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Review

Applications of DEM Particle Breakage Models in Mineral Industrial

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A B S T R A C T

Modeling processes are carried out in the mineral industry as well as in many areas depending on the development of computer technologies and software. Discrete Element Method (DEM) is used in modeling studies to explain the interaction of particles with other particles and communication equipment. The DEM provides the capability to simulate the movement of the granular media in a series of computational processes of each individual particle that consists of the granular media. It is becoming increasingly widely used to predict energy consumption, wear, particle breakage and particle size distribution in crushing and grinding processes that can be described in terms of granular materials using DEM. The selection of particle breakage models used by commercial software for modeling DEM particle breakage is important. In this study, it is summarized the studies have been carried out to understand the performance of particle breakage methods, which are Bonded Particle Model (BPM), Fast Breakage Model (FBM) and Particle Replacement Model (PRM), in the modeling of comminution equipment. In addition, the relationship between particle and breakage energies and theory of applied forces are described in detail for three breakage models existing in commercial DEM simulators

Keywords: *Bonded Particle Model, DEM, , Fast Breakage Model, Particle Breakage, Particle Replacement Method.*

Introduction

In industrial scale processing operations, size reduction is carried out by crusher and tumbling mills with rod/ball-autogenous/semi-autogenous grinding technologies. In plants, energy consumption during the grinding stage can be up to 80-90% of total energy consumption (Jeswiet and Szekeres, 2016). Modeling studies are carried out to reduce energy consumption in grinding. It has been proven that modeling size reduction processes with Discrete Element Method (DEM) are a useful technique for predicting wear, particle breakage and particle size distribution in addition to energy consumption.

The DEM was first proposed by Cundall and Strack (1979) as a numerical model to define the mechanical behavior of spheres or discs. The use of this method has become increasingly widespread in processes such as rock mechanics, crushing and grinding, which can be expressed in terms of granular materials, with the idea that particle motions explain the motion of the whole mass. In granular media, particles that move independently from each other and interact only at their contact points affect the behavior of the media. In granular media, DEM utilizes Newton's laws of motion for the motion of individual particles and contact laws for the contact between particles (Weerasekara et al., 2013).

In DEM, the motion of the particles is found by a series of calculations that follow the contact forces and the displacement of the particles during the collision. First, it is verified whether the particles are in contact with each other for particles i and j with radius R_i and R_j, respectively;

$$R_i + R_j > D \quad (1)$$

Where, D is the distance between the centers of the two particles. A spring and dashpot pair mechanism is assumed at each contact point. These pairs form the normal and tangential force components. In the tangential direction, there is also a sliding mechanism (Figure 1) (Cundall and Strack, 1979). Of these components, the Hertz model (Hertz, 1982) represents for the normal force component and the Mindlin model (Mindlin, 1949) for the tangential force component. Normal force;

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$$F_n = -k_n \Delta x + C_n v_n \quad (2)$$

where Δx is amount of overlap, k_n is the stiffness in the normal direction and C_n is the normal damping coefficient. $k_n\Delta x$ is the spring mechanism and $C_n v_n$ is the dashpot mechanism. The C_n value is derived from the coefficient restitution (ε), which is the ratio of the particle velocities before and after the collision (Cleary, 1998).

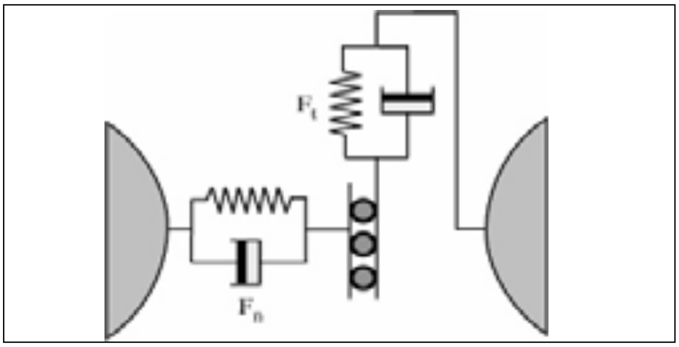

Figure 1. Contact forces between two particles in contact (Cleary, 2001a)

The tangential force is given by;

$$F_t = min(\mu F_n, k_t \int v_t dt + C_t v_t) \quad (3)$$

where μ is the friction coefficient, k_t is the stiffness in the tangential direction and C_t is the tangential damping coefficient. The total tangential force is restricted by the Coulomb frictional limit, above which the surface contact shears and the particles begin to slide over each other. (Cleary, 1998).

The simulation step in DEM consists of particle creation, contact of the particles with each other and with the geometry, and a series of time steps that depend on their position. After each time step, the position and contacts of the particles and geometry are updated. This process continues until all time steps are completed (Figure 2) (EDEM, 2023).

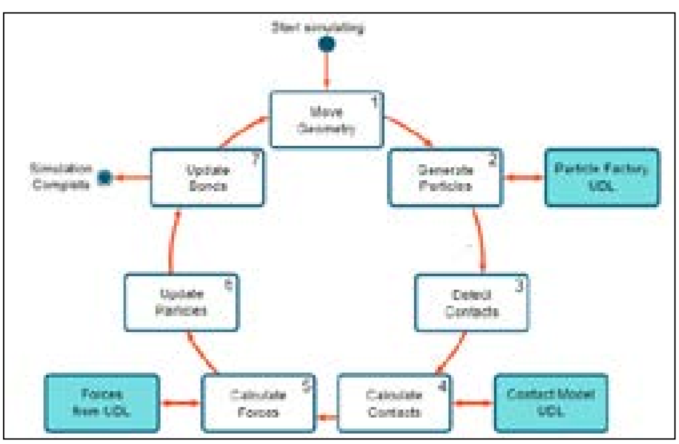

Figure 2. EDEM's simulation sequence

DEM was first used in mineral processing to model various mills with the assumption that particle motions explain the motion of the whole mass. Firstly, Mishra and Rajamani (1992) and Mishra and Rajamani (1994) used DEM to predict the media motion in ball mills in 2D. After that, Rajamani and Mishra (1996) also used DEM to describe particle motion in 2D semi-autogenous mills. As the application of DEM in 3D increases the accuracy of predictions, the use of DEM in different types of mills has increased significantly (Bian et al., 2017; Cleary, 1998; Cleary, 2001a; Cleary, 2001b; Cleary et al., 2003; Cleary, 2015; Datta and Rajamani, 2002; Djordjevic, 2005; Herbst and Nordell, 2001; Jayasundara et al., 2012; Morrison et al., 2009; Powell et al., 2011; Wang et al., 2012).

In modeling crushers with DEM; many researchers have studied it to define cone crushers (Delaney et al., 2015; Lichter et al., 2009; Quist and Evertsson, 2016), pressure crushers (Cleary and Sinnott, 2015; Refahi et al., 2010) and HPGR (Barrios and Tavares, 2016). The particle breakage mechanism of different types of crushers is also different. For the use of DEM in crushers, particle breakage is required to be included in the model. There are main three methods used by commercial software to simulate particle breakage in DEM: Bonded-Particle Model (BPM), Fast Breakage Model (FBM) and Particle Replacement Model (PRM). In the present work, the theory of these three models describing particle breakage were investigated in detail. According to the results of the investigation, the applicability of these models in comminution processes was determined.

1. Particle breakage models

1.1. Bonded Particle Model (BPM)

The BPM developed by Potyondy and Cundall (2004) aimed to simulate the mechanical behavior of rock by representing it as a cemented granular material. A system is developed with non-uniformly sized spherical particles connected to each other at the contact points. The mechanical behavior of this system is described by the motion of each particle and the force and moment acting at each contact. The interconnected particles are called fraction particles and the resulting cluster is called meta particles. In this study, they proposed a numerical model represented by a packing of dense spherical or circular particles that are tightly bonded together at the contact points and whose mechanical behavior is simulated by DEM using the two- and three-dimensional discontinuous programs PFC2D and PFC3D. Breakage was presented by broken bonds. It was found that particle size has a significant effect on the breaking strength of a material. In addition, similar results were obtained for their mechanical behavior.

A critical part of BPM is the determination of the size of the spheres that form the meta particle, also referred to as the parent particle. These include normal distributions (Antonyuk et al., 2006), mono size (Metzger and Glasser, 2012) and bi-modal (Quist and Evertsson, 2016) distributions (Figure 3).

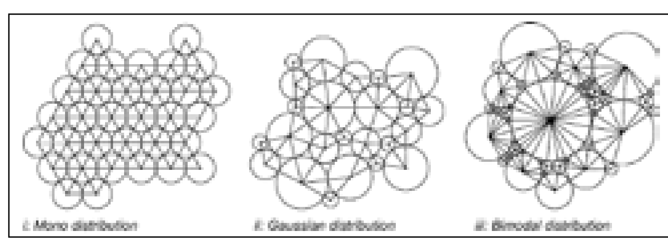

Figure 3. Schematic presentation of three types of packaging structures (Quist and Evertsson, 2016)

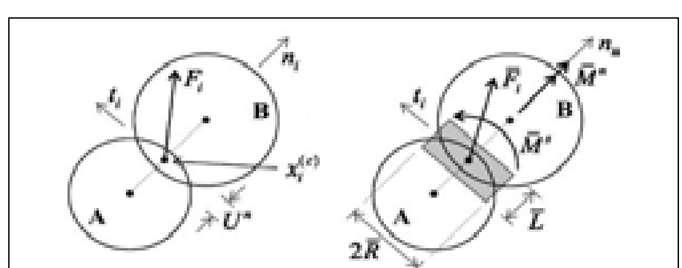

Figure 4. Illustration of BPM forces and moments (Potondy and Cundall, 2004)

Figure 4 shows, $F_{n,b}$, $F_{t,b}$ and M_{bn}, M_{bt} are the axial (normal) force and shear force and moments, respectively;

$$\delta F_{t,b} = -k_b^t A \Delta U_t$$
$$\delta M_b^n = -k_b^t J \Delta\theta_n \qquad (4)$$
$$\delta M_b^t = -k_b^n I \Delta\theta_t$$

where;

$$\Delta U_n = v_n \delta_t$$
$$\Delta U_t = v_t \delta_t$$
$$\Delta\theta_n = w_n \delta_t \qquad (5)$$
$$\Delta\theta_t = w_t \delta_t$$

where; k_{bn} and k_{bt} normal and shear bond stiffness. v_n, v_t, w_n and w_t are normal and tangential velocities and normal and tangential angular velocities, respectively. A, I and J are an area of the parallel bond cross section, moment of inertia and polar moment of inertia.

$$A = \pi R_b^2$$
$$I = \frac{1}{4}\pi R_b^4 \qquad (6)$$
$$J = \frac{1}{2}\pi R_b^4$$

Maximum tensile (σ_{max}) and shear stresses (τ_{max});

$$\sigma_{max} = \frac{F_{n,b,toplam}}{A} + \frac{2M_b^n}{J} R_b$$
$$\tau_{max} = \frac{F_{t,b,toplam}}{A} + \frac{2M_b^t}{J} R_b \qquad (7)$$

The maximum tensile stress exceeds the tensile strength (σ_c) or the maximum shear stress exceeds the shear strength (τ_c), the parallel bond is broken and the associated forces, moments and stiffeners are removed from the model (Potyondy and Cundall, 2004).

BPM has been used to model vertical roller mill (Liu et al., 2022), ball mill (Metzger and Glasser, 2013), cone crusher (Quist and Evertsson, 2016), gyratory crusher (Quist et al., 2011) in mineral processing by using DEM.

Quist and Evertsson (2016) simulated an industrial size cone crusher with a BPM with a bimodal particle distribution. They verified the simulation results with single particle breakage experiments. They determined that throughput, power, pressure and particle size distribution can be predicted using the BPM. Unfortunately, this process increases the computational workload considerably. The results are obtained in accordance with the results of the analytical models developed by Evertsson (2000).

1.2. Fast Breakage Model (FBM)

Potapov and Campbell (1996) proposed a model including polyhedral particles, called FBM. The paper described the extension of the existing two-dimensional technique to three dimensions in order to simulate the breakage of brittle solids. In this way, a fault can spread uniformly through the simulated material. The breakage occurred on a simulated particle formed by gluing polyhedral particles together with bonds between fitting parts. FBM is an instantaneous breakage model that utilizes Laguerre-Voronoi tesselation to break the particle into 2D polygonal or 3D polyhedral at the initial moment when the total energy of the collision is greater than the energy required for breakage (Jiménez-Herrera et al., 2018) (Figure 5).

The FBM has been used to describe breakage in the particle bed (Paluszny et al., 2016; Potapov and Campbell, 2000) and to simulate comminution equipment (Herbst and Potapov, 2004; Lichter et al., 2009). In comparing this model with two other breakage models, Jiménez-Herrera et al. (2018) reported that the FBM describes the interaction between the particle bed and the dropping ball very well, but is limited in describing the particle size distribution as well as the measured force deformation caused by single particle breakage. Moreover, mass conservation and the possibility of generating irregularly shaped particles have made the FBM a potentially powerful model for simulating large-scale communication systems (Jiménez-Herrera et al., 2018).

Lichter et al. (2009) simulated different cone crushers utilizing DEM's FBM breakage model to determine the flow rate, energy consumption and particle size distribution of the product. The approach, referred to currently as FBM, combines DEM components with Population Balance Modeling (PBM) components. The PBM method used 3D polyhedral particles. The contact energy of the particle is sufficient to break the particle and the particle is instantly broken into smaller sizes, the size distribution of which is calculated by PBM. According to the results of the study, the outputs were obtained in agreement with the experimental data (Figure 6).

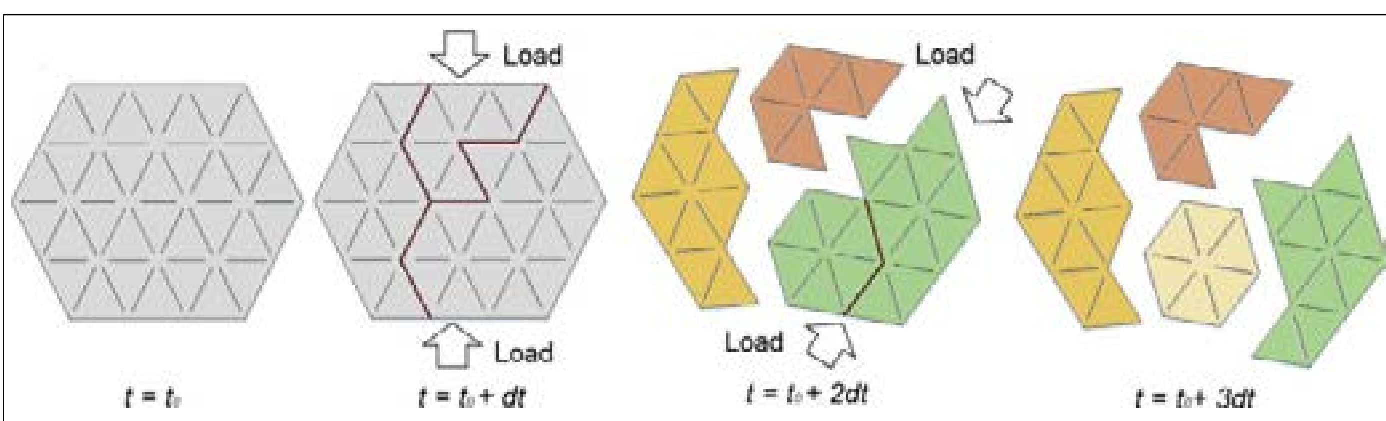

Figure 5. 2D illustration of particle breakage in FBM (shows multiple breakage phenomena) (Jiménez-Herrera et al., 2018)

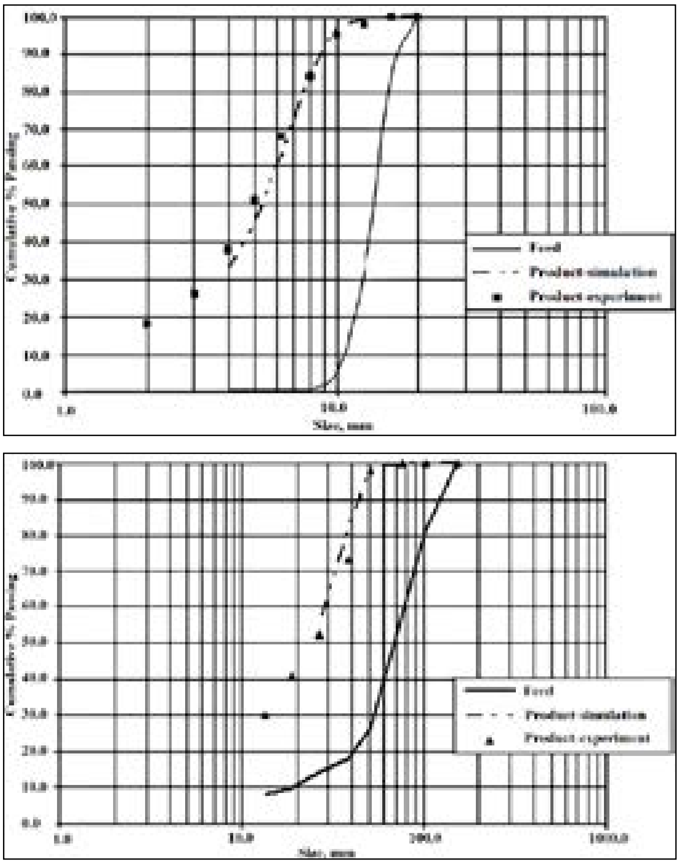

Figure 6. Simulated and experimental particle size distributions - HP100 Model (Right), B90 Model (Left) (Lichter et al., 2009)

The Vogel and Peukert model is based on a generalized dimensional analysis approach proposed by Rumpf (1973) and a detailed breakage mechanics model based on Weibull statistics (1951). The breakage probability P(E) is expressed in Equation 8 by combining these two different approaches in the Vogel and Peukert model (Vogel and Peukert, 2005).

$$P(E) = 1 - \exp\left[-S\left(\frac{d_i}{d_{i,ref}}\right)E_{cum}\right] \quad (8)$$

$$E_{cum} = E'_{cum} + E - E_{min}$$
$$E_{min} = e_{min,ref}\left(\frac{d_{i,ref}}{d_i}\right) \quad (9)$$

where E'_{cum} is the energy deposited in the grain just before the moment of stress, E is the total specific energy of the collision in an impact event. S, $d_{i,ref}$, $e_{min,ref}$ and d_i are the model parameters that define the breaking strength of the material, the reference size, the minimum energy required to break this reference size and the particle size, respectively.

1.3. Particle Replacement Model (PRM)

First proposed by Cleary (2001a and 2001b) to describe particle breakage in DEM, PRM is an instantaneous (over a period of time) replacement of particles with smaller sized progeny when the particles achieve the breakage requirement. Using PRM, the product size can be defined and a target size distribution can be obtained. Several studies have been conducted in which the particle can be a sphere or cluster sphere (Åström and Herrman, 1998; Barrios et al., 2020; Cleary, 2001b; Cleary and Sinnott, 2015; Tavares et al., 2021), superquadric (Delaney et al., 2015) or polyhedral cell (Arruda Tino and Tavares, 2022; Chen et al., 2024; Tavares et al., 2020) and is replaced by smaller particles of the same or different shape by breakage.

PRM is implemented in the Tavares UFRJ Fracture Model, which is available as a breakage model in Altair EDEM. In the model, the main condition for a particle to break and be replaced by its fragments is that the specific impact energy must be higher than the specific breaking energy of the particle. In Figure7, each parent particle is removed from the simulation when breakage occurs and replaced by a group of smaller sized particles. The majority of PRM utilizes spheres as the replacement particles for computational efficiency. The main drawback, however, is the mass loss that occurs when a large sphere is replaced by several smaller spheres. To obtain a breakage result close to a real breakage result, the spheres inside the parent particle are organized so that the largest spheres overlap in the direction perpendicular to the stress that caused the breakage. The remaining smaller spheres are then arranged in the remaining spaces, usually overlapping with the larger spheres. During replacement, the spheres are initially allowed to overlap in order to fill the volume of the original parent particle. But this overlap can be significant sufficient to lead to large artificial repulsive forces between them. A further reason for the unrealistic results of the simulation is mass loss (Jiménez-Herrera et al., 2018; Tavares and Chagas, 2021; Tavares et al., 2021).

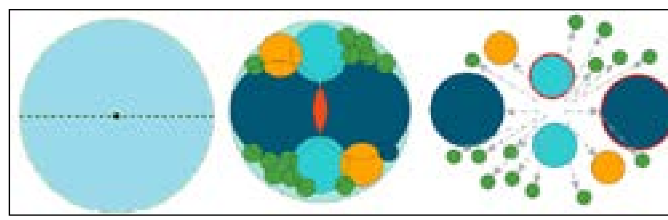

Figure 7. Schematic presentation of PRM; (a) main particle (b) particle replacement by size distribution (c) image after applying force (EDEM, 2023)

The potential explosions between the spheres are controlled by applying a dumping approach to limit the total contact force applied to each sphere (global damping strength) and also the time for which the reduced force is applied (global dumping time). Thus, part of the overlap can be defined in such a way that part of the breakage energy is released back into the particles (local damping strength). This method provides the possibility to control the kinematics of the particles and prevents the appearance of excessively high velocities of the particles, which would make the simulations unrealistic. In addition, in order to eliminate mass loss and provide accurate size distributions when generating realistic simulations, a size class called dummy particles is created, which defines particles up to 1/5 of the main size. Particles in this dummy size class are not allowed to break (EDEM, 2023).

The size, average value and standard deviation of each particle are assigned a specific breakage energy. This energy is determined according to the distribution given by Tavares and King (2002).

$$P(E) = \frac{1}{2}\left[1 + erf\,\frac{lnE^* - lnE_{50}}{\sqrt{2}\sigma}\right] \quad (10)$$

$$E^* = \frac{E_{max}E}{E_{max} - E} \quad (11)$$

where P(E) is the probability of breakage or cumulative distribution, E is the particle breakage energy distribution corresponding to the maximum stress energy it can endure in a collision, E_{max} is the upper cut-off value of the distribution, and E_{50} and σ are the median and standard deviation of the distribution, respectively.

The upper cut-off value is usually represented by the ratio E_{max}/E_{50}. When this ratio is equal to infinity, E* = E and Equation 10 becomes the lognormal distribution. Another parameter that affects the breakage probability is the particle size. The relationship between particle size and average breakage energy (Tavares and King, 1998; Tavares, 2022);

$$E_{50,i} = \frac{E_\infty}{1+k_p/k_s}\left[1 + \left(\frac{d_o}{d_i}\right)^{\varphi}\right] \tag{12}$$

where E_∞, d_0 and ϕ are model parameters to be determined by experimental data and d_i is the representative size of particles in size class i. k_p is the particle stiffness, k_s is the Hertzian stiffness of the surface in contact with the particle. The stiffness of a particle is significantly smaller than the stiffness of the surface of the testing machine or equipment in contact with the particle. In some cases, however, this is not the case, so a correction must be used. The strain energy, e, involved in an event used to deform the particle is given by (Tavares, 2022).

$$e = \frac{1}{1+k_p/k_s} \tag{13}$$

e is expressed as the ratio of the energy involved in a collision and distributed to the particles according to their stiffness. In the case of two particles of the same material in collision, Equation 13 gives e=0.5, since the energy is shared equally between them.

In a collision, when the specific strain energy is smaller than the breakage energy of the particle, the particle would not break, but would maintain internal fault-like damage that would make it more fracture prone in a future tensile phenomenon. This damage is described on the basis of a model based on continuous damage mechanics in which the specific breakage energy of the particle is reduced (Figure 8) (Tavares and King, 2002; Tavares, 2009);

$$E' = E(1-D) \tag{14}$$

$$D = \left[\frac{2\gamma}{(2\gamma-5D+5)}\frac{eE_k}{E}\right]^{\frac{2\gamma}{5}} \tag{15}$$

where E′ is the breakage energy of the particle after the collision phenomena, D is the damage exposed to the particle after a contact that does not lead to breakage. e′ is the specific energy involved in the additional collision (or effective impact energy) and γ is the damage accumulation coefficient, which characterizes the damage tolerance of a material before breakage. Equation 15 can be calculated iteratively, starting with D = 0.

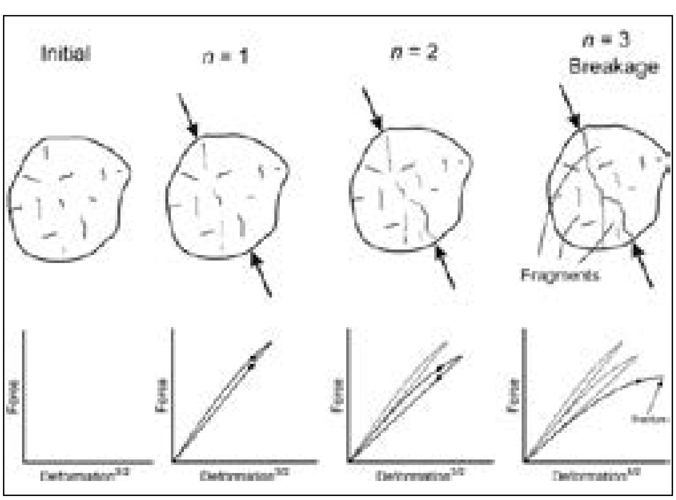

Figure 8. Illustration of the failure effect of particles caused by damage accumulation during repeated loading phenomena (Tavares, 2009)

The breakage level of particles can be expressed by a single parameter, t_{10}, which represents the ratio of particles finer than 1/10 of the parent particle size (Napier-Munn et al., 1996). Tavares (2009) stated the relationship between the specific tensile energy and the average breakage energy of particles in Equation 16.

$$t_{10} = A\left[1 - \exp\left(-b'\frac{eE_k}{E_{50b}}\right)\right] \tag{16}$$

where A and b′ are the model parameters obtained from the single particle breakage data and E_{50b} is the median breakage energy of the broken particles. In the case of the primary breakage function, where the ratio eE_k/E_{50b} can be assumed to be equal to one, Eq. 16;

$$t_{10} = A[1 - \exp(-b')] \tag{17}$$

The total particle size distribution is;

$$t_n(t_{10}) = \frac{100}{\int_0^1 x^{\alpha_n-1}(1-x)^{\beta_n-1}dx}\int_0^{t_{10}/100} x^{\alpha_n-1}(1-x)^{\beta_n-1}\,dx \tag{18}$$

where x is the cumulative mass (t_n) of the particles passing through the sieve of the corresponding size, calculated from the given distribution value t_{10}, α_n and β_n are the parameters of the model for the chosen values of t_n.

PRM has been studied to simulate particle bed breakage and single particle breakage processes (Arruda Tino and Tavares, 2022; Barrios et al., 2015; Barrios et al., 2020; Jiménez-Herrera et al., 2018; Tavares et al., 2020; Tavares et al., 2021; Tavares and Chagas, 2021).

Tavares and Chagas (2021) described the simulation of particle breakage using DEM and proposed a stochastic-randomized approach to produce realistic size distributions. In the study, a family of spherical particles for various values of t_{10} was created to represent the breakage of spherical particles simulated using DEM, based on data obtained from weight reduction tests. It is emphasized that the proposed model is successful in simulating the breakage of particles and is successful to generate real size distributions. A similar study was performed by Tavares et al. (2021) in the updated version of EDEM and again showed a strong performance in predicting particle breakage. In addition to these studies using EDEM, Tavares et al. (2020) and Arruda Tino and Tavares (2022) simulated particle breakage using polyhedral particles instead of spherical particles using Rocky DEM commercial software.

Arruda Tino and Tavares (2022) showed in their study that the results of the JK drop weight test, Los Angeles abrasion test and Bond breakage tests can be predicted by simulating them with DEM using polyhedral particles and Voronoi tessellation in the PRM. Comparison of experimental JK drop weight test results with its simulations revealed that the simulation is sensitive to the variables. It was found that there was good agreement for copper ore and granulite, but deviation for limestone. Considering the predictions of the breakage amenability parameter A*b, it overestimated the experimentally obtained values.

Tavares et al. (2020) reported that their model was able to accurately predict the breakage probability and particle size distribution for single and multiple impact cases. The simulation results were also analyzed for sensitivity to the coefficient restitution and it was observed that the contact parameter had a limited effect on the simulation results. It was also shown that as the number of layers increases, the amount of broken material is reduced in hard copper ore and the amount of broken material is increased in soft limestone.

Barrios et al. (2020) simulated particle breakage with a replacement model implemented in the EDEM commercial software. The model predicted some parameters based on single particle breakage tests on iron ore pellets. The predictions of the model were compared with results of experiments and showed agreement both in terms of the breakage probability and the particle size distribution obtained as a result of compression and impact. Figure 9 shows the results of experiments and simulations for the collision of a 3-layer particle bed with an 88 mm steel ball having an impact energy of 10 J. Figure 9 illustrates the generation of new particles as well as the expulsion of particles. The color in the images of simulations indicates the velocity at which the particles are thrown. The times are displayed in terms of t = 0, corresponding to the moment of contact of the dropped ball with the top of the particle bed.

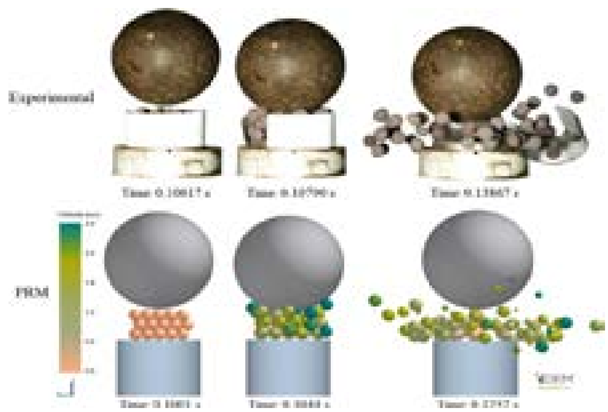

Figure 9. Comparison of snapshots of the experiment and DEM simulation of unconfined particle bed impact tests using Impact load cell (Barrios et al., 2020)

Jiménez-Herrera et al. (2018) compared three different particle breakage models in the commercial DEM simulators for modeling the particle bed. It was found that BPM is ideal because it can describe the force-deformation profile and the interaction with the ball dropped on the bed particles, but it cannot represent the material breakage distribution. FBM and PRM, on the other hand, could not adequately describe the force-deformation profile. The model patterns of BPM, PRM and FBM before, during and after single particle breakage simulation were analyzed and the results of the particle breakage simulation are shown in Figure 10. At BPM, the progression of breakage is apparent, following the breaks of the bonds. In the FBM plot, both the first breakage of the main particle and the following breakage of the progeny particles are shown, revealing the irregular shape of the particles. Figure 10 also shows two different moments after a particle reached the critical load for breakage in the PRM plot, revealing the very strong overlap that should be allowed in this model instantly after breakage.

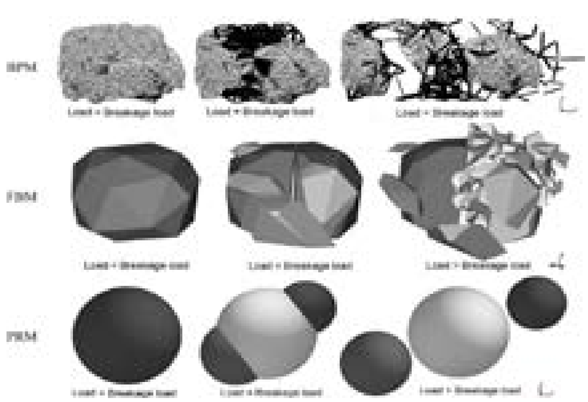

Figure 10. Schematic comparison of breakage models (Jiménez-Herrera et al., 2018)

There are some studies using PRM to model different comminution equipment such as HPGR (Barrios and Tavares, 2016; Rodriguez et al., 2021), cone crushers (Delaney et al., 2015), compression crushers (Cleary and Sinnott, 2015).

Cleary and Sinnott (2015) simulated the flow and breakage of material as it passes through the breakage chamber using the PRM breakage model of the DEM proposed by Cleary (2001a and 2001b) to study jaw, cone, gyratory, impact and double roll crushers. Using PFC3D software, energy, particle size, throughput and wear were estimated for the crushers. Figure 11 shows the breakage model of the collision of two high-velocity particles in 3D. The main particles in Figure 11a collide and distribute enough energy in the normal direction to break and then are replaced by the small particles in Figure 11b. These particles can then move independently (Figure 11c) and interact with other particles and the boundaries of the crusher, possibly breaking again if conditions are favorable.

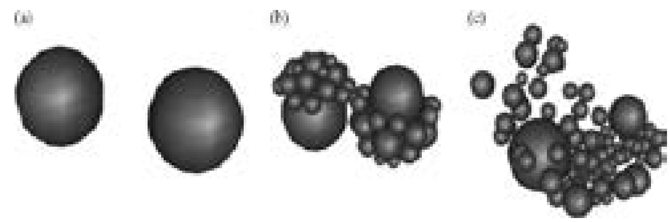

Figure 11. Schematic comparison of breakage models (Cleary and Sinnott, 2015)

Delaney et al. (2015) determined the flow and breakage process in an industrial cone crusher with the PRM breakage model using non-spherical particles. In the simulations, the model was developed using non-spherical particles known as superquadrics instead of spheres to improve reality, but this resulted in computational limitations. The study simulated the performance of the cone crusher and predicted particle size distribution, throughput, energy consumption and liner wear. The wear distribution is affected by the stress distribution and flow pattern and is different in the mantle and concave section. The wear is highest at the location of the plug point of the crusher.

3. Conclusion

This work is summarized the studies have been carried out to understand the performance of particle breakage methods in the modeling of comminution equipment. In addition, the relationship between particle and breakage energies and theory of applied forces are described in detail for three breakage models existing in commercial DEM simulators. Each model has advantages and disadvantages and a general summary is presented below.

In the Bonded Particle Model (BPM), non-uniform particle, called meta-particles, are formed by connecting each other at the contact points and the problem of mass conservation is eliminated by high packing density. This method, is based on particle flow dynamics. Realistic results are obtained in simulation processes, but it requires high computational effort and is not suitable for a large number of particles. However, it describes the force-deformation profile well.

Fast Breakage Model (FBM) is an instantaneous breakage model that uses Laguerre-Voronoi tessellation to breakage polyhedral particles. This model produces irregular shapes particles and provides mass conservation. It is not suitable for particle size distribution estimation and requires high computation effort. Unlike BPM, it does not describe the force-deformation profile well.

Simulating breakage with Particle Replacement Model (PRM) is based on the replacement of the parent particle by progeny particles when a load is applied. In order to obtain close to realistic results, the particles are placed on top of each other to ensure mass conservation while placing the particles that will result from the breakage processes in the main particle volume. However, this high overlap causes artificial repulsion forces. This is controlled by a damping approach. It also uses dummy particles to ensure mass conservation. It is suitable for simulations with a large number of particles. It does not describe the breakage probability and force-deformation profile well.

In summary, three methods provide mass conservation by applying different methods. PRM offers faster results than other methods when using a large number of particles. However, the model that best describes the force-deformation profile is BPM.

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