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A Note on Gourava Index for an Algebraic Structure Graphs

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Abstract

Molecular descriptors such as topological indices are widely employed in the construction of quantitative structure-activity relationships (QSAR), quantitative structure-property relationships (QSPR), and quantitative structure-toxicity relationships (QSTR). Gourava index is one of the very important topological indexes that have been recently defined. In this study, the Gourava index will be obtained for the zero-divisor graph of monogenic semigroups.

Keywords: *Gourava index, monogenic semigroup, topological index*
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1. Introduction and Preliminaries

Graph Theory and Algebra were two different mathematical areas until rings been studied in graph theory. Firstly, commutative rings were studied with the graph coloring [6]. Zero divisor graphs have become a highly common kind of graph for rings since this discovery. Especially commutative rings and semigroups have been studied in many researches [3],[4],[8], [9] and [10]. They are included mostly in graphs. parameters. The zero-divisor graph for monogenic semigroup is first defined. in [7]. Then some graph parameters for this graph have been obtained. Then the other popular graph for commutative rings is dot product graph [5]. Also, the dot product graph has been defined for zero-divisor graph of monogenic semigroup [1]. Besides graph parameters, it was determined to be a perfect graph.
The topological index is one of the graph parameters. It is such an important parameter since it can be used multidisciplinary. They can be defined on degree-based, distance-based or eccentricity-based. There are dozens of indices of each in literature. Especially, topological index is used for molecules in chemical graph theory. A molecular graph is defined as the vertex set is atoms and the edge set is the bonds of molecules.
Topological index are very important for graph theory and topology. So, recently, many topological indices and graph applications were studied, for example, in [2], [11], [18],[19], [20], [21] and [16]

Definition 1.1. *[13] For any graph G, a vertex-degree-based topological index is defined as*

$$TI(G)=\sum_{uv\in E(G)} F\left(du,dv\right)$$

Where du and dv are the degrees of the adjacent vertices u and v. Also F is a function as $F\left(du,dv\right)=F\left(dv,du\right)$

Also, topological indices promise to have far-reaching applications in drug design, cancer research and bonding theory etc. Among them, first degree based topological index is first Zagreb index developed in 1972 [15]. Further, the second Zagreb index [14] and F-index [12] were studied. Motivated by the definitions of the Zagreb indices and their wide applications, Kulli introduced the Gourava index of a molecular graph [17].
The Gourava index is also a vertex-degree-based topological index. It is defined on the edge set of the graph.

Definition 1.2. *[17] For any graph G, then the Gourava index*

$$GO(G)=\sum_{uv\in E(G)} (du+dv+dudv)$$

A monogenic semigroups ;

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Definition 1.3. *For nonnegative number n, S_M^n is a monogenic semigroup with elements $\{0,x,x^2,...,x^n\}$.*
Then the zero divisor graph of monogenic semigroup $\Gamma\left(S_M^n\right)$ is consisting of a set of vertices and edges as above;

$$V\left(\Gamma\left(S_M^n\right)\right)=\left\{x,x^2,...,x^n\right\}=S_M^n\setminus\{0\}$$
$$E\left(\Gamma\left(S_M^n\right)\right)=\left\{x^ix^j;i+j>n\right\}$$

Monogenic semigroup, Zero-divisor graph and generally topological index and specially Gourava index have been defined.

2. Main Result

In this section, Gourava index will be obtained for the zero divisor graph of monogenic semigroups.
Since Gourava index is a vertex-degree based topological index, we will interest the degrees of the vertex set of zero-divisor graph for monogenic semigroups.
The following proposition gives us the degree sequence of monogenic semigroup graphs.

Proposition 2.1. *[7] The degree sequence for the zero-divisor graph of monogenic semigroups $\Gamma\left(S_M^n\right)$*
$DS\left(\Gamma\left(S_M^n\right)\right)=\left\{1,2,...,\left\lfloor\frac{n}{2}\right\rfloor-1,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2,...n-1\right\}$

Conclusion 2.2. *For the zero-divisor graph for monogenic semigroup, the vertex set can be defined as $V\left(\Gamma\left(S_M^n\right)\right)=\left\{x^i:i\in\mathbb{N}^+\right\}$.*
$\forall x^i\in V\left(\Gamma\left(S_M^n\right)\right)$ for $i\in\left\{1,2,...,\left\lfloor\frac{n}{2}\right\rfloor\right\}$ then the degrees of vertices $dx^i=i$. For all other vertices in the vertex set of $\Gamma\left(S_M^n\right)$ the degrees of vertices $dx^i=i-1$ for $i\in\left\{\left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2,...,n\right\}$.
In other words if $i\in\left\{1,2,...,\left\lfloor\frac{n}{2}\right\rfloor\right\}$ then $dx^i=i$ else $dx^i=i-1$.

Theorem 2.3. *For any monogenic semigroup S_M, the Gourava index of the graph $\Gamma(S_M)$ is*

$$\begin{aligned}GO\left(\Gamma\left(S_M^n\right)\right)=&\frac{1}{2}\cdot\left(\left\lfloor\frac{n}{2}\right\rfloor-1\right)\left\lfloor\frac{n}{2}\right\rfloor\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)+\left\lfloor\frac{n}{2}\right\rfloor\left(n\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)-1\right)\\&+\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\sum_{k=i+1}^{n-i-2}\left[(n-i-1)+(n-i)k\right]\end{aligned}$$

Proof. for the vertex x^n is adjacent to the vertices $\left\{x,x^2,...,x^{n-1}\right\}$ then for the definition of index $du=n-1$ and $dv=1,2,...,\left\lfloor\frac{n}{2}\right\rfloor-1,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2,...n-2$.
Similarly, for the vertex $x^{n-1}\in V\left(\Gamma\left(S_M^n\right)\right)$ is adjacent to the vertices $\left\{x^2,x^3,...,x^{n-2}\right\}$ then for the definition of index $du=n-2$ and $dv=2,3,\ldots,\left\lfloor\frac{n}{2}\right\rfloor-1,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2,\ldots,n-3$.
However, let's be careful not to count the adjacent vertices twice.
If an algorithm can be obtained as above for the vertex x^{n-i} is adjacent to the vertices $\left\{x^{i+1},x^{i+2},...,x^{n-i-1}\right\}$.
If $x^{n-i}\sim x^{n-i-1}$ then $(n-i)+(n-i-1)\geq n+1$. Hence $n-i\geq\frac{n}{2}+1$ then $i\leq\frac{n}{2}-1$. If n is even or odd then $i=0,1,2,...,\left\lfloor\frac{n}{2}\right\rfloor-1$ since $i\leq\left\lfloor\frac{n}{2}\right\rfloor-1$.
So $i\in\left\{0,...,\left\lfloor\frac{n}{2}\right\rfloor-1\right\}$ then $n-i\in\left\{\left\lfloor\frac{n}{2}\right\rfloor+1,...,n\right\}$ from proposition $d\left(x^{n-i}\right)=n-i-1$.
Consequently, for the definition of index $du=n-i-1$ and $dv=i+1,i+2....,\left\lfloor\frac{n}{2}\right\rfloor-1,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2,...n-i-2$.
Let, the definition of the Gourava index is adapted to the above algorithm.

$$\begin{aligned}GO(G)=\sum_{uv\in E(G)}(du+dv+dudv)&=\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\sum_{\substack{x^{n-i}\sim v\\ dv=i+1,i+2,...,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor,...,n-i-2}}\\&=\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\sum_{dv=i+1,i+2,...,\left\lfloor\frac{n}{2}\right\rfloor,\left\lfloor\frac{n}{2}\right\rfloor,...,n-i-2}\left[(n-i-1)+(n-i)dv\right]\\&=\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\left[\sum_{k=i+1}^{n-i-2}\left[(n-i-1)+(n-i)k\right]+(n-i-1)+(n-i)\left\lfloor\frac{n}{2}\right\rfloor\right]\\&=\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\left[(n-i-1)+(n-i)\left\lfloor\frac{n}{2}\right\rfloor\right]+\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\sum_{k=i+1}^{n-i-2}\left[(n-i-1)+(n-i)k\right]\\&=\frac{1}{2}\cdot\left(\left\lfloor\frac{n}{2}\right\rfloor-1\right)\left\lfloor\frac{n}{2}\right\rfloor\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)+\left\lfloor\frac{n}{2}\right\rfloor\left(n\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)-1\right)+\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\sum_{k=i+1}^{n-i-2}\left[(n-i-1)+(n-i)k\right]\end{aligned}$$

□

3. An Example

Consider the monogenic semigroup S_M^6 given below (Figure 3.1) and calculate the Gourava index of $\Gamma(S_M^6)$ graph by applying the rule given in Theorem 2.3.

$$S_M^6 = \left\{x, x^2, x^3, x^4, x^5, x^6\right\} \cup \{0\}$$

From the definition of monogenic semigroup graphs, we have Figure 3.1. In line with this information, the S_M^6 graph is given below in Figure 3.1.

$$\begin{aligned} GO(\Gamma(S_M^6)) =&(1+5+1.5)+(2+5+2.5)+(2+4+2.4)+(3+5+3.5)+(3+4+3.4)+(3+3+3.3)+(3+5+3.5)\\ &+(3+4+3.4)+(4+5+4.5)\\ =&170. \end{aligned}$$

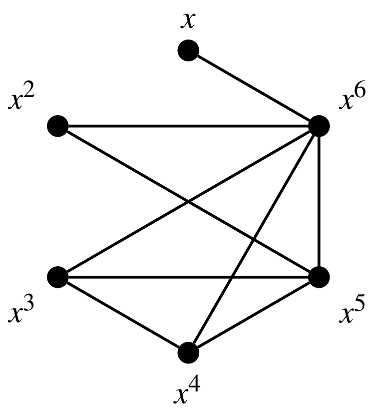

Figure 3.1: The graph of $\Gamma(S_M^6)$

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