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Long Memory Behavior in the Returns of Pakistan Stock Market: ARFIMA-FIGARCH Models[1]

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ABSTRACT: This study examines the weak-form market efficiency of Pakistan Stock Market namely Karachi Stock Exchange for the period 2010-2013. The efficiency of stock market has tested by using ARFIMA-FIGARCH models estimated under different distribution assumptions as Normal, Student-t, Skewed Student- t and GED distribution. According to findings of study, ARFIMA model do not support long memory behaviour for the stock market returns. However, FIGARCH model indicate that volatility of market returns has long memory. Moreover, in order to test the feature of long memory in the return and volatility of the stock market simultaneously, ARFIMA-FIGARCH models are estimated according to different distributions simultaneously. Predictable structure of volatility of Pakistan Stock Market display that this market is the weak-form market inefficiency. Consequently, it is possible to say that technical analysis related to this stock market may be valid. This implies that it is possible to predict future stock prices and extra ordinary gains could be obtained trading in this market.

Keyword: Weak-Form Efficient Market Hypothesis; Long Memory; ARFIMA-FIGARCH model; Volatility.
JEL Classifications: C13; C58; G10; G15; G17

1. Introduction

The concept of efficieny is very important in terms of financial markets. The Efficient Market Hypothesis has been a widely accepted phenomenon in the behavioral finance. In recent years, a lot of research have investigatd the efficiency of stock markets and its magnitude role in finance. The most essential theory trying to explain the formation process of stock market prices in financial markets was developed by Eugene Fama (1965) as Efficient Market Hypothesis. This hypothesis supports that a new information received in the market reaches to all of investors simultaneously and accordingly the prices of assets can not be estimated with regard to the historical prices and the asserts existing in an efficient market reflect all of information existing in the market for that moment. According to the hypothesis, it is not possible to obtain profits over the average by using this information. That is to say, Efficient Market Hypothesis is the application of Random Walk Hypothesis. The idea of random walk was firstly asserted by Jules Ragnault in 1863. Then, Louis Bachelier (1900) evaluated some remarkable insights and commentary about the hypothesis in PhD. dissertation. If asset prices have Random Walk process, the prices will fluctuate in response to spontaneous information for market and because it enters to stock markets accidentally, the price fluctuations also will become random. (Haque, 2011; Burton 1987, Çevik and Erdoğan, 2009).

The Efficient Market Hypothesis is evaluated in three different forms. i) Weak-form efficiency, ii) Semi-strong form efficiency, iii) Strong-form efficiency. In weak-form efficient market, the current market prices reflect all information involved in the historical prices of assets. In semi-form efficient market reflect not only the historical information but also the information disclosured to the public. Specifically, this form of the hypothesis holds that the analysis of any publicly available

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information is meaningless because all such information is already reflected in stock prices. The strongest version of market efficiency is Strong-form efficiency. It states all information in a financial market, whether public or private, is accounted for in a stock price. Not even insider information could give an investor the advantage. This degree of market efficiency indicates that profits exceeding normal returns cannot be obtained, regardless of the amount of research or information investors have access to. In its strong-form, the efficient market hypothesis holds that even investors with privileged information cannot use that information to developed profitable investing strategies(Atan et al. 2006, Balaban, 1995; Çevik and Erdoğan, 2009; Çevik, 2012).

The efficiency of financial markets is related to long term dependence of return prices. The decreasing tendency at a hyperbolic in autocorrelation functions of return and volatility which means the property of slow mean reversion and slow decay in the stock returns is defined Long Memory. In case the existence of long term dependency among price movements, there is positive autocorrelation among price movements and stock market prices will have a predictable structure and historical prices can be used price estimations in future(Baillie et al., 2007).

The many studies for testing Efficient Market Hypothesis in finance literature indicate the results supporting foreseeability of returns in contrast with the hypothesis. Firstly, Granger (1980) and Granger and Joyeux(1980) imply that ARMA time series models with fractional integration (ARFIMA) model long memory property of stock market returns. Balaban (1995) examines Turkish stock market for the period of 1988-1994. The author displays that Turkey' stock market is not efficient market. Bollerslev and Mikkelsen (1996) propose FIEGARCH model as FIGARCH model with EGARCH approach. They apply FIEGARCH model for long memory of volatility of S&P500 returns in U.S. stock market. Huang and Yang (1999) confirm the presence of long memory in the NYSE and NASDAQ indexes by using a modified R/S technique. Barkoulas et al. (2000) invesitagate Greece Stock Market by using weekly data for the peirod 1981-1990. They display that Greece Stock Market is a weak-form inefficient market. Resende and Teixeira (2002) investigate the existing of long memory property in Brazilian Stock Market by using ARFIMA model. They show that Brazilian Stock Market is efficient. Tolvi (2003) analyzes stock market of 16 OECD countries for the period 1960-1999 by using ARFIMA model. The author implies that Finland, Denmark and Ireland Stock Markets are weak-form inefficient markets. Wright(2002) examines long memory property of U.S. Stock Market by using Log-Periodogram method. Henry (2002) finds that Germany, Japan, South Korea and Taiwan Stock Markets have long memory and these markets are inefficient. Kılıç (2004) reveals that daily returns of Turkish Stock Market are not characterized by long memory by using non-parametric methods and FIGARCH model. Vougas (2004) finds that Athens Stock Market is an efficient market by using ARFIMA model for the period 1990-2000. Kumar (2004) displays that India Stock Market has long memory due to the presence of conditional heteroscedasticity in returns by using ARFIMA-GARCH models. Caporale and Gil-Alana (2004) report that S&P500 index has long memory property by using daily data for the period 1828-1991. Assaf and Cavalcante (2005) investigate long memory in return and volatility of Brazilian Stock Market by using Lo'(R/S) statistics and FIGARCH model. They can not find any evidence of long memory. Gil-Alana (2006) implies the presence of long memory for FTSE100, S&P500, Singapure All Shares, Nikkei, Hang-Seng, EOE and DAX indexes. Jayasuriya (2009) examines long memory in volatilities of some frontier markets and 23 emerging markets by using FIEGARCH model. Tan et al. (2010) find that Kuala-Lumpur stock market has long memory by using Geweke and Porter-Hudak (1983) model. Thupayagale (2010) displays mix evidence of long memory property for 11 African stock markets. Lopez-Herrera et al. (2012) investigate long memory behavior in the returns of the Mexican Stock Market for the period January 1983 to December 2009 by using ARFIMA models. The authors find that the Mexican Stock Market has long memory and it is weak-form inefficient. Macheshchandra (2012) examines the existence of long memory in the Indian stock market by using ARFIMA-FIGARCH models. He finds that ARFIMA model implies the absence of long memory in return series of the Indian Stock Market, however, FIGARCH model shows strong evidence of long memory in volatility of stock returns.

There is a controversy about the efficiency of emerging markets in finance literature. In this sense, the study provides a major contribution about an emerging market namely Pakistan Karachi Stock Exchange to applied finance literature. In this study, dual long memory properties in returns of Pakistan Karachi Stock Exchange, which is selected among emerging markets which have gradually increasing importance in global economy by using ARFIMA-FIGARCH models for the period of (2010-2013) are investigated and tested weak-form efficiency of the stock market.

2. Method

Granger (1980), Granger and Joyeux (1980) and Hosking (1981) suggested ARFIMA (autoregressive fractionally integrated moving average) approach to model financial time series characterized by long memory processes. These model differs from the standard ARIMA models in terms of the lag function of the residuals. A process ARFIMA (p, ξ, q) is generated as follows:

$$\psi(L)(1-L)^{\xi}(y_t-\mu)=\theta(L)\varepsilon_t \quad (1)$$

$$\varepsilon_t = z_t\sigma_t, \quad z_t \sim N(0,1)$$

$$(1-L)^{\xi}=\sum_{k=0}^{\infty}\frac{\Gamma(k-\xi)L^k}{\Gamma(-\xi)\Gamma(k+1)}.$$

Where, $\Gamma(\ )$ is a gamma function. From Eq.(1), $(1\text{-L})^{\xi}$ can be also written as a finite MA process with binominal expension (Hosking, 1981; Granger and Joyeux, 1980).

$$(1-L)^{\xi}=1-\xi L+\frac{\xi(\xi-1)}{2!}L^2-\frac{\xi(\xi-1)(\xi-2)}{3!}L^3+\ldots \quad (2)$$

where ε_t, is independent and identically distributed with σ^2 variance (i.i.d.). L indicates lag operator. In addition, $(1-L)^{\xi}$ is the operator of fractional difference. ξ shows fractional degree of integration. Integer value of ξ states traditional ARMA model. If, $0 < \xi < 0.5$, then it indicates that the process has positive dependency among distant observations showing the property of long memory, $-0.5 < \xi < 0$, then it shows that the process has negative dependency among distant observations named as anti-persistence. The process is stationary when $\xi = 0$, and the process has a unit root process when $\xi = 1$. $\psi(L) = 1 - \psi_1 L - \psi_2 L^2 - \cdots \psi_p L^p$ and $\theta(L) = 1 + \theta_1 L - \theta_2 L^2 - \cdots \theta_q L^q$ are autoregressive AR and moving average MA polynomials respectively. Hosking (1981) displayed that autocorrelation functions of fractional integrated processes slowly decrease at a hyperbolic rate.

According to Baillie, Bollerslev and Mikkelsen (1996), the effect of shocks on the volatility is not finite. This idea leads to the process of fractional integration in volatility. FIGARCH model has been suggested for the extended version of squared errors in ARFIMA model notation by Baillie et al. (1996). FIGARCH (p,d,q) model is introduced as follows:

$$\phi(L)(1-L)^d\varepsilon_t^2=\omega+[1-\beta(L)]v_t \quad (3)$$

$v_t=\varepsilon_t^2-\sigma_t^2$ are serially uncorrelated errors having zero mean. ε_t^2 are squared errors of GARCH process. The process of $\{v_t\}$ is integrated for conditional variance σ_t^2 as variations. It is assumed that all roots of $\phi(L)\ and\ [1-\beta(L)]$ stayed out of unit circle. If d=0, then the process of FIGARCH (p,d,q) is reduced to the process of GARCH (p,q). If d=1, then the process of FIGARCH becomes an integrated process of GARCH (IGARCH). Shocks have an infinite effect on prospective volatility in this process. The FIGARCH (p,d,q) process imposes an ARFIMA structure on ε_t^2. The Model (3) can be formed as follows (Baillie et al., 1996):

$$[1-\beta(L)]\sigma_t^2=w+[1-\beta(L)-\phi(L)(1-L)^d]\varepsilon_t^2, \quad (4)$$

Conditional variance of ε_t^2 is presented with;

$$\sigma_t^2=\frac{\omega}{[1-\beta(L)]}+\lambda(L)\varepsilon_t^2, \quad (5)$$

where,

$$\lambda(L)=1-\frac{\phi(L)}{[1-\beta(L)]}(1-L)^d. \quad (6)$$

Baillie et. al. (1996) indicate that when $0 \le d < 1$, the effect of a shock on conditional variance of FIGARCH (p,d,q) processes decreases at a hyperbolic rate. In the other words, long term dynamics of volatility can be evaluated with fractional integration parameter as d.

3. Empirical Results

In the study, whether the weak-form efficient market hypothesis is valid for Pakistan Karachi Stock Exchange will be tested through ARFIMA-FIGARCH model.

3.1. Data and Initial Analysis

Data consists of daily stock index data for the period of 2010-2013 after global economic crisis. Daily logarithmic returns in t-time related to the stock market of Pakistan (Karachi Stock Exchange) are;

$$R_t = \ln(P_t / P_{t-1}) x100 \ , \ t=1,2,\ldots.n. \qquad (7)$$

Where R_t shows index return in t-time, P_t is the closure price of index in t-time, P_{t-1} is the closure price of index in (t-1) time. Descriptive statistics of stock index returns of Pakistan (RPAK) are given in Table 1.

Table 1. Descriptive Statistics of Return Series

	RPAK
Mean:	0.078018
Standard Deviation:	1.1239
Skewness:	0.056774
Kurtosis:	1.9415
Minimum:	-4.7627
Maximum:	4.4837
J-B: **Prob.**	117.25 (0.0000)
ARCH (2):	12.134 *
ARCH (5):	6.5607 *
ARCH (10):	3.9208 *
Q(5):	9.07980
Q(10):	12.9233
Q(20):	22.6289
Q(50):	57.6394
Q^2(5):	41.1381*
Q^2(10):	53.4283*
Q^2(20):	60.1606*
Q^2(50):	104.202*
Lo R/S Test Statistics for Return	1.42344
Hurst-Mandelbrot R/S Test Statistics for Return	1.43744
Lo R/S Test Statistics for Squared Return	1.9898*
Hurst-Mandelbrot R/S Test Statistics for Squared Return	2.09225*
* shows statistical significantly at level 5% Lo R/S and Hurst-Mandelbrot R/S Test Statistics 95%, (0.809-1.862)	

According to the results in Table 1, skewness parameter of RPAK return series is positive and near to zero and the value of kurtosis are very low. According to relevant statistics, it can be said that RPAK return series show symmetric properties and platycurtic and fat tail compared to the normal. Furthermore, the statistic of Jarque-Bera having a relatively high value is also statistically significant as an indication related to return series non-normal distribution. Ljung-Box statistics (Q and Q^2) in various delays were estimated for independency test of return error and squared return error series. Especially squared return error series has not i.i.d. (independent and identically distributed) process because of RPAK squared return errors highly correlated up to 50^{th} delay.

Moreover, statistical value of Ljung-Box statistics (Q^2) in 50^{th} delay, in high degrees displaying extensive effect of volatility clustering in RPAK stock market returns, is also statistically significant. Form the Table 1, the results of ARCH-LM test in various lags imply the existence of significant ARCH effects in standardized errors.

At the 5% significance level, the null hypothesis of a short memory process is rejected if the modified Lo (R/S) and Hurst-Mandelbrot (R/S) statistic does not fall within the confidence interval [0.809, 1.862]. According to Table 1, test statistics present an evidence of long memory property in squared return series considered as the most popular proxy for volatilities in financial markets.

Furthermore, results of three different unit root tests as ADF (Augmented Dickey Fuller), PP (Phillips-Perron) and KPSS (Kwiatkowski, Phillips, Schmidt ve Shin) in order to determine whether stock market return series (RPAK) have the feature of stationarity I(0) are given in Table 2.

Table 2. Unit Root Tests for Return Series

Tests	RPAK
ADF	-15.6574*
PP	-23.997*
KPSS	0.083840
* indicates the refusal of unit root null hypothesis in the significance level at %5. (McKinnon Critical Value[-2.865], Kwiatkowski Critical Value [0.463000])	

According to the results in Table 2, while high negative results of ADF and PP tests indicates the refusal of unit root null hypothesis for all return series at the significance level of 5%, KPSS test statistics do not refuse null hypothesis showing I(0) process for all return series at the significance level of 5%. The results of unit root tests are supported stationary for return series.

3.2. Model Estimation for Pakistan Stock Exchange

3.2.1. ARFIMA (p, ξ, q) Model Estimation Results

In order to analyze long memory property in return series of Pakistan stock market and whether it is efficient market, firstly, ARFIMA models for different lags (p,q) under assumption of Normal(N), Student-t(ST), Skewed Student-t(SST) and GED(GED) distributions. The different ARFIMA(p, ξ, q) models as p,q=0,1,2 for RPAK return series were estimated and compared in terms of Akaike (AIC) and Schwarz (SIC) Information Criteria. Estimation results of most appropriate model for RPAK (ARFIMA(0, ξ, 1)) is displayed in Table 3.

As shown in Table 3, ARFIMA model don't support long memory behaviour in terms of distributions of N, ST, SST for RPAK returns. Long memory parameter in return, ξ isn't statistically significant except for GED distribution. This situation indicates a unpredictable behaviour of stock market index returns for Pakistan which is in consistent with Efficient Market Hypothesis. Diagnostic statistics in Table 3 especially represent positive asymmetry and platycurtic values for RPAK return series. Jarque-Bera statistics is also an indicator referring standardized errors have distributions different from normal distribution.

3.2.2. FIGARCH (p, d, q) Model Estimation Results

In this stage, the property of long memory in volatility(conditional variance) of Pakistan Stock Exchange is evaluated by using FIGARCH model approach. The most appropriate long memory FIGARCH Model results selected from models with different lags (p,q=0,1,2) are presented in Table 4.

Table 3. The Results of ARFIMA Model

(p,ξ,q)	RPAK(0,ξ,1)			
	N	ST	GED	SST
μ	0.080085 (0.050576) [0.1137]	0.059188** (0.032066) [0.0653]	0.018210* (0.003818) [0.0000]	0.083897* (0.039562) [0.0343]
ψ_1	-	-	-	-
ψ_2				
ξ	0.039135 (0.071973) [0.5868]	0.018313 (0.052499) [0.7273]	0.004270* (0.001764) [0.0157]	0.019580 (0.051915) [0.7062]
θ_1	-0.020149 (0.078823) [0.7983]	0.022023 (0.056958) [0.6991]	-0.013109* (0.0021664) [0.0000]	0.025118** (0.056397) [0.06562]
θ_2	-	-	-	-
v	-	-	0.995497* (0.074128) [0.0000]	3.460799* (0.50117) [0.0000]
ln(ζ)	-	-	-	0.048672 (0.039422) [0.2174]
Log(L)	-1142.127	-1107.340	-1097.518	-1106.726
AIC	3.080986	2.990161	2.963759	2.991199
SIC	3.105782	3.021156	2.994754	3.028393
Skewness	0.091185	0.043979	0.057094	0.043460
Kurtosis	1.8920	1.9626	1.9394	1.9639
J-B	112.00	119.65	117.00	119.80
Q(5)	7.77938	10.1412	9.08715	10.2086
Q(10)	11.2594	14.2970	12.8580	14.3898
Q(20)	21.5770	23.5256	22.7797	23.5661
Q(50)	56.5820	58.6312	57.7074	58.6990
$Q^2(5)$	45.5817*	46.2610*	42.3872*	48.3820
$Q^2(10)$	58.6471*	59.3785*	55.2265*	61.8419
$Q^2(20)$	65.1112*	66.2077*	62.0584*	68.6436
$Q^2(50)$	110.180*	110.287*	106.579*	112.761*
ARCH(5)	7.2650 [0.0000]*	7.4287 [0.0000]*	6.7619 [0.0000]*	7.8028 [0.0000]*
ARCH(10)	4.2981 [0.0000]*	4.4107 [0.0000]*	4.0492 [0.0000]*	4.6192 [0.0000]*
P(40)	124.7097 0.000000	54.4946 0.050730	60.8387 0.014149	50.3011 0.106136
P(50)	139.7366 0.000000	76.0269 0.007964	90.1398 0.000312	74.4140 0.011079
P(60)	143.2581 0.000000	75.6774 0.070686	83.4194 0.019900	75.6774 0.070686

* indicate statistically significant 5%. ( ) indicates standard error, [ ] indicates p-values. P(40), P(50) ve P(60) indicate, Pearson Goodness of Fit for 40, 50, 60 cells.

According to Table 4, long memory d parameter for FIGARCH model is significantly different from zero for return series and the volatility(conditional variance) of returns demonstrates long memory process. Furthermore, it can be said that return series demonstrate i.i.d. property according to Ljung-Box test statistics (Q and Q^2). The results of Pearson Goodness of Fit Test, which is the test for appropriateness of distribution, indicates that different distributions are also appropriate for RPAK return series. Moreover, tail parameter "v" is statistically significant for all of the distributions (ST, SST, GED).

Table 4. The Results of FIGARCH Model

p=1,q=1	FIGARCH			
	N	ST	GED	SST
ω	2.326957 (1.6119) [0.1493]	1.914974** (1.0052) [0.0572]	1.879901** (1.0235) [0.0666]	1.906445** (1.0102) [0.0595]
β_0	0.024066 (0.18666) [0.8974]	-0.047615 (0.24167) [0.8439]	-0.023555 (0.23075) [0.9187]	-0.048579 (0.24180) [0.8408]
β_1	0.397799 (0.27967) [0.1553]	0.247393 (0.26669) [0.3539]	0.281531 (0.27680) [0.3095]	0.246398 (0.26712) [0.3566]
d	0.442619 * (0.16296) [0.0068]	0.364587* (0.10605) [0.0006]	0.370840* (0.12441) [0.0030]	0.364239* (0.10613) [0.0006]
v		4.875755* (0.86067) [0.0000]	1.104947* (0.093065) [0.0000]	4.879404* (0.86259) [0.0000]
ln(ζ)				0.003042 (0.032704) [0.9259]
Log(α)HY				
Log(L)	-1108.987	-1088.865	-1080.081	-1088.862
AIC	2.991901	2.940497	2.916885	2.943177
SIC	3.016696	2.971492	2.947880	2.980371
Skewness	-0.011005	0.0070933	0.0073462	0.0075962
Kurtosis	1.3809	1.4149	1.4054	1.4152
J-B	59.130	62.064	61.237	62.094
Q(5)	4.39515	4.58522	4.56656	4.58299
Q(10)	11.9250	11.9800	12.0354	11.9820
Q(20)	17.8408	18.4327	18.4254	18.4393
Q(50)	52.8548	53.8824	53.8905	53.8947
$Q^2(5)$	1.28523	0.609293	0.725384	0.606738
$Q^2(10)$	2.53058	2.15759	2.19099	2.15471
$Q^2(20)$	8.25102	7.56156	7.64109	7.55544
$Q^2(50)$	31.1721	29.8157	30.0912	29.8103
ARCH(5)	0.25293 [0.9384]	0.12082 [0.9878]	0.14955 [0.9802]	0.12025 [0.9879]
ARCH(10)	0.24613 [0.9913]	0.21157 [0.9953]	0.22017 [0.9945]	0.21116 [0.9953]
P(40)	102.3441 0.000000	83.4194 0.000046	54.6022 0.049721	91.2688 0.000004
P(50)	124.8172 0.000000	109.2258 0.000002	86.7796 0.000711	117.9624 0.000000
P(60)	176.1613 0.000000	128.7419 0.000000	91.9677 0.003880	131.8065 0.000000

*, ** indicate statistically significant 5% and 10% respectively. ( ) indicates standard error, [ ] indicates p-values. P(40), P(50) ve P(60) indicate Pearson Goodness of Fit for 40, 50, 60 cells.

3.2.3. ARFIMA-FIGARCH Model Estimation Results

Long memory dynamics in conditional mean and volatility(conditional variance) were examined separately. In this stage, the existence of dual long memory property in conditional mean and volatility is investigated by using ARFIMA-FIGARCH model approach because the property of long memory in conditional mean and volatility of the returns may simultaneously be observed. Similarly, different combinations of ARFIMA (p, ξ, q)-FIGARCH (p, d, q) models are estimated for p,q=0,1,2 and the most appropriate model selected in terms of model selection criteria (AIC, SIC, Log L) is given in Table 5.

Table 5. The Results of ARFIMA-FIGARCH Model

	(1,ξ,1)-(1,d,1)			
	N	ST	GED	SST
μ	0.119433* (0.043198) [0.0058]	0.092906* (0.029925) [0.0020]	0.065490* (0.002669) [0.0000]	0.109692* (0.032541) [0.0008]
ψ_1	0.152725 (0.52680) [0.7720]	0.373709 (0.42190) [0.3760]	0.515603* (0.004095) [0.0000]	0.340729 (0.32318) [0.2921]
θ_1	-0.174689 (0.56011) [0.7552]	-0.350525 (0.48669) [0.4716]	-0.590265* (0.004801) [0.0000]	-0.305324 (0.33209) [0.3582]
ξ	0.026479 (0.068458) [0.6990]	-0.034661 (0.081500) [0.6707]	0.039083* (0.003412) [0.0000]	-0.041484 (0.073097) [0.5705]
ω	2.352459 (1.6320) [0.1499]	2.048206** (1.1998) [0.0882]	1.908569** (1.0939) [0.0815]	2.034579** (1.1044) [0.0658]
β_0	0.013734 (0.21346) [0.9487]	-0.050713 (0.24431) [0.8356]	-0.027355 (0.23364) [0.9068]	-0.078054 (0.23887) [0.7439]
β_1	0.374698 (0.31670) [0.2371]	0.245517 (0.27859) [0.3784]	0.278264 (0.28736) [0.3332]	0.211100 (0.26931) [0.4334]
d	0.444550* (0.16677) [0.0079]	0.385295* (0.11077) [0.0005]	0.378600* (0.12842) [0.0033]	0.378651* (0.10173) [0.0002]
v		4.929222* (0.89908) [0.0000]	1.107413* (0.087245) [0.0000]	4.866204* (0.86270) [0.0000]
$\ln(\xi)$				0.062214** (0.037545) [0.0979]
Log(L)	-1103.880	-1083.903	-1076.130	-1082.913
AIC	2.988924	2.937911	2.917015	2.937937
SIC	3.038515	2.993702	2.972806	2.999927
Skewness	0.013936	0.0087040	-0.0034015	0.017928
Kurtosis	1.3884	1.4939	1.4747	1.5041
J-B	59.783	69.192	67.422	70.172
Q(5)	3.81614	6.21142	8.04301*	6.24977
Q(10)	10.2630	14.5662	15.1409	14.6354
Q(20)	16.3900	20.1856	21.4688	20.2276
Q(50)	50.5959	54.4894	56.8421	54.3688
$Q^2(5)$	1.02918	0.773565	0.839474	0.664599
$Q^2(10)$	3.06774	2.75299	2.77071	2.75896
$Q^2(20)$	9.05167	8.25380	7.92720	8.31946
$Q^2(50)$	31.9357	29.8661	29.8897	29.8525
ARCH(5)	0.19518 [0.9644]	0.14571 [0.9813]	0.17655 [0.9714]	0.12416 [0.9870]
ARCH(10)	0.28721 [0.9841]	0.26157 [0.9889]	0.28018 [0.9855]	0.26138 [0.9890]
P(40)	81.6989 0.000074	62.1290 0.010683	41.1613 0.376163	59.1183 0.020360
P(50)	124.6828 0.000000	87.4516 0.000604	44.7097 0.647587	75.4892 0.008899
P(60)	117.4516 0.000009	74.2258 0.087438	61.9677 0.370730	85.6774 0.013216

*, ** indicate statistically significant 5% and 10% respectively. ( ) indicates standard error, [ ] indicates p-values. P(40), P(50) ve P(60) indicate Pearson Goodness of Fit for 40, 50, 60 cells.

According to the results of ARFIMA-FIGARCH model in Table 5, while long memory parameter ξ in returns is not statistically significant for return series of RPAK except for GED distribution. However, The parameter d which is the long memory parameter in volatility was found statistically significant for RPAK according to all of distributions. In addition, results of ARCH-LM test support that there is no ARCH effects in the errors for return series. Furthermore, tail and asymmetry parameters (v and $\ln(\xi)$) are statistically significant and these statistics show that other distributions(especially GED distribution) give better results compared to Normal distribution.

4. Conclusion and Remarks

In this study, dual long memory properties in Pakistan stock exchange returns have been examined by using ARFIMA-FIGARCH model types for the period of 2010-2013 and Weak-Form Efficient Market Hypothesis has been tested. The efficiency of stock markets which are one of the most important indicators of national economies is relatively a crucial issue. The Weak-Form Efficient Market Hypothesis supports that asset prices reflect all available information. According to hypothesis, asset prices accidentally fluctuate as random walk which is satisfied by unpredictable behaviour of asset returns. If return series have long memory property, they will indicate significant autocorrelations between distant observations. In case of long memory, returns are nor independent over time and future returns can be predicted by using past prices.

Firstly, ARFIMA model were estimated with different distributions (Normal, Student-t, GED, Skewed-Student t) for long memory property in return of Pakistan Karachi Stock Exchange and they were not considered as statistically significant for returns except GED distribution. Furthermore, FIGARCH model estimations for modelling long memory property in the volatility of return series were also obtained. Estimation results for FIGARCH model indicate that the volatility has long memory in returns of Pakistan Karachi Stock Exchange for all of the distributions. Moreover, in order to test of long memory property in the return and volatility of stock market simultaneously, ARFIMA-FIGARCH models were jointly estimated for different distributions. According to results, Pakistan stock market do not demonstrate dual long memory property except for GED distribution.

In the study, a significant evidence of fractional dynamics with long memory property in volatility of returns in an emerging market namely Pakistan Karachi Stock Exchange is presented. Price movements in this stock market are influenced by price realizations from current and historical returns. This significant finding contradicts the weak-form efficient market hypothesis. The practical implications of findings for the Pakistan Karachi Stock Exchange are therefore established. The investment strategies containing equity portfolios should depend on completely characterization of returns in emerging markets. The study strongly suggest that long memory dynamics in volatility occupy an important place in this characterization. From that point, investors should be aware of dynamics of financial market for their decisions. The findings of the study can be evaluated by policy-makers, investors and academicians. Furthermore, studies on the financial markets of countries, not only volatility of stock markets but also long memory dynamics in stock markets should be taken into account. On behalf of evaluating asymmetric effects in emerging markets, it would be interesting in the future to extend this study by using asymmetric long memory models which consider fat-tails error distribution also.

In conclusion, there is a controversy about the efficiency of emerging markets in finance literature. In this sense, the study provides a major contribution about an emerging market namely Pakistan Karachi Stock Exchange to applied finance literature. According to the findings obtained, predictable structure of volatility indicates that the Pakistan stock market is inefficient in weak-form. In this respect, it is possible to specify that technical analysis related to this stock market may be valid. This implies that it is possible to predict future stock prices and extra ordinary gains could be obtained trading in this market, contrary to what the efficient markets theory points out.

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