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Transboundary Pollution, R&D Spillovers, Absorptive Capacity and International Trade[1]

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ABSTRACT: In this paper, we study the effects of absorptive capacity and R&D spillovers on cross-border pollution in a game played by two regulator-firm hierarchies. By means of a tax per-unit of pollution, a subsidy per-unit of original research and a subsidy per-unit of absorptive research, the regulators can reach the first-best outcome. We show that, in addition to free R&D spillovers and absorptive research, competition of firms on the common market help non-cooperating countries to better internalize transboundary pollution. More importantly, opening borders increase the per-unit emission-tax and decrease the per-unit original research subsidy. Thus, when the investment-cost parameters are sufficiently high, the international trade increases the original research, production. Consequently, the emission ratio is lower.

Keywords: Transboundary pollution; R&D spillovers; absorptive capacity; international trade.
JEL Classifications: D62; H21; O32.

1. Introduction

The relation between openness to international trade and pollution can be explicated by the scale, technique, and composition effects. Copeland and Taylor (1995) evaluated this interaction by developing a static two country general equilibrium model with a continuum of goods differing in their pollution intensity of production. They showed that income effects created by income transfers or by trade in goods or pollution permits have important and often surprising effects on pollution, trade flows and welfare levels. Antweiler et al. (2001) developed an explicit model of international trade and pollution and they have moved from theory to empirical estimation by using data on sulfur dioxide concentrations.

By employing a two country general equilibrium model, Takarada (2005) examined the welfare effects of the transfer of pollution abatement technology under cross-border pollution. He demonstrated that technology transfer is not always welfare improving for the donor and the recipient even if pollution is transboundary. Péchoux and Pouyet (2003) considered an economy composed of tow country where each domestic market is served by a regulated protected monopoly. The authors showed that, for a low or high degree of complementarity, the gain in opening markets to international competition is larger under incomplete information than under complete information.

In the context of imperfectly competitive international markets, Conrad (1993) constructed a model of international oligopoly with negative externalities in production, in which optimal environmental policy replies to foreign emissions tax and subsidy programs can be calculated. Also, Spencer and Brander (1983) introduced a positive theory to explain such industrial strategy policies when the R&D rivalry between firms plays an important role. They showed that there are national incentives to subsidize R&D if export subsidies are not available. Liao (2007) extended the Spencer and Brander (1983) model by considering the presence of R&D spillovers between firms. Bjorvatn and Schjelderup (2002) introduced international spillovers in public goods provision and showed that such spillovers reduce, and in the limiting case of prefect spillovers, eliminate tax competition.

Using a non-cooperative and symmetric three-stage game played by two regulator-firm hierarchies, Ben Youssef (2009) showed that free R&D spillovers and the competition of firms on the common market help non-cooperating countries to better internalize transfrontier pollution. More importantly,

[1]We would like to thank Slim Ben Youssef for his rich comments.

international competition increases the per-unit emissions-tax and decreases the per-unit R&D subsidy.

Our work departs from the theoretical literature dealing with cross-border externalities and absorptive capacity by using a theoretical model where firms can invest both in original and absorptive R&D to reduce their emission/output ratio.

The investment in R&D not only generates innovation but also facilitates learning; this idea is pioneered by Cohen and Levinthal (1989). They showed that firms invest in R&D not only to generate new process and product innovation, but also to develop and maintain their broader capabilities to assimilate, and exploit existing information. When R&D spillovers are treated as exogenous, the theoretical models of research joint ventures focus on the effects of R&D cooperation. D'Aspremont and Jacquemin (1988, 1990) considered a duopoly model with R&D externalities and they found that, for large spillovers, duopolists cooperating spend more on R&D and produce more output. Kamien et al. (1992) showed that creating a competitive research joint venture increases firms' profits and social welfare. However, most of the recent models are characterized by an endogenous spillover parameter. Focusing on this approach, Kamien and Zang (2000) modeled a firm's effective R&D investment that requires it to engage in R&D in order to realize spillovers from other firms' R&D activity.

Using a very simple non-tournament model of R&D, Poyago-Theotoky (1999) showed that firms never disclose any of their information when they non-cooperate in R&D, whereas they will always choose to fully share their information in the cooperative equilibrium. Leahy and Neary (2007) specified a general model of the absorptive capacity process and showed that costly absorption both raises the effectiveness of a firm's own R&D and lower the effective spillover which it obtains from rival firms. This weakens the case for encouraging research joint ventures, even if there is complete information sharing between firms.

Milliou (2009) examined firms' incentives to protect their R&D investments, as well as the impact of protection on innovation and welfare. Hammerschmidt (2009) distinguished between two different components of R&D: one that produces original results (inventive R&D) and another that aims at improving a firm's absorptive capacity (absorptive R&D). She found that, when the R&D spillover parameter rises, firms invest more in R&D to strengthen absorptive capacity.

Ben Youssef and Zaccour (2009) considered a duopoly competing in quantities and where firms can invest in original and absorptive R&D to control their emissions. They showed that a regulator can reach the social optimal outcome by implementing a taxation and subsidy policy. There are no free R&D spillovers between firms, Ben Youssef (2010) established that the investment in absorptive research enables non-cooperating regulators to better internalize transboundary pollution.

Our research work differs from the existing literature by the fact that we investigate how internalization of cross-border pollution is very important when R&D spillovers and the absorptive capacity are higher and when markets are opened to international competition.

We consider a non-cooperative and symmetric three-stage game consisting of two identical regulator-firm hierarchies. Each firm produces one good sold on the domestic market in the third stage and they can invest in original and absorptive research to reduce its emission/output ratio, in the second stage. In the first stage, regulators announce non-cooperatively their per-unit emission tax and R&D subsidies so as to maximize their social welfare function. This game is solved backward to get a sub-game perfect Nash equilibrium.

In our model, we show that regulators can pushes their firms to reach the non-cooperative socially optimal levels of production and R&D by using three regulatory instruments, which are a per-unit emission tax, a per-unit original research subsidy and a per-unit absorptive research subsidy.

More importantly, we show that when R&D spillovers and the ability to absorb are equal to zero, transboundary pollution is partially internalized when markets are separated. The higher are the ability to absorb and the R&D spillovers, the greater is the transboundary pollution internalized by competing countries. Moreover, the internalization of transborder pollution by non-cooperating firms on the common market is very important when markets are opened to international trade. Opening borders increases the per-unit emission tax and decreases the per-unit original research subsidy. Consequently, the absorptive capacity is higher and the emission ratio is lower which leads to more production in common market.

The structure of the paper is as follows. In the next Section, we present the basic model when markets are separated and examine the impact of the R&D spillovers and the absorptive capacity in the internalization of transboundary pollution. Section 3 treats the common market case and shows how this contributes to internalize transboundary pollution. In Section 4 we compare the non-cooperative socially optimal values given by the two market regimes, and the last section concludes the paper.

2. Separate Markets

We consider a symmetric model composed two countries and two firms. Firm i located in country i is a regional monopoly and produces good i in quantity q_i sold on the domestic market having the following linear inverse demand function $p_i(q_i) = a - bq_i$ where $a, b > 0$. One reason for the market structure we adopt is that the markets of industries engaging in important R&D investments are oligopolistic.

The current production activity generates pollution and firms can invest in R&D to decrease their emissions per unit of output. We distinguish between two components of R&D efforts, namely, original R&D, denoted by x_i^o, which directly reduces the emission ratio and costs $k^o(x_i^o)^2$, where $k^o > 0$, and absorptive capacity R&D, denoted by x_i^a, which allows a firm to capture part of the original research developed by its competitor, and costs $k^a(x_i^a)^2$, where $k^a > 0$.

The innovation activity realized by firms is characterized by positive externalities which imply that a fraction of each firm's R&D level gratuitously spillovers to the other firm and by absorptive capacity is considered as the ability to absorb spillovers from rival firm. The effective R&D effort of firm i is $x_i = x_i^o + (\beta + lx_i^a)x_j^o$, where $0 \leq \beta < 1$ is the R&D spillover parameter and $l > 0$ is a learning or absorptive parameter. Thus, we impose that $0 \leq \beta + lx_i^a \leq 1$.

We normalize the emissions per unit of production to one without innovation, the emission/output ratio of firm i is $e_i = 1 - x_i^o - (\beta + lx_i^a)x_j^o$ and its emission of pollution is $E_i = \left[1 - x_i^o - (\beta + lx_i^a)x_j^o\right]q_i$.

Since firm i constitute a polluting monopoly, it is regulated. Each non-cooperating regulator maximizes his own social welfare function and uses three regulatory instruments that he announces at the first stage of the game: a per unit emission tax t_i^s inducing the non-cooperative socially optimal levels of production and pollution, a per unit original research subsidy r_i^{os} and a per unit absorptive research subsidy level r_i^{as} inducing the non-cooperative socially optimal levels of effective R&D and emission/production ratio. Firm reacts by investing in R&D at the second stage, and by offering their production on the market at the third stage. This game is solved backward to get a sub-game perfect Nash equilibrium.

Denoting the marginal cost of production by $\theta > 0$, the profit function of firm i is[2] $\pi_i^s = p_i(q_i)q_i - \theta q_i - k^o(x_i^o)^2 - k^a(x_i^a)^2$, and its profit net of taxes and subsidies is $V_i^s = \pi_i^s - t_i^s E_i + r_i^{os}x_i^o + r_i^{as}x_i^a$.

We make simple assumption about the original and absorptive research.

Assumption 1. *We assume that* $lim_{k^o,k^a\to\infty}\, x_i^{os} = lim_{k^o,k^a\to\infty}\, x_i^{as} = 0$.

This intuitive assumption is logical because when the investment cost parameters are relatively very high, it is socially optimal to not invest in R&D.

Under the presence of cross-border pollution which is a negative externality among countries. Thus, the damages caused to country i are $D_i = \alpha E_I + \gamma E_J$, where $\alpha > 0$ the marginal damage of the domestic pollution is, and $\gamma > 0$ is the marginal damage of the foreign pollution[3].

The consumer surplus in country i engendered by the consumption of q_i^s is $CS_i^s = \int_0^{q_i} p_i(u)du - p_i(q_i)q_i = \frac{b}{2}q_i^2$.

Hence, after simplifications, the social welfare function of country i is depend on the consumer surplus, damages and the profit of the domestic firm.

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[2] In what follows, the subscripts o and a refer to the original and absorptive research, respectively. The superscripts s and c refer to the separate markets and common market, respectively.

[3] The cross-border pollution is partially non-internalized, if damage functions are not linear with respect to total pollution, nor separable with respect to the pollution remaining at home and the one received from other countries.

$$W_i^s(q_i, q_j, x_i^o, x_j^o, x_i^a) = CS_i^s - D_i + \pi_i^s \tag{1}$$

Expression (1) will be used to determine the socially-optimal levels of production, original and absorptive research.

2.1 The firms' Behavior

In the first stage, the regulator announced the per-unit emission tax and the per-unit R&D subsidies, the firm reacts by choosing its optimal research and production levels in the second and third stages, respectively. The model is solved by backwards. At the third stage, the firm maximizes its net profit with respect to its production level, and at the second stage, it maximizes its net profit with respect to its R&D levels.

The third stage first order condition of firm i is:

$$\frac{\partial V_i^s}{\partial q_i} = 0 \tag{2}$$

The resolution of (2) gives:

$$q_i^{*s} = \frac{a - \theta - t_i\left[1 - x_i^o - (\beta + l x_i^a)x_j^o\right]}{2b} \tag{3}$$

We deduce the following:

$$\frac{\partial q_i^{*s}}{\partial x_i^o} = \frac{t_i}{2b}; \frac{\partial q_i^{*s}}{\partial x_i^a} = \frac{t_i l x_j^o}{2b}$$
$$\frac{\partial q_i^{*s}}{\partial x_j^o} = \frac{t_i(\beta + l x_i^a)}{2b}; \frac{\partial q_i^{*s}}{\partial x_j^a} = 0$$

Consider the case of a positive emission tax. When a firm increases its level of original or a absorptive research, then its emission ratio diminishes enabling it to expand its production. When the competing firm increases its original research, this has a positive effect on the production of the firm because of R&D spillovers and absorptive capacity, the emission ratio of firm decreases enabling it to expend its production.

The symmetric optimal level of production for each regulator is:

$$q_i^{*s} = \frac{a - \theta - t_i[1 - (1 + \beta + l x_i^a)x_i^o]}{2b} \tag{4}$$

The second stage first-order conditions of firm i are[4]:

$$\frac{dV_i^s}{dx_i^o} = \frac{\partial q_i^{*s}}{\partial x_i^o}\frac{\partial V_i^s}{\partial q_i} + \frac{\partial V_i^s}{\partial x_i^o} = 0 \tag{5}$$
$$\frac{dV_i^s}{dx_i^a} = \frac{\partial q_i^{*s}}{\partial x_i^a}\frac{\partial V_i^s}{\partial q_i} + \frac{\partial V_i^s}{\partial x_i^a} = 0 \tag{6}$$

In equilibrium, solving the first-order conditions yields the symmetric solutions which can be written as:

$$t_i^s q_i^{*s} + r_i^{os} - 2k^o x_i^o = 0 \tag{7}$$
$$l t_i^s q_i^{*s} x_i^o + r_i^{as} - 2k^a x_i^a = 0 \tag{8}$$

Where q_i^{*s} is given by (4).

2.2 The Socially Optimal Emission Tax and R&D Subsidies

At the first stage, by using the expressions of the optimal production quantity and R&D levels for firms determined at the third and second stages, each regulator i maximizes his social welfare given by (2) with respect to t_i^s, r_i^{os} and r_i^{as}. However, this direct method is not easy to do if the regulator looks directly for the optimal per-unit emission tax and per-unit R&D subsidies. Therefore, we will use a simpler method. Indeed, the regulator will choose the socially optimal production and the R&D levels, respectively in the third and second stages. Then, he determines the non-cooperative socially optimal

[4] The second-order conditions are verified in the Appendix when k^o and k^a are high enough.

emission tax and R&D subsidies by equalizing the obtained socially optimal quantities to those optimal for his firm. The model solved as if it was a two-stage game[5].

Expression (1) is equivalent to:

$$W_i^s = \frac{b}{2}q_i^2 - \alpha\left[1 - x_i^o - (\beta + lx_i^a)x_j^o\right]q_i - \gamma\left[1 - x_j^o - \left(\beta + lx_j^a\right)x_i^o\right]q_j + p_i(q_i)q_i - \theta q_i - k^o(x_i^o)^2 - k^a(x_i^a)^2 \tag{9}$$

In the third stage, when regulator i chooses his socially optimal production quantity, the parameter γ is eliminated by the derivation of W_i^s with respect to q_i. Then, the cross border pollution is not totally internalized. However, when he chooses his optimal level of original research in the second stage, the negative transboundary externality is partially internalized if the parameters of learning and R&D spillovers are different from zero. The higher are the absorptive and R&D spillovers, the more important the internalization of transboundary pollution is.

The third stage first-order condition of regulator i is:

$$\frac{\partial W_i^s}{\partial q_i} = 0 \tag{10}$$

Solving of (10) yields:

$$\hat{q}_i^s = \frac{a - \theta - \alpha\left[1 - x_i^o - (\beta + lx_i^a)x_j^o\right]}{b} \tag{11}$$

The symmetric expression of (11) is:

$$\hat{q}_i^s = \frac{a - \theta - \alpha[1 - (1 + \beta + lx_i^a)x_i^o]}{b} \tag{12}$$

The above production quantities are positive if and only if:

$$a - \theta > \alpha \tag{13}$$

Thus, the marginal domestic damage cost of pollution is lower than the maximum willingness to pay for the good minus its marginal cost of production.

The second stage first-order conditions of regulator i are[6]:

$$\frac{dW_i^s}{dx_i^o} = \frac{\partial \hat{q}_i^s}{\partial x_i^o}\frac{\partial V_i^s}{\partial q_i} + \frac{\partial \hat{q}_j^s}{\partial x_i^o}\frac{\partial V_i^s}{\partial q_j} + \frac{\partial W_i^s}{\partial x_i^o} = 0 \tag{14}$$

$$\frac{dW_i^s}{dx_i^a} = \frac{\partial \hat{q}_i^s}{\partial x_i^a}\frac{\partial V_i^s}{\partial q_i} + \frac{\partial \hat{q}_j^s}{\partial x_i^a}\frac{\partial V_i^s}{\partial q_j} + \frac{\partial W_i^s}{\partial x_i^a} = 0 \tag{15}$$

In equilibrium, the equations system (14)-(15) is simplified, and the symmetric solutions are given by:

$$b[\alpha + \gamma(\beta + lx_i^{as})]\hat{q}_i^s - \alpha\gamma(\beta + lx_i^{as})[1 - (1 + \beta + lx_i^{as})x_i^{os}] - 2bk^o x_i^{os} = 0 \tag{16}$$

$$\alpha l\hat{q}_i^s x_i^{os} - 2k^a x_i^{as} = 0 \tag{17}$$

When we substitute $\hat{q}_i^s$ by its symmetric equation, the equations (16)-(17) yield:

$$(a - \theta)[\alpha + \gamma(\beta + lx_i^{as})] - \alpha[\alpha + 2\gamma(\beta + lx_i^{a})][1 - (1 + \beta + lx_i^{as})x_i^{os}] - 2bk^o x_i^{os} = 0 \tag{18}$$

$$\alpha l x_i^{os}[a - \theta - \alpha + \alpha(1 + \beta + lx_i^{as})x_i^{os}] - 2bk^a x_i^{as} = 0 \tag{19}$$

The nonlinear system (18)-(19) verifies that when the parameters of learning and free spillover are equal to zero $(l = 0, \beta = 0)$, then γ does not appear from (18) and (19), transboundary pollution is partially internalized. Consequently, we can get the expressions of $\hat{x}_i^{os}$ and $\hat{x}_i^{as}$ explicitly. When $l \neq 0$ and $\beta \neq 0$, the higher are l and β, the more important the internalization of transboundary pollution is. Thus, we summarize in the following.

Proposition 1. *The investment in absorptive research and the R&D spillovers allow competing countries to better internalize transboundary pollution. The higher are R&D spillovers and the ability to absorb, the greater the transboundary pollution is internalized.*

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[5] These two stage games are standards since the model is symmetric, see Gibbons (1992).

[6] The second-order conditions are verified in the Appendix when k^o and k^a are high enough.

Resolving of the nonlinear system (18)-(19) yields the symmetric socially optimal R&D levels denoted by $\hat{x}_i^{os}$ and $\hat{x}_i^{as}$. We are not able to find the explicit solutions. However, we shall prove the existence of couple solutions which is unique. Thus, it is possible to establish the following result.

Proposition 2. *When k^o and k^a are sufficiently high, there is a unique couples of real solutions $\hat{x}_i^{os} > 0$ and $\hat{x}_i^{as} > 0$ that solve the nonlinear system (18)-(19).*

Proof. See the Appendix.

Notice that the inequality (13) and the assumption 1 ensure that the socially optimal levels of original and absorptive research, production, and pollution are positive, and that $0 \leq \beta + lx_i^a \leq 1$, when k^o and k^a are high enough.

The socially optimal original and absorptive research and production levels which are $\hat{x}_i^{os}$, $\hat{x}_i^{as}$ and $\hat{q}_i^s$ are decentralized by the use of the emission tax and the R&D subsidies, then from equations (4), (7), and (8) we obtained the optimal emission tax and R&D subsidies:

$$t_i^s = \frac{a - \theta - 2b\hat{q}_i^s}{1 - \left(1 + \beta + l\hat{x}_i^{as}\right)\hat{x}_i^{os}} \tag{20}$$

$$r_i^{os} = 2k^o\hat{x}_i^o - t_i^s\hat{q}_i^s \tag{21}$$

$$r_i^{as} = 2k^a\hat{x}_i^{as} - lt_i^s\hat{q}_i^s\hat{x}_i^{os} \tag{22}$$

Proposition 3. *Under autarky, each regulator can induce their firms to reach the non-cooperative socially optimal levels of production and R&D by using the three regulatory instruments, which are a per-unit emission tax, a per-unit original research subsidy, and a per-unit absorptive research subsidy.*

The proposition shows that necessity of the three regulatory instruments to push firms to implement the first best level of production and R&D.

Substituting (12) into (20) and using the assumption 1, we obtain:

$$\lim_{k^o,k^a \to +\infty} t_i^s = 2\alpha - (a - \theta) < 0 \leftrightarrow \frac{a - \theta}{2} > \alpha \tag{23}$$

Consider the case when k^o and k^a are high enough. Thus, when the marginal damage cost of pollution is sufficiently low, the tax is negative meaning that each regulator actually subsidizes pollution (or production because they are proportional) to correct the monopoly distortion.

From (18) and (19), we have:

$$\lim_{k^o,k^a \to +\infty} k^o\, \hat{x}_i^{os} = \frac{\alpha(a - \theta - \alpha) + \gamma\beta(a - \theta - 2\alpha)}{2b} \tag{24}$$

$$\lim_{k^o,k^a \to +\infty} k^a\, \hat{x}_i^{as} = 0 \tag{25}$$

Substituting (24) and (25) in (21) and (22), we get:

$$\lim_{k^o,k^a \to +\infty} r_i^{os} = \frac{(a - \theta - \alpha)^2 + \gamma\beta(a - \theta - 2\alpha)}{b} \tag{26}$$

$$\lim_{k^o,k^a \to +\infty} r_i^{as} = 0 \tag{27}$$

Then we state the following proposition.

Proposition 4. *If a-θ>2α, then when the investment-cost parameters are sufficiently high, the per-unit R&D subsidy for inventive research is higher than the one for absorptive research.*

Consider the case where k^o and k^a are high enough. When the marginal disutility of pollution is high enough, the regulator actually taxes the investment in original research because firms are tempted to overinvest in original research .On the other hand, when α is sufficiently low, the pollution is really subsidized, which may incite firms to underinvest in original research; to remedy this, the regulator subsidizes the investment in original research.

3. Common Market

When markets are opened to international competition, both firms produce homogeneous goods sold on the common markets.

The inverse demand function is $p\left(q_i, q_j\right) = a - \frac{b}{2}\left(q_i + q_j\right)$.

The firms' profits are $\pi_i^c = p\left(q_i, q_j\right)q_i - \theta q_i - k^o(x_i^o)^2 - k^a(x_i^a)^2$ and their net profits are $V_i^c = \pi_i^c - t_i^c E_i + r_i^{oc}x_i^o + r_i^{ac}x_i^a$, with t_i^c is the emission tax per-unit of pollution, r_i^{oc} is the subsidy per-unit of original R&D level and r_i^{ac} is the subsidy per-unit of absorptive R&D level.

As for the autarky case, we make the following assumption.

Assumption 2. $lim_{k^o,k^a \to \infty} x_i^{oc} = lim_{k^o,k^a \to \infty} x_i^{ac} = 0$

Total consumers' net surplus of the two symmetric countries is then given by:

$$CS_i^c = \int_0^{q_i+q_j} p\,(u)du - p(q_i,q_j)q_i = \frac{b}{8}(q_i+q_j)^2$$

Under common market, social welfare function in country i is:

$$W_i^c(q_i,q_j,x_i^o,x_j^o,x_i^a,x_j^a) = CS_i^c - D_i + \pi_i^c \tag{28}$$

3.1 The firms' Behavior

Each firm maximizes its net profit of taxes and subsidies with respect to its output, original and absorptive research levels in the third and second stage, respectively.

The first-order conditions of the firm's third stage are:

$$\frac{\partial V_i^c}{\partial q_i} = \frac{\partial V_i^c}{\partial q_j} = 0 \tag{29}$$

The resolution of system (29) yields:

$$q_i^{*c} = \frac{2\left(a-\theta+t_j\left[1-x_j^o-\left(\beta+lx_j^a\right)x_i^o\right]-2t_i\left[1-x_i^o-\left(\beta+lx_i^a\right)x_j^o\right]\right)}{3b} \tag{30}$$

The partial derivatives of the above expression are:

$$\frac{\partial q_i^{*c}}{\partial x_i^o} = \frac{2t_i[2-(\beta+lx_i^a)]}{3b};\frac{\partial q_i^{*c}}{\partial x_i^a} = \frac{4t_i l x_i^o}{3b}$$

$$\frac{\partial q_i^{*c}}{\partial x_j^o} = \frac{2t_i[2(\beta+lx_i^a)-1]}{3b};\frac{\partial q_i^{*c}}{\partial x_j^a} = -\frac{2t_i l x_i^o}{3b}$$

Consider the case of a positive emission tax. When a firm increases its level of original research (or a level of absorptive research), then its emission ratio decreases enabling it to expand its production. When the competing firm increases its original research, this has tow opposite effects on the production of the firm: because of R&D spillovers and absorptive capacity, the emission ratio of firm decreases enabling it to expend its production; the second effect is a negative one, thus obliging the firm to diminish its production because the competing one can increases its production due to the decrease of its emission/output ratio. When β and/or l are high enough, the first positive effect dominates. When the competing firm increases its absorptive capacity, its emissions ratio diminishes enabling it to expend its production, which obliges the firm to reduce its production.

The symmetric expression of (30) is:

$$q_i^{*c} = \frac{2(a-\theta-t_i[1-(1+\beta+lx_i^a)x_i^o])}{3b} \tag{31}$$

The first-order conditions of firm's second stage are:

$$\frac{dV_i^c}{dx_i^o} = \frac{\partial q_i^{*c}}{\partial x_i^o}\frac{\partial V_i^c}{\partial q_i} + \frac{\partial q_j^{*c}}{\partial x_i^o}\frac{\partial V_i^c}{\partial q_j} + \frac{\partial V_i^c}{\partial x_i^o} = 0 \tag{32}$$

$$\frac{dV_i^c}{dx_i^a} = \frac{\partial q_i^{*c}}{\partial x_i^a}\frac{\partial V_i^c}{\partial q_i} + \frac{\partial q_j^{*c}}{\partial x_i^a}\frac{\partial V_i^c}{\partial q_j} + \frac{\partial V_i^c}{\partial x_i^a} = 0 \tag{33}$$

In equilibrium, solving the system (32)-(33) yields:

$$\frac{3}{2}[2-(\beta+lx_i^{ac})]t_i^c q_i^{*c} + r_i^{oc} - 2k^o x_i^{oc} = 0 \tag{34}$$

$$\frac{3}{4}lt_i^c q_i^{*c} x_i^{oc} + r_i^{ac} - 2k^c x_i^{ac} = 0 \tag{35}$$

Where q_i^{*c} is given by (31).

The equations system contains two equations and two unknown variables which are the original and absorptive research levels (x_i^{oc} and x_i^{ac}).

3.2 The Socially Optimal Emission Tax and R&D Subsidies

Given that the socially optimal per-unit emission-tax and per unit R&D subsidies are reached in the first stage, regulators determine the socially optimal production, original and absorptive research levels in the third and second stages, respectively. Then, by equalizing the socially optimal quantities obtained to those chosen by the taxed and subsidized firm, they determine the socially optimal per-unit emission tax and per-unit R&D subsidies.

The first-order conditions of the regulators third stage are:

$$\frac{\partial W_i^c}{\partial q_i} = \frac{\partial W_j^c}{\partial q_j} = 0 \tag{36}$$

The resolution of system (36) yields:

$$\hat{q}_i^c = \left(\frac{2(a-\theta-\alpha) + \alpha x_i^o\left[3-(\beta+lx_i^a)x_j^o\right] + \alpha x_j^o[3(\beta+lx_i^a)-1]}{2b}\right) \tag{37}$$

The transboundary pollution is not effectively internalized because the above quantity does not depend on the marginal damage of the foreign pollution (γ).

The symmetric production quantities are given by expression (37) is:

$$\hat{q}_i^s = \frac{a-\theta-\alpha[1-(1+\beta+lx_i^a)x_i^o]}{b} \tag{38}$$

The socially optimal production quantity is positive if and only if:

$$a-\theta > \alpha \tag{39}$$

The first-order conditions of the regulator's second stage are[7]:

$$\frac{dW_i^c}{dx_i^o} = \frac{\partial \hat{q}_i^c}{\partial x_i^o}\frac{\partial V_i^c}{\partial q_i} + \frac{\partial \hat{q}_j^c}{\partial x_i^o}\frac{\partial V_i^c}{\partial q_j} + \frac{\partial W_i^c}{\partial x_i^o} = 0 \tag{40}$$

$$\frac{dW_i^c}{dx_i^a} = \frac{\partial \hat{q}_i^c}{\partial x_i^a}\frac{\partial V_i^c}{\partial q_i} + \frac{\partial \hat{q}_j^c}{\partial x_i^q}\frac{\partial V_i^c}{\partial q_j} + \frac{\partial W_i^c}{\partial x_i^a} = 0 \tag{41}$$

In equilibrium, the equations (40)-(41) are simplified by using (36), and the symmetric solutions are given by:

$$2b[\alpha+\gamma(\beta+lx_i^{ac})]\hat{q}_i^c - \alpha\gamma[3(\beta+lx_i^{ac})-1][1-(1+\beta+lx_i^{ac})x_i^{oc}] - 4bk^o x_i^{oc} = 0 \tag{42}$$

$$2b\alpha l\hat{q}_i^c x_i^{oc} + \alpha\gamma l[1-(1+\beta+lx_i^{ac})x_i^{oc}]x_i^{oc} - 4bk^a x_i^{ac} = 0 \tag{43}$$

By substituting $\hat{q}_i^c$ by its expression, the system (42)-(43) are equivalent to:

$$\begin{aligned}2(a-\theta)&[\alpha+\gamma(\beta+lx_i^{ac})] \\ &- \alpha\gamma[5\gamma(\beta+lx_i^{ac})+2\alpha-\gamma][1-(1+\beta+lx_i^{ac})x_i^{oc}] \\ &- 4bk^o x_i^{oc} = 0\end{aligned} \tag{44}$$

$$\alpha l x_i^{oc} + (2(a-\theta)+(\gamma-2\alpha)[1-(1+\beta+lx_i^{ac})x_i^{oc}]) - 4bk^a x_i^{ac} = 0 \tag{45}$$

From equations system (44)-(45), we observe that if $l=0$ and $\beta=0$, transboundary pollution is relatively internalized since γ appears from (44) but not from (45). Consequently, we can find $\hat{x}_i^{oc}$ and $\hat{x}_i^{ac}$ so explicit. When $l \neq 0$ and $\beta = 0$, the internalization of transboundary pollution is very important.

We can thus state the following.

Proposition 5. *The competition of firms on the common market enables non-cooperating countries to better internalize transboundary pollution. The higher β and/or l are, the greater is the transboundary pollution internalized.*

As in autarky, the resolution of the nonlinear equations system (44)-(45) contains two equations with two unknown variables which are the symmetric socially optimal R&D levels denoted by $\hat{x}_i^{oc}$ and $\hat{x}_i^{ac}$. However, we will prove the existence of couple solutions which is unique. Indeed, we have the following proposition.

Proposition 6. *When k^o and k^a are sufficiently high, there exists a unique couples of real solutions $\hat{x}_i^{oc}$ and $\hat{x}_i^{ac}$ that solve the non-linear equations system given by (44) and (45).*

Proof. See Appendix.

For determine the socially optimal emission tax and R&D subsidies, using (31), (34) and (35).

$$t_i^c = \frac{2(a-\theta)-3b\hat{q}_i^c}{2\left[1-\left(1+\beta+l\hat{x}_i^{ac}\right)\hat{x}_i^{oc}\right]} \tag{46}$$

[7] When k^o and k^a are sufficently high, we show in the Appendix that function W_i^s and W_i^c are strictly concave which imply that there is a unique maximum.

$$r_i^{oc} = 2k^o \hat{x}_i^{oc} - \frac{2}{3}(2 - \beta - l\hat{x}_i^{ac}) t_i^c \hat{q}_i^c \tag{47}$$

$$r_i^{ac} = 2k^a \hat{x}_i^{ac} - \frac{4}{3} l t_i^c \hat{x}_i^{oc} \hat{q}_i^c \tag{48}$$

Therefore, we can then establish the following.

Proposition 7. *Under a common market, regulators can push their firms to implement the non-cooperative socially optimal levels of production and R&D by means of a tax per-unit of pollution, a subsidy per-unit of original research, and a subsidy per-unit of absorptive research.*

By using the assumption 2, expressions (38) and (47), we obtain:

$$\lim_{k^o,k^a \to +\infty} t_i^c = \frac{3\alpha - (a - \theta)}{2} < 0 \leftrightarrow \alpha < \frac{a - \theta}{3} \tag{49}$$

When k^o and k^a are high enough, Thus when the marginal damage cost of pollution is low enough, the emission tax is negative.

Further, from (44) and (45), we have:

$$\lim_{k^o,k^a \to +\infty} k^o \hat{x}_i^{oc} = \frac{2\alpha(a - \theta - \alpha) + \gamma\beta(2a - 2\theta - 5\alpha) + \alpha\gamma}{4b} \tag{50}$$

$$\lim_{k^o,k^a \to +\infty} k^a \hat{x}_i^{ac} = 0 \tag{51}$$

By using (50) and (51) in (47) and (48), we get:

$$\lim_{k^o,k^a \to +\infty} r_i^{oc} = \frac{2(a - \theta - \alpha)\big(3[\alpha(\beta - 1) + \beta\delta] + (2 - \beta)(a - \theta)\big) + 3\gamma\alpha(1 - 3\beta)}{6b} \tag{52}$$

$$\lim_{k^o,k^a \to +\infty} r_i^{ac} = 0 \tag{53}$$

Expression (52) shows that when k^o and k^a are high enough and $a - \theta > 3\alpha$, then $r_i^{ac} > 0$.

Under a common market, we compare the subsidy levels of original and absorptive R&D in the following proposition.

Proposition 8. *When markets are opened to international trade, the per-unit R&D subsidy for original research is greater than the one for absorptive research.*

When k^o and k^a are high enough. Thus, when the investment-cost parameters are sufficiently high, the subsidy for original research is always positive.

4. Separate Markets versus Common Market

In the previous sections, we show that each regulator can induce firms to reach the socially optimal levels of production and R&D by means of the three regulatory instruments, which are a tax the per-unit emission, a subsidy per-unit of original research, and a subsidy per-unit of absorptive research. Then, in this section, we compare the non-cooperative the socially optimal values in autarky and common market.

Let us denote that if there is no cross-border pollution across countries, i.e., γ is nil, then expressions (18)-(19), (44)-(45), (12), (38), (1)and (28) prove that the socially optimal R&D level for original research and for absorptive capacity are equal which means that production, pollution, and social welfare are identical in autarky and common market.

In what follows, we will substitute the marginal damage of the domestic pollution by the marginal damage of the foreign pollution $(\alpha = \gamma)$, so we can be simplified calculations. We suppose that the condition (39) is verified, meaning the investment-cost parameters are sufficiently high and that k^o and k^a are high enough.

Proposition 9. *Opening borders to international competition increases the original research level, absorptive capacity and production, and reduces the emission/output ratio.*

Proof. See the Appendix.

Opening borders leads to a higher level of production is accompanied by diminishing the emission ratio and provided by an increase the level of original research.

Furthermore, when the marginal disutility of pollution is sufficiently high, the better internalization of transborder pollution generated by competition on the common market is realized by a raise of the original research level. Consequently, the absorptive research is significantly increases, which leads to reduce the emission ratio is low.

Proposition 10. *The tax per-unit of emission is higher in common market than in separate markets, while the subsidy per-unit of R&D for original research is lower.*

Proof. See the appendix.

When markets are opened to international competition, the per-unit emission tax increases whereas the per-unit R&D subsidy decreases. These results are interesting because one may think that, to give a competitive advantage to its domestic firm, each regulator reduces the per-unit emission-tax and increases the per-unit production subsidy, when markets are opened to international trade. Ben Youssef (2009) found similar results with a different model in which regulatory instruments are a tax per-unit of pollution and a subsidy per-unit of R&D level.

5. Conclusion

We have developed a non-cooperative three-stage game model composed by two regulator-firm hierarchies in presence of cross-border pollution, the R&D spillovers and the absorptive capacity. We study the impact of the R&D externality, the ability to capture part of original research developed from other firms and international trade on the internalization of the transboundary pollution. Firms have the possibility to invest in original and in absorptive research to reduce their emission/output ratio. Indeed, we evaluate the impact of international competition on the original research, by means the emission-tax and the R&D subsidies.

We show that free R&D spillovers, the investment in absorptive research and the common markets permit competing countries to better internalize transboundary pollution. The higher R&D spillovers and the ability to absorb are, the higher transboundary pollution internalized is. Accordingly, when regulators fully cooperate, then transboundary pollution is totally internalized and the first best outcome may be reached.

More importantly, when markets are opened to international trade, the internalization of transboundary pollution is very important when the regulator increases the tax per-unit of pollution and the profit of the domestic firm reduces by a very important investment in R&D when he diminishes the subsidy per-unit of original research.

Besides, opening borders leads firms to produce more and to invest more in R&D, then to reduce the emission ratio.

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Appendix

In separate markets case, the second-order conditions of the firms second stage

We consider the Hessian matrix:

$$H_V^S = \begin{pmatrix} \frac{d^2V_i^s}{dx_i^{o^2}} & \frac{d^2V_i^s}{dx_i^o x_i^a} \\ \frac{d^2V_i^s}{dx_i^o x_i^a} & \frac{d^2V_i^s}{dx_i^{a^2}} \end{pmatrix}$$

By using the first-order conditions given by (5) and (6), the matrix H_V^S is equivalent to:

$$H_V^S = \begin{pmatrix} f_1 - 2k^o & f_2 \\ f_2 & f_3 - 2k^a \end{pmatrix}$$

where $f_{i,i=1,2,3}$ are polynomial functions in x_i^o and t_i^s.

Since $lim_{k^o,k^a\to\infty}\, x_i^{os}$ and $\lim_{k^o,k^a\to+\infty} t_i^s$ are finite numbers, then f_i take finite values when k^o and k^a tend to $+\infty$:

Therefore, when k^o and k^a are sufficiently high:

a. $\frac{d^2V_i^s}{dx_i^{o^2}} < 0$ and $\frac{d^2V_i^s}{dx_i^{a^2}} < 0$

b. $det\, H_V^S = (f_1 - 2k^o)(f_2 - 2k^a) - f_2^2 > 0$

Thus, we have a maximum.

The second-order conditions of the regulators second stage, under autarky case

We consider the Hessian matrix:

$$H_W^S = \begin{pmatrix} \frac{d^2W_i^s}{dx_i^{o^2}} & \frac{d^2W_i^s}{dx_i^o x_i^a} \\ \frac{d^2W_i^s}{dx_i^o x_i^a} & \frac{d^2W_i^s}{dx_i^{a^2}} \end{pmatrix}$$

By using the first-order conditions given by (14) and (15), the Hessian is equivalent to:

$$H_W^S = \begin{pmatrix} g_1 - 2k^o & g_2 \\ g_2 & g_3 - 2k^a \end{pmatrix}$$

where $g_{i,i=1,2,3}$ are polynomial functions in x_i^{os} and x_i^{as}.

Since, we have the assumption 1 when $lim_{k^o,k^a\to\infty}\, x_i^{os} = lim_{k^o,k^a\to\infty}\, x_i^{as} = 0$, then g_i take finite values when k^o and k^a tend to $+\infty$:

Therefore, when k^o and k^a are sufficiently high:

a. $\frac{d^2W_i^s}{dx_i^{o^2}} < 0$ and $\frac{d^2W_i^s}{dx_i^{a^2}} < 0$

b. $det\, H_V^S = (g_1 - 2k^o)(g_2 - 2k^a) - g_2^2 > 0$

Thus, we have a maximum when k^o and k^a are high enough.

Proof of proposition 2

From (19), we deduce:

$$x_i^{as} = \frac{\alpha l[a - \theta - \alpha + \alpha(1+\beta)x_i^{os}]x_i^{os}}{2bk^a - \alpha^2 l^2 x_i^{os^2}} \tag{54}$$

Expression (18) can be written as:

$$\begin{aligned}\alpha(a - \theta - \alpha) + \gamma\beta(a - \theta - 2\alpha) + \gamma l(a - \theta - 2\alpha)x_i^{as} \\ + [\alpha(1+\beta)(\alpha + 2\gamma\beta) - 2bk^o]x_i^{os} + \alpha l[\alpha + 2\gamma(2\beta + 1)]x_i^{os}x_i^{as} \\ + 2\alpha\gamma l^2 x_i^{as\,2} x_i^{os} = 0\end{aligned} \tag{55}$$

Substituting (54) in (55), and multiplying by $\left(2bk^a - \alpha^2 l^2 x_i^{os^2}\right)^2$, we get a polynomial function of degree 5 in x_i^{os}; $A(x_i^{os}) = 0$. The constant term of A is $4b^2k^{a2}[\alpha(a - \theta - \alpha) + \gamma\beta(a - \theta - 2\alpha)] > 0$ and the coefficient of $(x_i^{os})^5$ is $-2bk^o l^4\alpha^4 < 0$.

We have $A(0) > 0$ and $\lim_{k^o,k^a \to \infty} A(x_i^{os}) = -\infty$, then $A(x_i^{os})$ admits at least one and at most five real and positive roots $\hat{x}_i^{os} > 0$. From expression (55) and inequality (13), we have $\hat{x}_i^{as} > 0$ when k^o and k^a are high enough.

Under a common market, the second-order conditions of the firms second stage

We consider the Hessian matrix:

$$H_V^c = \begin{pmatrix} \frac{d^2V_i^c}{dx_i^{o^2}} & \frac{d^2V_i^c}{dx_i^o x_i^a} \\ \frac{d^2V_i^c}{dx_i^o x_i^a} & \frac{d^2V_i^c}{dx_i^{a^2}} \end{pmatrix}$$

By using the first-order conditions given by (31) and (32), the matrix H_V^c is equivalent to:

$$H_V^c = \begin{pmatrix} f_1 - 2k^o & f_2 \\ f_2 & f_3 - 2k^a \end{pmatrix}$$

where $f_{i,i=1,2,3}$ are polynomial functions in x_i^{oc} and t_i^c.

Since $lim_{k^o,k^a \to \infty} x_i^{oc}$ and $\lim_{k^o,k^a \to +\infty} t_i^c$ are finite numbers, then f_i take finite values when k^o and k^a tend to $+\infty$.

Therefore, when k^o and k^a are sufficiently high:

a. $\frac{d^2V_i^c}{dx_i^{o^2}} < 0$ and $\frac{d^2V_i^c}{dx_i^{a^2}} < 0$

b. $det\, H_V^c = (f_1 - 2k^o)(f_2 - 2k^a) - f_2^2 > 0$

Thus, we have a maximum.

The second-order conditions of the regulators second stage, for the common market case

The Hessian matrix is:

$$H_W^c = \begin{pmatrix} \frac{d^2W_i^c}{dx_i^{o^2}} & \frac{d^2W_i^c}{dx_i^o x_i^a} \\ \frac{d^2W_i^c}{dx_i^o x_i^a} & \frac{d^2W_i^c}{dx_i^{a^2}} \end{pmatrix}$$

By using the first-order conditions given by (39) and (40), the matrix $\mathrm{H_W^S}$ is equivalent to:

$$H_W^c = \begin{pmatrix} g_1 - 2k^o & g_2 \\ g_2 & g_3 - 2k^a \end{pmatrix}$$

where $g_{i,i=1,2,3}$ are polynomial functions in x_i^{oc} and x_i^{ac}.

Since, we have the assumption 2 when $lim_{k^o,k^a \to \infty} x_i^{oc} = lim_{k^o,k^a \to \infty} x_i^{ac} = 0$, then g_i take finite values when k^o and k^a tend to $+\infty$.

Therefore, when k^o and k^a are sufficiently high:

a. $\frac{d^2W_i^c}{dx_i^{o^2}} < 0$ and $\frac{d^2W_i^c}{dx_i^{a^2}} < 0$

b. $det\, H_V^c = (g_1 - 2k^o)(g_2 - 2k^a) - g_2^2 > 0$

Thus, when k^o and k^a are high enough we have a maximum.

Proof of proposition 6

From (45), we deduce:

$$x_i^{ac} = \frac{\alpha l[2(a-\theta) + (\gamma - 2\alpha)(1-(1+\beta)x_i^{oc})]x_i^{oc}}{4bk^a - \alpha l^2(\gamma - 2\alpha)x_i^{oc^2}} \tag{56}$$

Expression (44) can be written as:

$$\begin{aligned}\alpha(2a - 2\theta - 2\alpha - \gamma) + \gamma\beta(2a - 2\theta - 5\alpha) + \gamma l(2a - 2\theta - 5\alpha)x_i^{ac} \\ + [\alpha(1+\beta)(2\alpha + 5\gamma\beta - \gamma) - 4bk^o]x_i^{oc} \\ + 2\alpha l(\alpha + 5\gamma\beta - 2\gamma)x_i^{oc}x_i^{ac} + 5l^2\alpha\gamma x_i^{ac2}x_i^{oc} = 0\end{aligned} \tag{57}$$

Substituting (56) into (54), and multiplying by $\left[4bk^a - \alpha l^2(\gamma - 2\alpha)x_i^{oc^2}\right]^2$, we get a polynomial function of degree 5 in x_i^{oc}; $B(x_i^{oc}) = 0$. The constant term of B is $8b^2k^{a2}[\alpha(2a - 2\theta - 5\alpha - \gamma) + \gamma\beta(2a - 2\theta - 5\alpha)] > 0$ and the coefficient of $(x_i^{oc})^5$ is $-4bk^o(\gamma - 2\alpha)l^4\alpha^2 < 0$.

We have $B(0) > 0$ and $\lim_{k^o,k^a\to\infty} B(x_i^{oc}) = -\infty$, then $B(x_i^{oc})$ admits at least one and at most five real and positive roots $\hat{x}_i^{oc} > 0$. From expression (55) and inequality (39), we have $\hat{x}_i^{as} > 0$ when k^o and k^a are sufficiently high.

Proof of proposition 8

To compare $\lim\limits_{k^o,k^a\to\infty} r_i^{oc}$ and $\lim\limits_{k^o,k^a\to\infty} r_i^{ac}$, we suppose that $\alpha = \gamma$. Then from (52) and (53), we get:

$$\lim_{k^o,k^a\to+\infty} r_i^{oc} = \lim_{k^o,k^a\to+\infty} r_i^{ac} = \frac{2(a-\theta-\alpha)[(2-\beta)(a-\theta) - 3\alpha(1-2\beta)] + 3\alpha^2(1-3\beta)}{6b}$$

When k^o and k^a are sufficiently high, then $\lim_{k^o,k^a\to+\infty} r_i^{oc} > \lim_{k^o,k^a\to+\infty} r_i^{ac}$ when $a - \theta > \alpha$.

Proof of proposition 9

From equations (24) and (49), we obtain $\lim_{k^o,k^a\to+\infty} k^o\,\hat{x}_i^{oc} - \lim_{k^o,k^a\to+\infty} k^a\hat{x}_i^{ac} = \frac{\alpha\gamma(1-\beta)}{4b} > 0$. Since k^o and k^a are high enough, then $\hat{x}_i^{oc} > \hat{x}_i^{os}$ which implies that $\hat{x}_i^{ac} > \hat{x}_i^{as}$ and $e_i^s > e_i^c \leftrightarrow \hat{q}_i^c > \hat{q}_i^s$.

Proof of proposition 10

Taking the equations (23) and (48), and making use of inequality $a - \theta > \alpha$ gives $\lim_{k^o,k^a\to+\infty} t_i^c - \lim_{k^o,k^a\to+\infty} t_i^s = \frac{(a-\theta-\alpha)}{2} > 0$. Since k^o and k^a are high enough, then $t_i^c > t_i^s$.

By using the inequality $a - \theta > \alpha$ and the expressions (26) and (52), we get

$$\lim_{k^o,k^a\to+\infty} r_i^{oc} = \lim_{k^o,k^a\to+\infty} r_i^{os} = \frac{2(a-\theta-\alpha)[(1+\beta)(a-\theta) - 3\alpha\beta] + 3\alpha^2(\beta-1)}{6b}$$

Since k^o and k^a are high enough and when the marginal damage of pollution is sufficiently low, then $r_i^{os} > r_i^{oc}$.