



Dependencies of Parameter and Load Torque Sensitivities of Electric Motor Outputs on Design Requirements

Elektrik Motoru Çıktılarının Parametre ve Yük Torku Duyarlılıklarının Tasarım Taleplerine Bağımlılıkları

Ata Sevinç¹

¹Kırıkkale University, 71451, Kırıkkale, TÜRKİYE

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Abstract

Sensitivity analysis is useful for parameter estimation and decision-making processes. It guides researchers in which input is more effective for controlling an output and which output provides more information to estimate a system parameter. This study is concerned with the sensitivities of the measurable states of electric motors to the model parameters and load torque. Its contribution is investigating the sensitivities' dependencies on design requirements. It has been revealed that some design requirements such as rated voltage, power, and speed have no effect on most sensitivities to the motor parameters. A similar study on transformers is also included with similar results. The results evoke some optimization choices about adjusting the sensitivities since the search space dimension is reduced. They also yield some educational benefits. Since such a theoretical study requires precise measurements or disturbance-free information, experimental work could not be very useful. Therefore, either mathematical model analysis or simulations have been used to calculate the sensitivities. For the same reason, design requirements are considered from the algorithms determining the model parameters of electric machinery according to desired operating conditions for simulation purposes. Dependencies have been found analytically for dc motors but numerically for ac motors, and transformers.

Key Words

"Sensitivity analysis, dc motors, ac motors, transformers, parameter estimation."

Öz

Duyarlılık analizi, parametre tahmini ve karar verme süreçleri için yararlıdır. Bu, bir çıkışı kontrol etmek için hangi girişin daha etkili olduğu, ve bir sistem parametresini tahmin etmek için hangi çıkışın daha fazla bilgi sağladığı konularında araştırmacıları yönlendirir. Bu çalışma, elektrik motorlarının ölçülebilir durum değişkenlerinin, model parametrelerine ve yük torkuna olan duyarlılıklarıyla ilgilenmektedir. Çalışmanın katkısı, duyarlılıkların tasarım taleplerine olan bağımlılıkları araştırmasıdır. Nominal gerilim, güç ve hız gibi bazı tasarım taleplerinin çoğunun, motor parametrelerine olan duyarlılıkların çoğunu etkilemediği ortaya çıkarılmıştır. Transformatörler üzerinde yapılan benzer bir çalışma da sunulmuş olup, benzer sonuçlar bulunmuştur. Sonuçlar, arama uzayının boyutu azaltıldığı için, duyarlılıkların ayarlanmasıyla ilgili bazı optimizasyon seçeneklerine yol açmaktadır. Aynı zamanda eğitime yönelik bazı faydalar da sağlar. Böyle teorik bir çalışma hassas ölçümler veya gürültüsüz bilgi gerektirdiğinden, deneysel çalışma pek faydalı olmazdı. Bu nedenle, duyarlılıkları hesaplamak için ya matematiksel model analizi ya da simülasyonlar kullanılmıştır. Aynı nedenden dolayı, simülasyon amaçları için istenen çalışma koşullarına göre elektrik makinelerinin model parametrelerini belirleyen algoritmalarındaki tasarım talepleri dikkate alınmıştır. Bağımlılıklar doğru akım motorları için analitik olarak, alternatif akım motorları ve transformatörler için ise sayısal olarak bulunmuştur.

Anahtar Kelimeler

"Duyarlılık analizi, dc motorlar, ac motorlar, trafolar, parametre tahmini"

1. Introduction

Even though many works in various fields use various sensitivity definitions, their analyses are conceptually similar. Sensitivity analysis is considering the relative effects of each factor on a variable together with the relative effects of the others. In this way, we see which parameter or input should be changed in which direction to achieve a desired variation in the target variable with minimum changes in the others. This helps us make decisions in many areas. As a decision-making model is an approximation of the real problem, a decision must be robust to uncertainties and this is also a subject of the sensitivity analysis. Therefore, sensitivity analysis has a wide range of uses in decision-making processes (Bujoreanu, 2011) in a variety of fields, including economics, engineering, and biology.

Basically, the sensitivity analysis is important for sensors used for measurements. Özçelik (2018) made a comparative sensitivity study on the optical sensitivities of phototransistor and photodiodes for the light control and measurement systems. The current responses are compared while the light intensity is varying.

The sensitivity analysis has been used for risk prioritization of failure mode and effect analysis (Ványi&Pokorádi, 2018). Cenikli and Akgüngör (2020) investigated into the effects of 5 traffic variables on traffic accidents with a sensitivity analysis based on factorial design method. Pastura *et al.* (2020) conducted a study on the sensitivity of the voltage distribution among the turns of the stator winding in an electric motor designed for aerospace use and supplied by wide band gap converters. They observed that non-uniform potential distributions are very sensitive to the inverter bus voltage magnitude and time-derivative of the voltage across the windings, but not to the switching frequency and duty cycle of the inverter. Identification of sensitive variables in control of exact tracking error dynamics with frequency response analysis has been shown to improve the controller's performance (Srinivasan *et al.* 2020). Considering each state variable's variation before and after the application of each switching command, a cycle state-variable sensitivity matrix has been derived for better control of a dc/dc converter (Wong *et al.* 2000). The sensitivity analysis is also useful to obtain nonsmooth limit cycles' characteristic multipliers, which determine the stability of the cycles on a compass gait biped robot (Hiskens, 2001). In another study (Gupta&Patra, 2005), the switching scheme of a dc/dc buck converter, which is based on energy, has been shown to be free from chaos with sensitivity analysis and phase plane analysis.

Although decoupling in multiple-input multiple-output control systems, such as the study of Brandstetter *et al.* (2017), is a similar problem, the sensitivity analysis is mostly concerned with the effects of the parameters. Parameter sensitivity can also be utilized for the stability of a dynamic system. Lima and Fernandes (2000) found the relations between the stability robustness of a 9-machine power system and the eigenvalue sensitivity and/or logarithmic sensitivity of the system matrix. Two indexes derived from them present reliable information about the stability and robustness of the system. Kazerooni and Tsay (1988) obtained the initial compliance of two robots by using a sensitivity function for their stability. Mert *et al.* (2020) made a sensitivity analysis to reveal the effects of the flowrate of the water inlet and low pressure drum vapor fraction on the outlet gas temperature; and the effects of the flowrate of the water inlet, low, intermediate, high steam pressures on the power produced. They found that high and intermediate pressures are essential for the produced power, and their effects to increase the exergy efficiency. Raturi *et al.* (2021) investigated the sensitivities of the daily heat gain of N similar partly enclosed photovoltaic thermal flat plate collectors with a series connection, with respect to the number of collectors, the angle of inclination, mass flow rate, and packing factor. They found that the sensitivities are ordered from largest to smallest, as listed here. Choudhary *et al.* (2023) analyzed the sensitivities of a solid oxide fuel cell variables to the compression ratio and turbine inlet temperature. They found the hybrid cycle performance is more sensitive to the turbine inlet temperature than the compression ratio.

Which measurement would be useful to estimate a system parameter requires sensitivity analysis especially if another parameter is also uncertain. Hung (2001) has used sensitivity analysis to get the gradient used in the steepest descent method for motor parameter estimations. Estimation of some important parameters in the integrated power system was also implemented with such an analysis (Prempraneerach *et al.* 2008). More than 30 parameters with perturbation as large as 50% were estimated with a gradient-based sensitivity analysis technique. Knudsen and Jensen (1995) defined a sensitivity approach for the predictability of parameters of nonlinear physical systems. It showed that dc servo motor parameters can be predicted precisely in the case of low parameter correlation and high parameter sensitivity. The approach was also used to define good input signal design. Feng *et al.* (2006) reduced unwanted sensitivity in a buck converter control by using a feedforward compensator. Li *et al.* (2020) analyzed the sensitivities of parameters defined for fast charging, energy and power states in a battery management system based on an electrochemical model for its terminal voltage and some other essential states. Apart from assessing which parameters' accuracies are crucial for the main functionalities, they found the identifiability of each parameter.

Apart from control inputs and parameters, some disturbance inputs may exist to estimate, for example, disturbance torque in motor control. Grignon *et al.* (2014) handled the problem of increasing the sensitivity of the error signal used by the estimator to the load torque disturbance while reducing the sensitivity to the other uncertainties in a dc motor. They designed a filter with high sensitivity to disturbance torque but low sensitivity to the other parameter changes with a method originally developed for fault detection (Liu&Zhou, 2007; Li&Zhou, 2009). Similar filters were defined in the studies of Prakosa *et al.* (2021) and Prakosa *et al.* (2022) to achieve the stability of a dc motor with disturbance and uncertainty by using the mixed-sensitivity synthesis method (Amin&Aijun, 2017). Rodriguez *et al.* (2022) modified an existing pseudo-global sensitivity analysis method to identify the static calculation model inputs that have the highest impact on the output estimations. They applied it to address the uncertainties in small power load estimations for

building energy audit calculations in order to help in choosing the best energy calculation model among different building scenarios and enables the auditors to focus on specific data collection to reduce the estimation uncertainty.

Design problems may also need sensitivity analysis. For this purpose, sensitivities of the desired quantities to the design criteria can be considered. Ribes-Mallada *et al.* (2011) analyzed the effects of small perturbations in design criteria for optimum performance of dc-dc converters. To change the armature winding of a dc motor, sensitivities of inner voltage drop and losses to winding parameters were used in the study of Karami-Shahnani *et al.* (2021). Their algorithm requires sensitivity analysis for the optimum in the first iteration. In the work of Boglietti *et al.* (2004), thermal sensitivities to design parameters have been used to obtain smaller totally enclosed fan-cooled induction motors. The design was shown very sensitive to the cooling air speed and heat transfer coefficient. Denizhan and Chew (2018) used sensitivity analysis for the optimized design of automotive engine hoods. They observed the changes in the optimization objective function while changing the bounds for some design parameters. However, they found that even though the changes in the bounds cause the design parameter values to change, their effects on the objective function is negligible. Mahmouditabar *et al.* (2020) optimized the design parameters of a permanent magnet motor of the flux switching type. They utilized the sensitivity analysis of the dimensional parameters of electromagnetic torque and signal to noise ratio.

The works mentioned here mostly utilize the sensitivities as fixed features of the system around operating points. This paper is concerned with the sensitivities at the design stage, either to achieve desired sensitivities or to see which sensitivities depend on which design requirements. Determining the model parameters for desired operating conditions for simulation purposes (Sevinç, 2019; Online Electric Motor and Transformer Design for Simulation Purposes, 2019) can also be considered a kind of design. This paper focuses on the sensitivities of the steady-state values of the measurable variables, such as currents and speed to the model parameters and load torque at the rated operating point for permanent-magnet (PM) dc motors and synchronous motors in the context of that type of design. A similar analysis for transformers is also comprised of the sensitivities of the steady-state primary current and secondary voltages to the transformer parameters. Determining the model parameters as in the work of Sevinç (2019), these sensitivities can be considered as functions of the requirements. The purpose of the article is to show on which requirements the sensitivities depend. Chang (2014) also considers design sensitivity analysis with the difference that constraint and objective functions' sensitivities to the design variables are calculated. This study, on the other hand, considers the design requirements in the context of dependencies, not directly sensitivities to them. The sensitivities here are calculated from the outputs to the parameters of the electric motors and transformers, not to the user requirements.

I need to clear up a misunderstanding: The algorithms in the work of Sevinç (2019) are certainly not parameter estimation algorithms. There are no motor parameters to estimate at the beginning of the algorithms. There are just user requirements about operating points. Those algorithms (Sevinç, 2019) result in what parameters satisfy the requirements similar to the design algorithms. That is why I mention them as a kind of design. However, I admit that they are not manufacturing-level designs giving physical features such as slot shapes, type of magnets or iron sheets, number of turns, and cross-sectional area of the windings.

The sensitivity calculations and dependency investigations are done analytically for PM dc motors due to their simplicity, but numerically for induction motors, PM synchronous motors (PMSM), wound-rotor synchronous motors (WRSM), and transformers due to the complicated stages to determine their parameters according to the desired operating conditions. There is no experimental work in this work for two reasons: First, the sensitivities are calculated better in disturbance-free conditions. Moreover, to detect their independencies accurately, disturbance-free conditions are necessary. Second, the independency of a design requirement could be detected by experimentally re-designing the motor after a change in that requirement. Therefore, such experimental work needs about 10 designs for each kind of motor. Obviously, such expensive work would not be reasonable just for detecting the independencies.

The benefits of the findings of this study are discussed. As the methods in the work of Sevinç (2019) for simulation purposes, this study is also concerned with the models used in simulations.

2. Definition

Sensitivity definitions vary widely among various fields. The definition which will be used in this paper is the relative sensitivity of a variable to a parameter or an independent variable in a dynamic system. The sensitivity of a variable v to a parameter or an independent variable p is defined as

$$S(v, p) = \frac{\partial v/v}{\partial p/p} = \frac{p}{v} \frac{\partial v}{\partial p} \quad (1)$$

(Hayward&Cruz-Hernández, 1998). It means that a very small proportional change in p corresponds to a proportional change in v that approximately equals the sensitivity times the change ratio in p while keeping all the other parameters constant. For example, if $S(v, p) = 0.4$, $p = 3.00$ and $v = 7.00$; and if all the other parameters remain the same, making $p = 3.15$ (5% increase) results in $v \approx 7.14$, which means $0.4 \times 5\% = 2\%$ increase approximately. It is also possible to express the sensitivity definition in (1) as logarithmic sensitivity (Hayward&Cruz-Hernández, 1998):

$$S(v, p) = \frac{\partial(\log v)}{\partial(\log p)} \quad (2)$$

Sensitivity can also be negative, which means that v decreases as p increases. It is not usually constant and may differ depending on many variables and parameters. We are usually concerned with the vicinity of a particular range. Even though (1) is said to be parameter sensitivity, the same definition can also be used if p is a variable such as a load torque. Furthermore, without concern with cause-effect distinction,

$$S(v, p) = \frac{1}{S(p, v)} \quad (3)$$

by the definition. i.e., the more sensitivity of v to p , the less sensitivity of p to v .

3. Sensitivities for DC Motors

The well-known PM dc motor model is

$$\begin{bmatrix} \dot{i}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -K_b/L_a \\ K_b/J_i & -B_f/J_i \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} 1/L_a \\ 0 \end{bmatrix} v_a - \begin{bmatrix} 0 \\ 1/J_i \end{bmatrix} T_L \quad (4)$$

where the parameters R_a and L_a are the armature resistance and inductance, K_b is the back emf or torque constant, B_f is the friction constant and J_i is the inertia; and the variables v_a and i_a are the applied armature voltage and current, ω is the angular rotor speed in rad/s, T_L is the load torque. The electrical equation of the motor is shown as an equivalent circuit in Figure 1.

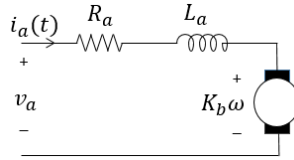


Figure 1. DC servo motor's equivalent circuit.

As the derivatives in (4) are zero at the equilibrium, the steady-state values are

$$i_a = \frac{B_f v_a + K_b T_L}{R_a B_f + K_b^2} \quad (5)$$

$$\omega = \frac{K_b v_a - R_a T_L}{R_a B_f + K_b^2} \quad (6)$$

It is noted that the steady-state values do not depend on L_a and J_i . The armature voltage is known since it is applied by the controller, but the load torque needs to be estimated like the parameters. Therefore, the sensitivities of i_a and ω to R_a , B_f , K_b , and T_L are concerned. Applying (1) to (5) and (6) separately, we find these sensitivities as follows:

$$S(i_a, R_a) = \frac{R_a}{i_a} \frac{\partial i_a}{\partial R_a} = \frac{-R_a B_f}{R_a B_f + K_b^2} \quad (7)$$

$$S(i_a, B_f) = \frac{B_f}{i_a} \frac{\partial i_a}{\partial B_f} = \frac{B_f K_b (K_b v_a - R_a T_L)}{(R_a B_f + K_b^2)(B_f v_a + K_b T_L)} \quad (8)$$

$$S(i_a, K_b) = \frac{K_b}{i_a} \frac{\partial i_a}{\partial K_b} = \frac{-2K_b^2}{R_a B_f + K_b^2} + \frac{K_b T_L}{B_f v_a + K_b T_L} \quad (9)$$

$$S(i_a, T_L) = \frac{T_L}{i_a} \frac{\partial i_a}{\partial T_L} = \frac{K_b T_L}{B_f v_a + K_b T_L} \quad (10)$$

$$S(\omega, R_a) = \frac{R_a}{\omega} \frac{\partial \omega}{\partial R_a} = \frac{-R_a B_f}{R_a B_f + K_b^2} - \frac{R_a T_L}{K_b v_a - R_a T_L} \quad (11)$$

$$S(\omega, B_f) = \frac{B_f}{\omega} \frac{\partial \omega}{\partial B_f} = \frac{-R_a B_f}{R_a B_f + K_b^2} \quad (12)$$

$$S(\omega, K_b) = \frac{K_b}{\omega} \frac{\partial \omega}{\partial K_b} = \frac{-2K_b^2}{R_a B_f + K_b^2} + \frac{K_b v_a}{K_b v_a - R_a T_L} \quad (13)$$

$$S(\omega, T_L) = \frac{T_L}{\omega} \frac{\partial \omega}{\partial T_L} = \frac{-R_a T_L}{K_b v_a - R_a T_L} \quad (14)$$

Although (7)-(14) seem to depend on 3 motor parameters, which are R_a , K_b and B_f , and 2 variables applied to the motor, which are T_L and v_a , the number of dependencies reduces to 2 at the rated operating point in terms of the design criteria used in the works of Sevinç (2019), and Online Electric Motor and Transformer Design for Simulation Purposes (2019). The requirements for the PM dc motor design algorithm for simulation purposes in the work of Sevinç (2019) are the armature voltage (v_a), rotor speed (n), output power (P_o), efficiency (η), mechanical to total loss rate ($k_{ml} = P_f/P_{loss}$, where P_f is the friction loss and P_{loss} is the total loss), electrical time constant (τ_{elc}), and mechanical time constant (τ_{mec}). Now the 3 motor parameters and 2 variables, hence the sensitivities, will be expressed in terms of these requirements.

According to the tabular formulae in the work of Sevinç (2019) giving the algorithm for operating values and model parameters,

$$R_a = P_{Cu}/i_a^2 \quad (15)$$

$$B_f = P_f/\omega^2 \quad (16)$$

$$K_b = P_m/(\omega i_a) \quad (17)$$

$$T_L = P_o/\omega \quad (18)$$

$$v_a = P_i/i_a \quad (19)$$

where P_m is the electromechanical power, P_{Cu} is copper loss and P_i is input power. As i_a and ω terms are canceled, all the sensitivities (7)-(14) can be written in power terms only:

$$S(i_a, R_a) = \frac{-(P_i - P_o - P_f)P_f}{(P_i - P_o - P_f)P_f + (P_o + P_f)^2} \quad (20)$$

$$S(i_a, B_f) = \frac{P_f P_m^2 - P_{Cu} P_f P_m P_o / P_i}{(P_{Cu} P_f + P_m^2)(P_f + P_m P_o / P_i)} \quad (21)$$

$$S(i_a, K_b) = \frac{P_{Cu} P_f P_m P_o - 2 P_f P_m^2 P_i - P_m^3 P_o}{(P_{Cu} P_f + P_m^2)(P_m P_o + P_f P_i)} \quad (22)$$

$$S(i_a, T_L) = \frac{P_m P_o}{P_m P_o + P_f P_i} \quad (23)$$

$$S(\omega, R_a) = \frac{-P_{Cu} P_o}{P_m P_i - P_{Cu} P_o} - \frac{P_f P_{Cu}}{P_f P_{Cu} + P_m^2} \quad (24)$$

$$S(\omega, B_f) = \frac{-P_f P_{Cu}}{P_f P_{Cu} + P_m^2} \quad (25)$$

$$S(\omega, K_b) = \frac{-2 P_m^2}{P_f P_{Cu} + P_m^2} + \frac{P_m P_i}{P_m P_i - P_{Cu} P_o} \quad (26)$$

$$S(\omega, T_L) = \frac{P_{Cu} P_o}{P_m P_i - P_{Cu} P_o} \quad (27)$$

Each power term in (20)-(27) except P_o can be expressed as a coefficient, which will be denoted with symbol c and the same subscript as that power, multiplied by P_o (Sevinç, 2019).

$$c_i = \frac{1}{\eta} \quad , \quad P_i = c_i P_o \quad (28)$$

$$c_f = k_{ml} \left(\frac{1}{\eta} - 1 \right) \quad , \quad P_f = c_f P_o \quad (29)$$

$$c_m = \frac{k_{ml}}{\eta} - k_{ml} + 1 \quad , \quad P_m = c_m P_o \quad (30)$$

$$c_{Cu} = k_{ml} - \frac{k_{ml}}{\eta} + \frac{1}{\eta} - 1 \quad , \quad P_{Cu} = c_{Cu} P_o \quad (31)$$

Canceling all P_o terms yields that sensitivities (20)-(27) can be expressed in terms of these coefficients only:

$$S(i_a, R_a) = \frac{-(c_i - 1 - c_f)c_f}{(c_i - 1 - c_f)c_f + (1 + c_f)^2} \quad (32)$$

$$S(i_a, B_f) = \frac{c_f c_m^2 - c_{Cu} c_f c_m / c_i}{(c_{Cu} c_f + c_m^2)(c_f + c_m / c_i)} \quad (33)$$

$$S(i_a, K_b) = \frac{c_{Cu} c_f c_m - 2c_f c_m^2 c_i - c_m^3}{(c_{Cu} c_f + c_m^2)(c_m + c_f c_i)} \quad (34)$$

$$S(i_a, T_L) = \frac{c_m}{c_m + c_f c_i} \quad (35)$$

$$S(\omega, R_a) = \frac{-c_{Cu}}{c_m c_i - c_{Cu}} - \frac{c_f c_{Cu}}{c_f c_{Cu} + c_m^2} \quad (36)$$

$$S(\omega, B_f) = \frac{-c_f c_{Cu}}{c_f c_{Cu} + c_m^2} \quad (37)$$

$$S(\omega, K_b) = \frac{-2c_m^2}{c_f c_{Cu} + c_m^2} + \frac{c_m c_i}{c_m c_i - c_{Cu}} \quad (38)$$

$$S(\omega, T_L) = \frac{c_{Cu}}{c_m c_i - c_{Cu}} \quad (39)$$

Since all the coefficients (28)-(31) depend on only mechanical to total loss rate k_{ml} and efficiency η , then all the sensitivities considered here are independent of the other requirements.

It should be noted that sensitivities (32)-(39) are valid under the desired operating conditions when determining the motor parameters according to [23-24]. Alternatively, under any steady-state operating conditions and with any motor parameters, the sensitivities are found as (32)-(39) after calculating the mechanical to total loss rate k_{ml} and efficiency η . An example to this claim can be shown with the dc motor parameter set in the works of Sevinç (2019), and Online Electric Motor and Transformer Design for Simulation Purposes (2019), which is determined according to $v_a = 100$ V, the speed $n = 2000$ rpm, $P_o = 1500$ W, $\eta = 0.900$, $k_{ml} = 0.5$, $\tau_{elc} = 0.15$ s, $\tau_{mec} = 0.25$ s rating requirements as $R_a = 0.300$ Ω , $L_a = 0.045$ H, $K_b = 0.454$ N·m/A = 0.454 V·s/rad, $B_f = 0.00190$ N·m·s/rad, $J_i = 0.000475$ kg·m². The rated load torque $T_L = 7.16$ N·m and armature current $i_a = 16.7$ A are the other values found with the same requirements.

Now, if we run the motor with completely different values, e.g., $v_a = 80$ V and $T_L = 8.00$ N·m, then the following sensitivities are obtained according to (7)-(14):

$$S(i_a, R_a) = -0.0027624$$

$$S(i_a, B_f) = 0.037437$$

$$S(i_a, K_b) = -1.0347$$

$$S(i_a, T_L) = 0.9598$$

$$S(\omega, R_a) = -0.073585$$

$$S(\omega, B_f) = -0.0027624$$

$$S(\omega, K_b) = -0.9237$$

$$S(\omega, T_L) = -0.070823$$

The operation with that voltage and load torque results in the speed $n = 1568.5$ rpm or $\omega = 164.25$ rad/s, and the armature current $i_a = 18.325$ A in the steady state. It also results in

$$P_i = (80 \text{ V}) \times (16.7 \text{ A}) = 1466.0 \text{ W},$$

$$P_o = (8.00 \text{ N·m}) \times (164.25 \text{ rad/s}) = 1314.0 \text{ W},$$

$$P_{loss} = (1466.0 - 1314.0) \text{ W} = 152.0 \text{ W},$$

$$P_{Cu} = (0.300 \text{ } \Omega) \times (16.7 \text{ A})^2 = 100.7 \text{ W},$$

$$k_{ml} = (152.0 - 100.7) / 152.0 = 0.337 \text{ and}$$

$$\eta = 1314.0 / 1466.0 = 0.896.$$

These k_{ml} and η values, which are different from the design requirements, yield following coefficients substituting them into (28)-(31):

$$\begin{aligned} c_i &= 1.1157 \\ c_f &= 0.039005 \\ c_m &= 1.0390 \\ c_{Cu} &= 0.076667 \end{aligned}$$

Using them in (32)-(39) yields exactly the same sensitivities as calculated from (7)-(14) above. This calculation verifies that all the sensitivities of the armature current and rotor speed of a PM dc motor to its parameters and load torque can be expressed in terms of just two quantities: k_{ml} , which is mechanical to total loss rate, and efficiency η . This claim should not be misunderstood as if the performance parameters of a PM dc motor can be functions of only two parameters. They are just sensitivities, not performance parameters, and the transient performance is not included in this analysis. It should also be emphasized that k_{ml} and η values are not constant and must be calculated for the considered operation.

This fact is useful to decide on a set of dc motor parameters for simulation purposes with optimal sensitivities of steady-state values at the rated operating point. Whatever the voltage, power, and speed of the motor, only a two-dimensional search is sufficient.

For example, let the search intervals be 0.5-0.9 for efficiency and 0.2-0.8 for k_{ml} . Let the purpose be to estimate the load torque from the armature current, and the armature resistance from the speed with the best sensitivities. i.e., $|S(i_a, T_L)/S(i_a, R_a)|$ and $|S(\omega, R_a)/S(\omega, T_L)|$ are required to be as large as possible. These two ratio values over the search grid are shown in Figure 2 and Figure 3 respectively. Although Figure 2 shows that the better efficiency and the larger $|k_{ml} - 0.5|$, the larger $|S(i_a, T_L)/S(i_a, R_a)|$ which eases the estimation of T_L from i_a , Figure 3 shows that the estimation of R_a from ω gets easier with lower choices of η and larger choices of k_{ml} since $|S(\omega, R_a)/S(\omega, T_L)|$ gets larger. To find an optimum choice, a weighted sum of the two sensitivity ratios with a multiplier (70 in the example) that brings both to nearly the same range on the latter is shown in Figure 4. Quite a wide range of coefficient selections instead of 70 gives similar results. As concluded from Figure 4, as k_{ml} is moving away from 0.5 and as η is approaching 1, better sensitivities are achieved to estimate simultaneously T_L from i_a , and R_a from ω . All the three figures are obtained with mesh command of MATLAB.

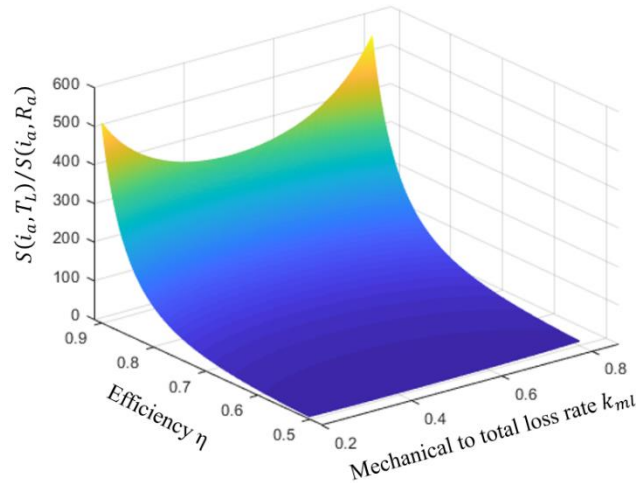


Figure 2. k_{ml} and η search to estimate T_L better from i_a with less deterioration from the change in R_a .

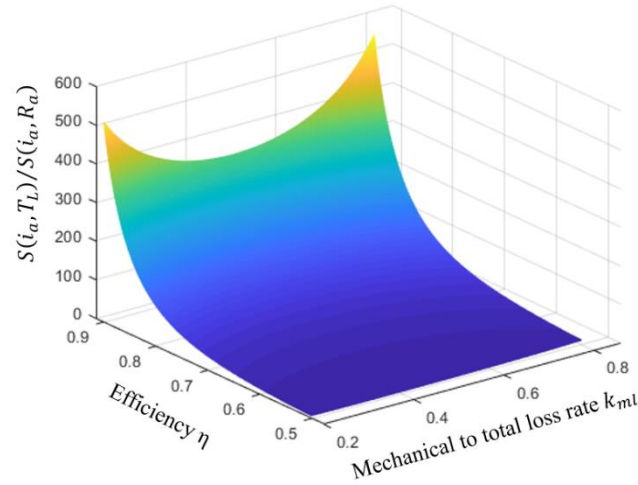


Figure 3. k_{ml} and η search to estimate R_a better from ω with less deterioration from the change in T_L .

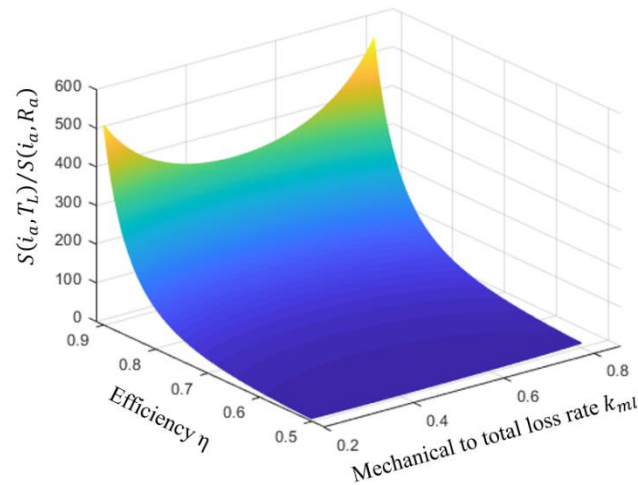


Figure 4. k_{ml} and η search to estimate T_L from i_a , and R_a from ω with less deterioration from each other.

4. Sensitivities for AC Motors

To find analytical expressions for sensitivities of steady-state currents and speed to induction and synchronous motor parameters determined according to desired criteria as in the work of Sevinç (2019) is highly complicated. To show their independence of some requirements analytically, those expressions' derivatives with respect to the requirements need showing as zero. However, taking the derivatives need much more complicated tasks. It takes many pages to show just one of them since the algorithms include some stages or numerical iterations. Alternatively, numerical computations are much faster, and showing the independencies numerically is quite reliable.

In this study, sensitivities of steady-state stator currents and speed to ac motor parameters and load torque are numerically analyzed at the rated operating points in the context of dependence on the design requirements for simulation purposes according to the algorithms given in the work of Sevinç (2019). Each sensitivity is calculated with 0.1% change in the parameter or load torque according to (1) approximately with separate simulations. After calculating all the sensitivities of measurable states to them, only one of the requirements which have been used to determine the model parameters is assigned with an obvious difference. The sensitivities are calculated again with the new parameters in the same way. If the difference is less than 10^{-9} , it is recorded as unchanged and independent of the requirement which has been changed in the repeated calculations. The same procedure is repeated for all the requirements given in the work of Sevinç (2019). As a result of the study, the requirements that can affect the sensitivities are listed.

As the steady-state currents and speeds are concerned, sensitivities to inertia (J_i) are always zero and all the sensitivities are independent of the mechanical time constant (τ_{mec}).

4.1. Induction Motor

There are 10 requirements in the algorithm to determine the parameters of an induction motor in the work of Sevinç (2019). It is better to explain the details of the procedure applied to the induction motor model in this study with numerical assignments. First, a set of motor parameters have been found as in the works of Sevinç (2019), and Online Electric Motor and Transformer Design for Simulation

Purposes (2019) with the following requirements: rms phase voltage $V_{s1}^{rms} = 231$ V, output power $P_o = 3000$ W, efficiency $\eta = 0.85$, rotor speed $n_r = 1470$ rpm, number of pole pair $n_{pp} = 2$, leakage coefficient $\sigma = 0.02$, mechanical time constant $\tau_{mec} = 2$ s, mechanical to total loss rate $k_{ml} = 0.3$, stator's share in copper loss $k_{cUsT} = 0.4$, and stator/rotor turn ratio $N_{St}/N_{Rot} = 3$.

These requirements yielded the following set of motor parameters, and load torque: Stator resistance $R_s = 0.95526$ Ω , stator inductance $L_s = 0.13493$ H, rotor resistance $R_r = 0.32492$ Ω , rotor inductance $L_r = 0.014993$ H, mutual inductance between the stator and rotor $M = 0.044526$ H, friction coefficient $B_f = 0.0067023$ N·m·s/rad, inertia $J_i = 0.013405$ kg·m², and load torque $T_L = 19.488$ N·m. These parameters belong to the following induction motor model with short-circuited rotor (Sevinç, 2019):

$$\frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{\sigma L_s} i_{sd} + \left(\omega_g + \frac{M^2}{\sigma L_s L_r} \omega_r \right) i_{sq} + \frac{M R_r}{\sigma L_s L_r} i_{rd} + \frac{M}{\sigma L_s} \omega_r i_{rq} + \frac{1}{\sigma L_s} v_{sd} \\ -\left(\omega_g + \frac{M^2}{\sigma L_s L_r} \omega_r \right) i_{sd} - \frac{R_s}{\sigma L_s} i_{sq} - \frac{M}{\sigma L_s} \omega_r i_{rd} + \frac{M R_r}{\sigma L_s L_r} i_{rq} + \frac{1}{\sigma L_s} v_{sq} \\ \frac{M R_s}{\sigma L_s L_r} i_{sd} - \frac{M}{\sigma L_r} \omega_r i_{sq} - \frac{R_r}{\sigma L_r} i_{rd} + \left(\omega_g - \frac{1}{\sigma} \omega_r \right) i_{rq} - \frac{M}{\sigma L_s L_r} v_{sd} \\ \frac{M}{\sigma L_r} \omega_r i_{sd} + \frac{M R_s}{\sigma L_s L_r} i_{sq} - \left(\omega_g - \frac{1}{\sigma} \omega_r \right) i_{rd} - \frac{R_r}{\sigma L_r} i_{rq} - \frac{M}{\sigma L_s L_r} v_{sq} \\ \frac{3 n_{pp}^2 M}{2 J_i} (i_{sq} i_{rd} - i_{sd} i_{rq}) - \frac{B_f}{J_i} \omega_r - \frac{n_{pp}}{J_i} T_L \end{bmatrix} \quad (40)$$

where v_{sd} , v_{sq} , i_{sd} , i_{sq} are d and q axis stator voltages and currents, i_{rd} and i_{rq} are d and q axis rotor currents referred to a reference frame rotating with angular velocity ω_g with respect to the stator, $\sigma = 1 - M^2/(L_s L_r)$ is leakage constant, and ω_r is electrical angular speed of the rotor. The electrical equations of the motor is shown as an equivalent circuit in Figure 5, where s is the slip, (N_{St}/N_{Rot}) is stator to rotor turn ratio, $r_1 = R_s$, $x_1 = 2\pi f_s \cdot (1 - \sqrt{1 - \sigma}) L_s$, f_s is the stator frequency, $r_2' = (N_{St}/N_{Rot})^2 R_r$, $x_2' = (N_{St}/N_{Rot})^2 2\pi f_s \cdot (1 - \sqrt{1 - \sigma}) L_r$, and $b_m = 1/(2\pi f_s \cdot (1 - \sqrt{1 - \sigma}) (N_{St}/N_{Rot}) M)$ is magnetizing susceptance.

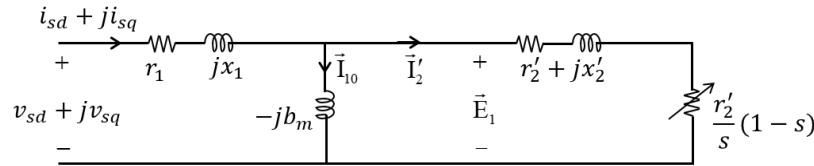


Figure 5. Induction motor's equivalent circuit with ignored iron losses (Sevinç, 2019).

For the base sensitivities which are to be compared for dependencies, first, the motor with the above parameters has been simulated with $v_{sd} = 0.474$ V and $v_{sq} = 326.68$ V as found from the algorithm (Sevinç, 2019) together with the load torque above. Then, the simulation has been repeated 7 times with the same voltages, parameters, and load torque with the exception that just one of the parameters or load torque above is changed 0.1% each time, excluding J_i . The sensitivities of d and q axis stator currents i_{sd} , i_{sq} , and rotor speed n_r to the changed parameter or load torque are calculated according to the approximate version of (1) with the changes to the first simulation. No sensitivity to J_i is calculated since it has no effect on the steady-state values. It should be noted that the set of $7 \times 3 = 21$ sensitivities explained in this paragraph is a base for the independencies of the design requirements.

After finding the base set of 21 sensitivities, only one of the requirements given in the first paragraph of this subsection is changed significantly. The motor parameter set, load torque, v_{sd} , and v_{sq} are found again with the algorithm in the work of Sevinç (2019). A new set of 21 sensitivities are calculated as explained in the previous paragraph according to these values. Comparing each sensitivity with the corresponding one in the base set, unchanged ones are described as an independent sensitivity of the changed design requirement.

This procedure has been repeated for all the 10 requirements given in the first paragraph of this subsection in the synchronous reference frame for accuracy.

The explained comparative study has shown that the sensitivities of the induction motor's stator currents and speed to its parameters and load torque are independent of V_{s1}^{rms} , P_o , n_r , n_{pp} , τ_{mec} , and N_{St}/N_{Rot} requirements. Then, it is possible to say that the sensitivities depend on only 4 quantities as shown in Table 1. As a result, if a motor with specific sensitivities is needed, the designer's optimization search space is reduced with these quantities.

Table 1. Design criteria dependencies for induction motor parameter and load torque sensitivities

Quantity affecting sensitivities	Symbol's meaning
η	Efficiency
σ	Leakage coefficient
k_{ml}	Mechanical to total loss rate
k_{CuSt}	Stator's share in copper loss

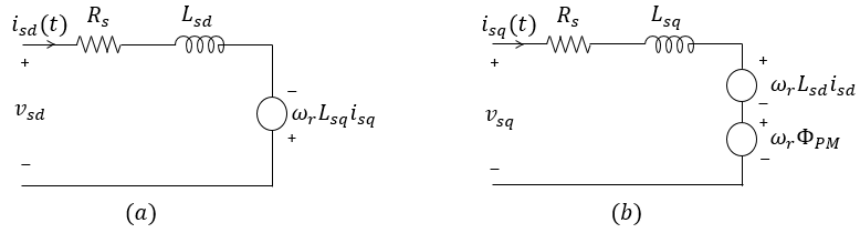
4.2. PMSM

There are 9 requirements in the algorithm to determine the parameters of a PMSM in [23]. First, a set of motor parameters have been found from the tool of Online Electric Motor and Transformer Design for Simulation Purposes (2019) with the following requirements: rms phase voltage $V_{s1}^{rms} = 220$ V, output power $P_o = 4000$ W, efficiency $\eta = 0.80$, rotor speed $n_r = 1500$ rpm, number of pole pair $n_{pp} = 2$, mechanical time constant $\tau_{mec} = 2$ s, saliency ratio $k_{dq} = L_{sd}/L_{sq} = 1.3$, mechanical to total loss rate $k_{ml} = 0.3$, and $\cos \varphi_1 = 0.7$.

These requirements yielded the following set of motor parameters, and load torque: Stator resistance $R_s = 1.9921$ Ω , d axis stator inductance $L_{sd} = 0.046896$ H, q axis stator inductance $L_{sq} = 0.036074$ H, permanent magnet's flux linkage on stator $\Phi_{PM} = 0.574332$ V·s, friction coefficient $B_f = 0.012159$ N·m·s/rad, inertia $J_i = 0.024317$ kg·m², and load torque $T_L = 25.465$ N·m. These parameters belong to the following PMSM model (Sevinç, 2019)

$$\frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{sq} \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_{sd}} i_{sd} + \frac{L_{sq}}{L_{sd}} \omega_r i_{sq} + \frac{1}{L_{sd}} v_{sd} \\ -\frac{R_s}{L_{sq}} i_{sq} - \frac{L_{sd}}{L_{sq}} \omega_r i_{sd} - \frac{\Phi_{PM}}{L_{sq}} \omega_r + \frac{1}{L_{sq}} v_{sq} \\ \frac{3n_{pp}^2(L_{sd} - L_{sq})}{2J_i} i_{sd} i_{sq} + \frac{3n_{pp}^2 \Phi_{PM}}{2J_i} i_{sq} - \frac{B_f}{J_i} \omega_r - \frac{n_{pp}}{J_i} T_L \end{bmatrix} \quad (41)$$

where v_{sd} , v_{sq} , i_{sd} , i_{sq} are d and q axis stator voltages and currents respectively, and ω_r is electrical angular speed of the rotor. The electrical equations of the motor is shown as an equivalent circuit in Figure 6.


Figure 6. PMSM's equivalent circuit. (a) d -axis, (b) q axis (Ahmed *et al.* 2021).

For the base sensitivities which are to be compared for dependencies, first, the motor with the above parameters has been simulated with $v_{sd} = -160.19$ V and $v_{sq} = 266.72$ V as found from the algorithm (Sevinç, 2019) together with the load torque above. Then, the simulation has been repeated 6 times for separate changes in each parameter and load torque except J_i similar to that explained for induction motors. After finding the base set of $6 \times 3 = 18$ sensitivities of i_{sd} , i_{sq} , and ω_r to them, only one of the requirements is changed significantly and the procedure is repeated 9 times for each requirement as explained for induction motors.

Table 2. Design criteria dependencies for PMSM parameter and load torque sensitivities

Quantity affecting sensitivities	Symbol's meaning
η	Efficiency
$k_{dq} = L_{sd}/L_{sq}$	Saliency ratio
k_{ml}	Mechanical to total loss rate
$\cos \varphi_1$	Power factor

Comparing each set of sensitivities with the base set, it has been observed that the sensitivities of the PMSM's stator currents and speed to its parameters and load torque are independent of V_{s1}^{rms} , P_o , n_r , n_{pp} , and τ_{mec} requirements. Then, it is possible to say that the sensitivities of i_{sd} , i_{sq} , and ω_r to the PMSM parameters and the load torque depend on only 4 quantities as shown in Table 2.

This result is obtained for the salient-pole rotor PMSM. It is also valid for the cylindrical-rotor PMSM with some extra notes: Even though d and q axis stator inductances are equal ($L_{sd} = L_{sq}$) in cylindrical rotor types, if sensitivities of i_{sd} , i_{sq} , and ω_r to L_{sd} and L_{sq} are handled separately in case of a cylindrical defect, all the sensitivities to L_{sd} are zero, thus, independent of all the requirements. In

addition, i_{sd} is zero in the steady state of the cylindrical rotor PMSM. Hence, the relative sensitivities of it are undefined, and there is no need for them.

4.3. WRSM

4.3.1. Salient-Pole rotor type

There are 12 requirements in the algorithm to determine the parameters of a WRSM in the work of Sevinç (2019). First, a set of motor parameters have been found from the tool of Online Electric Motor and Transformer Design for Simulation Purposes (2019) with the following requirements: rms phase voltage $V_{s1}^{rms} = 220$ V, output power $P_o = 4000$ W, efficiency $\eta = 0.80$, rotor speed $n_r = 1500$ rpm, number of pole pair $n_{pp} = 2$, mechanical time constant $\tau_{mec} = 0.05$ s, saliency ratio $k_{dq} = L_{sd}/L_{sq} = 1.3$, mechanical to total loss rate $k_{ml} = 0.05$, and $\cos \varphi_1 = 0.8$, the leakage coefficient between a stator phase and rotor windings $\sigma_f = 0.02$, rotor voltage $v_f = 24$ V, and the ratio of the rotor copper loss to the total loss $k_{rl} = P_{CuRot}/P_{loss} = 0.2$.

These requirements yielded the following set of motor parameters, and load torque: Stator resistance $R_s = 3.025$ Ω , d axis stator inductance $L_{sd} = 0.046629$ H, q axis stator inductance $L_{sq} = 0.035868$ H, rotor resistance $R_f = 2.88$ Ω , rotor inductance $L_f = 0.202542$ H, mutual inductance between the stator and rotor $M = 0.078551$ H, friction coefficient $B_f = 0.002026$ N·m·s/rad, inertia $J_i = 0.000101$ kg·m², and load torque $T_L = 25.465$ N·m. These parameters belong to the following WRSM model (Sevinç, 2019)

$$\frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_f \\ \omega_r \end{bmatrix} = \begin{bmatrix} \frac{1}{1.5M^2 - L_{sd}L_f} (R_s L_f i_{sd} - L_{sq} L_f \omega_r i_{sq} - R_f M i_f - L_f v_{sd} + M v_f) \\ -\frac{L_{sd}}{L_{sq}} \omega_r i_{sd} - \frac{R_s}{L_{sq}} i_{sq} - \frac{M}{L_{sq}} \omega_r i_f + \frac{1}{L_{sq}} v_{sq} \\ \frac{1}{1.5M^2 - L_{sd}L_f} (-1.5R_s M i_{sd} + 1.5L_{sq} M \omega_r i_{sq} + R_f L_{sd} i_f + 1.5M v_{sd} - L_{sd} v_f) \\ \frac{3n_{pp}^2 (L_{sd} - L_{sq})}{2J_i} i_{sd} i_{sq} + \frac{3n_{pp}^2 M}{2J_i} i_{sq} i_f - \frac{B_f}{J_i} \omega_r - \frac{n_{pp}}{J_i} T_L \end{bmatrix} \quad (42)$$

where v_{sd} , v_{sq} , i_{sd} , i_{sq} are d and q axis stator voltages and currents respectively, v_f and i_f are rotor voltage and current, and ω_r is the electrical angular speed of the rotor.

For the base sensitivities which are to be compared for dependencies, first, the motor with the above parameters has been simulated with $v_{sd} = -134.49$ V and $v_{sq} = 280.56$ V as found from the algorithm (Sevinç, 2019) together with the load torque above. Then, the simulation has been repeated 8 times for separate changes in each parameter and load torque except J_i similar to that explained for induction motors. After finding the base set of $8 \times 4 = 32$ sensitivities of i_{sd} , i_{sq} , i_f , and ω_r to them, only one of the requirements is changed significantly and the procedure is repeated 12 times for each requirement as explained for induction motors.

Comparing each set of sensitivities with the base set, it has been observed that the sensitivities of the WRSM's stator currents, rotor current and speed to its parameters and load torque are independent of V_{s1}^{rms} , P_o , n_r , n_{pp} , τ_{mec} , σ_f , and v_f requirements. Then, it is possible to say that the sensitivities of i_{sd} , i_{sq} , i_f , and ω_r to the motor parameters and the load torque depend on only 5 quantities as shown in Table 3.

Table 3. Design criteria dependencies for WRSM parameter and load torque sensitivities

Quantity affecting sensitivities	Symbol's meaning
η	Efficiency
$k_{dq} = L_{sd}/L_{sq}$	Saliency ratio
k_{ml}	Mechanical to total loss rate
$\cos \varphi_1$	Power factor
$k_{rl} = P_{CuRot}/P_{loss}$	Rotor copper loss to total loss rate

This result is obtained for the salient-pole rotor WRSM. It is also valid for the cylindrical-rotor WRSM with some extra notes:

- $i_{sd} = 0$ in steady state, hence, the relative sensitivities of it are undefined, and there is no need for them.
- Even though d and q axis stator inductances are equal ($L_{sd} = L_{sq}$) in cylindrical rotor types, if sensitivities of i_{sq} and ω_r to L_{sd} and L_{sq} are handled separately in case of a cylindrical defect, all the sensitivities to L_{sd} are zero, thus, independent of all the requirements.
- Sensitivities of the rotor speed to all the parameters and load torque are zero, hence, independent of all the requirements.
- Sensitivities of the rotor current to all the parameters and load torque, except R_f , are zero, hence, independent of all the requirements.
- Sensitivities of i_{sq} to R_s and L_f are also zero, hence, independent of all the requirements.
- Nonzero sensitivities are $S(i_{sq}, L_{sq})$, $S(i_{sq}, R_f)$, $S(i_{sq}, M)$, $S(i_{sq}, B_f)$, $S(i_{sq}, T_L)$, $S(i_f, R_f)$ only.
- $S(i_{sq}, R_f) = 1$, $S(i_f, R_f) \approx -1$, $S(i_{sq}, M) \approx -1$.

5. Sensitivities for Transformers

Although this paper's focus is mainly on electric motors, it will be better to extend the study to transformers as in the works of Sevinç (2019), and Online Electric Motor and Transformer Design for Simulation Purposes (2019). Transformer study has been implemented by numerical steady-state equivalent circuit calculations. There are 9 requirements in the algorithm to determine the parameters of transformers in the study of Sevinç (2019) excluding number of phases and their connection types. First, a set of transformer parameters have been found with the following requirements: Primary rms voltage per phase $V_1 = 230$ V, secondary rms voltage per phase $V_2 = 110$ V, frequency $f = 50$ Hz, secondary apparent power per phase $S_2 = 1500$ V·A, efficiency at rated resistive load $\eta = 0.80$, copper loss to total loss rate (P_{Cu}/P_{loss}) at rated resistive load $k_{Cu} = 0.55$, secondary copper loss to total copper loss rate (P_{Cu2}/P_{Cu}) at rated resistive load $k_{Cu2} = 0.4$, coupling coefficient $k = 0.98$, and $\alpha_{EV2} = 5^\circ$, which is leading angle of the parallel branch voltage with respect to the secondary voltage in T-equivalent circuit of the transformer referred to the stator as shown in Figure 7.

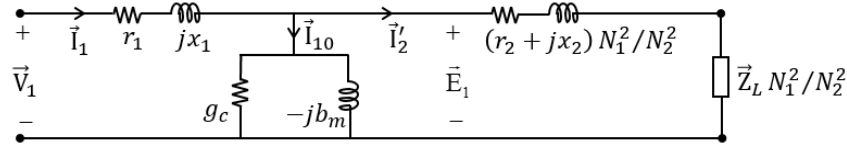


Figure 7. Transformer's equivalent circuit with load \vec{Z}_L referred to the primary (Sevinç, 2019).

According to the tool of Online Electric Motor and Transformer Design for Simulation Purposes (2019), these requirements yielded the following set of T-equivalent circuit parameters of the transformer: Primary resistance and leakage reactance $r_1 = 1.6404 \Omega$ and $x_1 = 2.3715 \Omega$, secondary resistance and leakage reactance $r_2 = 0.44367 \Omega$ and $x_2 = 0.74456 \Omega$, core loss conductance $g_c = 0.003826$ S, magnetizing susceptance $b_m = 0.008434$ S, and turn ratio $N_1/N_2 = 1.80$.

For the base sensitivities which are to be compared for dependencies, first, the transformer circuit with the above parameters has been analyzed with $V_1 = 230$ V and a rated resistive load of 8.067Ω as found from the algorithm (Sevinç, 2019) assuming the angle of the primary voltage is zero. Then, the analysis has been repeated 7 times for separate changes in each parameter similar to that explained for induction motors. After finding the base set of $7 \times 4 = 28$ sensitivities of real and imaginary components of the primary current and secondary voltage to the parameters, only one of the requirements is changed significantly and the procedure is repeated 9 times for each requirement as explained for induction motors.

Comparing each set of sensitivities with the base set, it has been observed that the sensitivities of the transformer's primary current and secondary voltage to its parameters are independent of V_1 , V_2 , f , and S_2 requirements. Then, it is possible to say that the sensitivities depend on only 5 quantities as shown in Table 4 at resistive loads.

Table 4. Design criteria dependencies for transformer parameter sensitivities at resistive load

Quantity affecting sensitivities	Symbol's meaning
η	Efficiency
$k_{Cu} = P_{Cu}/P_{loss}$	Ratio of copper loss to total loss
$k_{Cu2} = P_{Cu2}/P_{Cu}$	Ratio of secondary copper loss to total copper loss
k	Coupling coefficient
α_{EV2}	Leading angle of the parallel branch voltage with respect to the secondary voltage in T-equivalent circuit of the transformer

6. Conclusions

The sensitivity analysis presented in this paper differs from the existing sensitivity analyses focused on the design stages in literature. Whilst they are concerned with constraint and objective functions' sensitivities to the design variables, the presented one in this paper investigated into the dependencies of the sensitivities on basic design requirements. For example, it is possible to find research on the sensitivities of losses or efficiency to winding parameters in literature. The corresponding analysis in this paper on the other hand, concentrated on the sensitivities of the output variables to the resulting winding parameters satisfying the efficiency demand in design, and checked if those sensitivities change with another set of winding parameters resulting from a repeated design for another efficiency demand.

This study has shown that many of the output variables' sensitivities to system parameters and load torque are independent of many of the variables which are used as requirements for designing electric machinery or deciding its parameter set at rated operating conditions. The requirements affecting the sensitivities are identified and listed for PM dc motors, induction motors, PMSM, WRSM, and transformers. This work has revealed that some main design requirements of electric machinery such as rated values of voltage, power, and speed have no effect on the most used sensitivities. The main quantities affecting the sensitivities are power loss-related ones such

as efficiency or proportion of a specific type of loss. Especially PM dc motors' sensitivities depend on only two design requirements: Efficiency and mechanical to total loss rate. That number is 4 for PMSM and induction motors, and 5 for WRSM and transformers.

The main benefit of these findings is to reduce the optimization search space dimension for desired sensitivities. Such optimization search may be needed to design electric machinery easy to estimate their parameters and load torque. Such a search space has been illustrated to estimate the armature resistance and load torque simultaneously for PM dc motors.

The findings of this study has also an educational benefit. Sensitivity homework or remote exam questions can be asked students with very different parameters which are derived for very different rated voltage, power, and speed values, yet their results are the same, easy for the lecturer to asses. This benefit is effective, especially in remote education as in the pandemic or earthquakes.

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