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Application of The Homotopy Perturbation Method to the Neutron Diffusion Equation

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Homotopy Perturbation Method, neutron diffusion equation, second order differential equations, spherical and cylindrical coordinates.

Abstract: The Homotopy Perturbation Method (HPM) has been shown to be effective in solving both linear and nonlinear differential equations in mathematics, making it useful in a wide range of applications in the fields of physics and engineering. In this study, the Homotopy Perturbation Method was applied to the neutron diffusion equation for a one-dimensional time-independent approach. The Laplace operator of the neutron diffusion equation was considered for Cartesian, spherical and cylindrical coordinates. The critical radius values obtained for three different systems were calculated for all possible values of the relevant material parameter B. The results show that the solution of the neutron diffusion equation is agree with the literature.

Homotopi Perturbasyon Metodunun Nötron Difuzyon Denklemine Uygulanması

Anahtar Kelimeler:
Homotopi Pertürbasyon Yöntemi, nötron difüzyon denklemi, ikinci derece diferansiyel denklemler, küresel ve silindirik koordinatlar

Özet: Homotopi Pertürbasyon Yönteminin (HPM), matematikte hem doğrusal hem de doğrusal olmayan diferansiyel denklemlerin çözümünde etkili olduğu, fizik ve mühendislik alanlarındaki geniş bir uygulama yelpazesinde faydalı olduğu gösterilmiştir. Bu çalışmada, tek boyutlu zamandan bağımsız yaklaşım için nötron difüzyon denklemine Homotopi Pertürbasyon Yöntemi uygulanmıştır. Nötron difüzyon denkleminin Laplace operatörü kartezyen, küresel ve silindirik koordinatlar için dikkate alındı. Üç farklı sistem için elde edilen kritik yarıçap değerleri, ilgili malzeme parametresi B'nin tüm olası değerleri için hesaplandı. Sonuçlar nötron difüzyon denkleminin çözümünün literatürle uyumlu olduğunu göstermektedir.

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1. INTRODUCTION

The Homotopy Perturbation Method introduced by Ji-Huan He in 1998, combines elements of the traditional perturbation method with Homotopy in topology. Extensive research has been conducted, both analytically and numerically, investigating its effectiveness in handling linear and nonlinear differentials as well as integral equations (Yener, 2009). In particular, HPM has demonstrated successful applications in the solution of the Laplace equation and demonstrated rapid convergence of solution series in a wide variety of situations (Eş, 2022).

HPM is a technique that provides reliable results for many equations and has attracted a great deal of interest from researchers over past years. The solution of nonlinear equations (Daghan et al., 2017), solution of partial differential equations (Ozpınar, 2020), solution of hyperbolic equations (Çiçek and Mondalı, 2022), solution of higher order differential equations (Es, 2022) solution of neutron diffusion equation (Shqair, 2019), solution of neutron diffusion equation in spherical and cylindrical coordinates (Dababneh et al., 2011), solution of diffusion equations of systems with different geometries (Koklu et al., 2016), have been studied with HPM.

In this study, the derivation of the neutron diffusion equation in the reactor for three different geometries (cube, sphere and infinite cylinder) has been carried out using the HPM method. In order to analyze the accuracy of the calculations, the reactor radius obtained from the neutron diffusion equation was calculated for many values of the material parameter buckling value B between zero and one.

In literature the critical size is found according to some certain buckling value in nuclear reactor studies. In respect to mathematical perspective, buckling value is taken into account as a number in terms of reactor media from zero to one. (Khasawneh, 2009; Dababneh, 2010).

It is well-known that the material parameter and the critical size is an inverse relationship in reactor engineering. When the material buckling is increased the critical size decrease up to a certain value. So, the results are in line with this expectation.

After calculating critical size for all different geometrical systems for many materials buckling values B, the graphs of the flux equations for all geometrical systems are also drawn to verify the roots that refers to the critical size of the assumed system. It is shown that the roots in the graphs are also consistent with the results of the calculations. In this way, it can be seen from the tables and graphs that the Homotopy Perturbation Method gives consistent and accurate results.

2. MATERIALS AND METHODS

2.1. Homotopy Perturbation Method and Applications

In this section, the homotopy perturbation method will be introduced to obtain analytical or approximate solutions of linear or nonlinear differential equations (He, 2000).

A is the general differential operator, f(r) is an analytical function and, Γ: Ω is the limit of boundary. A nonlinear differential equation given by;

$$A(u) - f(r) = 0, \qquad r \in \Omega \tag{1}$$

If B is the boundary condition,

$$B\left(u, \frac{\partial_U}{\partial_n}\right) = 0, \quad r \in \Gamma \tag{2}$$

A is a differential operator, L is a linear operator and N is a nonlinear operator. If they are substituted into Eq. (1)

$$L(u) + N(u) = f(r), \qquad r \in \Omega \tag{3}$$

is found. Here $v(r,p)$= Ω x[0, 1]→R Homotopy can be occurred. $p \in [0,1]$ and u_o are the initial approximations that satisfies the boundary conditions;

$$H(v,p) = (1-p)[\,L(v) - L(u_0)\,] + p[\,A(v) - f(r)\,] = 0, \quad r \in \Omega, \tag{4}$$

The Eq. (4) is formed. From the Eq. (1),

$$\begin{aligned} H(v,p) &= L(v) - L(u_0) - p[L(v) - L(u_0) - A(v) + f(r)] = 0 \\ &= L(v) - L(u_0) + pL(u_0) - p[L(v) - A(v) + f(r)] = 0 \end{aligned} \tag{5}$$

And it is obtained that

$$L(v) - L(u_0) + pL(u_0) - pL(v) = 0 \tag{6}$$

Eq. (3) can be rewritten as

$$L(u) = -N(u) + f(r) \tag{7}$$

It can be put into Eq. (6)

$$L(v) - L(u_0) + pL(u_0) - p[-N(v) + f(r)] = 0 \quad (8)$$

So the Eq. (4) becomes as;

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \quad (9)$$

As the parameter p changes from 0 to 1, the function $v(r,p)$ also changes from u_o to u_r.

$$H(v,0) = L(v) - L(u_0) = 0 \quad (10)$$

and

$$H(v,1) = L(v) + N(v) - f(r) = 0 \quad (11)$$

Here, when p=0, equation (4) becomes a simple linear differential equation, and when p=1, the original nonlinear differential equation we discussed is obtained. In the Homotopy perturbation method, where p is a very small parameter, the solution of Eqs. (4) and (9) can be written as a power series of p.

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \cdots = \sum_{n=0}^{\infty} p^n v_n \quad (12)$$

The approximate solution of the Eq. (1) is found as;

$$u = \lim_{p \to 1}(v_0 + pv_1 + p^2 v_2 + \cdots) = v_0 + v_1 + v_2 + \cdots = \sum_{n=0}^{\infty} v_n \quad (13)$$

This case is proposed by Ji Huan He to clarify HPM and many researchers in mathematics and physic are used the method. (Dababneh, 2010; Shqair, 2022).

2.1.1. HPM Application to the Cube

Neutron diffusion theory is a methodology used to estimate the distribution of neutrons within a nuclear reactor by employing the diffusion equation, like molecular transport. This theory involves solving the diffusion equation to approximate the spatial, distribution of neutrons. It was extensively employed in the design of many early reactors to model and understand neutron behaviour within the reactor core.

The Homotopy Perturbation Method is applied to the neutron diffusion equation written for a general time-independent, steady state, one-dimensional geometry with vacuum boundary conditions (Lamarsh and Baratta, 2001).

$$\nabla^2 \phi(r) + B^2 \phi(r) = 0 \quad (14)$$

It is the best-known form of the neutron diffusion equation. Here the gradient operator can be reorganised for the geometry of the system.

The neutron diffusion equation can be written for the one-dimensional cartesian geometry;

$$\frac{d^2\phi(x)}{dx^2} + B^2\phi(x) = 0 \quad (15)$$

Here B^2 is the material buckling known as the $B^2 = \frac{\nu \sum_f - \sum_a}{D}$. Here $\sum_f$ is the macroscopic fission cross section, $\sum_a$ is the macroscopic absorption cross section, D is the diffusion length ν is the number of delayed and prompt neutrons numbers in a fission chain reaction. It is known that the neutron diffusion equation is widely used in nuclear reactor calculation. The parameters for material buckling are related with the material of the reactor core. The exact values of them can be found in the evaluated nuclear data library center or the others. However, in our study we want to show the application of the HPM method solving for three different geometrical reactor core systems. In this reason when the critical radius calculations are done, the material parameter B value is taken into consideration in the range of zero to one. So, we can check the Homotopy Perturbation method used in the neutron diffusion equation solution. The Homotopy is applied to the equation $\frac{d^2\phi(x)}{dx^2} + B^2\phi(x) = 0$ as following:

$$H(\phi, p) = x^2\phi''(x) + p\, B^2\phi(x) = 0 \qquad p \in [0,1] \tag{16}$$

The variation of p from zero to unity corresponds to the variation of Eq. (16); which is an initial approximation obtained when *p*=0. The basic assumption of HPM is that the solution of Eq. (16) can be expressed as a power series in p,

$$\phi(x) = \phi_0(x) + p\,\phi_1(x) + p^2\,\phi_2(x) + \dots \tag{17}$$

The Eq. (17) is substituted into Eq. (16) and then the power series of *p* is found as;

$$\begin{aligned} p^0 &= \phi_0''(x) = 0 \\ p^1 &= \phi_1''(x) + B^2\,\phi_0(x) = 0 \\ p^2 &= \phi_2''(x) + B^2\,\phi_1(x) = 0 \\ &\vdots \\ p^k &= \phi_k''(x) + B^2\,\phi_{k-1}(x) = 0 \end{aligned} \tag{18}$$

It is assumed that $\phi_0(0) = C$. So, it becomes $\phi_0''(0) = 0$. The initial condition of $\phi_0(x)$ is applied to Eq. (18) and it is found that,

$$\phi_1''(x) = -B^2C \tag{19}$$

$\phi_1(x)$ can be obtained by integrating of the Eq. (19) as;

$$\phi_1(x) = -B^2C\,\frac{x^2}{2} \tag{20}$$

When the resultant equation (20) is substituted into Eq. (18), the other fluxes can be obtained as,

$$\begin{aligned} \phi_1(x) &= -B^2C\,\frac{x^2}{2} \\ \phi_2(x) &= B^4\,C\,\frac{x^4}{2.3.4} \\ \phi_3(x) &= -\,B^6\,C\,\frac{x^6}{2.3.4.5.6} \\ &\vdots \\ \phi_k(x) &= C - p\frac{B^2C}{2}\,x^2 + p^2\frac{B^4C}{2.3.4}x^4 - p^3\,\frac{B^6C}{2.3.4.5.6}\,x^6 + \dots \end{aligned} \tag{21}$$

When the approximation of the results with the limit is taken

$$\lim_{p\to 1}\left[C - p\frac{B^2C}{2}x^2 + p^2\frac{B^4C}{2.3.4}x^4 - p^3\frac{B^6C}{2.3.4.5.6}\,x^6 + \cdots\right] \tag{22}$$

If the series expansion of Eq. (22) is made and the general form of the equation is written:

$$\phi(x) = \sum_{k=0}^{\infty}\frac{(-1)^k(Bx)^{2k}.C}{(2k)!} \tag{23}$$

2.1.2. HPM Application to the Sphere

The neutron diffusion equation is organized for r dependent;

$$\nabla^2 \phi(r) + B^2\phi(r) = 0 \tag{24}$$

If the Laplacian operator (∇^2) is written for spherical coordinates;

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi(r)}{dr}\right) + B^2\phi\,(r) = 0 \tag{25}$$

After arranging,

$$\frac{1}{r^2}(\,2r\phi'(r) + r^2\phi''(\,r)) + B^2\phi(r) = \,0$$

The Eq. (25) is multiplied by r^2 and the transformation of $x = Br$ is done;

$$x^2\,\phi''(x) \,+\, 2x\,\phi'(x) \,+\, x^2\phi\,(x) \,=\, 0 \tag{26}$$

Thus, the equation dependent on the x variable is obtained. By constructing the HPM to Eq. (26)

$$H(\,\phi, p) = x^2\phi''(x) + 2x\,\phi'(x) + px^2\phi\,(x) = 0 \qquad p \in\, [0,1] \tag{27}$$

By using the initial boundary value of p as 0, the flux function is found as

$$\phi(x) = \,\phi_0(x) + \,p\,\phi_1(x) + \,p^2\phi_2\,(x) + \cdots \tag{28}$$

From the Eq. (28), it is shown that $\phi_0(x) = C$. Now the Eq. (28) is substituted into Eq. (27). Then one can obtain,

$$H(\phi,p) = 2x\big(\phi_0{}'(x) + p\phi_1{}'(x) + p^2\phi_2{}'(x)\big) + \mathrm{x}^2\big(\phi_0{}''(x) + \mathrm{p}\phi_1{}''(x) + \mathrm{p}^2\phi_2{}''(x)\big) + \mathrm{px}^2(\phi_0(x) + \,p\,\phi_1(x) + \,p^2\phi_2\,(x)) \tag{29}$$

The power series of p is organized as;

$$\begin{aligned} p^0 &= 2x\phi_0{}'(x) + \mathrm{x}^2\phi_0{}''(x) = 0 \\ p^1 &= 2x\phi_1{}'(x) + \mathrm{x}^2\phi_1{}''(x) + \mathrm{x}^2\phi_0(x) = 0 \\ p^2 &= 2x\phi_2{}'(x) + \mathrm{x}^2\phi_2{}''(x) + \mathrm{x}^2\phi_1(x) = 0 \\ p^3 &= 2x\phi_3{}'(x) + \mathrm{x}^2\phi_3{}''(x) + \mathrm{x}^2\phi_2(x) = 0 \\ &\;\;\vdots \\ p^k &= 2x\phi_k{}'(x) + \mathrm{x}^2\phi_k{}''(x) + \mathrm{x}^2\phi_{k-1}(x) = 0 \end{aligned} \tag{30}$$

The assumption $\phi_p(x) = \mathrm{Ax}^2 + \mathrm{Bx} + \mathrm{C}$ is substituted into Eq. (30) and the series solution of the flux function can be ordered as

$$\begin{aligned} \phi_1(\mathrm{x}) &= -\frac{\mathrm{C}}{6}\mathrm{x}^2 \\ \phi_2(\mathrm{x}) &= \frac{\mathrm{C}}{120}\mathrm{x}^4 \\ \phi_3(\mathrm{x}) &= -\frac{\mathrm{C}}{5040}\mathrm{x}^6 \\ &\;\;\vdots \\ \phi_{\mathrm{k}}(\mathrm{x}) &= C - p\frac{\mathrm{C}}{6}\mathrm{x}^2 + \mathrm{p}^2\frac{\mathrm{C}}{120}\mathrm{x}^4 - \mathrm{p}^3\frac{\mathrm{C}}{5040}\mathrm{x}^6 + \cdots \end{aligned} \tag{31}$$

The approximate result is found by taking the limit of the Eq. (31)

$$\phi(\mathrm{x}) = \lim_{\mathrm{p}\to 1}[C - p\frac{\mathrm{C}}{6}\mathrm{x}^2 + \mathrm{p}^2\frac{\mathrm{C}}{120}\mathrm{x}^4 - \mathrm{p}^3\frac{\mathrm{C}}{5040}\mathrm{x}^6 + \cdots] \tag{32}$$

The limit value in Eq. (32) is substituted into Eq. (29) and the sum of the exponent series come together and $x = Br$ is used in the general form of the flux equation

$$\phi(\mathrm{Br}) = \sum_{\mathrm{k}=0}^{\infty}\frac{(-1)^{\mathrm{k}}\mathrm{C}}{(2\mathrm{k}+1)!}(\mathrm{Br})^{2\mathrm{k}} \tag{33}$$

2.1.3. HPM Application to the Infinite Cylinder

The neutron diffusion equation is used in Eq (24). However, the Laplacian is formed in cylindrical coordinates only for r

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi(r)}{dr}\right) + \mathrm{B}^2\phi(r) = 0 \tag{34}$$

$$\frac{1}{r}\left(\phi'(r) + r\phi''(r)\right) + \mathrm{B}^2\phi(r) = 0$$

Again, r is the radius of the cylinder and B is the buckling of the system. The Eq. (34) is multiplied by r^2, then the neutron diffusion equation can be rewritten as,

$$\phi'(r) + r\phi''(r) + \mathrm{rB}^2\phi(r) = 0 \tag{35}$$

Now the well-known transformation $x = Br$ is applied to Eq. (35) and the derivations are performed,

$$\mathrm{r}^2\mathrm{B}^2\phi''(x) + Br\phi'(x) + \mathrm{r}^2\mathrm{B}^2\phi(x) = 0 \tag{36}$$

Here the variables of the Eq. (36) transform r to x.

$$\mathrm{x}^2\phi''(x) + x\phi'(x) + \mathrm{x}^2\phi(x) = 0 \tag{37}$$

The solution of the Eq. (37) is done by constructing the Homotopy Perturbation method as following,

$$H(\phi, p) = x\phi'(x) + \mathrm{x}^2\phi''(x) + \mathrm{px}^2\phi(x) = 0 \qquad p \in [0,1] \tag{38}$$

Within this context, the parameter 'p' ranges between 0 and 1. The initial solution of Eq.–(31) is derived by solving the condition when 'p' equals 0. The solution to Eq.-(32) manifests as a power series dependent on the variable 'p'

$$\phi(x) = \phi_0(x) + p\,\phi_1(x) + p^2\phi_2(x) + \cdots \tag{39}$$

The series of the flux equation as shown in Eq. (33) is applied to Eq. (32) and the power series of p is listed below,

$$\begin{aligned} p^0 &= \mathrm{x}^2\phi_0''(x) + x\phi_0'(x) = 0 \\ p^1 &= \mathrm{x}^2\varphi_1''(\mathrm{x}) + x\phi_1'(x) + \mathrm{x}^2\phi_0(x) = 0 \\ p^2 &= \mathrm{x}^2\varphi_2''(\mathrm{x}) + x\phi_2'(x) + \mathrm{x}^2\phi_1(x) = 0 \\ &\vdots \\ p^k &= \mathrm{x}^2\varphi_k''(\mathrm{x}) + x\phi_k'(x) + \mathrm{x}^2\phi_{k-1}(x) = 0 \end{aligned} \tag{40}$$

For zero value of p, it is known the initial value of $\phi_0(\mathrm{x}) = C$ by solving Eq. (38) as finite.

$$\phi_p(x) = Ax^2 + Bx + C \tag{41}$$

The Eq. (41) is substituted into Eq. (40) and the series of the neutron fluxes are founded as;

$$\begin{aligned}\phi_1(\mathrm{x}) &= -\frac{\mathrm{C}}{4}\mathrm{x}^2\\ \phi_2(\mathrm{x}) &= \frac{\mathrm{C}}{4.16}\mathrm{x}^4\\ \phi_3(\mathrm{x}) &= -\frac{\mathrm{C}}{4.16.36}\mathrm{x}^6\\ &\vdots\end{aligned} \tag{42}$$

$$\phi_{\mathrm{k}}(\mathrm{x}) = C - p\frac{\mathrm{C}}{4}\mathrm{x}^2 + \mathrm{p}^2\frac{\mathrm{C}}{4.16}\mathrm{x}^4 - \mathrm{p}^3\frac{\mathrm{C}}{4.16.36}\mathrm{x}^6 + \cdots$$

The power series of p can be combined as

$$\phi(\mathrm{x}) = C - p\frac{\mathrm{C}}{4}\mathrm{x}^2 + \mathrm{p}^2\frac{\mathrm{C}}{4.16}\mathrm{x}^4 - \mathrm{p}^3\frac{\mathrm{C}}{4.16.36}\mathrm{x}^6 + \cdots \tag{43}$$

the approximate result of the Eq. (37) can be obtained by taken limit of Eq. (43)

$$\begin{aligned}\phi(\mathrm{x}) &= \lim_{\mathrm{p}\to 1}\left[C - p\frac{\mathrm{C}}{4}\mathrm{x}^2 + \mathrm{p}^2\frac{\mathrm{C}}{4.16}\mathrm{x}^4 - \mathrm{p}^3\frac{\mathrm{C}}{4.16.36}\mathrm{x}^6 + \cdots\right]\\ &= C - \frac{\mathrm{C}}{4}\mathrm{x}^2 + \frac{\mathrm{C}}{4.16}\mathrm{x}^4 - \frac{\mathrm{C}}{4.16.36}\mathrm{x}^6 + \cdots\end{aligned} \tag{44}$$

The enclosed form of the Eq. (44) can be written by displaying the change of variable $x = Br$.

$$\phi(\mathrm{Br}) = \sum_{\mathrm{k}=0}^{\infty}\frac{(-1)^{\mathrm{k}}\mathrm{C}}{4^{\mathrm{k}}.\mathrm{k}!.\mathrm{k}!}(\mathrm{Br})^{2\mathrm{k}} \tag{45}$$

3. RESULTS AND DISCUSSION

The radius of different geometric systems can be calculated from the flux equations obtained by solving the neutron diffusion equation by employing the Homotopy Perturbation Method. The parameter B can only take values between zero and one for any combination of materials. The flux equation provides us with an understanding of how material in a medium affects its radius, which is based on a range from 0 to 1 for all possible values. In this case, the radius values are based on the [0, 1] range of B, which includes all possible values regardless of the specific material in the environment. The Table 1 shows the results of the radius r calculated against varying B values. The calculations were performed using the Mathematica program. In the radius calculations according to the B values, it was observed that the radius value decreased as the B value increased. While this decrease was very fast in the first steps, it was observed that a slower decrease was observed over time.

The graphs present the angular flux roots derived from the cubic geometrical system, as delineated in Eq. (23). Specifically, the graphs delineate the correspondence between radius and Buckling B values.

The cubic flux equation as shown in Eq. (23) demonstrates the relation between material parameter B and the radius of the system. The root of the flux equation gives the radius as following graphs as shown in Figure 2. The change of the neutron flux depending on B and r is presented in the Figure 3 by three-dimensional graph.

The roots of the angular flux for spherical geometrical system obtained in Eq. (33) are demonstrated in the graphs. The graphs are drawn for radius to Buckling B values respectively.

The roots of the flux equation calculated for spherical system in Eq. (33) is displayed in the Figure 4

The graphs depict the angular flux roots for an infinite cylindrical geometric system, as obtained from Eq. (45). They display the association between radius and Buckling B values, respectively.

Lastly, the flux solution of the neutron diffusion equation is demonstrated as graphs with varying values of B in Figure 6. The roots can be localized from the graphs for all B values.

The results of r corresponding to the changing values of B are shown in Figure 7 in a three-dimensional graph

Table 1. Radius Values for Different Geometric Systems.

B	Cube (cm)	Sphere (cm)	Cylinder (cm)
0.1	15,708	31,415	24,05
0.2	7,853	15,707	12,025
0.3	5,235	10,471	8,016
0.4	3,926	7,853	6,012
0.5	3,141	6,283	4,81
0.6	2,617	5,235	4,008
0.7	2,243	4,487	3,435
0.8	1,963	3,926	3,006
0.9	1,745	3,490	2,672

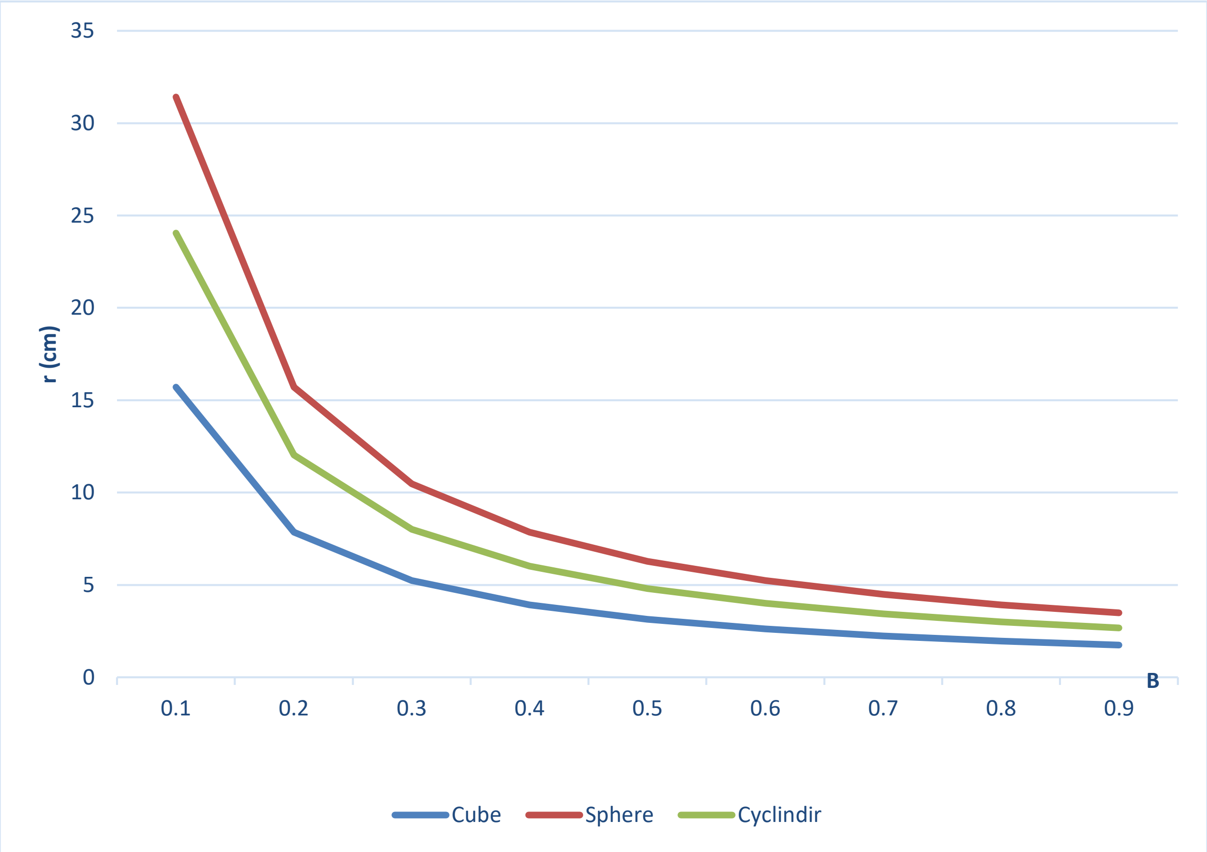

Figure 1. Comparison of All Radii Corresponding to all B for Different Geometric Systems.

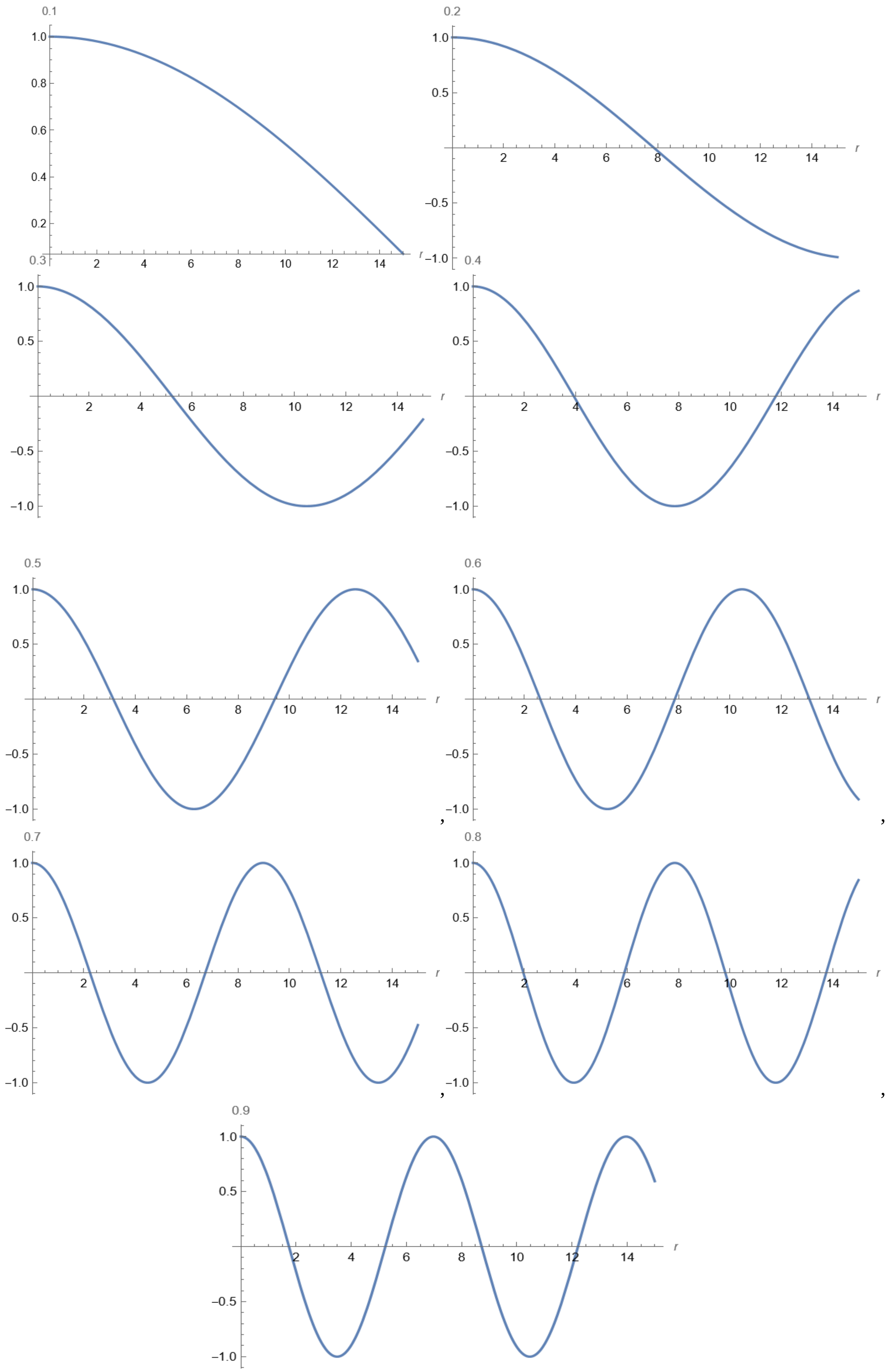

Figure 2. The Roots of the Flux Equation for Cubic System with B=0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9

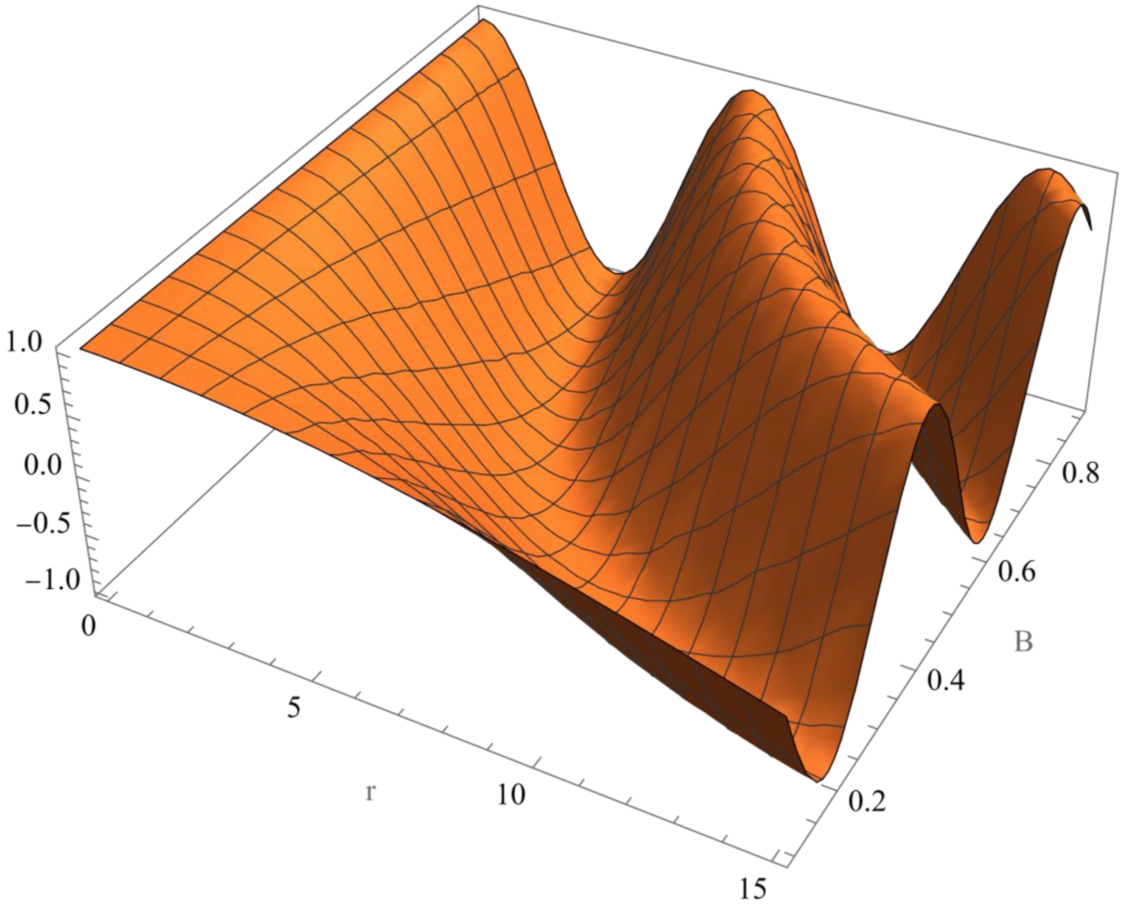

Figure 3. Flux Equation of Cubic System Graph for Changing r and B in 3D

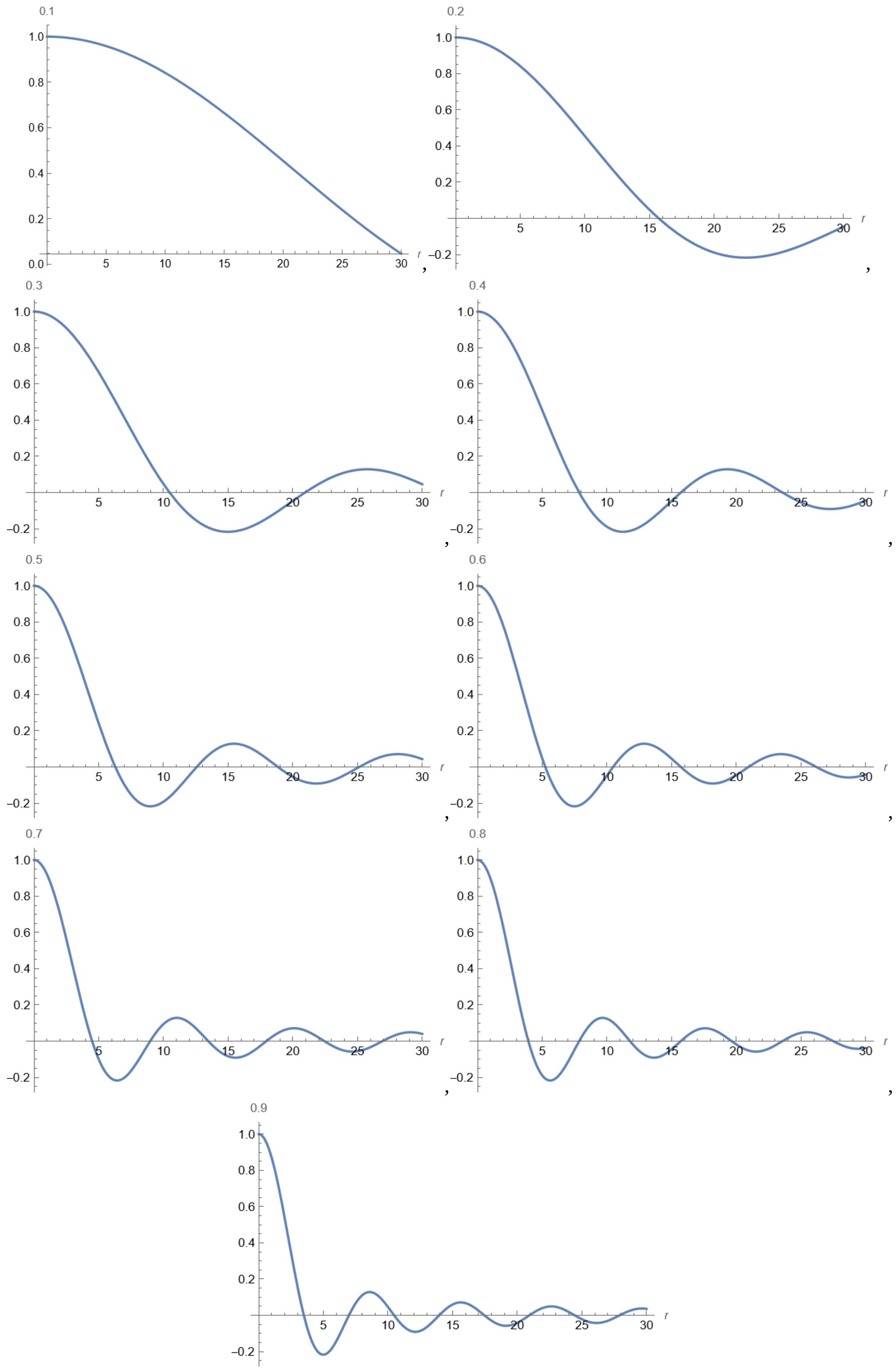

Figure 4. The roots of the Flux Equation for Spherical System with B=0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9

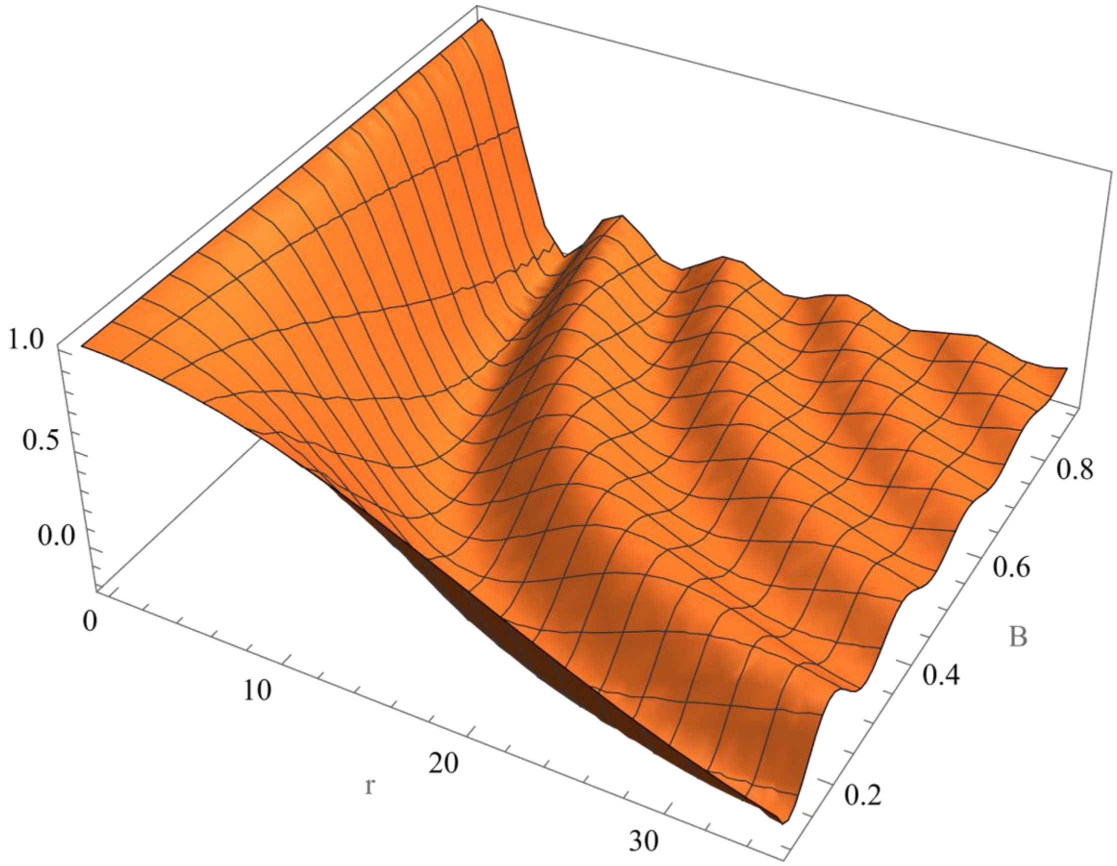

Figure5. Flux Equation of Spherical System Graph for Changing r and B in 3D The neutron flux in spherical geometry for various B and r is shown in Figure 5 as a three-dimensional graph

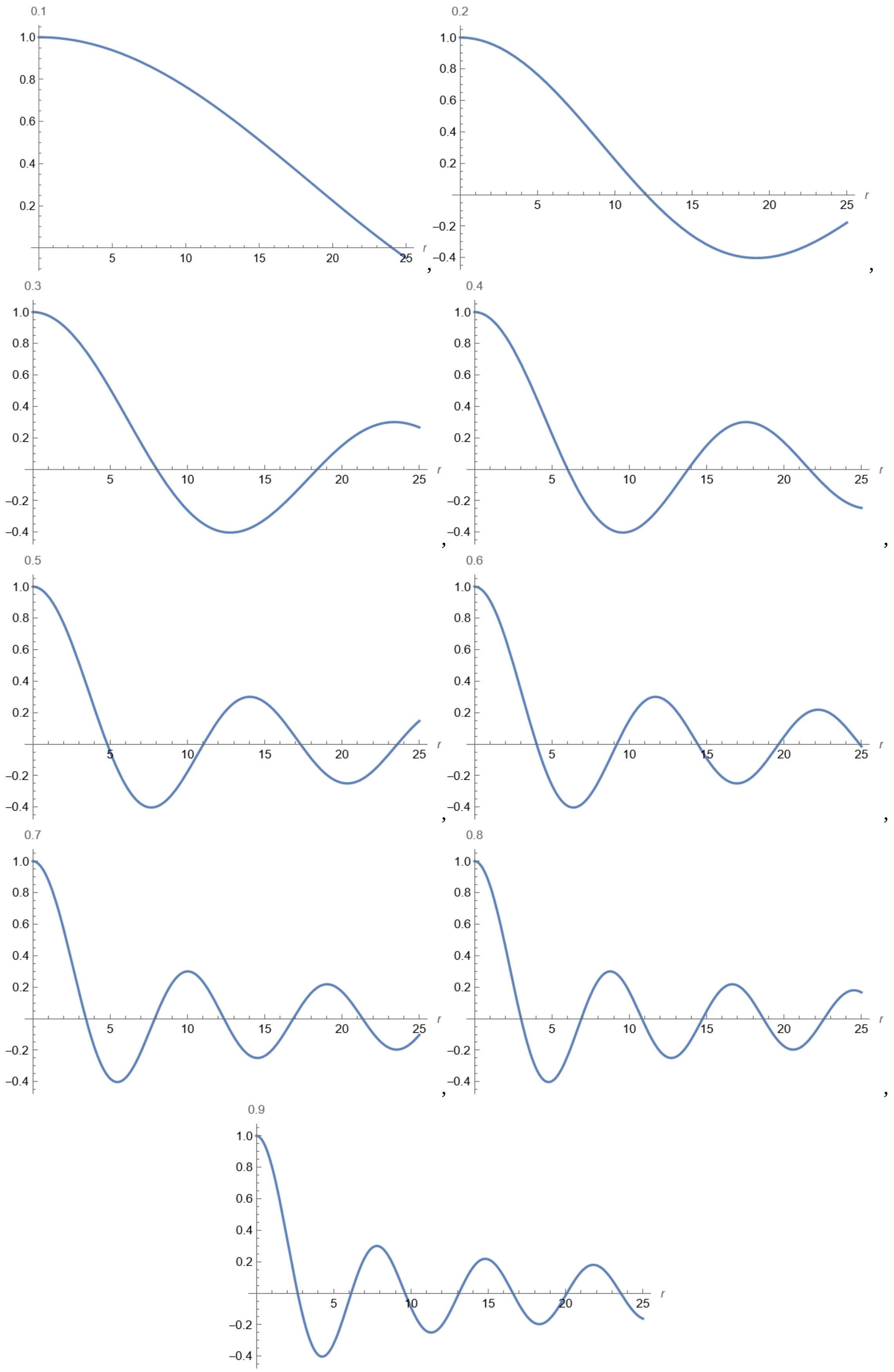

Figure 6. The Roots of the Flux Equation for Infinite Cylindrical System with B=0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9

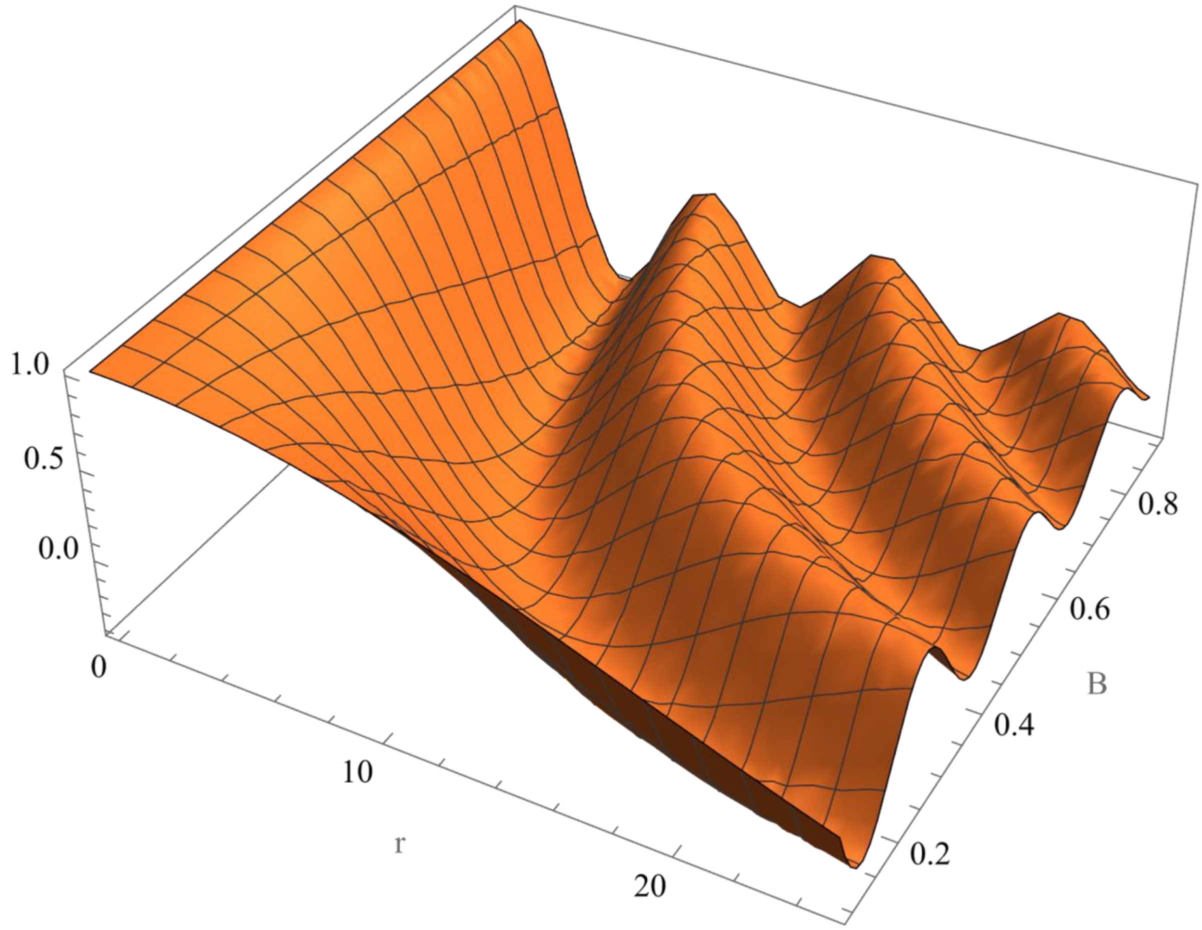

Figure7. The Flux Equation of Infinite Cylindrical System Graph for Changing r and B in 3D.

4. CONCLUSION

The Homotopy perturbation method is used for the solution for the neutron diffusion equation in three different geometric systems. In each geometric system contains own mathematical structure. When making the calculations, the Homotopy perturbation method had been tried the performance of solving spherical, cylindrical and cartesian coordinates systems. The root of the neutron diffusion equation implies the critical radius of the one-dimensional geometric system. It is well-known that when the material parameter increases, the critic radius of the system decreases. The behaviour of the roots meets the expectation of the neutron diffusion equation. The critical sizes for each geometrical system are listed in tables and also the results are shown in graphs. All values obtained from the solution of the neutron diffusion equation with the Homotopy Perturbation method are in line with the literature results mentioned in introduction section that are calculated for the certain material buckling values.

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