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Measuring Liquidity Risk in an Emerging Market: Liquidity Adjusted Value at Risk Approach for High Frequency Data

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ABSTRACT: The present paper introduces an enhanced liquidity adjusted intraday value at risk measure named the LIVaR applied to a sample of listed securities in an emerging market; namely the Tunis Stock Exchange (BVMT). Very specific econometric tools were used to perform models that suit the statistical properties of the data and to obtain a more realistic and efficient measure. This methodology was applied to intraday data. It was found that in the BVMT, the liquidity risk is very high. It represents about 25% of the total cost supported by a day trader for the most active stocks of the considered sample. It can also reach more than 40% for the less liquid ones. These results reveal how thin the Tunis stock market is.

Keywords: Liquidity; intraday value at risk; spread; ACD; Monte Carlo simulation.
JEL Classifications: C41; G17

1. Introduction

In the recent years and with reference to the successive market crises (especially that of the year 2008), the issue of considering liquidity as a risk factor has been frequently revisited. Thus, it has become crucial to study the impact of liquidity variations on the market movements. This leads to a growing literature interested mainly in either establishing liquidity adjusted pricing model (Pastor and Stambaugh (2003), Archarya and Pedersen (2005), Sadka (2006), Davivongs and Pavabutr (2012)) or incorporating liquidity risk in a well-known risk management tool namely; the value at risk (Bangia et al. (1999), Le Saout (2002), Al Janabi (2008), Aktas et al. (2012). Indeed, the latter is a risk indicator of the maximum loss of a financial asset or portfolio at a time horizon and a certain probability. Despite the critics addressed to it, the value at risk remains the approach currently used by regulators and market professionals given its ease of calculation and direct interpretation. In this respect, the value at risk framework has been continuously evolving. It began with a JP Morgan's (1996) Risk Metrics based on the normality assumption corresponding to the asset's returns. This restrictive assumption has been abandoned in favour of more sophisticated techniques such as the historical and Monte Carlo simulations having as an objective to state a more realistic framework and obtain a more efficient risk valuation.

In this field of research, not only mature markets were explored but also the emerging ones. Recently, Harvey (2012) advocated the idea that there existed more than one reason to urge the interest in emerging markets. He asserted that it is more advantageous to invest in the emerging markets as they lead to higher expected yields and according to Bekaert et al. (2007a) they offer higher opportunities for growth. Indeed they are more volatile and then riskier which is not only due to

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market environment but also to particular conditions such as political instability and companies governance problems. In this context, studies dealing with risk valuation are of great help especially for regulators since many markets need structural and regulatory changes in order to improve their trading environment (transparency, order rooting, and liquidity) and attract foreign investors given that in many emerging markets, stability depends on foreign order flow. Santoso et al. (2010), for instance, declared that from 60% to 70% of the invested funds in Indonesia stock market are from foreign sources. Thus in this case, foreign portfolio adjustment directly affects market movements. The risk related to international order flow migration makes market regulators more concerned about liquidity and risk valuation.

The present paper fits in this framework. We propose a liquidity adjusted value at risk measure performed in an intraday time horizon which requires Ultra high frequency data. The challenge is to take into account the intraday particularities of the trading process including liquidity, volatility and trading cost. It is organized as follows: Section 1presents a short literature review. Section 2 exposes the methodology. Results are exposed and discussed in section 3 and the last section concludes the paper.

2. Literature Review

In microstructure literature, stockholders face two types of risk: a market, and a liquidity risk. Both types feed uncertainty about the portfolio liquidation value. Bangia et al. (1999) divided the latter into two types: uncertainty about future returns (the market risk), and uncertainty about liquidity. The former is related to returns variation. The latter, in turn, is decomposed in two types: endogenous and exogenous liquidity. The first is commonly supported by all the traders interested in stocks. It reflects the market characteristics such as depth, and spread while the second is related to the position taken by the trader. When considering the two types of risk, Moulton (2003) asserted that the total supported risk depends on the adequacy between desired quantity and the trading price. Bangia et al. (1999), however, defined the trading risk as a result of circumstances which determines the part of each risk on the total uncertainty. As a matter of fact, in a period of a steady market, a trader aiming at buying a large quantity will face a higher level of endogenous liquidity risk, increased with higher market risk. Furthermore, in a high volatility period, large trades face high market risk that increases with a weak level of endogenous liquidity risk.

The most important question raised by several microstructure studies is how to estimate the two components of liquidity risk, namely; the endogenous and the exogenous ones. In this respect, Bangia et al. (1999) provided a pioneering work considered as the starting point of many researches. Each time they begin with an associated criticism. The often cited one is related to the exogenous liquidity risk measure. Indeed, these authors argued that relative spread represents the cost of an immediate transaction. However, when the order quantity exceeds the depth displayed by the best limits, the execution cost will exceed the half of the quoted spread. It includes both the exogenous and the endogenous liquidity risk. In this case, the latter is the difference between the price impact and half the spread. Accordingly, it can be assumed that both endogenous and exogenous liquidity risks are very much linked and the traded quantity determines the importance of each one. Nevertheless, Shamroukh (2000) suggested to take the spread’s volatility (exogenous liquidity risk) into account unless its effect is insignificant compared to that of price. The second critic addressed to Bangia et al. (1999) is relative to the empirical methodology. They based their exogenous liquidity risk measure on a historical simulation which is considered very simplistic since the value at risk corresponds to a percentile of the empirical distribution of the spread.

Similarly, the endogenous liquidity estimation was the subject of different studies as Jarrow and Subramanian (1997), Almgren and Chris (2001) and Bertsimas and Lo (1998). The first one calculated the optimum portfolio liquidation value during a fixed time horizon. It compared the cost of a block trade and progressive liquidation. Bertsimas et al (1998), on the other hand, offered the hypothesis of the fixed liquidation delay. They made dynamic trading strategies aiming at minimizing the execution cost during an exogenous time horizon. This strategy resulted in orders’ sequences drawn up in different quantities that are fixed according to the market conditions. For a risk-averse agent, it is optimal to split the desired volume into fractions and to trade one fraction per period. The authors modelled the price impact as a linear function of the traded volume. Unfortunately, these two studies did not take the market risk into consideration. Hisata and Yamai (2000) generalized the model of Almgren and Chriss (2001) by considering the impact cost minimization when trading quantities in relation to market liquidity. When using the value at risk approach, they considered a non-linear relation between the traded volume and the market impact variable. First of all, they divided the market impacts into permanent and temporary. The latter refers to the total traded volume. Secondly, they formulated an optimum execution strategy which consists in minimizing the liquidation cost. Thirdly, they calculated the VaR (value at risk) adjusted to liquidity. This study pioneered in two major points: it considered the objective of minimizing the trading cost, and implemented a non-linear relation between volume and market impact. However, its major deficiency lies in that it assumed a simple valuation of the impact cost.

In another respect, Longstaff (2001) defined liquidity as the quantity that could be traded during a period of time. He offered a model of partial equilibrium which consists in choosing a strategy with restricted management's margins. He concluded that an agent must trade as often as possible to reduce his/her trading cost. Pereira and Zhang (2007) went further and suggested that it is more advantageous to trade different quantities during different periods. Thus, allowing the traded volume to be exogenous. They claimed that the ex-ante liquidity premium which required holding an illiquid stock reflects the ex-post price that will be paid to compensate the loss of its sale. Progressive liquidation was also considered by Haberle and Persson (2000) who proposed a model without a price impact since they asserted that traders exchange quantities lower or equal to the market depth. In this case only exogenous liquidity risk must be measured.

According to Le Saout (2001), periods of extreme variations of both returns and liquidity are rather frequent. He pointed out that large spreads are not observable during the periods of extreme price variations. Therefore, VaR adjusted to liquidity as calculated by Bangia et al. (1999) overestimates the risk. On the other hand, they used the relative spread to measure liquidity even though it only represents the tightness of the market and not the price impact. Le Saout (2001) also proposed to use value weighted spread to take into account the market resiliency. He gave the example of the stock Saint Gobin quoted on the Paris stock exchange to show that for a quantity of 5000, the endogenous liquidity components represent 17.13 % of the total risk.

In the same vein, this paper is about measuring liquidity risk in an emerging market: the Tunis Stock exchange (BVMT). The focus on the latter is attributed to the little microstructure literature that deals with it despite the growing foreign interest in it. In fact, foreign ownership was up to 20% at the end of December 2012. The market capitalization was about 10.5M$ and the daily traded volume reached 4.1m$. Its attractiveness is explained by its transparency and the modern automated trading system inspired from Euronext named UTP (Universal Trading Platform). Between 1996 and 2008, Tunis Stock Exchange changed the trading platform twice. This was due to the authority's pursued objective to offer better trading environment with an acceptable transparency degree and attracting foreign investors especially when extending the trading session duration from one hour and a half to four hours for continuously traded stocks.

In this paper we propose an intraday value at risk measure adjusted to liquidity. The objective is to investigate whether liquidity risk is particularly high in BVMT. The Ultra high frequency data availability allows us to make this possible in an intraday horizon from a reconstituted limit order book. Indeed, this research is conducted in order to validate the following hypotheses:

H1: In BVMT, a day trader faces a very high liquidity risk;

H2: liquidity risk is higher for less liquid stocks.

3. Data and Methodology

The Tunis Stock Exchange is an emerging market in the MENA region. It attracted little attention especially from the microstructure field. The present research fits in this framework as it seems to be the first to focus on the intraday risk valuation. We suggest a very precise methodology that could be used even in the context of mature markets. We tried to take into account every liquidity dimension corresponding not only to trading cost but also to the time opportunity cost.

When focusing on liquidity, the choice of the sample is determinant. That is why we applied a parsimonious procedure to the 14 stocks quoted within the continuous session all along the year 2006. In order to choose the most adequate sample: We classified them according to the daily number of trades, daily traded volume, quotation frequency, time duration between successive trades and daily price variation. The first (second) half the total number of stocks represents the most (less) liquid ones.

We retained the three most liquid stocks (STB, STPIL and SFBT) as well as the first three stocks among the least liquid ones (UIB, MG, and BIAT).

During the year 2006, for continuous sessions, trading begins at 10:00 am until 11:30. During this time interval, we reconstituted the order book for the six stocks of the sample.

Moreover, we have chosen the year 2006 because it represents a favourable context to study liquidity and risk determinants. Market professionals confirmed that it is characterized by a great enthusiasm even "euphoria" as the market index namely TUNINDEX increased by almost the half. Market capitalization has increased by 43% to reach 5.491 million dinars (3.6 M$).

3.1. Econometric Models

The literature review exposed above was based on three among the four liquidity dimensions, namely; tightness, resiliency and depth, when incorporating liquidity cost in the risk measuring. According to Kyle (1985), tightness also named market width refers to the spread which defines the cost of an immediate transaction. Resiliency represents the speed at which the price will reach a new equilibrium after a trade occurrence. Depth, on the other hand, corresponds to the volume that could be traded at the current price. In the same respect, Harris (1990) introduced another dimension that is immediacy i.e. the time dimension of the trading process, which refers to the possibility of trading a desired quantity very quickly and at an acceptable cost. Unfortunately, this dimension was not explored by the risk valuation framework. Therefore, we aim at filling this gap when proposing an empirical model that will make it possible to take into account the fourth liquidity dimensions that could be interpreted as the time opportunity cost. Biais et al (1995) argued that a large group of traders stay outside the market waiting for an acceptable opportunity to intervene and satisfy their needs for liquidity. Time between successive trades informs us about traders' interest in trading. It also expresses the time opportunity cost since large durations imply a loss for the traders who have been waiting for long time to have the possibility to find convenient counterparts. However, in microstructure literature, durations have been considered as an indicator of the trading intensity (Bauwens and Giot (2002)). Very recently, Rouetbi and Mamoghli (2013) used durations between limit order submissions as an indicator of liquidity renewal frequency. The time duration between successive trades has not been approached in the risk valuation literature despite its importance for order driven markets except by Dione et al (2006) who pioneered in this context but they omitted the liquidity adjustment of the proposed IVaR. We have been inspired by this study which we propose to extend. In this very context, the present work proposes trade durations as a risk amplifying factor introduced in returns' volatility equation. This specification may make the analysis more complete and thus reliable.

Moreover, when measuring liquidity risk we suggest an empirical model based on a well-known one in risk management that is value at risk. By definition, it expresses the maximum loss supported by a trader when liquidating his/her position within a fixed time interval. The employed methodology was inspired by such studies as Bangia et al. (1999), Le Saout (2001) who applied a value at risk approach using an aggregated intraday data. Dionne et al. (2006), however, carried out the first research that proposed an intraday measure of the value at risk using high frequency data: the IVaR (intraday vale at risk). As a matter of fact, the current work combines the empirical contribution of the three previously cited studies to validate the H1 hypothesis above. Moreover, we propose to test whether liquidity risk is higher for less liquid stocks i.e. H2 hypothesis. In this context, we suggest the use of two subsamples: more and less liquid stocks. Accordingly and in order to validate the value at risk measure, we have to perform a back testing. In this vein, we propose to divide the sample period into two: the first six months' data will be dedicated to the model's estimation and forecasting. The second half of the period will be devoted to back testing and validating the value at risk approach.

The challenge, here, is to model market and liquidity risks when taking into account each of the related dimensions of the trading process. To achieve this goal, a set of specific econometric tools is necessary. Time dimension requires the use of a duration model i.e. ACD (Autoregressive Conditional Duration). Price variations (returns) are modelled by an Ultra high frequency GARCH i.e. UHF-GARCH. The quantity dimension, however, needs to be carefully approached. The traded volume is related to the impact of the trading cost. Therefore, it is naturally linked to the endogenous cost of liquidity. We fixed the traded quantity at 100 stocks. We suppose that traders desiring to trade large quantities would use several transitions at minimum 100 stocks per trade. Thus returns' variation will naturally take into account the market price impact and depth.

To fulfil such a purpose the following steps are necessarily followed:

Firstly, we propose to model the time durations between MID prices variations. In this respect, we will use the Bauwens et al. (2004) procedure to choose the ACD model that will efficiently fit the econometric properties of the data. This will be achieved during the estimation period (the first six months of the year 2006).

Secondly, the conditional duration calculated will be introduced as an explanatory variable into the returns' volatility equation allotting to the latter an EGARCH model as in Dionne et al. (2006). We will use the in-sample estimation result to predict the value at risk calculations. The back testing period will be used to validate the model.

Thirdly, we will introduce the exogenous liquidity risk by using the empirical density of the spread. In this respect, we will extend the procedure suggested by Bangia et al. (1999).

3.2. Estimates of the duration model

Table 1displays the characteristics of the durations between MID prices variations caused by a minimum traded number of stocks fixed at 100. This choice will enable us to easily calculate the trading and the price impact cost since such quantity is achievable at the best bid or ask price. In the same way, agents wishing to exchange large quantities are supposed to intervene through several orders carrying at most 100 stocks each. Thus, our intraday VaR will account for the strategy of order split aiming at limiting the undergone trading cost. This interpretation supports that of Hisata and Yamai (2000), Almgren and Chriss (2001) proposals. Moreover, it appears to be in perfect harmony with the temporal horizon considered (intraday).

Table 1. Descriptive statistics of the trade durations:

This table reports descriptive statistics associated with the time duration between MID price variations for the most liquid stocks of the BVMT: STB, SFBT, STPIL and less liquid ones UIB, MG, BIAT. Sample period: January to June 2006. Durationsare measured in seconds.

		STB	STPIL	SFBT	UIB	MG	BIAT
Observations' number		6244	4050	2295	1498	1237	991
Observations' number by 30 minutes interval	1st half-an-hour	36%	27%	31%	30%	27%	24%
	2nd half-an-hour	35%	29%	34%	33%	34%	32%
	3rd half-an-hour	29%	43%	34%	37%	39%	45%
Raw durations	Mean	116	161	198	233	225	265
	Sigma*	209.93	281.76	278.64	363.22	348.89	401.02
	Id**	1.81	1.75	1.41	1.56	1.55	1.51
	min	1	1	1	1	1	1
	max	25	44	62	58	67	69
Adjusted durations	Mean	1	1	1	1	1	1
	Sigma*	1.65	1.59	1.26	1.33	1.33	1.32
	Id**	1.59	1.57	1.25	1.33	1.33	131
	min	0.006	0.005	0.004	0.003	0.003	0.003
	max	28.75	22.73	14.61	12.75	12.83	13.09

*sigma is the standard deviation; **the dispersion indice: standard deviation/mean.

After adjusting raw durations for intraday seasonality, we carried out the estimated model of GGLACD1 (1.1): logarithmic autoregressive conditional duration model which allots to the standardized duration a Gamma generalized law. Indeed, Bauwens et al. (2004) and Bauwens and Giot (2002) highlighted the flexibility of this type of models and its superiority compared to the simple autoregressive conditional duration model (ACD) when studying events related to the trading activity. This model expresses the conditional expected duration φ_i as a function of the lagged duration $x_i$on logarithm:

$$x_i = e^{\varphi_i}\varepsilon_i \qquad (1)$$

$$\varphi_i = \omega + \propto \ln(x_i) + \beta(\varphi_{i-1}) \qquad (2)$$

x_i: the intraday duration at the time i;
φ_i: the conditional expected duration;
ε_i: the random variable called standardized duration.

Table 2 indicates that the durations are over-dispersed; the dispersion index I_d is higher than the unit. The exponential law cannot then describe the behaviour of the standardized durations. In the same way, it is clear that model GGLACD1 (1.1) could fit the data. Indeed, the stability condition is checked (α+β<1) and the autocorrelations of the residuals are not significant for all the stocks of the sample. The Si is lower than the unit.

Moreover, the three parameters of the model are positive and significant, which confirms the activity clustering over time. Periods of short (long) durations are followed by periods of short (long) durations. With reference to the test goodness of fit we notice that specification GGLACD1 (1.1) is valid for all the stocks of the sample. Thus we can use this model to reproduce the statistical behaviour of the time durations.

Table 2. Estimation results of the model GGLACD1(1,1):
This table gives the estimation results of GGLACD1 (1.1) model : $x_i = e^{\varphi_i}\varepsilon_i$, $\varphi_i = \omega + \propto \varepsilon_{i-1} + \beta\ \varphi_{i-1}$ where x_i dessaisonnalizedduration observed at the time i. φ_i denotes the conditional expected duration. ε_i denotes standard duration.

		STB	STPIL	SFBT	UIB	MG	BIAT
Coefficients	ω P-value	0.171 (0.021)	0.228 (0.032)	0.092 (0.044)	0.192 (0.029)	0.173 (0.041)	0.139 (0.036)
	α P-value	0.242 (0.022)	0.301 (0.023)	0.252 (0.046)	0.272 (0.025)	0.257 (0.035)	0.216 (0.028)
	β P-value	0.701 (0.044)	0.640 (0.054)	0.721 (0.133)	0.621 (0.064)	0.712 (0.077)	0.582 (0.110)
The GG Parameters	γ P-value	0.140 (0.049)	0.115 (0.055)	0.426 (0.071)	0.366 (0.062)	0.281 (0.064)	0.389 (0.085)
	ν P-value	2.567 (0.018)	3.111 (0.029)	2.786 (0.001)	2.985 (0.001)	5.123 (0.002)	2.724 (0.001)
Residual	Mean	0.981	0.991	0.990	0.964	0.956	0.975
	sigma	1.656	1.768	1.345	1.408	1.484	1.406
	I_d	1.689	1.867	1.358	1.460	1.553	1.442
	min	0.002	0.001	0.002	0.003	0.002	0.003
	max	24.126	30.280	15.175	12.656	19.364	16.454
χ2(19)*	Static P-value	27.431 (0.095)	39.583 (0.004)	37.189 (0.008)	56.310 (0.000)	52.653 (0.000)	66.834 (0.000)
S_i **	S_1	0.980	0.005	0.860	0.031	0.608	0.395
	S_5	0.766	0.111	0.543	0.153	0.472	0.717
	S_{10}	0.625	0.556	0.517	0.623	0.351	0.962
	S_{50}	0.841	0.747	0.643	0.722	0.769	0.647

Note: The dispersion index I_d (standard deviation/mean) of residuals; I_d ≈N (0,1) test statistic for overdispersion of residuals under exponential null hypothesis; *Results of the goodness of fit test; **Sᵢi={1,5,10,50} the Ljung box ratio of the residual corresponding to the quintile 95% of χ2(i).

3.3. Modelling Returns

Returns relative to the MID price variations are modelled by using a GARCH model adapted to the ultra-high frequency data. The conditional return variance is defined by the following equation:

$$h_i = x_i^{\gamma}\sigma_i^2 \tag{3}$$

h_i is the conditional variance;
x_i is the duration between successive MID price variations caused by trades with 100 stocks minimum volume; σ_i^2 is the return variance per unit of time. γ is a coefficient to be estimated. According to Dionne et al (2006), it corresponds to the impact of durations on the conditional volatility.
The conditional variance of the durations per unit of time is modelled as follows: Average equation: ARMA (1.1):

$r_i = c + dr_{i-1} + fe_{i-1}$ (4)

Conditional variance: EGARCH (1.1):

$$\log(\sigma_i^2) = \omega + a\left\{\frac{e_{i-1}}{\sqrt{h_{i-1}}} - E\left(\frac{e_{i-1}}{\sqrt{h_{i-1}}}\right)\right\} + b\frac{e_{i-1}}{\sqrt{h_{i-1}}} + \beta\log(\sigma_{i-1}^2) \quad (5)$$

Where $e_i = z_i\sqrt{h_i}$, and $z_i = \{z_i, i \in \mathbb{Z}\}$ is a white noise z_i are i.i.d with a zero mean and a unit variance. For γ = 0 we will find the equation of the UHF-GARCH introduced by Engle (2000). The estimation results of the model are presented in table III below.

Table 3 indicates that the EGARCH (1.1) model is valid for all the stocks of the sample. Coefficients of the conditional variance of the returns (β) are lower than the unit.
The parameter a is positive and statistically significant for all stocks of the sample: a shock on returns amplifies the stock's volatility. However the impact of these shocks is asymmetric because the sign of the parameter B is not identical for all securities of the sample.

It is positive for STB, MG and BIAT: negative shocks cause more volatility than positive ones. The opposite is true for STPIL, UIB and SFBT for which parameter B is significantly negative.

Table 3. Estimation results of the EGARCH(1,1) for the returns:

This table gives the estimation results of EGARCH (1,1). Mean equation: $r_i = c + dr_{i-1} + e_i + fe_{i-1}$ where $r_i$the returns at time i and $e_i$the corresponding residual. The conditional variance equation: $\log(\sigma_i^2) = \omega + a\left\{\frac{e_{i-1}}{\sqrt{h_{i-1}}} - E\left(\frac{e_{i-1}}{\sqrt{h_{i-1}}}\right)\right\} + b\frac{e_{i-1}}{\sqrt{h_{i-1}}} + \beta\log(\sigma_{i-1}^2)$. The volatility is modeled by the product of the conditional variance and the time duration between MID price variation$h_i = x_i^\gamma\sigma_i^2$.

	Mean equation					
	STPIL	**STB**	**SFBT**	**UIB**	**MG**	**BIAT**
C P-value	**0.0016** (0.042)	**0.016** (0.000)	**0.011** (0.000)	**0.017** (0.001)	-	-
d P-value	**0.598** (0.000)	**0.349** (0.000)	**0.412** (0.000)	**-0.878** (0.000)	**0.477** (0.000)	**-0.135** (0.000)
f P-value	**-0.720** (0.000)	**-0.565** (0.000)	**-0.604** (0.000)	**0.788** (0.000)	**-0.695** (0.000)	**-0.214** (0.000)
	Volatility equation					
ω P-value	**-2.633** (0.000)	**-1.501** (0.000)	**-2.618** (0.000)	**-1.302** (0.000)	**-1.899** (0.000)	**-0.658** (0.000)
a P-value	**0.580** (0.000)	**0.376** (0.000)	**0.426** (0.000)	**0.421** (0.000)	**1.125** (0.000)	**0.184** (0.000)
B P-value	**-0.059** (0.000)	**0.023** (0.000)	**-0.124** (0.000)	**-0.161** (0.000)	**0.438** (0.000)	**0.057** (0.000)
β P-value	**0.356** (0.000)	**0.550** (0.000)	**0.422** (0.000)	**0.668** (0.000)	**0.163** (0.000)	**0.795** (0.000)
γ P-value	**0.056** (0.000)	**0.039** (0.000)	**0.050** (0.000)	**0.028** (0.000)	**0.047** (0.000)	**0.017** (0.000)
P-value of Q(20)	**0,304**	**0,426**	**0,239**	**0,380**	**0,560**	**0,496**
P-value of Q^2(20)	**0,486**	**0,540**	**0,593**	**0,871**	**0,758**	**0,648**
Wald test	**5488,63**	**72,720**	**943,193**	**453,621**	**97,543**	**743,52**

The coefficient γ is positive and statistically significant for all the stocks of the sample. An increase in the duration induces an increase in the stock volatility. This result confirms the asymmetric informational theory predictions especially those of Admati and Pfleiderer (1988) who stated that deceleration of the trading activity signals the presence of information which results in higher volatility. It also confirms the findings of those of Rouetbi and Mamoghli (2013) who asserted that MID price volatility enhances market activity i.e. liquidity supply and demand.

3.4. The Monte Carlo simulation

As it is mentioned in the first section of present paper, the total risk supported is decomposed into market and liquidity risks. Empirically speaking, the first could be modelled by the UHF-GARCH displayed above. According to Bangia et al. (1999), liquidity risk is divided into endogenous and exogenous risk. The first is related to the trade size. It is the impact cost of the transaction which is an increasing function in the volume traded. Kumar (2003) previously indicated that it can be controlled by reducing the traded quantity. The exogenous liquidity risk, on the other hand, reflects the characteristics of the market such as the spread. It is commonly supported by all the traders. To estimate both market and endogenous liquidity risks, we followed the methodology of Dionne et al. (2006) which consists in generating returns series by simulation based on the four preceding equations.

Exogenous liquidity risk is measured by the relative spread. Then, the Kornish-Fisher approximation is necessary to calculate the corresponding risk component. We justify the choice of this variable by the fact that we fixed at a minimum of 100 stocks the volume per trade which is easily achievable at the best limits. Hence, the use of weighted spreads or other price impact measures becomes needless.

Time durations between MID prices variations will enable us to take into account the market risk and at the same time the endogenous liquidity. The calculation of return during these durations naturally incorporates the impact cost since it is caused by trading a minimum quantity (100). From equation (2) we simulate the durations during the forecasting period. EGARCH model specified above will be used to generate the returns during these durations.

The obtained returns are irregularly spaced in time. However, Giot and Grammig (2002) insisted that the VaR estimation requires regular data. Thus, we must regroup them into fixed time intervals to make the estimation possible.

The Monte Carlo simulation is made as follows:

- Firstly, we draw random numbers from Gamma generalized law that will enable us to build the scenarios for the durations from equation (2);
- Secondly, we draw Gaussian random numbers[2] which will serve to generate the series of returns and volatility using the EGARCH equations (4) and (5).Then we can generate different scenarios for future durations, returns and volatility. These outputs are irregularly spaced in time.
- Thirdly, we regroup forecasted variables into fixed time intervals;
- Finally, the preceding steps will be repeated until obtaining the desired number of observations.

3.5. IVaR calculation and backtesting

The IVaR corresponds to a percentile of the simulated returns associated with a selected confidence interval.

With a confidence level (1-α), the IVAR is defined as:

$$\mathrm{P}(\mathrm{y_k} < -IVAR(\alpha)/\Omega_\mathrm{k}) = \alpha \qquad (6)$$

y_k/k={1,.....N} where N is the realized length of the process.

Where y_k is the sum by interval of the tick by tick returns generated by simulation:

$$y_k = \sum_{i=1}^{\tau(k)-1} r_i \qquad (7)$$

$\tau(k)$ is the number of observations per time interval T:

$$\sum_{i=1}^{\tau-1} x_i \leq T \qquad (8)$$

Ω_k refers to the information set available for durations and returns.

The durations generated by simulation are not expressed in seconds since the time of the day effect is removed. They are gathered by interval so that their sum does not exceed T which is fixed at 30 for all the stocks of the sample.

For STPIL, we generated 2020 observations gathered in 228 intervals from which we determined the percentile of the simulated returns corresponding to the selected confidence level.

The IVaR series are then confronted with the returns carried out during the testing period. Graph 1 displays the realized returns during the back testing period and the forecasted IVAR (99.5%) for the stock STPIL during the 228 time intervals.

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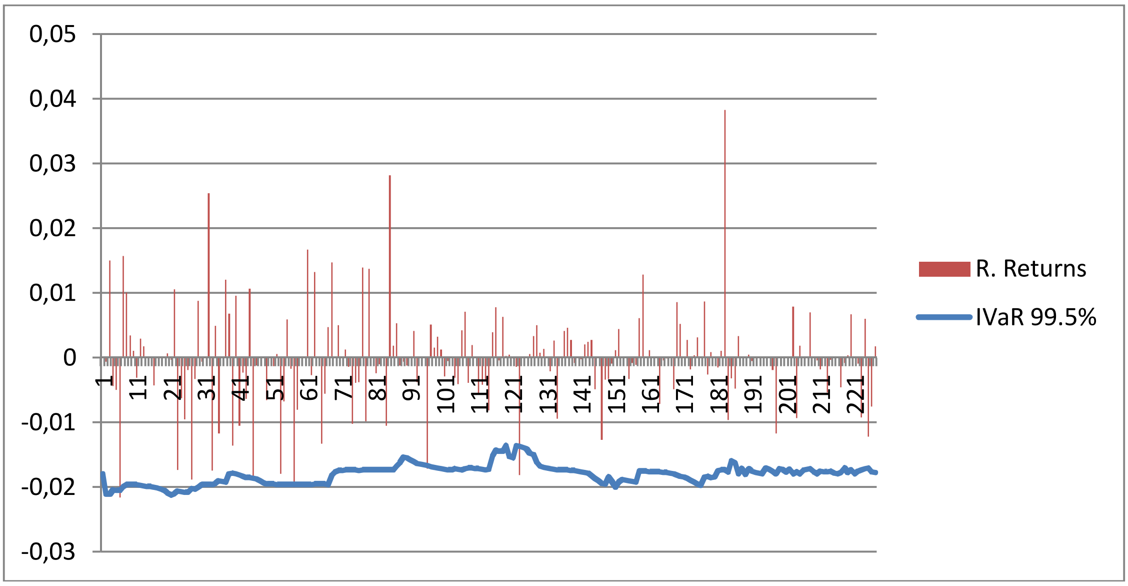
[2] Returns are not normal but it is the case for the variable z_i.

This graph shows that the realized loss is often lower than that calculated by the IVaR model except for two observations. However, the graphic analysis is not enough to validate the model. Indeed, statistical tests should be made to empirically validate the value at risk measure. It is a question of checking whether the expected loss is lower than that realized for (1-α) % of cases during the backtesting period.

Graph 1. Intraday returns and the IVaR: the case of STPIL
This graph exhibits the returns (R. Returns) recorded at the back testing period and the maximum loss calculated at 99.5% confidence level.



Thus, α represents the maximum level of tolerance. If the real proportion of going beyond is higher than α, the model must be rejected. The first test used is the test of unconditional cover also known as the test of Kupiec. It defines a dichotomic variable which is equal to the unit when the realized loss is lower or equal to IVaR, and to zero otherwise.

$I_t \approx Bernouilli(\propto)$ where α is the theoretical proportion. $\hat{\propto}$ refers to the real proportion of going beyond the realized loss; the test of Kupiec can then be formulated as follows:

H_0 : $\hat{\propto} = \propto$;

H_1 : $\hat{\propto} \neq \propto$

The likelihood ratio could be written as follows:

$$LR = 2[\ln(\hat{\propto}^m\ (1-\hat{\propto})^{n-m}) - \ln(\propto^m\ (1-\propto)^{n-m})] \qquad (9)$$

Where n is the number of observations and m the number of exceptions (the number of times where IVaR is lower than the realized loss).

The failure rate of IVaR measure is then definite by $\hat{\propto} = \frac{m}{n}$.

The ratio LR is distributed as a $\chi^2(1)$.

The results of the test are displayed in table 4below:

The independence test confirms the validity of IVaR measure for our sample. It acts well for all the stocks of the sample for a confidence level of 99.5%. We notice a failure of the model for the 99% level. However, the Kupiec test is not enough to validate or not the IVaR estimation. Accordingly, we suggest a complementary test namely; the exceptions independence test:

H_0: the exceptions are independent.

H_1: the exceptions are not independent.

The ratio of probability of the test is written as follows:

$$LR_{ind} = 2(\ln[l(I, \hat{z}_{01}, \hat{z}_{11})] - \ln[l(I, \hat{\propto}_1)] \approx \chi^2(1) \qquad (10)$$

Where:

$$\hat{z}_{01} = \frac{Number of interval\ without\ overtaking\ followed\ by\ an\ overtaking}{Nomber of successive\ intervals\ without\ overtaking} \qquad (11)$$

And

$$\hat{z}_{11} = \frac{Nomber\ of successive\ intervals\ withan\ overtaking}{n} \quad (12)$$

With reference to table (5) for 99.5% confidence level, the estimated VaR is valid for all the stocks of the sample. These results confirm those of the preceding test. It also confirms the failure of the IVAR measure at a 99% confidence level.

Table 4. Back testing results

This table displays the tests' results used in order to confirm the validity of the IVAR measure.

The likelihood ratio of the Kupiec test: $LR = 2[\ln(\hat{\alpha}^m\ (1-\hat{\alpha})^{n-m}) - \ln(\alpha^m\ (1-\alpha)^{n-m})]$. Where n is the number of observations, and m the number of exceptions (the number of times where IVAR is lower than the realized loss). The ratio LR is distributed as a $\chi^2(1)$.

The likelihood ratio of the second test named independence test:

$\mathrm{LR_{ind}} = 2(\ln[l(I, \hat{z}_{01}, \hat{z}_{11})] - \ln[l(I, \hat{\alpha}_1)] \approx \chi^2(1)$.Where:

$$\hat{z}_{01} = \frac{\text{Number of interval without exceptions followed by an exception}}{\text{Nomber of successive intervals without exceptions}}$$

and $\hat{z}_{11} = \frac{\text{Nomber of successive intervals withan overtaking}}{n}$

	STPIL	STB	SFBT	UIB	MG	BIAT
Kupiectest						
99%	0.036	0.636	4.46	0.014	7.87	0.392
P-value	0.000	0.005	0.34	0.174	0.212	0.343
99,5%	0.531	2.220	0.17	0.732	0.384	0.015
P-value	0.000	0.000	0.013	0.032	0.019	0.063
Independence test						
99%	0,329	24.15	-	0.364	-	8.24
P-value	0.000	0.290		0.390		0.000
99,5%	0,393	0.288	1.62	0.270	1.79	2.778
P-value	0.000	0.000	0.000	0.000	0.000	0.000

After calculating the intraday VaR, measuring the price risk and the endogenous liquidity risk, we propose to include the exogenous liquidity risk component.

3.6. The IVaR adjustment to the exogenous liquidity risk:

Bangia et al. (1999) were the first to introduce the exogenous liquidity risk. They included a component related to the spread to the market risk premium. This component is expressed as follows:

$$ELC = \frac{1}{2}[p_t * (\bar{s} + \hat{z}_{\alpha}\tilde{\sigma})] \quad (13)$$

ELC: The exogenous liquidity cost;

p_t The MID price recorded at the time t. It is the average of best bid and ask prices;

$\bar{s}$The average relative spread (spread/MID);

$\tilde{\sigma}$ The spread's volatility;

$\hat{z}_{\alpha}$The percentile from the spread's empirical distribution corresponding to the (1-α) confidence level.

To obtain as much a realistic measure as possible, it is necessary to take into account the empirical properties of the series. In this vein, Bangia et al. (1999) explained that a thick tail of the spread's distribution reflects a lack of liquidity. The advantage of their method is that it allows the spread to be leptokurtic. However, several authors, such as Ernst, Stange and Kaserer (2008) and Herlemont (2009) among others, suggested applying the approximation of Cornish Fisher instead of using the empirical distribution of the spread. From a quintile of the Gaussian distribution, it is possible to calculate z_{kf}, the adjusted one, by introducing the corresponding coefficient of asymmetry as well as the excess Kurtosis.

$$z_{kf} = z_{\alpha} + \frac{1}{6}(z_{\alpha}^2 - 1)\gamma + \frac{1}{24}(z_{\alpha}^3 - 3z_{\alpha})\kappa - \frac{1}{36}(2z_{\alpha}^3 - 5z_{\alpha})\gamma^2 \quad (14)$$

Where z_{α}indicates the α% percentile of the standard normal distribution;

γ is the coefficient of asymmetry (skewness);

κ is the excess Kurtosis which is equal to 3 in a Gaussian distribution.

With reference to equation (14), the exogenous liquidity cost (ELC) could be expressed as follows:

$$ELC = \frac{1}{2}\left[P_t * (\bar{s} + z_{kf}\tilde{\sigma}_s)\right] \quad (15)$$

Table 5 indicates that the spread's distributions are leptokurtic for all the stocks of the sample. The kurtosis coefficients largely exceed 3. The distributions present thick tails which reflects the lack of liquidity. Similarly, all the coefficients of asymmetry are different from 0. They are rather positive. The empirical distributions are all spread out on the right (the skewness coefficients are all >0). Thus spread's series are far from normal.

Table 5. Spread's descriptive statistics

This table presents the descriptive statistics of the average relative spread (bid ask spread/MID price).

	STPIL	STB	SFBT	UIB	MG	BIAT
Mean	0.53%	2.89%	0. 77%	8.60%	14.78%	4.26%
Standard deviation	0.0061	0.085	0.0145	0.172	0.3959	0.0385
Skewness	3.414	2.375	9.020	1.956	3.215	1.479
Kurtosis	23.389	5.407	51.123	8.870	10.406	11.729
Jarque-Bera	440928.7	231775.4	963761.7	37847.7	14666.3	124036.8

Using the formula of Cornish-Fisher, we calculated the cost of the exogenous liquidity and adding it to the risk calculated by the Monte Carlo simulation; we obtained the total cost of intraday liquidation.

The IVaR adjusted to liquidity named LIVaR could be expressed as follows:

$$LIVaR = P_t(1 - \exp(\tilde{\mu}_r + IVaR(r)\tilde{\sigma}_r)) + \frac{1}{2}P_t(\bar{s} + z_{kf} * \tilde{\sigma_s}) \quad (16)$$

Wherep_t is the average MID price recorded at the beginning of the estimation period. $\tilde{\mu}_r$ and $\tilde{\sigma}_r$are, respectively, the average and the standard deviation of the estimated returns obtained by Monte Carlo simulation.

The estimation results of market and liquidity risks are illustrated in table (6) below:

Table 6. LIVaR decomposition

This table shows the estimation results of the liquidity adjusted value at risk measures expressed by the equation: $\mathrm{LIVAR} = \mathrm{P_t}(1 - \exp(\tilde{\mu}_r + \mathrm{IVAR}(r)\tilde{\sigma}_r)) + \frac{1}{2}\mathrm{P_t}(\bar{s} + z_{kf} * \tilde{\sigma_s})$. Where P_t is the average MID price recorded at the beginning of the estimating period. $\tilde{\mu}_r$ and $\tilde{\sigma}_r$ are, respectively, the average and the standard deviation of the estimated returns obtained by Monte Carlo simulation.$\mathrm{ELC} = \frac{1}{2}[\mathrm{P_t} * (\bar{s} + z_{kf}\tilde{\sigma}_s)]$ represents the exogenous liquidity cost. Where z_{kf} is the α% quintile from normal distribution adjusted to skewness and kurtosis of the empirical spread series.

	STPIL	STB	SFBT	UIB	MG	BIAT
Initial Price (MID in dinar)	37.900	6.575	44.480	15.740	24.730	42.110
Market component* (in dinar) $\mathrm{P_t}(1 - \exp(\tilde{\mu}_r + \mathrm{IVAR}(r)\tilde{\sigma}_r))$	0.450	0.477	0.550	0.538	0.472	0.373
Liquidity component (in dinar) $\frac{1}{2}\mathrm{P_t}(\bar{s} + z_{kf} * \tilde{\sigma_s})$	0.146	0.219	0.309	0.356	0.315	0.288
VaR adjusted to liquidity (in dinar)	0.596	0.696	0.859	0.894	0.787	0.661
ELC percentage	24.50%	31.47%	35.97%	39.42%	40%	43.57%

*We have calculated the IVAR at 99.5% confidence level.

**The dinar is the Tunisian national currency

Ernst et al. (2008) attested the superiority of the Kornish-Fisher approximation compared to the historical simulation used by Bangia et al. (1999) in measuring the exogenous liquidity risk. For this reason we adopted this procedure. As a matter of fact, we preferred it to that adopted by Lei and Lai (2007) which consists in a moving average of the values recorded five days before during the same time interval. The authors were inspired by Heuda and Wynendaele (2001) who introduced the spread average to the market component of the VaR. This formulation seems to be very simplistic since it does not account for the spread's variation in time.

The results of estimation displayed in table VI clearly show that the liquidity cost constitutes more than the quarter of the total cost supported by a day trader. It is to be mentioned that for the stock STPIL, the cost of the exogenous liquidity represents 24.5% of the total liquidation cost. For the least liquid stocks, this proportion is around 40%. For BIAT, its share reaches 43.57%. Although the quantity per transaction is rather weak, the liquidation cost in a day time horizon is relatively high especially for less liquid stocks.

These findings indicate that liquidity risk is found to be very high at the intraday level. It is, also, relatively higher for less liquid stocks. Both hypotheses (H1 and H2) are then confirmed. Within a value at risk framework, we can say that omitting liquidity risk will lead to underestimate the embedded risk at about 25% for most liquid stocks and more than40% for less liquid ones. Liquidity is found to be a preponderant risk component at the intraday level. Day trading success which depends on a high volatility level and very liquid securities is not possible in the Tunis stock exchange setting. It is the liquidity cost that hampers the profitability of such trading style.

Further, this study results' are not surprising as they confirm those found in the emerging markets literature based on averaged intraday data or even monthly ones. Indeed, they seem to be in perfect harmony with those of Bangia et al. (1999) who asserted that ignoring liquidity risk in emerging markets leads to an underestimated risk measure at about 25%-30%. Moreover Al Janabi (2008) used daily data from emerging Gulf Cooperation Council (GCC) markets to corroborate the already cited results. Within a value at risk framework, he claimed that without incorporating liquidity, the risk estimation would fail to reflect the total embedded risk. Roy (2004), however, applied Bangia et al. (1999) hypothesis in the Indian bond market and showed that liquidity risk represents about 16% of the total daily supported risk. His findings, however, contrast with Bekaert et al. (2007) who used daily data from 18 emerging markets to shed light on liquidity- returns relation using a VAR specification (Vectorial autoregressive model). They came to the conclusion that liquidity foreshadows returns. But when including liquidity as a risk indicator, results become dependent on the country integration degree. In fact, for the fully integrated models, countries with better liquidity face a higher level of trading cost. They added that liquidity risk decreases when markets are liberalized. However, Angelidis and Benos (2006) collected data from Athens Stock Exchange and stated that for high capitalization stocks, correcting the VaR at liquidity is not necessary since liquidity risk represents only 3.4% of the total risk. But for low capitalization ones, this figure is about 11%. Hence, it should be integrated in the value at risk measure.

Finally, and in the light of this literature, our results do confirm the necessity of taking into account the liquidity risk in the risk valuation tools.

4. Conclusion

This paper contributes to the literature of risk valuation in emerging markets in many ways: Firstly, researches on liquidity have often focused on the mature markets. Emerging markets were paid less interest. Moreover, within this category, only the Asian market was always the subject of several studies. MENA region has attracted a noticeable lack of attention especially in the microstructure field. This situation urged the present study to be carried out to address liquidity risk estimation in this setting namely; The Tunis Stock Exchange (BVMT).

Secondly and empirically speaking, we proposed an intraday measure of liquidity risk resulting from a value at risk framework named LIVaR. This one seems to be the most complete in the microstructure literature since it considers every dimension of the liquidity process (tightness, resiliency, time, and depth). In addition, we suggested a rather complex modelling which takes into account empirical specificities of each type of data. In this respect, we used an autoregressive conditional duration model reflecting the dynamics of the durations. The latter is considered as an explanatory variable in the return's volatility equation as modelled by an EGARCH (1.1). The estimation of both specific models was done during the estimation period. The resulting parameters were used to calculate and forecast the maximum loss supported by a day trader named the IVaR. To perform this measure, the Monte Carlo simulation was used. Indeed, the back testing validated this measure for a confidence level of 99.5%. In fact, the Value at Risk obtained involves the market risk and the endogenous liquidity one since we considered the time duration between successive trades with a minimum of 100 stocks per trade. As an approximation of the time opportunity cost, duration was also integrated in the returns volatility equation. It was considered as an amplifying factor of the stock's volatility. The corresponding coefficients were positive for all stocks of the sample. The long waiting time outside the market caused the investors a loss which, in turn, resulted in increased returns' volatility that will automatically amplify the loss value and then the VaR level. The last step, in this respect, consisted in integrating the exogenous liquidity cost adjusted to the empirical characteristics of the spread. We used the approximation of Cornish Fisher which is preferable to the historical simulation used by Bangia et al. (1999). We chose the relative spread instead of the weighted one (Giot and Grammig (2002)), or of XML (Gomber et al. (2004)) to measure exogenous liquidity cost. This choice was attributed to the fixed minimum quantity per trade achieved by trades at the best quotes. It seems to be the case that the procedure used is the most complete since it inspired different studies. It can be used to measure liquidity risk in both emerging and mature markets.

Thirdly and at the theoretical level, we focused on the significant portion of liquidity risk within the total risk supported by an intraday liquidation. This study seems to pioneer in performing this task in emerging markets especially when considering the intraday horizon. In Tunis Stock Exchange, it represents almost the quarter and near the half of the total cost for the most and the least liquid stocks respectively. These findings reveal an interesting feature of the market: that is its thinness which results in important volatility and a lack of liquidity. Then liquidity is proved to be very precious in the BVMT.

In a word, better insights about the emerging markets will be obtained if further research is carried out. Indeed, a plan to extend the LIVaR application performing a comparative study between different emerging markets or between emerging and developed ones seems to be of great relevance.

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