

Lepton Flavor Violation of Tau Decays into Lepton-Gamma in the Minimal Supersymmetric Type-I Seesaw Model

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Abstract

Experiments have shown that the violation of the lepton flavor so far is only in the neutrino sector (neutrino oscillation). Therefore, we predict it to happen in the charged lepton sector. We present a study of the lepton flavor violation (LFV) of tau lepton decays in two different channels. Tau into muon-gamma and Tau into electron-gamma. The prediction is performed in the Minimal Supersymmetric Standard Model (MSSM) extended by Seesaw Type-I Model (MSSM-Seesaw Type-I Model). The predicted calculations of the upper limit of branching ratio of the two channels are in the order of $\sim 10^{-8} - 10^{-9}$, which is in coincidence with the sensitivity of the future colliders (FCC-ee/CEPC).

Keywords: MSSM, Beyond Standard Model (BSM), Lepton Flavor Violation (LFV), Seesaw Model.

Kısıtlı Minimal Süpersimetri Tip-I Seesaw Modelinde Tau'nun Lepton-Gamma Bozunma Lepton Lezzeti İhlali

Özet

Deneyle, şu ana kadar lepton aromasının ihlalinin yalnızca nötrino sektöründe (nötrino salınımı) olduğunu göstermiştir. Bu nedenle bunun yüklü lepton sektöründe gerçekleşeceğini tahmin ediyoruz. Tau bozunmalarının lepton lezzet ihlaline (LFV) ilişkin iki farklı kanalda bir çalışma sunuyoruz. Tau müon-gamaya ve Tau elektron-gamaya dönüşür. Tahmin, Tahterevalli Tip-I Modeli (MSSM- Tahterevalli Modeli) ile genişletilen Minimal Süpersimetrik Standart Modelde (MSSM) gerçekleştirilir. İki kanalın dallanma oranına ilişkin tahmin edilen hesaplamalar $\sim 10^{-8} - 10^{-9}$ mertebesinde ve bu, gelecekteki çarpıştırıcıların (FCC-ee/CEPC) hassasiyetiyle örtüşmektedir.

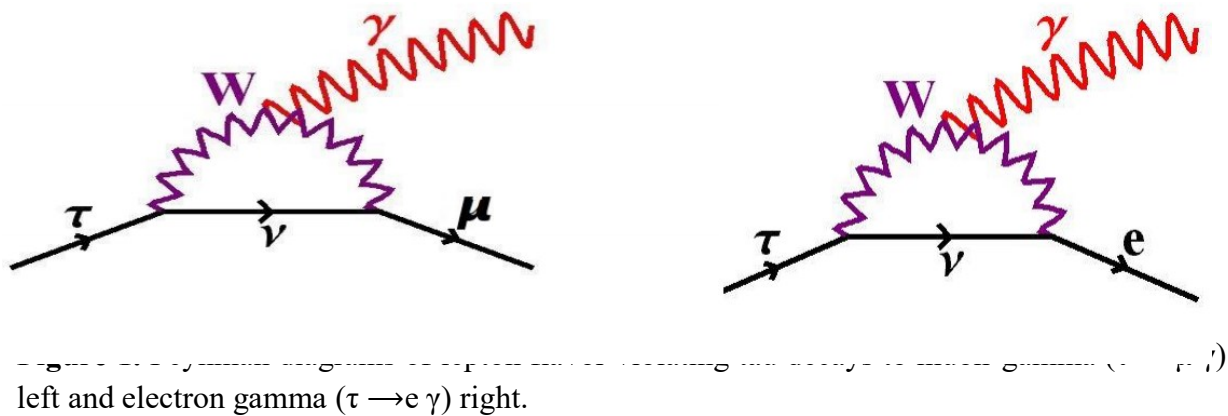
Anahtar Kelimeler: MSSM, Standart Modelin Ötesi (BSM), Lepton Lezzet İhlali (LFV), Seesaw Modeli.

1. Introduction

Collisions of electrons and nuclei in the cosmic rays and in particle accelerators are done at the beginning of the Thirties(1930s), causing the discovery of many particles. Some of them are predicted, others are completely unexpected but discovered by chance. At the beginning it was thought that all of these particles are fundamentals [1]. Later physicists have found hundreds of new particles, which we know nowadays that most of them are not fundamental. Several theories are developed to explain quite well the physical phenomena in nature; the most successful theory is called the Standard Model of particle physics (SM). The SM defines the block of matter as elementary particles called fermions and the force carrier as elementary particles called bosons. Fermions contains two classes six quarks and six leptons where there are four bosons which are exchanged by three of fundamental forces (the strong force, the weak force and the electromagnetic force) [2]. Unfortunately, the SM could not explain all the physical phenomena in nature like Gravity, Dark matter, Dark energy, the origin of neutrino mass, Hierarchy problem, Flavor problem, Unification of forces and other phenomena. Which in turn led to the emergence of the Beyond Standard Model theories (BSM). The Flavor problem is one of the phenomena which SM could not explain it, so the lepton number is conserved in SM but the lepton flavor is not conserved in nature, which is proved by confirming neutrino oscillation experimentally, besides neutrino mass differences and mixing angles [3]. However, the neutrino sector is the only place where the lepton flavor violation (LFV) has been seen so far. Consequently, determining whether or not LFV also exists in the charged lepton sector will be a difficult task for the current and future tests [4]. Based on the discovery of neutrino oscillations, the standard model (SM) with massless neutrinos must be expanded because lepton flavors are not conserved [3]. However, recent advancements in supersymmetric (SUSY) theories, particularly SUSY Grand Unified Theories (GUTs) & variants of Minimal Supersymmetric (MSSM), have made these theories more viable. Lepton flavor violation (LFV) processes have attracted more attention, including: $\tau \rightarrow e \gamma$, $\tau \rightarrow \mu \gamma$, $\mu \rightarrow e \gamma$, $Z \rightarrow l^+ l^-$, $H \rightarrow l^+ l^-$..etc. [5]. LFV processes in tau lepton decays (Z boson and Higgs boson decays) are predicted by several models: Models with heavy neutrinos, Supersymmetry and Extended gauge models [23]. Supersymmetry (SUSY) is considered as the most compelling theory from other theories [24]. Although, the Large Hadron Collider (LHC) has not detected any signs of supersymmetry up to now [25]. SUSY solves the hierarchy problem, provides signs to a dark matter and interprets neutrino masses when it is extended with right-handed Majorana neutrinos [26]. We will focus in our study on tau lepton decays into different lepton flavor ($\tau \rightarrow l \gamma$) where l either electron or muon. The observation of the LFV processes demand theories which interpret large mixing and tiny masses of left-handed neutrinos (Standard Model neutrinos) by the existence of heavy neutrinos [27]. The Seesaw mechanism is a useful SUSY extension for investigation of the tiny masses of neutrinos [28], this means that the new physics scale is about 10^{14} GeV [29]. Which is the best extension being to impose SUSY on the seesaw mechanism [28]. The study considers the Universal Gaugino Masses model which is extended by type-I seesaw model (three right-handed Majorana neutrinos and their supersymmetric partners).

2. Beyond Standard Model (BSM)

The term beyond standard model (BSM) describes theoretical physics frameworks that make an effort to explain phenomena that are not currently covered by the Standard Model (SM) of particle physics [6]. The Standard Model, which defines the fundamental particles and their interactions, is a very successful theory, yet it has some restrictions as mentioned in the introduction and unanswered issues (Gravity, Dark Matter, Hierarchy Problem, Neutrino Masses and) [7]. Several BSM ideas have been put forward to overcome these issues and broaden the standard model. Several prominent extensions consist of (Supersymmetry (SUSY), Grand Unified Theories (GUTs), String Theory and Extra Dimensions and variants of Minimal Supersymmetric (MSSM) [8]. There are many different methods and theories that have been investigated. It's vital to remember that while these theories might provide workarounds for the problems with the Standard Model, experimental proof is necessary to confirm their predictions [9].



2. LFV in MSSM-Seesaw Type-I Model

The study is performed in the scope of the minimal supersymmetric standard model (MSSM) extended by Seesaw Type-I model by adding three right-handed neutrinos, along with their SUSY partners [10]. We focus on two decay channels ($\tau \rightarrow e + \gamma$ and $\tau \rightarrow \mu + \gamma$), where it is anticipated that LFV effects in the charge lepton sector may be detectable as shown in Figure 1. The seesaw mechanism is a straightforward and alluring addition to introduce the neutrino masses [3]. The charged lepton sector encounters lepton flavor violations (LFVs) due to a new additional Yukawa matrix for right-handed neutrinos, just like the quark sector in the Standard Model (SM) [11]. The LFV processes, however, only take place via a loop containing neutrino, and they are suppressed by small neutrino masses. Thus, just the neutrino oscillations can be used to observe the LFV [4]. Due to considerable experimental advancements in recent years, the permitted ranges of the model parameters have altered significantly. The most significant development is that a Higgs boson, whose mass is now determined to be approximately 126 GeV, was discovered at the Large Hadron Collider (LHC) in July 2012 [12].

The super-potential in this case is written as following:

$$W = -y_u \hat{u}_R \hat{H}_u \hat{Q} + y_d \hat{d}_R \hat{H}_d \hat{Q} + y_l \hat{l}_R \hat{H}_d \hat{L} + \mu \hat{H}_u \hat{H}_d + \frac{1}{2} M_R \hat{\nu} \hat{\nu} + y_\nu \hat{\nu} \hat{L} \hat{H}_u \quad (1)$$

- y_u : Yukawa coupling constant for up quarks.
- y_d : Yukawa coupling constant for down quarks.
- y_ν : Yukawa neutrino coupling.
- \hat{Q} : left quark superfield.
- y_l : Yukawa coupling constant for leptons.
- μ : Higgsino mass.
- \hat{u}_R : Right up quark superfield.
- \hat{d}_R : right down quark superfield.
- \hat{l}_R : Right leptons superfield.
- M_R : Heavy Majorana neutrinos masses (right-handed neutrinos mass).
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The soft super-symmetry breaking results in the mass terms of the supersymmetric field particles as well as the triplet coupling constants between the supersymmetric fermions and the Higgs field. The mass terms in the supersymmetric leptons sector is written as the following equation [13].

$$-\mathcal{L}_{soft, lepton} = \sum_{i=gen} m_{\tilde{L}_i}^2 \tilde{L}_i^\dagger \tilde{L}_i + m_{\tilde{l}_{Ri}}^2 \tilde{l}_{Ri}^\dagger \tilde{l}_{Ri} + m_{\tilde{\nu}_{Ri}}^2 \tilde{\nu}_{Ri}^\dagger \tilde{\nu}_{Ri} + \sum_{i,j=gen} A_{ij}^l y_{ij}^l \tilde{l}_{Ri} H_d \tilde{L}_j + A_{ij}^\nu y_{ij}^\nu \tilde{\nu}_{Ri} H_u \tilde{L}_j + h.c. \quad (2)$$

- $m_{\tilde{L}_i}^2$, $m_{\tilde{l}_{Ri}}^2$, $m_{\tilde{\nu}_{Ri}}^2$: The square of mass terms of supersymmetric leptons.
- i, j : Indicates to generation number.
- $T_{ij}^l = A_{ij}^l y_{ij}^l$, $T_{ij}^\nu = A_{ij}^\nu y_{ij}^\nu$: A_{ij}^l , A_{ij}^ν : The terms of the trilinear coupling.
- **h.c.**: Hermitian conjugation.

We assume that Majorana mass is greater than M_{SUSY} ; hence, the appropriate neutrinos should be integrated out and should not contribute to M_{SUSY} in any way [14]. This will produce the Weinberg operator, which combines two left-lepton superfields with two up-Higgs superfields [15].

$$\frac{1}{2} W_{op} \hat{L} \hat{L} \hat{H}_u \hat{H}_u \quad (3)$$

The huge M_R suppresses the Weinberg operator, which violates the lepton-number. After soft symmetry-breaking produces a Majorana mass of left-handed light neutrinos [15]:

$$m_\nu = -\frac{v_u^2}{2} (y_\nu)^T (M_R)^{-1} y_\nu \quad (4)$$

The low neutrino masses imply that the M_R scale is very high. After the electroweak symmetry is broken, this term yields the neutrino mass matrix. where $VEV \simeq 174$ GeV [16].

Using the Constrained MSSM, we assume global conditions at the Grand Unified Theory (GUT): $m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$

- Soft Gaugino masses combine to a common value: $M_1 = M_2 = M_3 = m_{1/2}$.
- The square of the symmetry-breaking masses of both supersymmetric fermions and Higgs doublets combine to a common value: $m_{\tilde{l}_R}^2 = m_{\tilde{L}}^2 = m_{\tilde{\nu}}^2 = m_0^2 I$ & $m_{\tilde{H}_u}^2 = m_{\tilde{H}_d}^2 = m_0^2$
- Terms of linear triplet couplings to a common value: $A_l = A_d = A_u = A_\nu = A_0$.
- The ratio of the Higgs vacuum expectation values $\tan\beta$ and the sign of the Higgs mixing term (Higgs mass term) $\text{sign}(\mu)$ are at the electroweak scale.
- All the values of the soft super-potential breaking terms at the electroweak scale are obtained through the Renormalization Group Equations (RGEs).

3. Renormalization Group Equations and Lepton Flavor Violation:

The two parameters of supersymmetric Seesaw Type-I model, are Yukawa neutrino coupling Matrix y_ν and the right-handed neutrino mass matrix M_R [17]. Left-handed sleptons mass matrix receives an additional contribution from RGEs into LFV occurs in the left-handed sleptons sector [11]. Right-handed sleptons mass matrix does not receive any contribution to the logarithmic approximation. Linear triplet coupling terms are suppressed by the charged leptons masses:

$$\Delta m_{\tilde{L}}^2 = -\frac{1}{8\pi^2} m_0^2 \left\{ 3 + \frac{A_0^2}{m_0^2} \right\} Y_\nu^\dagger Y_\nu \log\left(\frac{M_{GUT}}{M_R}\right) \quad (5)$$

$$\Delta T_l^2 = \frac{-3}{8\pi^2} A_0 Y_l Y_\nu^\dagger Y_\nu \log\left(\frac{M_{GUT}}{M_R}\right)$$

$$\Delta m_{\tilde{e}}^2 = 0$$

$$; \frac{A_0}{m_0} = \text{const} = a_0 \Rightarrow A_0 = a_0 m_0$$

Models with large mixing in y_ν are achieved [18]: $y_\nu = D_u U_{PMNS}^\dagger$

- y_ν : Yukawa neutrino coupling Matrix.
- D_u : Diagonal Yukawa coupling matrix for top quarks.
- U_{PMNS} : leptonic mixing matrix.

The mixing matrix appears in the Dirac couplings in this scenario, with the Majorana matrix being diagonal, analogous to the quark case. The PMNS matrix then controls the mixing in y_ν [19].

$$(M_{R1}, M_{R2}, M_{R3}) = (4.0 \times 10^9 \text{ GeV}, 4.0 \times 10^9 \text{ GeV}, 5.9 \times 10^{14} \text{ GeV})$$

The soft SUSY breaking terms A_0 , m_0 , and $m_{1/2}$ as well as $\tan\beta$ are free parameters [19].

4. Results and Discussion

The final variables in the research are: y_ν , M_R , $m_{1/2}$, A_0 , m_0 , $\text{sign}(\mu)$ and $\tan\beta$ [20]. In our calculations, the soft symmetry breaking terms are constrained by several theoretical and experimental conditions, such as the lightest supersymmetric particle of the used model is the neutralino [21]. For heavy Majorana neutrinos masses we will consider the degenerate case as follows: $(M_{R1}, M_{R2}, M_{R3}) = (4.0 \times 10^9 \text{ GeV}, 4.0 \times 10^9 \text{ GeV}, 5.9 \times 10^{14} \text{ GeV})$ [19].

The energy scale of Great Unified Theory (GUT) is fixed to be $M_{\text{GUT}} = 2 \times 10^{16} \text{ GeV}$. The supersymmetric breaking scale is fixed to be $M_{\text{SUSY}} = 10^3 \text{ GeV}$.

We calculate the upper limit of the branching ratio for the two channels $\text{BR}(\tau \rightarrow e \gamma)$ and $\text{BR}(\tau \rightarrow \mu \gamma)$.

First, we calculate the upper limit of the branching ratio BR as a function of m_0 , so we tune the other parameters to get the best value of BR. The tuned parameters values are $\tan(\beta)=5$, $A_0=0 \cdot m_0$, $\mp 2 \cdot m_0 \text{ GeV}$, $m_{1/2}=200 \text{ GeV}$, $\text{sign}(\mu)>0$, $M_{\text{SUSY}}=1000 \text{ GeV}$.

We tune the A_0 parameter value. The best calculated values of BR for $\text{BR}(\tau \rightarrow e \gamma)$ are at $A_0 = \mp 2 \cdot m_0 \text{ GeV}$, $m_0 = 200 \text{ GeV}$. While the best calculated values of BR for $\text{BR}(\tau \rightarrow \mu \gamma)$ are also at $A_0 = \mp 2 \cdot m_0 \text{ GeV}$, $m_0 = 200 \text{ GeV}$. By increasing values of m_0 the values of $\text{BR}(\tau \rightarrow e \gamma)$ and $\text{BR}(\tau \rightarrow \mu \gamma)$ are decreasing as shown in Figure 2.

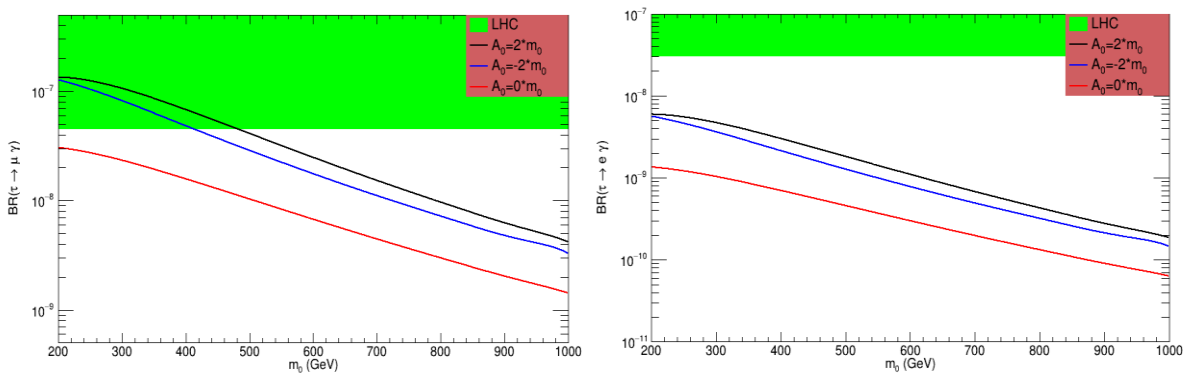


Figure 2. The upper limit of $\text{BR}(\tau \rightarrow e \gamma)$ and $\text{BR}(\tau \rightarrow \mu \gamma)$ as function of m_0 , $\tan(\beta)=5$, $A_0=0 \cdot m_0$, $\mp 2 \cdot m_0 \text{ GeV}$, $m_{1/2}=200 \text{ GeV}$, $\text{sign}(\mu)>0$, $M_{\text{SUSY}}=1000 \text{ GeV}$. $\text{BR}(\tau \rightarrow e \gamma)$ “Left panel”. $\text{BR}(\tau \rightarrow \mu \gamma)$ “Right panel”. Black line for $A_0 = 2 \cdot m_0$, blue line for $A_0 = -2 \cdot m_0$ and red line for $A_0 = 0 \cdot m_0$.

Secondly, we calculate the upper limit of the branching ratio BR as a function of m_0 , so we tune the other parameters to get the best value of BR. The tuned parameters values are $\tan(\beta)=5$, $A_0=2*m_0$ GeV, $m_{1/2}=200, 300, 400, 500$ GeV, $\text{sign}(\mu)>0$, $M_{\text{SUSY}}=1000$ GeV.

We tune the value of the $m_{1/2}$ parameter. The best calculated values of BR for $\text{BR}(\tau \rightarrow e \gamma)$ are at $m_{1/2}=200$ GeV. While the best calculated values of BR for $\text{BR}(\tau \rightarrow \mu \gamma)$ are also at $m_{1/2}=200$ GeV. By increasing values of m_0 the values of $\text{BR}(\tau \rightarrow e \gamma)$ and $\text{BR}(\tau \rightarrow \mu \gamma)$ are decreasing as shown in Figure 3.

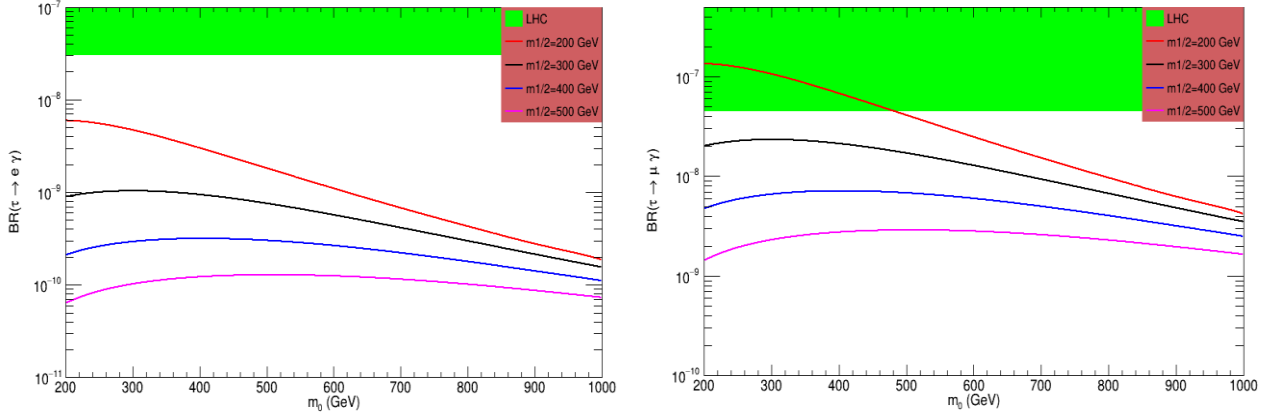


Figure 3. The upper limit of $\text{BR}(\tau \rightarrow e \gamma)$, $(\tau \rightarrow \mu \gamma)$ as function of m_0 , $\tan(\beta)=5$, $A_0=2*m_0$ GeV, $m_{1/2}=200, 300, 400, 500$ GeV, $\text{sign}(\mu)>0$, $M_{\text{SUSY}}=1000$ GeV. The best value of $\text{BR}(\tau \rightarrow e \gamma)$ “Left panel” at $m_{1/2}=200$ GeV (red line). The best value of $\text{BR}(\tau \rightarrow \mu \gamma)$ “Right panel” at $m_{1/2}=200$ GeV (red line).

Thirdly, we calculate the upper limit of the branching ratio BR as a function of $\tan(\beta)$, so we tune the other parameters to get the best value of BR. The tuned parameters values are $m_0=500$, $A_0=2*m_0$ GeV, $m_{1/2}=500$ GeV, $\tan(\beta)=[5 - 55]$, $\text{sign}(\mu)>0/ \text{sign}(\mu)<0$, $M_{\text{SUSY}}=1000$ GeV.

We tune the value of the $\text{sign}(\mu)$ parameter. The best calculated values of BR for $\text{BR}(\tau \rightarrow e \gamma)$ are at $\text{sign}(\mu)>0$ and $\tan(\beta)=55$. While the best calculated values of BR for $\text{BR}(\tau \rightarrow \mu \gamma)$ are also at $\text{sign}(\mu)>0$ and $\tan(\beta)=55$. By increasing values of $\tan(\beta)$ the values of $\text{BR}(\tau \rightarrow e \gamma)$ and $\text{BR}(\tau \rightarrow \mu \gamma)$ are increasing as shown in Figure 4.

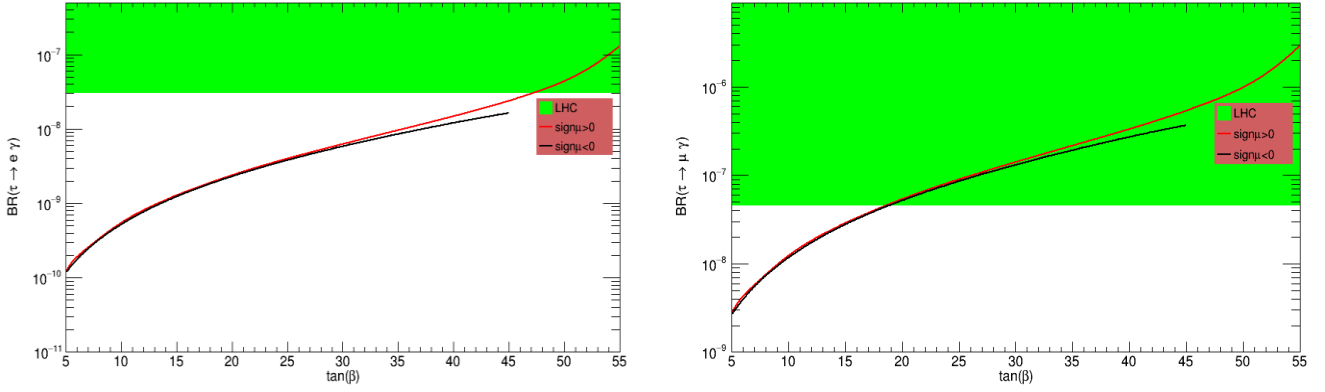


Figure 4. The upper limit of $BR(\tau \rightarrow e \gamma)$ and $BR(\tau \rightarrow \mu \gamma)$ as function of $\tan(\beta)$, $m_0=500$ GeV, $A_0=2*m_0$ GeV, $m_{1/2}=500$ GeV, $\text{sign}(\mu)<0/ \text{sign}(\mu)>0$, $M_{\text{SUSY}}=1000$ GeV. The best value of $BR(\tau \rightarrow e \gamma)$ “Left panel” at $\text{sign}(\mu)>0$ (red line). The best value of $BR(\tau \rightarrow \mu \gamma)$ “Right panel” at $\text{sign}(\mu)>0$ (red line).

Fourthly, we calculate the upper limit of the branching ratio BR as a function of $\cos(\theta)$, so we tune values of other parameters to get the best value of BR . The tuned parameters values are $m_0=m_{1/2}=500$ GeV, $A_0=2*m_0$ GeV, $\tan(\beta)=5$, $\text{sign}(\mu)>0/ \text{sign}(\mu)<0$, $M_{\text{SUSY}}=1000$ GeV.

We tune the value of the $\cos(\theta)$ with respect to $\text{sign}(\mu)$ parameter. The best calculated values of BR for $BR(\tau \rightarrow e \gamma)$ are at $\text{sign}(\mu)>0$ and $\cos(\theta)=1$. While the best calculated values of BR for $BR(\tau \rightarrow \mu \gamma)$ are also at $\text{sign}(\mu)>0$ and $\cos(\theta) =1$. By increasing values of $\cos(\theta)$ the values of $BR(\tau \rightarrow e \gamma)$ and $BR(\tau \rightarrow \mu \gamma)$ are increasing as shown in Figure 5.

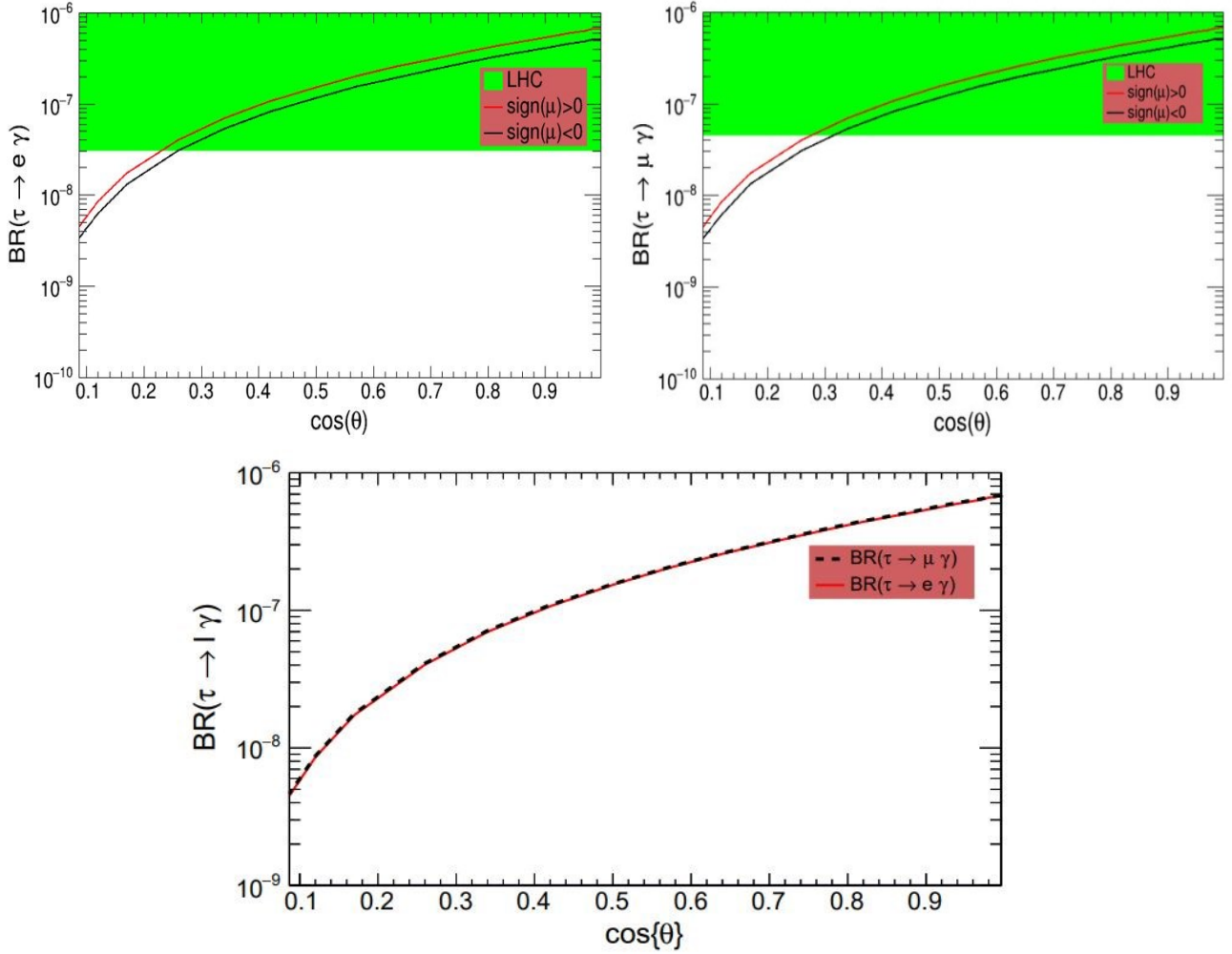


Figure 5. The upper limit of $BR(\tau \rightarrow e \gamma)$ and $BR(\tau \rightarrow \mu \gamma)$ as function of $\cos(\theta)$, $m_0=m_{1/2}=200$ GeV, $A_0=2*m_0$ GeV, $\text{sign}(\mu)>0/ \text{sign}(\mu)>0$, $M_{\text{SUSY}}=1000$ GeV. The best value of $BR(\tau \rightarrow e \gamma)$ “Left panel” at $\text{sign}(\mu)>0$ (red line). The best value of $BR(\tau \rightarrow \mu \gamma)$ “Right panel” at $\text{sign}(\mu)>0$ (red line). Bottom figure shows the upper limit of BR for the two channels only when $\text{sign}(\mu)>0$.

Last and not least, we calculate the upper limit of the branching ratio BR as a function of m_0 , final tuned parameters values are $A_0=2*m_0$ GeV, $m_{1/2}=200$ GeV, $\tan(\beta)=5$, $\text{sign}(\mu)>0$ and $M_{\text{SUSY}}=1000$ GeV. We tune the value of the m_0 parameter .

Best calculated values of $BR(\tau \rightarrow e \gamma)$ are at $m_0=200$ GeV and the best calculated values of $BR(\tau \rightarrow \mu \gamma)$ are also $m_0=200$ GeV. By increasing values of m_0 the values of $BR(\tau \rightarrow e \gamma)$ and $BR(\tau \rightarrow \mu \gamma)$ are decreasing as shown in Figure 6.

By comparing the results of the upper limit of the branching ratio for the two considered channels $BR(\tau \rightarrow e \gamma)$ and $BR(\tau \rightarrow \mu \gamma)$ as shown in the figure below, we conclude here that the upper limit value for the muon-gamma channel is better than that for the electron-gamma channel. Thus, the probability to detect LFV process in muon-gamma channel is most likely.

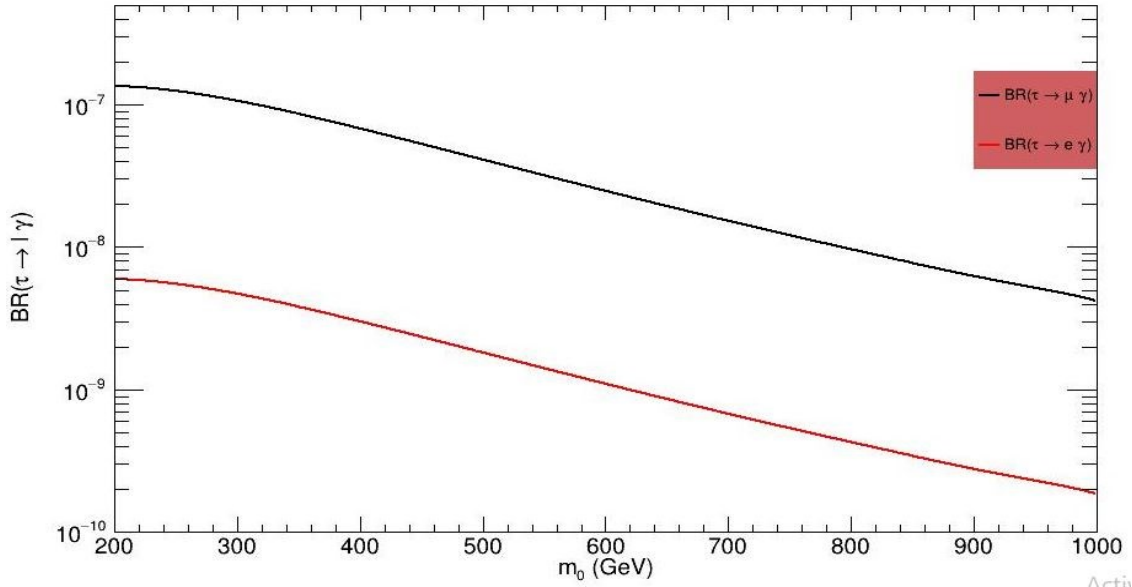


Figure 6. The upper limit of $BR(\tau \rightarrow \ell, \gamma)$ as function of m_0 . Where $\tan(\beta)=5$, $A_0=2*m_0$ GeV, $m_{1/2}=200$ GeV, $\text{sign}(\mu)>0$, $M_{SUSY}=1000$ GeV. The best value the upper limit of branching ratio BR is for $(\tau \rightarrow \mu \gamma)$ channel (black line), where the one for the $(\tau \rightarrow e \gamma)$ channel is less probable (red line).

5. Conclusion

In this article, lepton flavor violation (LFV) of the tau lepton decays $(\tau \rightarrow \ell \gamma)$, where $\ell = \mu$ or e , in the minimal supersymmetric standard model (MSSM) extended by the Seesaw Type-I model are studied in this article. In supersymmetric models, the Seesaw-I mechanism can be realized by adding three right-handed neutrinos. Universal Gaugino masses with the degenerate case scenario is considered. Because of light neutrino masses, Majorana neutrinos scale is about $(10^{13} - 10^{14})$ GeV and Yukawa coupling of neutrinos Y_ν is ~ 1 . From numerical calculations we found that the branching ratios BRs of tau lepton decays $(\tau \rightarrow \ell \gamma)$ are decreasing when increasing the values of universal scalar mass m_0 and Gaugino masses ($m_{1/2}$). While the values of BRs are increasing when the values of trilinear parameter A_0 , $\tan(\beta)$ and $\cos(\theta)$ are increasing. The best values of $BR(\tau \rightarrow \ell \gamma)$ are at input parameters: $m_0 = 200$ GeV, $m_{1/2} = 200$ GeV, $\tan(\beta)=5$, $\text{sign}(\mu) > 0$, and $\cos(\theta)=0.61$. The obtained results for tau decays $(\tau \rightarrow e \gamma)$, $(\tau \rightarrow \mu \gamma)$ are close to the experimental limits in BABAR and BELL. We observed the best value for $\tau \rightarrow e \gamma$ which is around $\sim 4 \times 10^{-9}$, while the experimental limit in BABAR is around $< 3.3 \times 10^{-8}$. We obtained also the best values for $\tau \rightarrow \mu \gamma$ which is around $\sim 1.3 \times 10^{-8}$, while the experimental limit in BELL is around $< 4.2 \times 10^{-8}$. Furthermore, the obtained values of BR of $BR(\tau \rightarrow \ell \gamma)$ for some parameters are in the range of the expected experimental upper limit for the LHC [22] as shown in figure(2-5). Also, the values of $BR(\tau \rightarrow \ell \gamma)$ are coincidence with the sensitivity of future colliders (FCC-ee/CEPC) [22]. Thus, the lepton flavor violation of the tau lepton decays in the MSSM-Seesaw Type-I model can be realized and can be observed in future colliders.

Ethics in Publishing

There are no ethical issues regarding the publication of this study.

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