlü kanıtlar sunabilmiştir. Ancak, öğrencilerin çözümlerine ilişkin öneri vermekte zorlanmışlardır. Düzeltmelerde, matematik öğretmen adaylarının fark etme becerileri sınıf tartışmasının katkısıyla daha iyi hale gelmiştir. Bu çalışma, özel öğretim yöntemleri derslerinde kullanılabilecek bir yaklaşım örneği sunarak, matematik öğretmen adaylarının öğrenci düşünüşünü fark etme becerilerinin ortaya çıkarılmasına ve geliştirilmesine olanak tanımaktadır.

Anahtar Sözcükler: *Fark etme becerisi, Matematik öğretmeni adayları, Olasılık, Öğrenci düşünüşü*

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INTRODUCTION

Teaching involves analyzing and assessing student thinking, which is recognized as one of the teacher's practices (Ball et al., 2008; Jacobs et al., 2010). By observing students' problem-solving techniques in a conversation setting or in the student's written response, teachers may identify the thinking patterns and strategies that their students employ. Teachers can create instructional strategies that support learning based on these outcomes (Lee & Lee, 2023). According to current trends in teacher education, it is more important for teachers to be aware of the thoughts that students have and to provide relevant feedback to students than knowledge about the problem-solving procedures that students use (Bergman et al., 2023; Ivars et al., 2020; Jacobs et al., 2010; van Es & Sherin, 2021). The component of teacher competency that entails the cognitive capacity to detect and analyze the significant features of the students' thinking in order to make instructional decisions is the teacher's professional noticing of students' mathematical thinking (Jacobs et al., 2010; Mason, 2002; van Es & Sherin, 2002). Professional noticing of children's mathematical thinking, proposed by Jacobs et al. (2010), requires teachers' evaluation of students' answers from the perspective of mathematical learning, beyond determining whether students' answers are correct or incorrect. This evaluation allows for the determination of pedagogical methodologies (Wilson et al., 2013).

Recent studies show that the ability to notice things is not a natural talent. Instead, it is a skill that can be learned through work experience and training (Star & Strickland, 2008; van Es & Sherin, 2008). Teacher education programs should provide opportunities for prospective teachers to understand how and in what ways to notice students' thinking (Amador et al., 2021; Star & Strickland, 2008; Stockero et al., 2017; van Es & Sherin, 2008). Thus, the objective of this study is to examine the noticing abilities of prospective teachers by analyzing the manner in which they attend to, interpret, and respond to students' solutions when incorporating a discussion environment into their teacher education program.

In the current study, our focus was on prospective mathematics teachers' ability to notice students' solutions. We used Jacobs et al.'s (2010) proposed construct for professional noticing of children's mathematical thinking to frame prospective mathematics teachers' noticing skills. This allowed us to look at prospective mathematics teachers' noticing of students' mathematical thinking in the context of students' solutions related to probability, which is a specific area of mathematics. Probability, as a mathematical concept, provides an essential basis for learning higher-level statistical topics (Gal, 2005). Additionally, the prevalence of chance in daily life serves as one of the rationales for the inclusion of probability in elementary education curricula. (Batanero et al., 2014).

The Background and Rationale for Research

Noticing Skills

Numerous researchers emphasize the significance of noticing skills in mathematics education and examine this concept. Mason (2002), for instance, proposed the concept of professional noticing and defined it as the ability to recognize and respond to significant aspects of one's profession. van Es and Sherin (2002) presented the idea of learning to notice. This concept has three components: identifying notable classroom situations, using this information to explain classroom interactions, and relating specific classroom situations to learning and teaching principles. Recently, van Es and Sherin (2021) have revised this concept and incorporated the element of acquiring additional knowledge through teacher-student interaction. Jacobs et al. (2010) developed the concept of professional noticing of children's mathematical thinking, which the current study is also based on. They define this concept as "how and to what extent teachers notice children's mathematical thinking" (p. 171). Jacobs et al. (2010) proposed three related skills: 1) attending to the student's solution strategy; 2) interpreting student comprehension; and 3) deciding how to respond to student reasoning. Moreover, these three abilities are interconnected. The quality of the teacher's comments and their ability to respond to students are both influenced by their ability to recognize the mathematical properties of the students' strategies.

Stockero et al. (2017) classified noticing studies as: *1) noticing among instances* and *2) noticing within an instance*. The form of *noticing among instances* proposed by Stockero et al. (2017) involves teachers selecting significant classroom video events and interpreting or reporting what they notice. Previous research (e.g., Sherin & van Es, 2009; van Es & Sherin, 2002) has examined teachers' capacity to notice by analyzing what they deem significant while observing classroom video excerpts. On the other hand, teachers and prospective teachers are given an example of student thinking and asked to analyze it in the second category (*noticing within an instance*). The work of Jacobs et al. (2010) provides a significant example within the context of an instance study. The researchers requested that teachers and prospective teachers examine the strategies utilized in video clips or written solutions of students. In this regard, the current study followed a similar approach to the studies of *noticing within an instance*.

When we examined the literature, we found that studies on noticing skills in the context of student thinking focused on students' solution strategies (e.g., Callejo & Zapetera, 2017; Fernández et al., 2013). Accordingly, we analyze and use the answers students provide to probability problems to investigate how prospective teachers attend to, interpret, and respond based on mathematical elements in the current study. As a result, we assured that prospective teachers could concentrate entirely on student thinking, excluding outside factors such as physical conditions or classroom management. Besides, teachers' capacity for noticing varies depending on the content of mathematics. Probability is an essential field of study in mathematics that has applications in a variety of scientific disciplines, including economics and education (Batanero & Álvarez-Arroyo, 2024). However, both children and adults exhibited comparable inadequate performance and misconceptions in probabilistic reasoning regarding the fairness of chance games (Batanero & Álvarez-Arroyo, 2024). They may fail in probabilistic reasoning tasks due to false beliefs and/or the inability to recall the probability of the draw (Supply et al., 2023). Therefore, this study focuses on both students' thinking processes and prospective mathematics teachers' ability to notice students' solutions. It aims to reveal the mathematical and cognitive dimensions of the probability concept and noticing skill.

Teaching and Learning about Probability

Students gain an intuitive comprehension of the concept of probability when they are able to make predictions and decisions regarding everyday probabilistic situations. Together with scientific knowledge in formal education, this comprehension can foster the development of new and accurate understandings in students (Kazak, 2012). According to the Turkish Ministry of National Education (TMoNE) (2018), 8th graders are able to identify possible outcomes of an event and events with different chances, examine events with equal probabilities, and calculate the chances of simple events. However, without education, students lack the intuitive comprehension necessary to understand advanced probability situations, which can lead to misconceptions (Fischbein & Schnarch, 1997). Students' beliefs and misperceptions about the uncertainty in probability, the concept of equiprobability, sample space identification, probability types, and proportional reasoning generally can result the misconceptions.

Probability encompasses a degree of uncertainty, meaning that the appropriate selection may not consistently lead to the anticipated or intended result. Consequently, students may perform calculations involving quantities while holding incorrect beliefs in the face of uncertainty (Falk et al., 2012). Moreover, when the probability calculations begin, students may struggle with the concepts of equiprobability and sample space determination. Students may overgeneralize under the false assumption of equiprobability, supposing that the removal of one of the names of two boys and three girls indicates that either a girl or a boy will appear, implying a probability of 1/2 (Tarr, 2002). On the other hand, if two events are the same, such as rolling two dice, students may view (1, 2) and (2, 1) as the same and count just one. It results incorrect probability estimations in determining the sample space (Callaert, 2004).

There are various forms of probability, including classical, frequentist, and subjective probability. The classical probability is the ratio of the number of favorable events of an event to all possible states; the frequentist probability is the probability determined by the frequency with which the event occurs in a large number of tried situations; and the subjective probability is determined by the subjective thoughts and beliefs of the individuals about the probability of an event (Batanero & Álvarez-Arroyo, 2024). It's crucial to recognize that as the number of trials rises, the frequentist probability will start to resemble the classical probability. However, students may consider that each trial will yield a unique outcome, making it impossible to determine the true probability (Konold & Miller, 2005). Park and Lee (2019) noted that some prospective teachers also held this misconception. Prospective teachers likely rejected the frequentist probability and interpreted the probability's outcome as arbitrary (Park & Lee, 2019). In the teaching and learning of probability, the coordination of the two perspectives, known as "modeling" is a challenging task (Kazak & Pratt, 2021; Park & Kim, 2023).

Probability continues to be a challenge for numerous individuals, including children and adults. They frequently attribute incorrect probabilistic reasoning to a lack of understanding of proportionality (Bryant & Nunes, 2012). For instance, the proportions of yellow and blue marbles in the bags help to assess the probability of obtaining a yellow marble. However, individuals struggle with probabilistic reasoning tasks due to false beliefs or forgetfulness of the denominator, a condition known as denominator neglect (Falk et al., 2012).

In conclusion, research indicates that students struggle with probabilistic thinking and have a variety of misconceptions. Accordingly, teaching probability is not an easy field (Batanero & Álvarez-Arroyo, 2024; Park and Kim, 2023; Supply et al., 2023). Therefore, in this study, we believed that supporting prospective mathematics teachers in this regard and providing them with awareness about students' thinking would aid in probability teaching and learning.

Significance of The Research

Teacher education programs should give prospective teachers opportunities to understand what and how they will notice student thinking (Star & Strickland, 2008; Stockero et al., 2017). This study would provide actual student solutions to the prospective mathematics teachers, allowing them to employ their noticing skills. Moreover, within the context of noticing skill, the prospective teachers would analyze and evaluate student understanding and could determine the appropriate pedagogical method based on this evaluation individually. Then, the prospective teachers would discuss their thinking on students' solutions in a classroom environment. The discussion environment facilitates the ability of prospective teachers to analyze, interpret, and suggest instructional strategies (Sherin & van Es, 2009; Sherin & Han, 2004; Ulusoy & Cakıroglu, 2021). It would expose prospective teachers to diverse perspectives and inspire them through classroom discussions and individual practices. This process is one of the study's contributions, helping prospective teachers realize their lack of or incorrect information and complete it. The current study's methodological approach, which involves an initial individual evaluation followed by a discussion, may set it apart from previous studies on noticing skills.

Studies examining prospective teachers' ability to notice within the context of content-specific noticing have acquired prominence in the literature in recent years (e.g., Copur-Gencturk & Rodrigues, 2021; Copur-Gencturk & Tolar, 2022; Ulusoy, 2020). In fact, Walkoe (2015) emphasizes the importance of focusing on a specific area of mathematics for teachers' ability to recognize the development of student understanding. A number of studies have looked at how well teachers can notice pattern generalization (Callejo & Zapatera, 2017; Lee & Lee, 2023), measurement (Girit-Yildiz et al., 2023), fractions (Ivars et al., 2020), exponential expressions (Ulusoy, 2020), and rational numbers (Warshauer et al., 2021). In these studies, researchers utilized written cases or video clips involving students' solution strategies. Combining student cognition with subject-specific mathematical components, they examined the noticing skills of teachers or prospective teachers and obtained subject-specific results. We anticipate that the current study will add to the existing literature and broaden the scope of previous research on content-specific noticing within the context of probability.

Prospective teachers must possess an established understanding of probability and the capacity to recognize students' misconceptions. Furthermore, prospective teachers have to anticipate student responses, respond correctly, and possess the capability to remedy them if needed (Lee & Lee, 2023). In a similar way, in order to effectively teach probability, it is necessary to anticipate the informal ideas and challenges that students bring to the classroom (Batanero & Álvarez-Arroyo, 2024). However, there are still very few publications that focus on the cognitive and interconnected components of teacher didactic knowledge in probability, which consist of how teachers conceptualize their students' learning, anticipate their difficulties and misconceptions, and devise instructional strategies that address these obstacles (Batanero & Álvarez-Arroyo, 2024). Consequently, it is essential to investigate whether prospective teachers possess a consistent ability to recognize students' misconceptions regarding probability and if their prospective education adequately equips them to address these issues (Park & Lee, 2019). This research presents actual student answers to prospective mathematics teachers with the aim of supporting their conceptions about student thinking. The prospective mathematics teachers would be required to identify and interpret student strategies in these solutions, as well as provide pedagogical suggestions to enhance student reasoning. The goal of this study at this point is to reveal and evaluate their attention and interpretation skills, as well as their instructional suggestions. In this context, the research questions of the study are as follows: 1) How do prospective mathematics teachers attend to and interpret student thinking in student solutions on probability? and 2) What instructional suggestions do prospective mathematics teachers have to respond to students' thinking?

METHOD

This study designed a qualitative investigation to reveal and assess prospective mathematics teachers' ability to notice students' thinking about probability learning. Qualitative research permits a comprehensive, theoretical structure to investigate a problem or topic, accompanying the interpretations and meanings of participants (Creswell, 2009).

We specifically employed the case study, a qualitative research methodology. Case studies seek answers to inquiries about the researched topic by examining one or more cases (Merriam, 2009). Researchers (Creswell, 2009; Merriam, 2009; Yin, 2009) define the case as a contextually bounded phenomenon. Merriam (2009) asserts that a phenomenon must possess a certain bound to be the case. Limiting the time for data collection, the number of participants, or the topic under investigation can provide this boundary. We restricted the content to the subject of probability, limited the number of participants to prospective mathematics teachers who took the course on the methods for teaching probability, and limited the time frame to seven weeks. We used a holistic approach, as stated by Yin (2009), in explaining the prospective mathematics teachers' noticing of students' thinking on probability in findings.

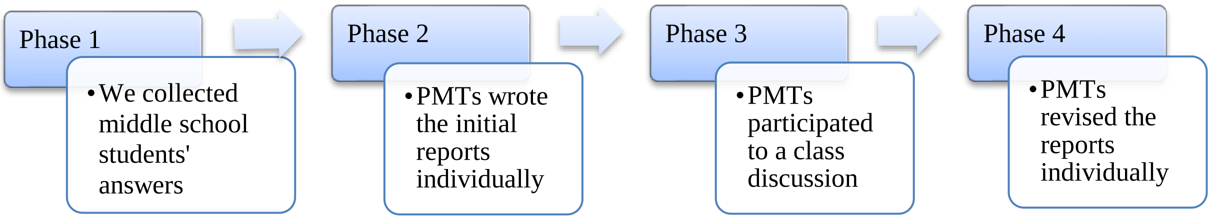
Participants

We conducted this research with the 29 prospective mathematics teachers (PMTs) (25 females and 4 males) enrolled in an elementary mathematics teacher training program, and they were in the fourth year (final year) of the program. PMTs who successfully finish this program will be qualified to instruct mathematics to students in the middle school grades (ages 11-14). At the time of the study, they had already completed a method course on probability teaching. Thus, we used purposive sampling, which is defined as sampling with a specific purpose (Merriam, 2009). The cumulative grade point averages of the PMTs ranged from 2.50 to 3.50 out of 4. Prior to collecting data from PMTs, however, it was necessary to use actual students' responses in the survey questions. We derived these solutions from the responses of sixty-two middle school students (13–14 years old) who participated in a probability course. We obtained the necessary ethical approvals for the investigation. Furthermore, all participants in the study were volunteers.

Data Collection

The study comprises a three-step procedure for collecting data (see Figure 1). Phase 1 involved assigning three problems to middle school students who had studied probability. We gave them one hour's lesson (40 minutes). We collected using the students' solutions. Next, we conducted an analysis and created three tasks for the PMTs over a period of approximately four weeks, as detailed in the following section. The next phases took place in successive weeks. In Phase 2, the PMTs answered each task's questions in the initial written responses. This phase lasted about 100 minutes, and the PMTs studied individually in a classroom. In Phase 3, the researcher facilitated a class discussion and asked such questions as, "In the student's solution, what did you notice, what was interesting, can you give details, how do you relate them to mathematical concepts, what would you suggest to remedy the student's mistake, and does anyone have different suggestions?". We recorded this class discussion on video. In these discussions, PMTs contributed by sharing their individual responses. This phase lasted about 60 minutes and took place in the classroom. The following week, during Phase 4, we provided each PMT with the opportunity to individually revise their initial written report. In addition, each prospective teacher was required to justify the changes they made to their revised report by relating them to what they noticed during the class discussion. This phase lasted about 60 minutes, and the PMTs studied individually in the classroom. PMTs' written comments on the two reports, the initial and revised reports, served as the primary data source for this study. Figure 1 shows the study's data collection process:

Figure 1.
The Data Collection Process



Preparation of Tasks for PMTs

We adapted probability problems from sample questions for national exams found on the TMoNE (General Directorate of Assessment, Evaluation, and Examination Services, n.d.) website and the textbooks published by TMoNE. We determined a criterion for problem selection that satisfied the five objectives of the middle school curriculum (TMoNE, 2018). The first problem is connected with "identifies the possible outcomes of an event and distinguishes the outcomes with more, equal, and less probabilities"; the second problem is connected with "explains that the probability value of each output is the same in events with equal chance and that this value is 1/n (equiprobability)"; the third problem is related to "understands that the probability value is between 0 and 1 (including 0 and 1) and calculates the probability of a simple event occurring". Thus, we ensured content validity by associating the problems with objectives. We finalized the problems (see Appendix) by consulting an expert in mathematics education and an expert in language. While the students were solving the problems, the second researcher asked them about any points they didn't understand. Therefore, we provided to ensure reliability issues. We collected and analyzed the students' solutions at this phase. We classified them as correct, partially correct, incorrect, correct answer (without solution), incorrect answer (without solution), and left-blank (see Table 1).

Table 1.
Classification of Middle School Students' Answers

Categories		First problem (#)	Second problem (#)	Third problem (#)
Correct solution		18	16	14
Incorrect solution		20	18	9
Partially correct solution	Incomplete	-	6	5
	Incorrect solution-correct answer	7	-	-
	Correct solution-incorrect answer	-	-	7
Correct answer (without solution)		8	-	-
Incorrect answer (without solution)		5	16	18
Left blank		4	6	9
Total		62	62	62

According to Table 1, while the proportions of correct and incorrect solutions for the first and second problems are roughly the same, the number of correct solutions exceeds the number of incorrect solutions for the third problem. The results indicated that students had misconceptions regarding probability, sample space determination in probability calculation, and certain vs. impossible events. Table 2 presents the subcategories of incorrect solutions for the problems:

Table 2.
The Subcategories of Incorrect Solutions

Problem	Sub-categories
First problem	Selecting small numbers
	Considering the difference between sales quantity and defective product quantity (additive thinking)
	Simplification errors in proportions
Second problem	Subtracting or adding the numbers without proportioning
	Incorrectly determining the sample space

We identify three subcategories within the category of partially correct solutions (see Table 1). In the incomplete subcategory, the solution remained incomplete despite accurate probability calculations. For example, in the second problem, some students correctly calculated the probability order but failed to determine the equiprobability. In the third problem, some students either only calculated the probability or correctly determined the impossible-to-certain events. In the subcategory of incorrect solution-correct answer, the correct response was provided by coincidence despite the solution process indicating erroneous reasoning. For example, in the first problem, some students arbitrarily selected the answer that showed the largest difference between the number of sales and the number of defective products and then provided the correct response based on the size of the numbers. The correct solution-incorrect answer subcategory encompassed both accurate probability calculations and misconceptions related to the concepts of impossible-certain events. In the third problem, for instance, some students correctly calculated the probability but defined all cases except the impossible event as certain. There are examples of student solutions in the Appendix.

For each probability problem, we have selected one of the correct, incorrect, and partially correct solutions, which will require the reasoning and noticing skills of PMTs. We devised tasks for PMTs in this manner. Each task contains a probability problem, three students' solutions, and three questions to which the PMTs must provide written responses. The questions are as follows: 1) Describe the student's strategy in detail by associating it with mathematical concepts. 2) Evaluate the student's strategy and provide a detailed explanation. 3) Pretend to be the student's teacher. How do you facilitate student learning when a solution is partially incorrect or founded on a misunderstanding? Or, if the student's answer is correct, how would you enhance their understanding? (see Appendix).

Data Analysis

We used Jacobs et al.'s (2010) professional noticing of children's mathematical thinking framework to assess the PMTs' noticing skills on student solutions. We first coded our data using the framework. Then we identified the need for some modifications. In a two-way conference, we determined our final codes and devised a rubric (see Table 3).

Next, we analyzed the data using one of the qualitative analysis methodologies, content analysis. Content analysis, a type of qualitative analysis, converts explanations (sentences, paragraphs, etc.) deemed meaningful in and of themselves into codes (Strauss & Corbin, 1994). The unit of analysis was PMTs' statements or explanations, each deemed meaningful in its own right, in their initial and revised reports. Therefore, we coded each PMT's writings based on how well they demonstrated the three skills of attending, interpreting, and responding. Table 3 demonstrates the use of a rubric to level each of the three skills. We examined the evidence in the responses of PMTs to understand their ability to notice student thinking.

Table 3.
Rubric for Analyzing PMTs' Responses

	Robust evidence (2)	Limited evidence (1)	Lack of evidence (0)
Attending	PMT explains most of the mathematical elements in the student strategy.	PMT explains some of the mathematical elements in the student strategy.	PMT uses general statements.
Interpreting	PMT makes meaningful and correct comments by referring to most of the mathematical elements of the student strategy.	PMT makes comments by referring to some of the mathematical elements of the student strategy.	PMT makes general comments.
Responding (Instructional suggestions)	PMT provides conceptual and mathematical suggestions.	PMT provides conceptual and partially mathematical suggestions.	PMT provides general or nonmathematical suggestions.

We used triangulation to ensure the validity of the research. In this study, we gathered data from two distinct sources: the individual reports and the class discussions. We utilized cross-checking to ensure coding reliability (Creswell, 2009). Therefore, we independently coded the complete data set with the codes to ensure the accuracy of the coding. Then, using Miles and Huberman's (1994) formula, we completed independent coding and derived a reliability percentage of 90%. We went over the disputed codes and discussed what they meant until we got a total agreement. Furthermore, to enhance reliability, we provided a detailed explanation of the research process and verified our findings with the scans of students' solutions and direct quotations throughout the study.

Ethical Procedures

The Human Research Ethics Committee of Trakya University's report E-29563864-050.04.04-272275, dated 15.06.2022, and the Ministry of National Education's report E-87085441-44-68502300, dated 17.01.2023, both confirm that the research does not pose an ethical problem. We informed the participants about the research prior to its implementation. They participated in the study voluntarily. We reported the names of participants according to ethical rules.

FINDINGS

PMTs' Attending, Interpreting, and Responding Skills in Their Initial Reports

Table 4 presents the results of the PMTs' levels of evidence, which were obtained from their initial reports regarding different types of students' answers.

Table 4.
Frequencies of PMTs' Levels of Evidence in Their Initial Reports

Task	Type of students' answer	Attending			Interpreting			Responding		
		(0)	(1)	(2)	(0)	(1)	(2)	(0)	(1)	(2)
1	I	4	6	19	10	10	9	18	6	5
	PC	12	9	8	19	6	4	17	5	7
	C	4	14	11	11	10	8	23	6	0
2	I	13	8	8	11	6	12	18	5	6
	PC	5	12	12	8	12	9	18	6	5
	C	8	3	18	12	6	11	21	5	3
3	I	7	17	5	9	7	13	19	4	6
	PC	5	5	19	5	10	14	14	1	14
	C	7	5	17	8	9	12	21	0	8
Total		65	79	117	93	76	92	169	38	54
%		25	30	45	36	29	35	65	14	21

Note. (0) Lack of evidence, (1) Limited evidence, (2) Robust evidence, I: Incorrect solution, PC: Partially correct solution, C: Correct solution

Table 4 shows that approximately half of the PMTs (45%) were able to provide answers supported by robust evidence to questions about attending skills. In this regard, the PMTs were able to explain the students' solution strategies using mathematical properties. In terms of interpretation, the majority of PMTs (64%) were able to provide answers supported by limited and robust evidence. In this sense, they were able to mathematically explain why students' solution strategies were correct, incorrect, or partially correct. In contrast to attending competence, however, there was a rise in the lack of evidence in interpretation. According to Table 4, responding is the ability for PMTs to provide the least amount of evidence. More than half of the PMTs' recommendations (65%) lacked evidence. Thus, the majority of PMTs provided non-mathematical or general suggestions. There are few recommendations that are supported by robust evidence (21%). PMTs were able to offer very few conceptual and mathematical suggestions.

The tables below provide examples of PMTs' explanations of their attending, interpreting, and responding abilities for students' incorrect, partially correct, and correct answers.

Attending

Table 5 provides examples of responses from PMTs, which were deemed to be at varying levels in terms of their ability to attend to the incorrect solution in the second task.

According to Table 5, PMT22 made a general comment about the student's errors but was unable to describe the student's strategy. Thus, he provided a lack of evidence. PMT17, on the other hand, stated that the student expressed the chances as percentages and the favorable number of marbles as a percentage of the total number of marbles while calculating these percentages. However, PMT17 did not discuss the specific strategy for determining the proportion of blue marbles. Thus, she utilized some mathematical concepts and was able to provide limited proofs. PMT21 reported that the student discovered equal probability due to the equality of yellow and black and demonstrated this mathematical reasoning using mathematical symbols. She also noticed and explained the unique strategy employed by the blue marbles. PMT21 detailed most of the mathematical elements in the student's strategy by using examples and mathematical notations. Therefore, we considered her response robust evidence.

Table 5.
Examples of PMTs' Attending Comments to an Incorrect Student's Answer

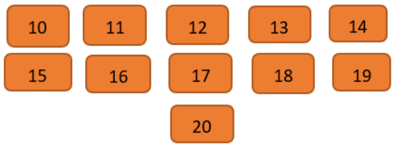
<table>
<tr><td>A student's incorrect solution</td><td>2nd Problem: Ece and Can want to play with the marbles they have. Can creates a table that determines the numbers and colors of the marbles. While Can is drawing the table, Sena puts the marbles in the bag. Ece asked Can to find out:
a. Chance of each marble (yellow, blue, black, white, and green) randomly selected from the bag.
b. Which of them has an equal chance? Order the values of probabilities.
What did Can find when he answered the questions correctly?
<table><tr><th>Colors</th><th>Number</th></tr><tr><td>Yellow</td><td>22</td></tr><tr><td>Blue</td><td>36</td></tr><tr><td>Black</td><td>22</td></tr><tr><td>White</td><td>24</td></tr><tr><td>Green</td><td>1</td></tr></table>
Yellow and Black has equal chance, yellow-black=83%
Yellow Blue Black White Green
83% 64% 83% 81% 104%</td></tr>
<tr><td>Level</td><td>Examples of PMTs' attending comments</td></tr>
<tr><td>A(0)</td><td>This student has a lot of misconceptions and mistakes about the subject (PMT22).</td></tr>
<tr><td>A(1)</td><td>This student correctly determined that the marbles are equally likely to be black or yellow. However, while writing the ratio for each marble, he subtracted the number of marbles from the total number of marbles and stated it as a percentage. He stated that yellow and black are equally likely because their percentages are the same. The student did not order the values of probability (PMT17).</td></tr>
<tr><td>A(2)</td><td>He assumed that there would be an equal chance of drawing yellow and black marbles based on the quantity of marbles. He calculated the percentages of all the other colors, excluding blue, and then represented them as [105 - (the number of marbles by color)]/100. For instance, he calculated as $\frac{(105-24)}{100}$ = %81 for the white ones. His approach in the blue marble was to $\frac{(100-36)}{100} = \%64$ (PMT21).</td></tr>
</table>

Interpreting

Table 6 provides examples of responses from PMTs, which were categorized based on their ability to interpret the partially correct solution in the third task.

According to Table 6, PMT10 made an overall comment and stated that only the student employed the correct method. We would have expected her to provide a detailed explanation of why this strategy was correct. In addition, PMT10 missed the student's strategy error. Therefore, she presented insufficient evidence. PMT6 explained the ratio used by the student in the chance calculation and stated that he found the values to be accurate. Additionally, she stated that the student made an error in determining a certain event, but she did not elaborate. Despite her failure to clarify the error, she recognized it and accurately evaluated the student's strategy by considering calculations. Thus, she was able to make some mathematical interpretations and provided limited evidence. On the other hand, PMT8 stated that the student did not have difficulty calculating the probability but that he made an error in determining a certain event. PMT8 elaborated by analyzing the majority of the mathematical concepts in the student's strategy. Therefore, we considered her interpretation robust evidence.

Table 6.
Examples of PMTs' Interpreting Comments to a Partially Correct Student's Answer

A student's partially correct solution	3rd Problem: The digits 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20 are written on identical cards and placed in a bag. The number of a card is randomly selected from the bag; Determine the probability of each of the following events:  a. One-digit number b. Even number c. Odd number d. Zero e. Two-digit number f. Three-digit number Determine which events are certain and which are impossible. a. One-digit number = impossible b. Even number = 6/11 = certain c. Odd number = 5/11 = certain d. Zero = impossible e. Two-digit number = 11/11 = certain f. Three-digit number = 0/11 = none = impossible
Level	Examples of PMTs' interpreting comments
I(0)	The method the student used to calculate the chance is correct and sufficient to arrive at a solution (PMT10).
I(1)	The student's solution is right. He calculated the probability by expressing the ratios as "favorable situation/all possible situations" and discovered that all of the ratios were accurate. However, while finding the impossible event true, he made a mistake in the concept of the certain event (PMT6).
I(2)	The student has some misconceptions. There is no problem in calculating chance values. He calculated the probabilities of all the items correctly from the ratio of the favorable number of cases to the number of all cases. He also found impossible events true. He knows that if the probability value is equal to zero, it will be an impossible event. However, his understanding of the term "certain event" was incorrect. He called every probability item except zero a certain event. He is not aware that the probability value must be equal to 1 for it to be a certain event (PMT8).

Responding

Table 7 presents examples of instructional suggestions for PMTs, which were deemed to be at varying levels based on the responses to the incorrect solution in the first task.

Table 7 reveals that PMT19 provided a broad recommendation to remedy the student's misunderstanding. In PMT19's proposal, misconceptions are not addressed, nor is the material or support provided. We expected him to provide a detailed explanation of the topics outlined in this proposal, the teaching methods, and the impact on addressing students' misconceptions. Consequently, PMT19 provided a lack of evidence. PMT7, on the other hand, correctly identified the misconception and suggested emphasizing the concept of equivalent fractions in order to eliminate it. In equivalent fractions, she explained, the difference between the numerator and denominator may be distinct, but it can indicate that the ratio is constant. We expected PMT7 to demonstrate her highlighted points and explain their connection to probability. We deemed the evidence insufficient to address mathematical elements, despite the possibility of his suggestion working. PMT1 first provided an illustration to help the student recognize his error. Then, she created questions specific to the problem and had the potential to stimulate the student's thinking. PMT1 effectively addressed the student's misconceptions by crafting examples and questions that complemented the concept of probability. Therefore, we considered her suggestions robust evidence.

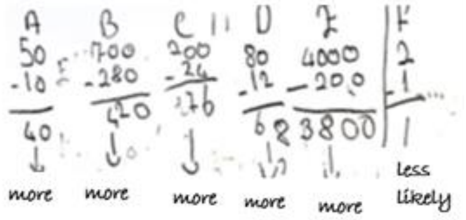
Table 7.

Examples of PMTs' Responding Suggestions to an Incorrect Student's Answer

A student's incorrect solution

1st Problem: This year, Nur will graduate from high school, and her family desires to buy Nur a computer as a graduation gift. Nur will select between four brands.
Listed below are the number of computers sold by each brand and the number of computers returned as defective products belonging to that brand. Which brand, according to the table, is Nur most likely to purchase a defective product from?

Brands	A	B	C	D	E	F
Number of defective products	10	280	24	12	200	1
Sale amount	50	700	200	80	4000	2



Level	Examples of PMTs' suggestions
R(0)	We must review and re-explain some topics that the student profoundly understands. Additionally, we should provide manipulatives and support (PMT19).
R(1)	He has a wrong idea. In order to figure out the chances, he took the favorable outcome out of the case. This made the results wrong. We can get rid of this mistaken idea by using equivalent fractions. Sometimes the difference between two fractions with the same probability can be bigger, but what's important is the ratio, not the difference, as shown by the other cases (PMT7).
R(2)	The student lacks a clear understanding of the concept of sample space. First, the student needs to understand that he made a mistake, as the number of products sold is not the same as before. For instance, 7 of 15 sales of one brand are defective, and 10 of 18 sales of the other brand are faulty. I would pose a question to the student: "How can you make a comparison?" When the student deducts the defective sales from the total sales, they consistently arrive at the same result. In this case, the student sees that even though the sales amounts are different, the difference is still the same, so she/he recognizes her/his mistake. In this scenario, I would pose the following question: "Is their probability the same?, If the number of sales isn't the same, does having the same number of defective products provide the correct answer?" After the student notices her/his mistake, I can teach that the probability is the ratio of favorable cases to the total cases. This way, the student can compare each of them with the probability (PMT1).

The significant number of lack-of-evidence suggestions for correct solutions was one of the study's highlights (see Table 4). Although PMTs can provide limited and robust attentions and interpretations on student solutions, their suggestions lack the necessary evidence to advance the student's learning. The PMTs frequently promoted asking diverse questions, yet they failed to provide guidance on how to execute them effectively. These are broad ideas, and it is unclear how to improve the understanding of students with specific questions. For instance, PMT22 made the following suggestion to the correct student's solution in the first task:

> *"The student has already reached the correct conclusion based on the concept of ratio. I would ask the students a variety of questions to help them learn more about the subject (PMT22)."*

PMT22 stated that she would pose various questions, but the nature of these questions was unclear. She did not specify whether there would be similar or higher-level questions or what types of questions would be high-level, and PMT22 made a very general suggestion. None of the PMTs presented a robust level of suggestion for the correct solution in the first task. For instance, the student could be asked to solve the question using a different strategy, or he could be encouraged to devise alternative strategies by posing questions such as, "How would you solve it if the number of errors were given?". Conversely, PMT8 suggests responding to the correct student's solution in the second task in the following manner:

> *"I believe the student has a solid grasp of the subject. He didn't have any misunderstandings. I would reexamine the solutions using a different example and assess whether the subject is well understood (PMT8)."*

The example provided by PMT8 in her proposal was not clear. She added that she would use an example to ensure the student's comprehension. Consequently, PMT8 provided a lack of evidence. However, the expectation here was to propose a subject-specific problem that would enhance the student's comprehension. For instance, comparing the probabilities of the same colors in two distinct bags, posing problems (find numbers such that when I remove a color, the probability of the remaining colors is the same, PMT10), or problem-solving (I chose the ball and did not place it back in the bag; how are the changes for this color and the other colors? Does the probability of this color decrease, increase, or remain constant? PMT21) may be utilized. PMT14's suggestion for the correct solution in the third task is:

> *"Because I thought the student understood the subject well, I would give him more difficult problems to solve to help him move forward (PMT14)."*

PMT14 also recommended solving non-specific examples, in line with the previous recommendations. For this endeavor, there were a few robust-level suggestions (see Table 4). For instance, one could pose questions that necessitate the fulfillment of both conditions, like calculating the probability of receiving an even number less than 20 or an odd number greater than 20 (PMT25) or determining which numbers should be eliminated to determine the probability of an odd number in a specific event (PMT1).

PMTs' Attending, Interpreting, and Responding Skills in Their Revised Reports Following the Class Discussion

The PMTs had the opportunity to revise their initial report following the class discussion. Table 8 displays the levels of evidence that the PMTs revised reports, pertaining to various types of student responses.

Table 8.
Frequencies of PMTs' Levels of Evidence in Their Revised Reports

Task	Type of students' answer	Attending (0)	Attending (1)	Attending (2)	Interpreting (0)	Interpreting (1)	Interpreting (2)	Responding (0)	Responding (1)	Responding (2)
1	I	5	3	21	5	8	16	5	13	11
	PC	8	10	11	12	9	8	10	8	11
	C	3	9	17	9	9	11	13	11	5
2	I	14	10	5	8	8	13	10	8	11
	PC	3	12	14	8	10	11	9	9	11
	C	6	4	19	14	4	11	10	12	7
3	I	5	10	14	9	9	11	11	9	9
	PC	4	5	20	4	8	17	8	3	18
	C	6	5	18	7	10	12	14	6	9
Total		54	68	139	76	75	110	90	79	92
%		21	26	53	29	29	42	35	30	35

Note. (0) Lack of evidence, (1) Limited evidence, (2) Robust evidence, I: Incorrect solution, PC: Partially correct solution, C: Correct solution

Table 8 shows that more PMTs were able to respond to questions about attending and interpreting skills with robust evidence than in the initial reports. Additionally, the proportion of cases lacking evidence has decreased. In other words, PMTs were able to mathematically explain students' solution strategies and determine whether they were correct, incorrect, or partially correct. Table 8 indicates that the instructional strategy suggestion, the skill for which PMTs provide the least amount of evidence in their initial reports, has undergone improvements. The rate of recommendations lacking evidence dropped from 65% in the initial report to 35% in the revision. Moreover, the number of recommendations that involve both limited and robust levels has increased. According to Table 8, the majority of PMTs could provide conceptual and mathematical recommendations. Table 9 presents the examples that show the shift in PMTs' noticing abilities alongside their respective justifications.

Table 9.
Examples of PMTs' Shifted Noticing Skills (For the First Task)

Noticing skill	Solution type	Initial report	Revised report
Attending	Partially correct solution	A(0): The student's method was not mathematically correct. The student was undecided and discovered the last correct answer; however, this correct answer was a bit of a coincidence (PMT28).	A(2): In the first case, the student chose option F and focused on the brand with the least faulty product. He is unaware that the brand with the least defective product should be proportionate to the sales amount, and vice versa. He focused on numbers. Although he initially favored the F brand, he shifted his preference to the E brand in the second instance, focusing on the total sales amount. Each time, he believes that the next faulty product may be related to him; occasionally, he employs a percentage, indicating that he tries to use a different strategy each time (PMT28). Rationale: I did not elaborate on why the student's answer was incorrect. Therefore, I revised it (PMT28).
Interpreting	Correct	I(0): The student's strategy in the solution is right. However, he was unable to apply this method to his solution. He followed the correct procedure in his technique, which involved proportioning the number of faulty items to the sales amount. However, he neglected to apply the denominator equating procedure for certain brands when equating the denominators of these brands in the fraction comparison (PMT12).	I(1): In his strategy, the student has only gone a long way. Instead of equating all the fractions to a common denominator and sorting immediately, he made the order numerous times by equating the denominators individually while comparing the fractions. At the same time, the student understood the concept of writing the favorable cases divided by all possible cases while calculating the chance (PMT12). Rationale: I was unaware of the student's method of comparing fractions. As I recognized that his technique of comparing all fractions independently was valid, I revised my argument (PMT12).

Table 9. (Continuıed)
Examples of PMTs' Shifted Noticing Skills (For the First Task)

Noticing skill	Solution type	Initial report	Revised report
Responding (Instructional suggestions)	Incorrect	R(0): I believe there is a gap in the student's prior knowledge, and activities in fraction comparison and percentage calculations can fill it in (PMT14).	R(2): I present examples of numbers with the same difference. When he recognized his solution was incorrect, I would begin with a simple chance calculation example. "For example, the first bag has two pink and two yellow balls. The second bag has one pink and one yellow ball. If we draw a ball, which one is more likely to be pink?" According to the student's reasoning, the solution is the second bag because 4-2=2, 2-1=1. I'd teach him that half of both bags' balls are pink and that probabilities should be equal. Then I'd show him how to complete the chance calculation and help him construct a multiplicative relationship (PMT14). Rationale: I provided some illustrative examples to my first suggestion. I've updated it to specifically explain what these are and how I can use them (PMT14). Rationale: I provided some illustrative examples in my first suggestion. I've updated it to specifically explain what these are and how I can use them (PMT14).

The rationales presented by the PMTs in their revised reports also were held in various discussion sections. Here are some discussion excerpts that support this while PMTs discuss the correctness of the student's strategy (interpreting):

> *PMT13: She understood the problem but was unable to solve it using the right strategy. With his subjective thinking, he arrived at the conclusion.*
>
> *Researcher: What mathematical concepts did the student employ here?*
>
> *PMT29: He didn't try to construct a mathematical ratio. He believes that if there were only 10 defectives, the chance would be lower. When there are 280 defective products, the likelihood increases.*
>
> *PMT21: He looks at the number of faulty products rather than the ratio and always thought that she would receive the faulty product in the next purchase. He held this belief until he encountered the final brand. The fact that it says F in the first place is due to the minimum number of faulty products. After examining all of them, he observes a significant difference in product E and concludes that it makes more sense. In other words, it also considers the number of product sales.*
>
> *PMT13 initiated the conversation with a comprehensive explanation. The researcher then posed a question that prompted the class to focus on mathematical concepts. Then, PMT29 commented mathematically, but he was not able to assess the whole reasoning of the student. PMT21 clarified that the student had chosen the option that had the highest number of defective products and provided adequate explanations for her sales. Thus, the discussion began with a general comment and then became increasingly specialized, focusing on mathematical properties.*

During the discussion, we also observed that PMTs' lacking knowledge was completed, as in the following excerpt as they described the solution strategy (attending):

> *PMT24: He organized the numbers in order of size. He stated that larger numbers indicate more chances. The student's reasoning was correct, but he didn't explain how he knew what our sample space was.*
>
> *PMT12: The teacher requested a chance calculation from the student, but he instead wrote the numbers. While calculating the likelihood of receiving a green, he wrote down the numbers, which led to a problem with sample space comprehension. He failed to include the number of greens in all possible cases. He arrived at the right order but failed to address the concept of probability.*

We expected PMT24 to observe the student's perception of the sample space based on the likelihood of selecting a green marble. PMT12 clarified that the student incorrectly calculated the sample space by subtracting the number of green cases from the total probability of being green.

During the discussion, the PMTs generally showed a tendency to ask questions intended to make the students recognize their misconceptions, as in the following excerpt as they discussed the instructional suggestions (responding):

> *PMT5: The student thought without regard for possibility. I recommend conducting chance trials with a large number of repeats. We move closer to classical probability as we increase the number of results.*
>
> *Researcher: This concept could be an effective method of teaching classical probability. How can we prevent him from disregarding probability in his thinking?*
>
> *PMT7: The question allows us to obtain equivalent fractions. For instance, when we contrast brand A with brand C, we find that the product and sales quantities are different, but we still need to equalize the sales amounts. He realizes he made a mistake there.*
>
> *PMT11: I presumed it was intended to help her comprehend her mistake. There is one in F; it focuses on one, or I felt it was the closest. Simplifying A yields a 1/5. As a result, there is at least one in both. Then he realizes that the number of sales is important. Therefore, one has a score of 5, while the other has a score of 2. This implies that he believes we should also examine the number of sales.*

Despite PMT5's assertion that the student lacked an understanding of probability, her suggestion could potentially provide a theoretical approach to the concept. By posing the question, the researcher aimed to focus the conversation on the concept of probability. On this point, PMT7 and PMT11 suggested expanding and simplifying the data in the question, as well as using equivalent fractions to help the student recognize the error. Using a similar strategy, we discovered that PMTs tended to prompt students with misconceptions to recognize their errors first, particularly at the phases where we requested instructional strategy recommendations.

CONCLUSIONS AND DISCUSSION

The purpose of this study is to investigate prospective mathematics teachers' abilities to attend to, interpret, and respond to students' understandings of probability. We expect them to analyze and discuss the strategies of students in the written cases, each with a unique understanding of probability. This study also examined how the class discussion supported PMTs' noticing skills.

The PMTs exhibited partial or robust evidence of attending to and interpreting students' strategies for solving probability-related problems during the individual analysis process before the class discussion.

The PMTs in this study typically tended to go further by simply identifying and making general statements about the students' strategies. In addition to describing the conceptions and misconceptions presented in the written cases, PMTs also considered and interpreted the reasoning of the students, which is consistent with previous research (e.g., Alsawaie & Alghazo, 2010; van Es et al., 2017; Girit-Yildiz et al., 2023; Ulusoy, 2020). By contrasting and comparing students' correct, incorrect, and partially correct solutions, the PMTs were able to understand the majority of the mathematical components. For instance, numerous PMTs noticed students' misunderstandings of the idea of probability and impossible-certain events. However, according to certain studies (e.g., Jacobs et al. 2010; Sánchez-Matamoros et al. 2019), many prospective teachers find it difficult to retain the mathematically significant elements of children's problem-solving procedures. In the current investigation, the prior knowledge of the probability of PMTs may have contributed to the better attending and interpreting skills in PMTs. The PMTs were exposed to probability-teaching strategies in methods courses, and they had the opportunity to observe an actual class on probability in middle schools in the context of the teaching practicum course before this investigation. In order to pay attention and interpret the subject-specific aspects of instruction, one must not only have the ability to concentrate on the essential aspects of a complicated classroom setting but also have a mathematical knowledge of teaching (Schlesinger et al., 2018; Zeeb et al., 2023). Presenting only students' written solutions could also be a contributing factor to the attending and interpreting skills of the PMTs. Since it is opposed to whole-class videos or scenarios, it may have encouraged a more concentrated and comprehensive examination of students' mathematical thinking in this study. Because prospective teachers find it difficult to concentrate on numerous facets of a complex classroom setting (Santagata et al., 2007; Star & Strickland, 2008). However, the PMTs performed less well at interpreting students' strategies than they did at attending. The PMTs made general comments (e.g., the student's strategy is correct; the student has some misconceptions) without providing mathematical justifications, as Rotem and Ayalon (2023a) indicated. As stated by Barnhart and van Es (2015) and Sánchez-Matamoros et al. (2019), prospective teachers' responses did not guarantee that they could interpret student understanding using the same mathematical evidence, even when they provided robust evidence in attending to students' strategies.

In the initial reports, the PMTs struggled to decide how to respond to students' reasoning, and they mostly provided general instructional suggestions as in prior research (Jacobs et al. 2010; Krupa et al. 2017). This is because they primarily suggested general instructional actions without referencing mathematical elements (e.g., Barnhart & van Es, 2015; Jacobs et al., 2010; Sánchez-Matamoros et al., 2019; Thomas et al., 2022). For instance, they recommended utilizing engaging activities and manipulatives, encouraging students' collaborative efforts, and reteaching the topic for students who provided incorrect or partially correct answers. However, they were unable to demonstrate how their problems, activities, or materials remedied students' misconceptions about probability and helped students' understanding of it. Furthermore, PMTs found it more challenging to suggest instructional strategies for students who had correctly solved the problem. When a student's solution was correct and you were the student's teacher, the PMTs explicitly asked what you would do to help the student progress further. They frequently suggested posing unique problems for these students. However, they struggled to pinpoint specific or challenging mathematics problems that could enhance the students' understanding of probability. According to Jacobs et al. (2022), teachers showed greater expertise in selecting follow-up questions than the next problems. Some PMTs stated that they would ensure students' understanding by asking similar problems. However, the reliability of using familiar problems to assess students' knowledge remains debatable.

Following the class discussion, the PMTs had the chance to attend to, interpret, and decide how to respond to students' understandings once again in the revision of their initial reports. When compared to the initial reports, PMTs' evidence for noticing skills was better with the support of the class discussion. The PMTs, in particular, provided more mathematics-specific instructional suggestions. They offered largely partial and robust evidence for responding in the revised reports. Prospective teachers' ability to pay attention to and analyze student thinking improved, but not their ability to respond (Jacobs et al., 2010; Schack et al., 2013). This suggests that PMTs needed a conducive environment where they could engage in discussions about student thinking, with a particular emphasis on mathematical suggestions. This is because studies (e.g., Barnhart et al., 2025; Fernández et al., 2024; Rotem & Ayalon, 2023b) emphasize devoting more time to discussing the meaning of attending to, interpreting, and responding to students' ideas. Furthermore, even if students do not actively participate in the debate, the discussion setting provides insight into their thinking for PMTs (Girit-Yildiz et al., 2023; Guner & Akyuz, 2020). In the current study, the class discussion facilitated the PMTs' efforts to address deficiencies, correct errors, provide mathematical explanations and suggestions, and pose questions that prompt students to recognize and overcome their misconceptions.

The limitation of this study could be that the written cases were limited to our collected students' solutions because student answers and performance on a task differ depending on group variables. Another limitation of this study is that we did not examine a different data set to determine whether there was an increase in the level of noticing skills. The goal of this study was to establish a professional development environment that could assist PMTs in identifying and filling in their knowledge gaps. Consequently, we examined the gains of this environment during the revision phase. The aim of the study was not to directly improve the PMTs' ability to notice.

The findings have substantial implications for prospective teacher noticing research and the design of effective educational settings in teacher training programs. The class discussion, in particular, allowed PMTs to discover what and how peers know, think, interpret, and suggest instructional approaches based on student mathematical thinking (Sherin & van Es, 2009; Sherin & Han, 2004; Ulusoy & Cakıroglu, 2021). Additionally, this study aims to support prospective teachers in focusing on student thinking and giving effective feedback to students. It is believed that this study will enhance the effectiveness of teaching practicum courses, thereby enhancing the competence of prospective teachers upon graduation. In addition, revealing the skills of noticing student solutions used in this study and providing the opportunity to improve these skills through class discussion will constitute an example of an approach that can be used for teaching methods courses. The results of this study may influence countries that are starting to emphasize chance in their primary probability curricula and enhance their teacher preparation programs, as indicated by Park and Lee (2023).

Future studies should investigate whether the professional development environments created in different subjects influence the noticing skills of prospective teachers and how this type of environment should be designed to the specific content. Focusing on different data sets over a longer period of time can provide concrete insights into the development of prospective teachers. Quantitative studies can also support the findings. Future research can also focus on prospective teachers' noticing abilities within group work and how the interactions in the groups affect their noticing skills.

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APPENDICES

Appendix 1: Prepared Tasks for PMTs

The first task

1st Problem: This year, Nur will graduate from high school, and her family desires to buy Nur a computer as a graduation gift. Nur will select between four brands.

Listed below are the number of computers sold by each brand and the number of computers returned as defective products belonging to that brand. Which brand will Nur be most unlikely to purchase a defective product if she chooses according to the table?

Brands	A	B	C	D	E	F
Number of defective products	10	280	24	12	200	1
Sale amount	50	700	200	80	4000	2

The following are three students' answers to the above problem:

Student1 (correct solution) *:

$A \Rightarrow \frac{1}{5} = \frac{5}{25}$

$B \Rightarrow \frac{4}{10} = \frac{2}{5} = \frac{10}{25}$

$C \Rightarrow \frac{12}{100} = \frac{6}{50} = \frac{3}{25}$

$D \Rightarrow \frac{6}{40} = \frac{3}{20}$

$E \Rightarrow \frac{5}{100} = \frac{1}{20} = \frac{3}{60}$

$F \Rightarrow \frac{1}{2}$

E

Student3 (incorrect solution) *:

A	B	C	D	E	F
50	700	200	80	4000	2
-10	-280	-24	-12	-200	-1
40	420	176	68	3800	1
more	more	more	more	more	less likely

Student2 (partially correct solution) *:

Of course, it would be F. If he chooses A, there are 10 faulty computers. Maybe the 11th will come to him. 280 faulty devices in B. Too much. I think it's 50% faulty. In C, 24 faulty computers are found, and the 25th wrong one comes to him. There are 12 faulty computers in D. She can get it. And so, E is. At least one person got the F. It comes out at 0.01%. ... But I think it would be E. There are 200 faulty computers. 4,000 people took it.

1) Describe each student's strategy in detail by associating it with mathematical elements.
Student1:
Student2:
Student3:

2) Evaluate the student's strategy and provide a detailed explanation.
Student1:
Student2:
Student3:

3) Pretend to be the student's teacher. How do you facilitate student learning when a solution is partially incorrect or founded on a misunderstanding? Or, if the student's answer is correct, how would you enhance their understanding?
Student1:
Student2:
Student3:

Appendix 1: Prepared Tasks for PMTs (Continued)

The second task

2nd Problem: Ece and Can want to play with the marbles they have. Can creates a table that determines the numbers and colors of the marbles. While Can is drawing the table, Sena puts the marbles in the bag. Ece asked Can to find out:

a. Chance of each marble (yellow, blue, black, white and green) randomly selected from the bag.

b. Which of them has equal chance? Order the values of probabilities.

What did Can find when he answered the questions correctly?

Colors	*Number*
Yellow	22
Blue	36
Black	22
White	24
Green	1

The following are three students' answers to the above problem:

Student1 (partially correct solution) *:

Ordering:
36 24 22 22 1
blue>White>yellow=black>green

Have equal chance: yellow and black

The chance of selecting green marble: 1/104

Student3 (correct solution)*

Yellow: 22/105, blue: 36/105, black 22/105,
White: 25/105, green: 1/105

Have equal chance: yellow and black

Ordering:
blue>White>yellow=black>green

Student2 (incorrect solution) *:

Yellow and Black has equal chance, yellow-black=83%

Yellow	Blue	Black	White	Green
83%	64%	83%	81%	104%

1) Describe each student's strategy in detail by associating it with mathematical elements.
Student1:
Student2:
Student3:

2) Evaluate the student's strategy and provide a detailed explanation.
Student1:
Student2:
Student3:

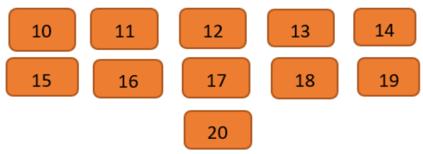
3) Pretend to be the student's teacher. How do you facilitate student learning when a solution is partially incorrect or founded on a misunderstanding? Or, if the student's answer is correct, how would you enhance their understanding?
Student1:
Student2:
Student3:

The third task

3rd Problem: The digits 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20 are written on identical cards and placed in a bag. The number of a card picked at random from the bag;
Find out the chance of the each of the following events:

a. One-digit number
b. Even number
c. Odd number
d. Zero
e. Two-digit number
f. Three-digit number

Determine which events are certain and which are impossible events.



The following are three students' answers to the above problem:
Student3:

Appendix 1: Prepared Tasks for PMTs (Continued)

Student1 (partially correct solution) *:

a. One-digit number = impossible
b. Even number = 6/11 = certain
c. Odd number = 5/11 = certain
d. Zero = impossible
e. Two-digit number = 11/11 = certain
f. Three-digit number = 0/11 = none = impossible

Student2 (correct solution) *:

A = impossible = 0/11
B = 6/11
C = 5/11
D = impossible = 0/11
E = certain = 11/11
F = 0/11 = impossible

Student1 (incorrect solution) *:

a. One-digit number 0% impossible
b. Even number 60% certain
c. Odd number 40% certain
d. Zero 0% impossible
e. Two-digit number 100% certain
f. Three-digit number impossible

1) Describe each student's strategy in detail by associating it with mathematical elements.
Student1:
Student2:
Student3:

2) Evaluate the student's strategy and provide a detailed explanation.
Student1:
Student2:
Student3:

3) Pretend to be the student's teacher. How do you facilitate student learning when a solution is partially incorrect or founded on a misunderstanding? Or, if the student's answer is correct, how would you enhance their understanding?
Student1:
Student2:
Student3:

TÜRKÇE GENİŞLETİLMİŞ ÖZET

Öğretmen adaylarının öğrencilerin problem çözme süreçleri hakkında bilgi sahibi olmalarının ötesinde, öğrenci düşünüşlerinin farkında olmaları ve öğrencilere uygun geri bildirimler vermelerinin daha değerli olduğu vurgulanmaktadır. Bu noktada öğretim yeterliliği boyutlarından biri olan öğrencilerin matematiksel düşünüşlerini fark etme becerisi önem kazanmaktadır. Bu beceri, öğrenci düşünüşünü tanımlama ve yorumlamayı sağlayan bilişsel beceriden ve bu değerlendirmenin sonucunda öğretimsel kararlar almayı sağlayan pedagojik süreçlerden oluşmaktadır. Fark etme becerisi kavramından hareketle bu çalışmanın amacı, öğretmen adaylarının tanımlama ve yorumlama becerilerini, öğretimsel önerilerini ortaya çıkarmak ve değerlendirmektir. Bununla birlikte öğretmenlerin fark etme becerisi çalışılan matematik konusuna da özgüdür. Mevcut çalışma olasılık bağlamında yürütülmüştür. Çünkü olasılık hem öğrencilerin hem de öğretmen adaylarının zorlandığı ve kavram yanılgıları yaşadıkları bir matematik konusudur. Dolayısıyla bu konunun ele alınmasıyla öğretmen adaylarının farkındalıklarını desteklemek ve böylece geleceğin öğretmenlerinin öğrencilerine de yardımcı olmak hedeflenmiştir.

Bu çalışmada nitel tasarımlardan durum çalışması kullanılmıştır. Katılımcılar İlköğretim Matematik Öğretmenliği Programındaki 4.sınıf öğrencileridir. Araştırma dört aşamalı bir veri toplama sürecini içermektedir. Çalışmanın ilk aşamasında 8.sınıf düzeyinde 62 öğrenciye üç tane olasılık problemi sorularak cevaplar elde edilmiştir. Soruların seçiminde müfredattaki olasılık öğrenme alanındaki toplam beş kazanımı karşılaması kriter olarak belirlenmiştir. Elde edilen öğrenci çözümleri içerik analizi ile doğru yanıt, doğru çözüm, kısmen doğru çözüm, yanlış yanıt, yanlış çözüm (kavram yanılgısı içeren) ve boş cevaplar kategorilerine ayrılmıştır. Öğrencilerden toplanan verilere göre doğru çözümler ve yanlış çözümler yaklaşık aynı orandadır. Öğrencilerin olasılık, olasılık hesaplamada örnek uzay belirleme ve kesin-imkansız olay kavramlarına ilişkin yanılgıları tespit edilmiştir. Ardından her soru için doğru çözüm, yanlış çözüm ve kısmen doğru kategorilerden öğrenci çözümleri kullanılarak öğretmen adayları için görevler oluşturulmuştur. İkinci aşamada, görevlerde öğretmen adaylarına fark etme becerisi kavramına ilişkin tanımlama, yorumlama ve öğretimsel strateji önerme bileşenlerine ait sorular sorulmuştur ve onlardan cevapların yer aldığı bir rapor yazmaları istenmiştir. Bu verilerin analizi için alan yazındaki fark etme becerisi ile ilgili önceki çalışmalar temel alınarak bir rubrik geliştirilmiştir. Bu rubrikte her bir fark etme becerisine ilişkin yüksek düzeyde, orta düzeyde ve düşük düzeyde kanıt olmak üzere üç düzey bulunmaktadır. Öğretmen adayları, bu süreçte öğrencilerin olasılıkla ilgili problemleri çözme stratejilerini tanımlama ve yorumlama konusunda orta veya yüksek düzeyde kanıtlar sunmuştur. Genel olarak öğretmen adayları, öğrencilerin stratejilerini belirlemenin ve bunlarla ilgili genel açıklamalar yapmanın da ötesinde performans sergilemiştir. Öğrencilerin doğru, yanlış ve kısmen doğru çözümlerini karşılaştırarak, matematiksel bileşenlerin çoğunu belirleyebilmiştir. Örneğin, çok sayıda aday, öğrencilerin olasılık fikri ve imkânsız-kesin olaylar hakkındaki yanlış anlamalarını gözlemleyebilmiştir. İlk yansıtma raporlarında, adaylar öğrencilerin muhakemelerine nasıl yanıt vereceklerine karar vermekte zorlanmıştır ve çoğunlukla düşük düzeyde öğretim önerileri sunmuştur. Bunun nedeni, ağırlıklı olarak matematiksel öğelere atıfta bulunmadan genel öğretim eylemleri önermeleridir. Örneğin, ilgi çekici etkinliklerden ve manipülatiflerden yararlanmayı, öğrencilerin işbirlikçi çabalarını teşvik etmeyi ve özellikle yanlış ve kısmen doğru öğrenci çözümleri için konuyu yeniden öğretmeyi önermişlerdir. Ancak, önerdikleri problemlerin, etkinliklerin ve materyallerin öğrencilerin olasılık hakkındaki yanılgılarını nasıl gidereceğini ve öğrencilerin olasılığı anlamalarına nasıl yardımcı olacağını temellendirememişlerdir. Ayrıca, adaylar doğru çözümler yapan öğrenciler için öğretim stratejileri önermede daha fazla zorluk yaşamışlardır. Öğretmen adaylarına "öğrencinin çözümü doğru olsaydı ve siz öğrencinin öğretmeni olsaydınız, öğrenciyi bir adım daha ilerletmek için ne yapardınız?" diye sorulmuştur. Adaylar da genellikle bu öğrenciler için daha zorlayıcı problemler oluşturmayı önermiştir. Ancak, zor problemleri ve bunların öğrencilerin olasılık anlayışını nasıl geliştirdiğini açıklayamamışlardır. Bazı adaylar da benzer soruları sorarak öğrencilerin anlamalarını sağlayacaklarını belirtmişlerdir. Ancak, bilindik problemlerin kullanılmasının öğrencilerin bilgilerini değerlendirmede güvenilir bir yol olup olmadığı tartışmalı bir konu olabilir.

Üçüncü aşamada bir sınıf tartışması yapılmıştır ve tartışma sonrasında dördüncü aşamada her öğretmen adayına bireysel yazılı raporunu düzeltme imkânı verilmiştir. Öğretmen adayları tartışma sürecinde edindikleri bilgileri ve tecrübelerini kullanarak ilk raporlarını revize etmişlerdir. Bu raporlarda, ilk raporlarla karşılaştırıldığında, sınıf tartışmasının desteğiyle adayların fark etme becerilerine ilişkin kanıtlarının daha iyi düzeyde olduğu belirlenmiştir. Adaylar özellikle daha çok matematiğe özgü öğretim önerileri sunmuştur. İlk raporlarda öğretim stratejileri önerme becerilerine ilişkin cevaplarında %35 oranında orta ve yüksek düzey kanıt bulunurken, revize raporlarda bu oran %65'e yükselmiştir. Buradan, öğretmen adaylarının matematiksel önerilere daha fazla odaklanabilecekleri ve öğrenci düşünüşünü tartışabilecekleri bir ortam gerektirdiğini çıkarmak da mümkündür. Ayrıca tartışma ortamı, öğretmen adayları tartışmaya aktif olarak katılmasalar bile adaylara öğrenciler hakkındaki düşüncelerine ilişkin içgörü sağlar (Guner & Akyuz, 2020). Mevcut çalışmadaki sınıf tartışması, eksik bilgileri tamamlama, hataları düzeltme, açıklama ve öneriler üzerinde matematiksel olarak detaylandırma ve öğrencilerin kavram yanılgılarını gidermeye yönelik soru sormada öğretmen adaylarını desteklemiştir.

Bu çalışma ile öğrenci düşünüşüne odaklanma ve öğrenciye etkili dönüt verebilme konularında öğretmen adaylarına destek olmak hedeflenmiştir. Böylece öğretmenlik uygulaması derslerinin daha etkili geçmesine, dolayısıyla öğretmen adaylarının mezun olduğunda daha yeterli öğretmenler olmasına katkı sağlayacağı düşünülmektedir. Ayrıca bu çalışmada kullanılan öğrenci çözümlerine yönelik öğretmen adaylarının fark etme becerilerini ortaya çıkarma ve sınıf tartışması yoluyla bu becerilerini iyileştirme fırsatı sağlama, her özel öğretim yöntemleri dersi için kullanılabilir bir yaklaşım örneği oluşturacaktır.