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Research Article

A study about mental calculation strategies with natural numbers in elementary school students using the broken calculator

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Abstract

This article reports a study about mental calculation in a group of elementary school students. Mental arithmetic, nowadays, is recognized as a subject of interest by several authors and educational systems. The latter have tried to design programs to develop it according to the flexibility of strategies, mathematical concepts, properties of the number system, and number sense. Several authors point out that tasks or activities should be incorporated with the intention of attending to this priority and not under a memoristic or strict teaching. In this sense, this qualitative research aimed to explore and describe the processes, strategies, operations, and skills (Rathgeb-Schnierer and Green 2019) used by six students studding sixth grade primary school in Puebla, Mexico. These students were 11 years old at the time they used the Broken Calculator application as a milieu to solve tasks; and it is shown that different strategies are developed when faced with the same tasks: some are predominant, others with a simple level of complexity, and how different calculation trains are elaborated according to the mastery of the strategies.

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Introduction

This exploratory study addresses mental arithmetic strategies using an app. It aims to analyze and describe the skills of sixth grade students when they confront tasks with the help of the *Broken Calculator* app. In this framework, six elementary school students were interviewed with the intention of recognizing the mental calculation strategies they have as well as their acquired knowledge.

Given the interest in the development of mental calculation skills, several teachers evidenced attempts to promote it in their classrooms as a personal decision, but which may be restricted by the conditions of the institutional environment (Weiss et al., 2019), the lack of knowledge of strategies (Mochón & Vázquez, 1995) and the use of inflexible rules (Lemonidis & Kaifa, 2014).

Mental arithmetic has been an important element in the training of basic education students. Such is the priority, that the *Secretaría de Educación Pública* [Ministry of Public Education] in Mexico (SEP, 2017) established as main purpose to use in a flexible way estimation, mental calculation and written calculation in operations with natural, fractional and decimal numbers. In those purposes, three important elements to achieve within the classroom are

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mentioned: estimation, written calculation (algorithmic calculation) and mental calculation (Jurić & Pjanić, 2023; Mochón & Vázquez, 1995).

Literature review

During the last decades, the importance of mental calculation has been recognized and like a necessity to include it in the mathematics curricula (Lemonidis, 2016). Since the 1980s, Hope (1987) mentioned that mental arithmetic should be part of the curriculum, as it promotes greater understanding of the structure of numbers and knowledge of number sense (Lemonidis & Kaiafa, 2014). However, this skill is rarely exploited, because it is not as evident as sports skills. For his part, Gómez (2005) argued that for the nineteenth century, the aim was to train versed calculators with a mastery of methods in the face of different situations. In the case of England, The Programme of National Numeracy Strategy in Primary Education was introduced in 1999. In 2000, the USA classified pencil and paper algorithms as a content that should be given less emphasis. By 1993 in Mexico, the SEP mentioned as a purpose in primary education to use mathematics as an instrument, both to solve problems and to estimate results, and even made recommendations to promote estimation calculation (SEP, 1993). It was at the secondary level where calculus estimation was proposed as a support for the study of whole numbers and decimals; however, there is no systematic and continuous practice (Cortés et al., 2004).

This analysis is closely related to the findings of Mochón and Vázquez (1995), who consider that students need active instruction in mental calculation and estimative calculus, since they have been neglected in the teaching of mathematics in Mexico. Similarly, Cortés et al. (2014) identified that few books in secondary education include this topic, have little didactic material and focus on estimation, also understood as approximation. Estimative calculus (or estimation) is that which does not seek to give exact answers to a problem, but rather its purpose is to give an answer close to the correct result of a problem (Mochón & Vázquez, 1995). Also, these students have knowledge of how place value is affected by arithmetic operations, the mathematical structure of the problem is adjusted and reformulated (Flores et al., 1990).

Within a study of teaching practice in Mexico, conducted by Weiss et al., (2019) in specific subjects, they observed that, for the case of mathematics, teachers allocate enough time to the exercise of algorithms and mental calculation in fourth and fifth grade; as it is recognized that formal education is decisive in arithmetic development and mental calculation (Jurić & Pjanić, 2023). It was also found that the problems posed were almost always isolated and for the purpose of applying knowledge already taught, despite the fact that authors such as Jurić and Pjanić (2023) indicate that the teaching of mental arithmetic should be carefully planned, since number sense is not developed by repeating algorithms (Yang et al., 2008).

Similarly, it was observed that little space is allotted for students to develop procedures autonomously. The teaching of mental arithmetic is minimal and little mentioned in the study program. Contrary, Gómez (2005) argues that mental arithmetic was taught, but not with such importance, and therefore there was no adequate approach or resources. Relationships with flexibility were unknown, nor with other mathematical knowledge and concepts that some studies already investigated (Barrera et al., 2018; Rathgeb-Schnierer and Green, 2019; Threlfall, 2009;).

Mental calculation is considered an indispensable segment of mathematical skills, important in and out of school (Juric & Pjanic, 2023), but erroneously executed in the classroom. Mochón and Vázquez (2005) mention that some exercises were taken as erroneous models and led to the idea that mental arithmetic were rules to be memorized as if they were shortcuts. Gómez (2005) argues the need for alternative calculation methods, since they are based on the properties of operations and the principles of the base ten numbering system, with the intention of obtaining exact results.

Mochón and Vázquez (1995) have defined mental calculation as a series of mental procedures that a person performs without the aid of paper and pencil, and that allow to obtain the exact answer to simple arithmetic problems. The original data of the problem are decomposed or replaced by others with which the subject works more comfortably to obtain the answer.

Theoretical Framework

Some common characteristics exist within the definitions of mental arithmetic. The most important ones deal with how it should serve to give exact answers, it makes use of decomposition strategies and the management of properties of the decimal number system in a cognitive way (Gómez, 2005; Mochón & Vázquez, 1995).

For his part, Lemonidis (2016) states that:

> *Mental calculation is calculation done mentally and using strategies. It produces a precise answer. Usually, it takes place without the use of external media such as paper and pencil, although it can be done with a paper and pencil, to make "jottings" that support the memory (p. 7)*

On the other side, written calculus is also known as algorithmic calculus and the following is mentioned: The distinction between algorithmic calculation and mental calculation does not consist in the fact that the former is written and the latter does not support the use of pencil and paper. Algorithmic calculus always uses the same technique for a given operation, whatever the numbers. On the other hand, when a mental calculation task is proposed, no single way of proceeding is expected (Ministry of Education, 2006).

Other authors such as Alsina (2007) and Rathgeb-Schnierer and Green (2019) recognize that children who have fewer memory resources have lower performance in calculation tasks and that one should not opt for repetition or mindless practice. They consider that understanding that changes and adapts to be used in problem solving should be preferred, in relation to connections between concepts, ideas, processes (Barrera-Mora et al., 2018) and easily remembered approaches (Threlfall, 2009). The above implies that a better choice of the appropriate strategy involves skills such as counting, mastery of tables, numerical combinations, offsets, and decompositions (Valencia, 2013).

Mental calculation, moreover, must be holistic, since the person maintains the identity of being whole numbers, even if decomposition is performed on them (Mochón & Vázquez, 1995). It is not fixed like the algorithm, but progressive (Threlfall, 2009) and, at the same time, it is variable since the problem can be solved in many correct ways. That is, mental calculation is flexible (Rathgeb-Schnierer & Green, 2019; Threlfall, 2002) since a single individual can use different strategies to solve problems, same that, together with memory, students begin from their early years, becoming more flexible and expanding in relation to knowledge and experiences (López, 2014). It is also constructive since the final result is built through partial results according to the chosen strategy. These should be mentioned in the classroom so that students understand that a question has several ways to be answered (Jurić & Pjanić, 2023; Mochón & Vázquez, 1995).

Rathgeb-Schnierer and Green (2019) and Threlfall (2002) assert that flexibility emphasizes deep conceptual development of numbers, operations, their relationships, and strategic means, should not be conveyed through methods for learning. They argue that teaching, from a flexible model, should emphasize from personal knowledge to the context of particular calculations, being analytical, from the perspective of students with operations and strategies at different levels of complexity (Carvalho & Rodrigues, 2021) and gradually expanding the range of numbers for problem solving, (Abd Algani et al., 2021). Rathgeb-Schnierer and Green (2017) conclude that there is consensus that mental calculation should be flexible, with two main characteristics: knowing different forms of solution and having the ability to adapt them appropriately to solve a problem optimally (Hickendorff, 2022).

Based on the characteristics mentioned about mental calculation and other research where the strategies developed by a student have been explored, as well as the favorable results of the Broken Calculator app (Rodríguez & Juárez, 2019; Sánchez et al., 2020), the interest was born to learn more about the development of strategies in this context and the implications of the application with different tasks and to use the calculator in a non-conventional way, taking as a reference the work of Goupil (2012). In addition, to make a technological environment that promotes students' concentration (Eleftheriadi et al., 2021) interacting with the calculator as a premeditated obstacle (also known as *milieu*) by the teacher (D'Amore and Fandiño-Pinilla, 2002).

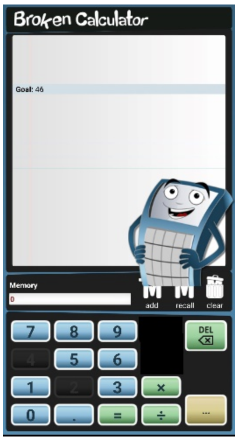

Figure 1. Image of broken calculator.

Method

Research Model

This research considered the challenges proposed previously for the development of mental calculation, so it introduced the Broken Calculator application, which in turn served as a *milieu*. Thus, the proposed technological environment challenged students to solve tasks as an a-didactic situation (D'Amore & Fandiño, 2002). Under the qualitative and **multiple case study approach** with **descriptive-exploratory** scope, the research question was resolved:

What strategies and skills of mental calculation are developed in elementary school students with the use of the decomposed calculator?

The research work has a phenomenological approach (Hernández et al., 2014). This approach explores, describes, and understands the processes of knowledge acquisition of the subjects. Making use of the classification proposed by Lemonidis, this helped us to discover the elements in common. Based on this question, the aim of the present study was to analyze the mental calculation strategies performed by sixth grade students with the basic operations of natural numbers using the Broken Calculator.

Participants

The subjects who participated in the study were three boys and three girls from an elementary school in Puebla City, Mexico. The age of the students was 11 years old. The students were selected by the teacher of the group according to their previous evaluations; three female students: one with high, medium, and low performance, three male students: with high, medium, and low performance.

Instrument and Data Collection

The instrument used to collect the information was a semi-structured interview. In it, the importance of the research question was considered as a real doubt that falls in the scientific field; also, under the natural observation of the events and the students (Schettini & Cortazzo, 2016), as a natural way to approach the case studies and that they could describe their mental calculation strategies.

The semi-structured interview allowed us to know, through the students' words, the processes they used, as well as to point out situations that could not be evaluated through writing, in order to describe situations that they considered important (Sánchez, 2013).

Classification

Table 1 is based on research literature gathered by Lemonidis (2016). He considers it third level because it is targeted for children who retrieve known numerical facts and mentally process them to compute another fact.

Table 1. Strategies compiled by Lemonidis

Strategy	Description	Examples	
		Addition (38 + 25)	**Subtraction (63–25)**
Division strategy (1010)	It is called this because the numbers added or subtracted are split into multiples of ten and units. This strategy is sometimes called partitioning method.	38+25: 5+8=13 30+20=50, 63	63-25: 13-5=8, 50-20=30, 38
Stringing strategy (N10)	According to this method, we keep the first term stable, split the second term into units and tens, and add or subtract units and tens successively from the first term.	38+25: 38+5=43, 43+20=63	63-25: 63-5=58, 58-20=38 (Subtraction) or 63-25: 25+8=33, 33+30=63, 38 (Addition)
Bridging through multiples of 10 (A10)	It can be considered a subcategory of the stringing strategy. In this case, we keep the first term stable, and add or subtract parts of the second term to get to the nearest ten.	38+25: 38+2=40, 40+23=63	**Subtraction** 63-25: 63-3=60, 60-20=40, 40-2=38 **Addition** 63-25: 25+**5**= 30, 30 + **33** =63, 38
Holistic: Compensation (N10C)	It can be regarded as a stringing strategy (N10) that moves to the nearest ten.	38 +25: 40+25= 65, 65 – 2 =63	
Levelling	This strategy is based on the property of levelling, where what is added to one term is subtracted from the other to have the same result	6 + 8: 7 + 7 = 14	
Direct modelling	Students model the problem and count the total number of objects, the number of groups or the number of the objects in every group.	**Multiplication**: They count in the model the total number of objects.	**Division** They count in the model the number of groups (quotitive) or the number of objects in each group (partitive).
Counting (3er level)	Every form of count strategy, skip counting forwards or backwards, repeated addition or subtraction, doubling and halving strategies.	**Multiplication** 5 x 15: 15, 30, 45, 60 or 5 x 15: 2 x 15=30, 30 + 30 = 60, 60 + 15 = 75	**Division** 75÷5: 15, 30, 45, 60, 75 or 180÷4: 180÷2 = 90 90 ÷ 2 =45
Direct retrieval	They use a known multiplication or division fact or a derived fact.	**Multiplication** 8×11=88, 5×12=60	**Division** 120÷6=20 because 6÷20=120
Partitioning a number based on place value.	One number is partitioned based on the place value of the arithmetic system.	**Multiplication** 7 x 15= (7x5) +(7x10) = 35 + 70 = 105	
Mental image of pen and paper algorithm	Children think and mentally perform the method of the standard written algorithm.		
Hybrid	Combine strategies from two or three basic groups of strategies to calculate multiplication tables. For example, this might be direct retrieval + counting all, or another such combination.		

Note: Strategies compiled by Lemonidis from the third level with examples and at the end is included the Hybrid strategy that belongs to strategies for multiplication table calculation, also mentioned by the same author.

Results

Task 1

In this first task, (see table 2) it was observed that, despite being buttons of a calculator, they had some inconveniences when writing the numbers they thought; some wrote in their attempts the number "1 + 1" and in subsequent movements they omitted the sign and wrote figures such as 111, 1111, etc. However, they had no problems in deducing the operation of the "DEL" key to erase some digits. The above could be due to the students' familiarity with the use of smartphones and not so with calculators.

In this first task the students had an attitude of testing and exploring the application. We could observe that such an attitude is favorably related to the functioning of *milieu*, since the researcher did not give recommendations to the students on how to solve the tasks and they remained interested, trying to correct their mistakes. In some cases, and later tasks, we could observe a tendency to change the strategies used to reduce the number of steps, this happened by students' initiative, which explains Valencia (2013) that mental calculation seeks to work comfortably with initial data that are easier to calculate. In the case of student **E3**, he mixed two strategies, also with the idea of avoiding the repeated addition of 7 times "+1" for this reason, he preferred to use three subtractions starting from 60. On the other hand, student **E1** changed strategy and reduced the number of steps performed, besides that in this first activity there was greater use of the division strategy (1010).

Table 2. Task 1: Set 57, with active keys 1, 0, +,-, =.

Student	Strategy used	Description
E1	*Started with division strategy (1010), changed to holistic strategy: Leveling*	2 attempts: 10 + 10 + 10 + 11 +11 +11 = 66, on the second attempt she changed the quantities 11 + 11 + 11 + 11 + 11 + 1 +1 = 57
E2	*Division strategy (1010)*	Two attempts: 10 + 10 +10 + 10 +10 + 1 + 1 + 1+ 1 +1 +1 +1 = 57
E3	*Division strategy (1010) and Estimation* (Mochón & Vázquez, 1995)	He made two attempts, in the first one he made a mistake and in the second one he approached the goal (the student mentioned going over the goal), then he subtracted: 10 + 10 + 10 + 10 + 10 + 10 – 1 – 1 – 1=57
E4	*Division strategy (1010)*	She made several attempts, also using decimals. Then corrected using: 10 + 10 + 10 + 10 +10 + 1 + 1 + 1 + 1 + 1 + 1 + 1
E5	*Holistic: Compensation (N10C)*	On his last attempt he placed 10 + 10 + 10 + 10 + 10 + 10 – 1 – 1 – 1, he said that "it's less steps". I plan to subtract to get there faster
E6	*Division strategy (1010)*	She made two attempts, in the last one he placed: 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 + 1 +1

Task 2

The first student mentioned in the interview that she used mental calculation and that for her it means "a way to do faster without writing operations", to know that the result sought was 60, but she already knew the table of 12, so she quickly placed in the broken calculator "12x5 ="; the student **E2**, found the result doing a multiplication x10 that she already knows and then added the result of 5 x 2 = 10, so she would put together both factors (10 and 2) to press "5 x 12 =".

The case of student **E3** stands out, since he performed the addition operation quickly, and when trying to describe it he mentioned that he only added it by imagining the operation, but in the calculator he placed a multiplication with the figures provided as an indication of the clause of the didactic contract to use as operators the numerical data present (D'Amore, 2021), when trying to specifically use "45 and 15". In turn, he used "- 99", since he could not place "- 100" because he did not have the digit "0" to approach the goal and until the last step he stopped to count with fingers the difference between the number he had and the goal as a reformulation of his estimate (Flores et al., 1990), he ended up rightly placing "- 21", but this counting strategy Lemonidis categorizes it in a lower level (2nd). In the case of student **E4**, mastery of the counting strategy was observed, since he mentally verified that the repeated sum of 15 gave 60, in order to place "15 x 4 = 60".

In the cases of students **E5** and **E6**, their strategies are not directly classified in the categories proposed by Lemonidis, nor do they fall into the counting classification, but they represent an ingenious alternative to solve the task, which demonstrates flexibility of operations and adaptability (Rathgeb-Schnierer & Green, 2019); on the other hand, their cases are particular because, in contrast to the group, they did not consider multiplication as the first option.

Table 3. Task 2: Enter 45 + 15, without the 0, +, keys

Student	Strategy used	Description
E1	*Bridging through multiples of 10 (A10)* and *multiplication: direct recovery.*	First, she mentally solved 45 + 5 = 50; 50 + 10 = 60, when she had the answer, she used keys "12 x 5", because she said she knew the table.
E2	*Partitioning a number based on place value*	(5 x 10) + (5 x 2) = 50 + 10 = 60, recovered the factors and wrote 5 x 12 =60
E3	*Mental image of pen and paper algorithm* *Estimation* and *Counting with fingers (2nd level).*	He said out loud that the answer is 60; however, to write it on the broken calculator he estimated 45 x 15 =675 and subsequently subtracted -99 several times, until he was as close to 60 as possible. In the last step, before pressing, he counted with his fingers to be sure. that 81 – 21 = 60.
E4	*Division Strategy (1010)* *Estimation* *Estimation and counting (3rd level)*	She wrote 45 x, then changed to 15 x. She quickly solved the operation = 60 and described it as 40 + 10 + 5 + 5. She wrote 45 x, then changed to 15 x. She said that he checked on the calculator that 15 x 4 = 60. Under the counting category this means: 15 x 4: 15, 30, 45, 60.
E5	They are not categorized.	Finding the answer 60, she described it as 5 + 5 = 10, and transformed the tens according to their place value 40 is 4 tens, 10 ones is 1 one, so 4 + 1 + 1 = 6 tens, and 6 tens is 60. Subtraction of units: 69 – 1 = 68 68 – 8 = 60. The student made two subtractions since by mistake he had written -1, so he subtracted – 8 to reach the goal.
E6	*Division Strategy (1010)*	The task answer was mentioned correctly and asked to describe how she found the answer, she mentioned a decomposition 45 + 5 = 50, 50 + 10 = 60 Subtraction of units: 61 – 1 = 60

Task 3

In this task, the + sign was omitted, since in the pilot it was the operation that predominated and did not meet the objective of the research. For this task, there was a greater tendency to use estimation since the *Broken Calculator* allows approaching from a higher or lower number to the goal, and a tendency to use multiplication since the "+" sign is not present.

Student **E1** had a more complex response than the rest on moving between categories of strategies and using them accurately and demonstrating certainty in their execution. For his part, student **E2** demonstrated mastery of the Direct modeling strategy by making a quick count of the times that "5 + 5 + ... are 90", and then converting it into a product just as student **E6** did (45 x 2).

Table 4. Task 3: Place 90 without using 9, 0 or +

Student	Strategies used	Description
E1	*Hybrid*	12-3= 9, (Subtraction) 9 x 5 = 45. 45 x 2 = 90 (Direct recovery)
E2	*Direct modeling*	Mentally he verified that 5 + 5 + 5…+ 5 = 90 Then he counted that 5 is repeated 18 times, so 5 x 18 = 90 His process was fast, he demonstrated efficiency.
E3	*Estimation* (Mochón & Vázquez, 1995) and *counting*.	Start by placing 12 x 12= 144 as a way to go over the target amount and then use subtraction to get closer. 12 x 12 = 144, 144 – 21 = 123, 123 – 21 = 102, 102 – 12 = 90
E4	*Estimation* and *counting*	She made several movements, in which estimation through multiplication predominated: 4 x 6, 4 x 7, 28 x 5, 24 x 3, 44.5 x 2. Finally, she placed 45 x 2 = 90. She explains that this she did it because 45 + 45 = 90.
E5	*Estimation*	He wrote a number higher than the target so he could use the "-" button; he then subtracted 111-11 = 100. He noted his mistake and corrected his second attempt to 111 – 21 = 90.
E6	*Counting*	In a single movement she wrote 45 x 2 =90. She explains that she did it because she mentally added 45 + 45 = 90.

Task 4

This task was not solved by all five students; however, those who had made an estimate in the previous tasks were more creative and exploratory than those who had not; their opinions about the task varied, but they agreed that there is a way to solve the challenge, they just could not find a way to do it. **E1**, planned a strategy that was accurate and demonstrated the use of two strategies to solve the task.

Table 5. Task 4: 46 without numbers 4, 2, +, -.

Student	Strategies used	Description
E1	*Division strategy* (1010) and *Direct recovery*.	On her first attempt, she wanted to add, but changed her strategy as she did not have +. Then she mentally added 40 + 40 + 40 + 6 + 6 + 6 = 138 138 ÷ 3 = 46 (justify her answer because 46 x 3 = 138).
E2	*They were estimation attempts with different quantities	Tried several multiplications with natural numbers, but did not achieve the result
E3		He tried various ways but did not achieve the result; He believes that this is because he "doesn't know much about mathematics" but that there is a result.
E4		She tried to multiply with the 5's table and add. Then with other factors, but none of them give him the answer. She chose to use decimals in different ways. The closest attempt he had was 5.1 x 9 = 45.9 (She said he would be one number short).
E5	**They planned a strategy to divide exactly but didn't find an answer.	He tried to place 90 between 2, resulting in 45, but he couldn't try 92 ÷ 2 because the 2 was broken. He said "I don't have a number to get 46"
E6		She pressed 92, but she saw that she didn't have the number 2 to divide. Then she pressed "158 ÷", however, she did not press anything else, she hesitated, she erased her progress to start over and she no longer knew what to do. In the interview she said that the task became complicated, and she kept thinking of a number to multiply or divide that would result in 46.

Discussion and Conclusion

The broken calculator study revealed that, within a group, we can find a diversity of strategies employed, and cases of students who efficiently employ more than one strategy for task resolution. For Lemonidis (2016) this is an indication of greater mastery and skill. Cases were also found that employed counting strategies and that, while not considered mental calculation, these students were able to perform better with the broken calculator as a *milieu* and with an exploratory attitude of the appeal.

The Broken Calculator application meets the characteristics of challenging students to provide answers with the resources on the screen. For the student, this environment proved to be more attractive, in which different strategies could be manifested and in which they acted autonomously, as the task seemed to be affordable to solve despite their failed attempts (Yang et al., 2008). The use of milieu, for mental calculation, is consistent with the recommendation of Yang et al., (2008) and Gomez (2005), who argue that mental calculation cannot be developed with the repetition of algorithms, as it requires adaptability from the students.

This study showed that students had the opportunity to create their own train of calculations corresponding to their level of numerical knowledge and operations (Carvalho & Rodrigues, 2021), and tended to rely on the operations they mastered and at the same time felt self-confident to solve more complex tasks as they interacted with the environment and exercised their working memory (Lopez, 2014). Also, despite the errors found, some students mentioned accepting that there was an answer, but they did not know what it was.

In the development of the activity, the affirmation of Mochón & Vázquez (2005) was reaffirmed, who state that the subject works more comfortably with the decomposition of the original data of the problem when substituted and that they are also easy to remember (just like task 1 and 2). We could see this in the decrease in the number of steps performed in the tasks each time they made a new attempt.

However, the tasks were increasing in complexity, and this demanded the use of more memory resources, being the student **E1**, the one who solved the problems more easily, also showed more planning and use of more elaborate strategies. For their part in task 2, students **E5** and **E6** also gave an answer that does not fall into Lemonidis' classification, but which refer to the concept of flexibility and ability to adapt them adequately to solve a problem (Hickendorff, 2022; Rathgeb-Schinner & Green, 2017). These last concepts were not the objective of the research, but the students' answers remain as a background for future research that, under the vision of the didactics of mathematics, has interest in exploring these characteristics in depth.

Also, as future research, we can rescue the signs of estimation in relation to mental calculation, because although they are not the same, there is a close relationship. This research could be useful to study in the future the relationship between them or the change of strategies before the proposed *milieu* throughout many sessions or different school levels.

Finally, it is worth mentioning that in this work specific tasks were used due to time limitations, but more or different tasks could be used, since the possibilities for investigating characteristics of mental calculation and its development such as number sense, flexibility and adaptability are very broad, so we suggest to narrow the focus of interest for future research, because of the possibilities of the calculator and the students' processes are diverse.

Limitations of Study

Due to the nature of the study, it was conducted with students without considering as variables the time or number of attempts to solve each task. Therefore, some interviews were longer, which led to limiting the specific number of participants and tasks.

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