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Explicit Formulas for Optimum Parameters of Viscoelastic-type Tuned Mass Dampers

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Abstract
Tuned mass dampers (TMDs) are passive vibration control devices that are attached to a primary system to reduce the dynamic vibrations under exciting motion. The Voigt-type TMD, which is the most widely used one, is known as a standard model of dynamic vibration absorber (DVA). The purpose of this study is to improve the vibration control performance of passive control devices by using viscoelastic-type tuned mass dampers (V-TMDs). The study adopts the Zener model to represent the viscoelastic behavior of V-TMD. In this study, the fixed-point method is used to determine the optimum parameters of a V-TMD. The displacement amplification factor (DAF) of the coupled system is obtained in the frequency domain. The optimal parameters of the V-TMD system attached to an undamped single degree-of-freedom (SDOF) main system are obtained by minimizing the DAF (symbolized with β) under the effect of base excitation. The optimum parameters, such as damping ratio (ξ) and stiffness ratio (κ) of the coupled system are derived, and explicit expressions corresponding to the optimum parameters are presented for engineering design. Moreover, the change in DAF values for different mass ratios (μ) is also discussed. It is proven that V-TMD is very effective in reducing the amplitudes of vibration. The study also provides valuable insights for engineering practitioners who want to design and implement V-TMDs for vibration control applications because accurate expressions, which are simple and easy to use, are derived in order to obtain optimum parameters, and step-by-step procedures are explained.

1. Introduction

Vibration control is an important engineering field to improve the performance and safety of structural systems. To reduce the dynamic vibrations of the structure, several techniques are available. The concept of using tuned mass dampers (TMDs) is one of the several techniques, and it is a recent one. TMDs are secondary oscillators that are attached to the primary structure by parallel springs and viscous dampers. The main purpose is to transfer the vibrational energy of the primary oscillator to the secondary oscillator to increase the damping capacity of a structural system. It is usually assumed that the parameters of the primary oscillator are known. Therefore, the mathematical problem of tuning the mass damper is to select the parameters appropriately through proper calibration of the damping ratio and tuning frequency.

In the nineteenth century, the concept of the TMD was proposed by [1] which considers a vibration control device without any inherent damping. [2] extended Frahm's absorber by introducing a certain amount of damping. Details regarding the design and theory of TMDs and closed-form expressions for optimum absorber parameters are presented in [3]. The dynamic analyses of structures equipped with the conventional Voight type vibration absorber are presented in many papers in order to obtain optimum parameters. The optimum damping and tuning frequency ratios of the conventional TMD are obtained by [4] using the

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numerical searching technique. Various mean square responses, such as relative displacement, velocity of the main mass, and force transmitted to the support, are minimized. Simple expressions for optimum absorber parameters are derived in [5, 6] for undamped and damped SDOF main systems under harmonic and white noise random excitations. Minimization of various response parameters is considered. The optimum tuning frequency and damping ratio of the TMD system are obtained by [7,8] using a numerical iteration searching technique. The steady-state response of damped systems is minimized to a minimum level through a curve-fitting method. [9] proposed a method for determining the optimum parameters of TMD by selecting the tuning and damping ratios that result in equal and large damping ratios in the first two modes of vibration. The responses of several single and multiple degree-of-freedom structures, with or without TMDs, subjected to different ground excitations are presented. Exact algebraic solutions are derived by [10] for the optimum tuning ratio and damping coefficient, assuming an undamped primary system and a hysterically damped primary system. Algebraic exact expressions for the resonance and antiresonance frequencies have been obtained. [11] derived the approximate optimum stiffness and a damping ratio using an extension of design formulas for $H\infty$ optimization under the assumption of damped structures. Ground and force excitations are considered. [12] derived explicit expressions for the optimum mass ratio, damper damping, and tuning frequency of the TMD system attached to a viscously damped SDOF primary system. An algorithm for particle swarm optimization (PSO) is presented, covering both external force and base acceleration. A hybrid passive optimal control method is proposed to find the optimal damping coefficients of viscous dampers (VDs) and a TMD in shear building structures by [13]. In the optimization problem, the damping coefficients of TMD and VDs are taken as design variables. The variation of the upper limits of damping coefficients, the variation of story mass and stiffness are investigated for a six-story shear building model with VD&TMD.

The classical Maxwell or Kelvin-Voight models are the most popular DVA models, where the spring and viscous damper are arranged in series or in parallel, respectively. In recent years, various types of viscoelastically damped DVAs have attracted much attention in order to obtain a considerably reduced structural response to wind or earthquake motions. The design of structures with viscoelastic tuned mass dampers (V-TMDs) together with the optimization of parameters is an important problem from a practical viewpoint. There is little research about the optimum design of V-TMDs. [14] presented a semi-analytical iterative approach to obtain the optimal parameters of a viscoelastically damped TMD. The primary structure is assumed to be linear and undamped. It has been demonstrated that superior vibration absorption is obtained by using a viscoelastically damped TMD compared to an equivalent viscously damped TMD. [15] developed a numerical approach, which is called a generalized optimality criteria approach, to minimize the maximum amplitude magnification factor of a three-element DVA. The proposed method handles the primary system damping. Simultaneous equations for the design problem are solved using numerical computing software. As reported, the three-element DVA is more effective than a conventional DVA of twice its mass. Exact solutions for the optimum parameters of the three-element DVA are derived by [16] through numerical analysis. It has been demonstrated that the optimized three-element type of DVA is more advanced than the conventional Voigt type of DVA. Using algebraic manipulation, the optimum tuning and damping parameters are obtained for the three-element type of DVA. The H2 optimization problem for a damped and undamped primary system with a three-element type of DVA is discussed by [17]. The Newton-Raphson method is used as a numerical approach for the solution of a damped primary system. It has been proven that the three-element type of DVA is superior to the conventional Voight type of DVA. [18] studied the three-element DVA for the damped primary system. The criteria of the equivalent linearization method are utilized, and the damped structure is replaced by an equivalent undamped one. The approximate analytical solution of the DVA's parameters is obtained from the results of the undamped structure. [19] also demonstrated that the three-element TMD produces better performance than the TMD. Optimum tuning, damping, and stiffness ratios are selected as the design variables for the three-element TMD, including the damped primary system. A simulated annealing algorithm is utilized for the solution.

This study investigates the application of the fixed-point approach to the Zener type of V-TMD. The Zener model (sometimes called the standard linear solid model and/or three-element model) is more accurate than the Maxwell or Kelvin-Voight models in predicting real viscoelastic material response. The standard Voight or Maxwell models are not adequate to describe the rheological behavior of viscoelastic dampers [20]. The Zener model is a combination of a viscous damper and two springs. The objective of the present study is to consider the dynamic analysis of a structure equipped with Zener type V-TMD and the optimization analysis of the structure, including parameter optimization. Numerous studies have been conducted on the Voigt type of TMD, and the analytical expressions for the optimum tuning and damping parameters have already been obtained. However, there are only a few papers that have been published on the viscoelastic Zener type of TMD. A literature review reveals that published works deal with the issue of parameter optimization in V-TMDs. The present study uses simplified mathematical operations, which in turn give rise to the simple analytical expressions of the optimum parameters of the V-TMD. In addition, accurate expressions that are simple and easy to use have not been derived until now for the optimum damping and stiffness ratios. The maximum DAF is efficiently minimized, and precise approximate solutions are obtained for the case where the primary oscillator is undamped. Precise expressions for the optimum parameters of a V-TMD, such as damping and stiffness ratios, are derived. In addition, the change in DAF values for different mass ratios is also discussed. The proposed procedure clearly explains the step-by-step instructions that are used to obtain optimum parameters and informs the readers associated with the process.

2. Methodology

2.1. Derivation of Equations of Motion for the Coupled System

Figure 1 shows a Zener type of V-TMD attached to an undamped SDOF main structure. It is assumed that the secondary oscillator is connected to the primary oscillator through the Zener model. By considering base excitation (y_0), the equations of motion are derived.

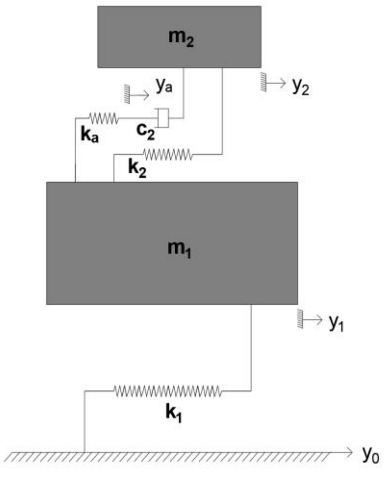


Figure 1. Zener type of V-TMD connected to an undamped structure.

The mass and stiffness parameters of the main structure are m_1 and k_1, respectively, and m_2 is the mass of the viscoelastic vibration absorber. Damping for the main structure is not considered here as it would help to reduce vibration and thus hide any additional damping that we expect to arise due to the viscoelastic damper. In order to develop and solve problems with the existence of fixed points, this assumption is a necessary condition.

The dynamic behavior of the system can be expressed by the following ordinary differential equations:

$$-y_2 k_2 - y_a k_a + y_1(k_1 + k_2 + k_a) + m_1 \frac{d^2 y_1}{dt^2} = y_0 k_1 \quad (1)$$

$$m_2 \frac{d^2 y_2}{dt^2} + c_2 \left(\frac{dy_2}{dt} - \frac{dy_a}{dt}\right) + k_2 (y_2 - y_1) = 0 \quad (2)$$

$$c_2 \left(\frac{dy_a}{dt} - \frac{dy_2}{dt}\right) + k_a (y_a - y_1) = 0 \quad (3)$$

where y_1 and y_2 are the displacements of the masses of the main structure and viscoelastic vibration absorber, respectively. The viscosity of the dashpot component is c_2, k_a and k_2 are the elasticity parameters in the model. An additional internal degree of freedom y_a is introduced, connected by a spring with a coefficient k_a and by a dashpot with a coefficient c_2.

$y_a(\omega)$ and $y_2(\omega)$ are obtained after frequency domain transformation of Eqns. (1) and (3) as follows:

$$y_a(\omega) = \frac{k_2 k_a y_1(\omega) - i\omega c_2 (k_1 y_0(\omega) - (k_1 + k_2 + k_a - \omega^2 m_1) y_1(\omega))}{k_2 k_a + i\omega c_2 (k_2 + k_a)} \quad (4)$$

$$y_2(\omega) = \frac{k_1(-i\omega c_2 - k_a) y_0(\omega) + (k_a(k_1 + k_2 - \omega^2 m_1) + i\omega c_2 (k_1 + k_2 + k_a - \omega^2 m_1)) y_1(\omega)}{k_2 k_a + i\omega c_2 (k_2 + k_a)} \quad (5)$$

Substituting Eqns. (4) and (5) into the Fourier transform of Eqn. (2), the DAF expression can be obtained with complex numbers as follows:

$$\beta(\omega) = \frac{k_1(k_a(k_2 - \omega^2 m_2) - i\omega c_2(-k_2 - k_a + \omega^2 m_2))}{-i(-ik_2 k_a + \omega c_2(k_2 + k_a))(-k_1 + \omega^2 m_1) + \omega^2(-k_a(k_1 + k_2 - \omega^2 m_1) - i\omega c_2(k_1 + k_2 + k_a - \omega^2 m_1))m_2} \quad (6)$$

where ω is the circular frequency. The absolute value of the equation, which is in terms of the complex numbers, is obtained as follows:

$$\beta(\omega) = \sqrt{\frac{k_1{}^2(k_a{}^2(k_2 - \omega^2 m_2)^2 + \omega^2 c_2{}^2(k_2 + k_a - \omega^2 m_2)^2}{k_a{}^2(k_2(-k_1 + \omega^2 m_1) + \omega^2(k_1 + k_2 - \omega^2 m_1)m_2)^2 + \omega^2 c_2{}^2((k_2 + k_a)(-k_1 + \omega^2 m_1) + \omega^2(k_1 + k_2 + k_a - \omega^2 m_1)m_2)^2}} \quad (7)$$

The following non-dimensional parameters are introduced for convenience:

Squared ratio of the excitation:

$\boldsymbol{\Omega} = \omega^2 / \omega_1{}^2$

Mass ratio:

$\boldsymbol{\mu} = m_2 / m_1$

Damping ratio:

$\boldsymbol{\xi} = c_2 / (2 m_2 \omega_2)$

Using these notations, DAF can be expressed as follows:

$$\beta(\Omega) = \sqrt{\frac{(((-\mu\Omega k_1 + k_2)^2 k_a{}^2 + 4\mu\zeta^2\Omega k_1 k_2(-\mu\Omega k_1 + k_2 + k_a)^2)}{((\mu(-1+\Omega)\Omega k_1 - (-1+\Omega+\mu\Omega)k_2)^2 k_a{}^2 + 4\mu\zeta^2\Omega k_1 k_2(\mu(-1+\Omega)\Omega k_1 - (-1+\Omega+\mu\Omega)(k_2 + k_a))^2))}} \quad (8)$$

2.2. Derivation of Tuning Parameters

The DAF function is plotted in Figure 2 for different damping ratios. In the example, the mass ratio is 0.1, and the stiffness values are k_1=1 N/m, k_2=0.0545 N/m and k_a=0.0578 N/m.

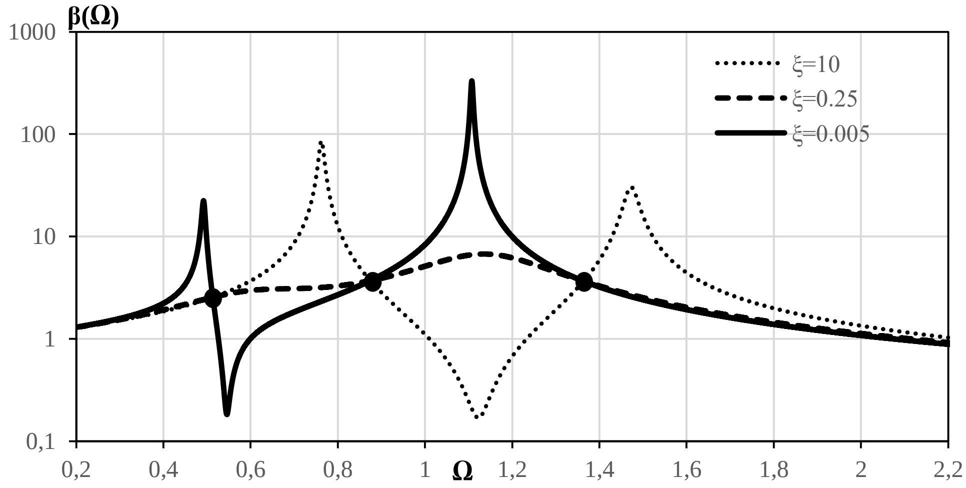


Figure 2. DAF function for different damping ratios

As can be seen from Figure 2, the response amplitude is independent of the damping ratio at three points (each one is represented by a solid circle). For classical TMDs, such fixed points have also been detected, and their number is two. For extracting the optimum parameter values of the traditional TMDs, classical methods are based on these fixed points. For the Zener type of V-TMD, there are three fixed points. The procedure for optimization is summarized through step-by-step instructions as follows:

Step 1:

Determine the optimum value for spring stiffness k_2 according to the three fixed points at which the curves intersect.

Step 2:

Determine the optimum value for spring stiffness k_a with respect to the k_2 value, which is calculated in Step 1.

Step 3:

Determine the damping ratio that gives the best symmetric DAF curve regarding the central fixed point.

2.2.1. Determination of Optimum k_2 Stiffness

In order to determine the three fixed points, the squared frequencies for which the DAF has the same values have been searched. For $\xi = 0$ and $\xi = +\infty$, the DAF yields the following equations:

$$\beta(\Omega_{\zeta_0}) = \sqrt{\frac{(-\mu\Omega k_1 + k_2)^2}{(\mu(-1+\Omega)\Omega k_1 - (-1+\Omega+\mu\Omega)k_2)^2}} \quad (9)$$

$$\beta(\Omega_{\xi_\infty}) = \sqrt{\frac{(-\mu\Omega k_1 + k_2 + k_a)^2}{(\mu(-1+\Omega)\Omega k_1 - (-1+\Omega+\mu\Omega)(k_2+k_a))^2}} \quad (10)$$

According to [16], the fixed points are settled at the same height with the following relation:

$$\beta(\Omega_{l,c,r}) = \sqrt{\frac{1+\mu}{\mu}} \quad (11)$$

By considering the equality of Eqn. (9) and (11), the explicit expression for the optimum k_2 stiffness value is derived as follows:

$$k_2 = \frac{\mu(1+\mu-\sqrt{\mu(1+\mu)})k_1}{(1+\mu)^2} \quad (12)$$

2.2.2. Determination of Optimum k_a Stiffness

The optimum k_a stiffness value is obtained by substituting the k_a stiffness value into Eqn. (10) and considering the equality with Eqn. (11). As a result, the following explicit expression is introduced:

$$k_a = \frac{2(\mu(1+\mu))^{3/2} k_1}{(1+\mu)^3} \tag{13}$$

The ratio of stiffness of springs (k_2 / k_a) is defined as the stiffness ratio. The optimum stiffness ratios are tabulated in Table 1 for different mass ratios. As tabulated, the optimum stiffness ratio of the V-TMD increases with the increase in mass ratio of the structure.

Using the optimum k_2 and k_a stiffness values, the corresponding DAF curve is plotted in Figure 3 for 10 % mass ratio.

Table 1. Optimum stiffness ratios for different mass ratios

Mass ratio (μ)	Stiffness ratio (κ)
0.01	0.221
0.02	0.326
0.03	0.412
0.04	0.488
0.05	0.558
0.10	0.863
0.15	1.131
0.20	1.379
0.25	1.618
0.30	1.849

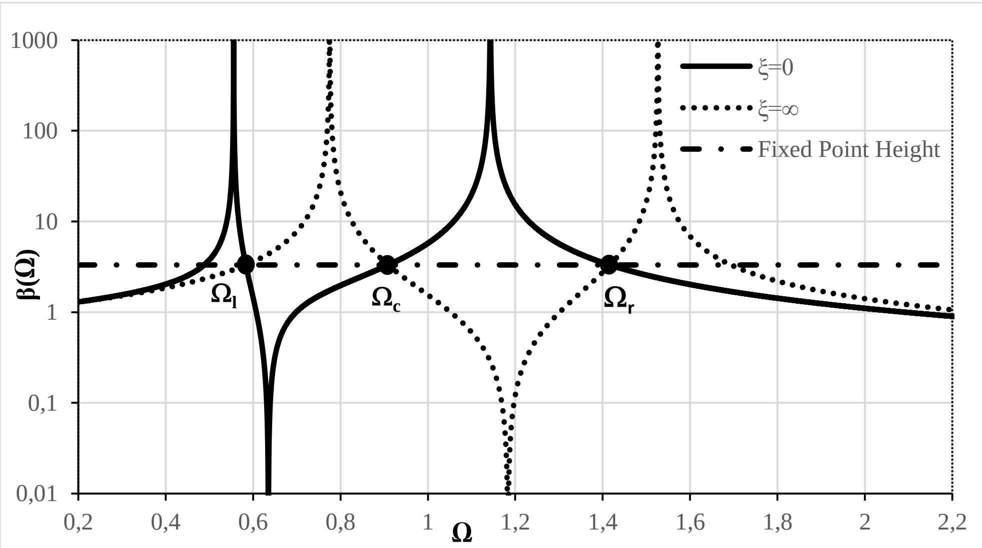


Figure 3. β(ξ=0) and β(ξ=∞) curves for 10% mass ratio

It is proved that the values of the three fixed points namely the left, center, and right fixed points at which an intersection occurs satisfy Eqn. (11).

For the % 10 mass ratio, closed-form solutions of the left, center, and right fixed points' squared frequencies are derived as follows:

$$\Omega_{r,l} = 1 \pm \frac{\mu(2+\mu)}{\sqrt{\mu(1+\mu)^2(2+\mu)}} \qquad \Omega_c = \frac{1}{1+\mu} \tag{14}$$

Using the non-dimensional parameter, the frequencies of the three fixed points are presented in Table 2 for the cases μ = 0.10, $\omega_1 = \sqrt{\frac{k_1}{m_1}}$ rad/s, k_1 = 1 N/m, m_1 =1 kg. Results are compared with [14] and [15]. The accurate and consistent performance of the presented approach has been indicated. As tabulated in Table 2, the calculated results are accurate to two decimal places for the frequency values of the three fixed points.

Table 2. Comparison of the left, central, and right fixed points' frequencies

Frequency values	Present Study	Reference Study [14]	Reference Study [15]
ω_{left}	0.764	~ 0.76	~ 0.76
ω_{center}	0.953	~ 0.95	~ 0.95
ω_{right}	1.190	~1.19	~ 1.18

2.2.3. Determination of the Optimum Damping Ratio (ξ)

The symmetry of the DAF curve regarding the central fixed point is directly related to the damping ratio effect. The best symmetry with regard to the central fixed point corresponds to the optimum value for the damping ratio. One way to check this symmetry is to make the slopes of the DAF curve at the left and right fixed points opposite each other. To the best of the author's knowledge, an analytical (or closed-form) expression for the optimum damping ratio has not been derived yet.

In order to get an explicit expression for the optimum value of the damping ratio, the following procedure is proposed:

The squared frequency of a central fixed point has already been calculated as 1/(1+μ), so it takes a value less than 1 for any mass ratio. The square root of 1/(1+μ) has a value that is less than 1. So as to obtain the optimum damping ratio, the square root of 1/(1+μ) must be equal to the height values of the three fixed points (see eqn. (11)) and must satisfy the following expression:

$$\beta(\Omega_{\zeta}) = \beta(\Omega_{l,c,r}) = \beta(\frac{1}{1+\mu}) = \sqrt{\frac{1+\mu}{\mu}} \qquad (15)$$

Here, Ω_{ξ} is the squared frequency that will be used to calculate the optimal damping ratio. Thereby, the symmetry of the DAF curve is precisely satisfied. An explicit expression of the optimum damping ratio for different mass ratios is derived as follows:

$$\xi_{opt} = \sqrt{\frac{\mu(16+20\mu-3\mu\sqrt{1/(1+\mu)}-12\sqrt{\mu(1+\mu)}+\sqrt{\mu}(4+5\mu)}{16+31\mu+16\mu^2}} \qquad (16)$$

and a change in the optimal damping ratio with different mass ratios is shown in Figure 4. The optimum damping ratio of the V-TMD increases with the increase in the mass ratio of the structure, as expected.

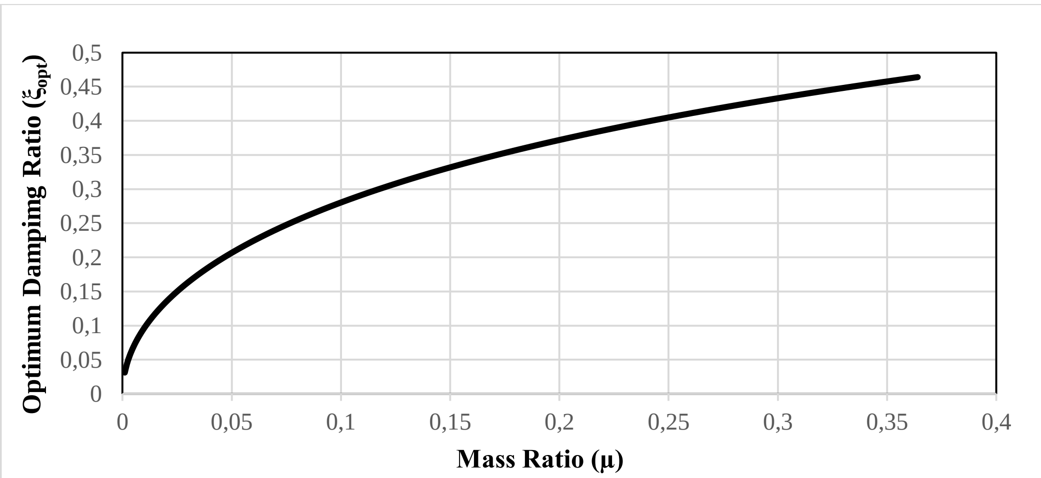
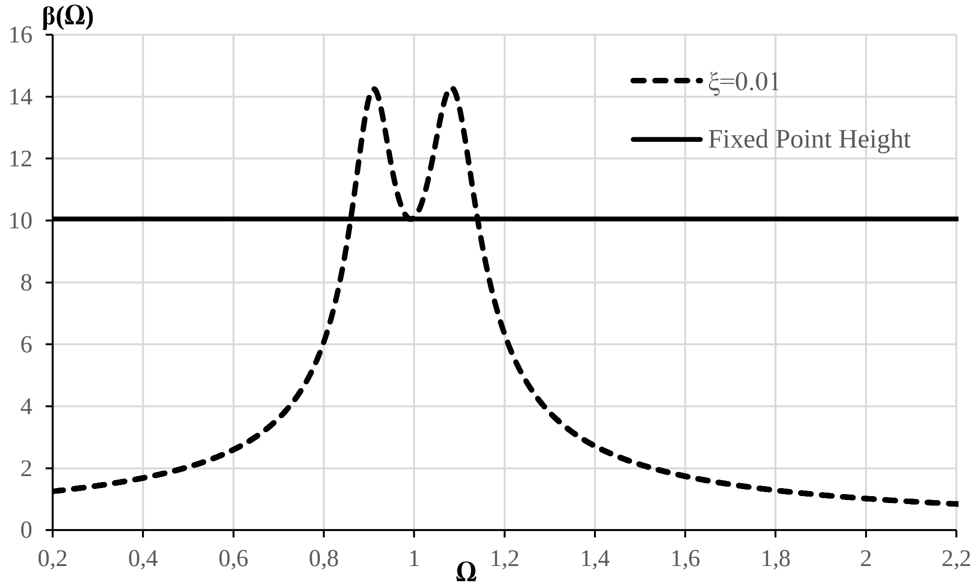


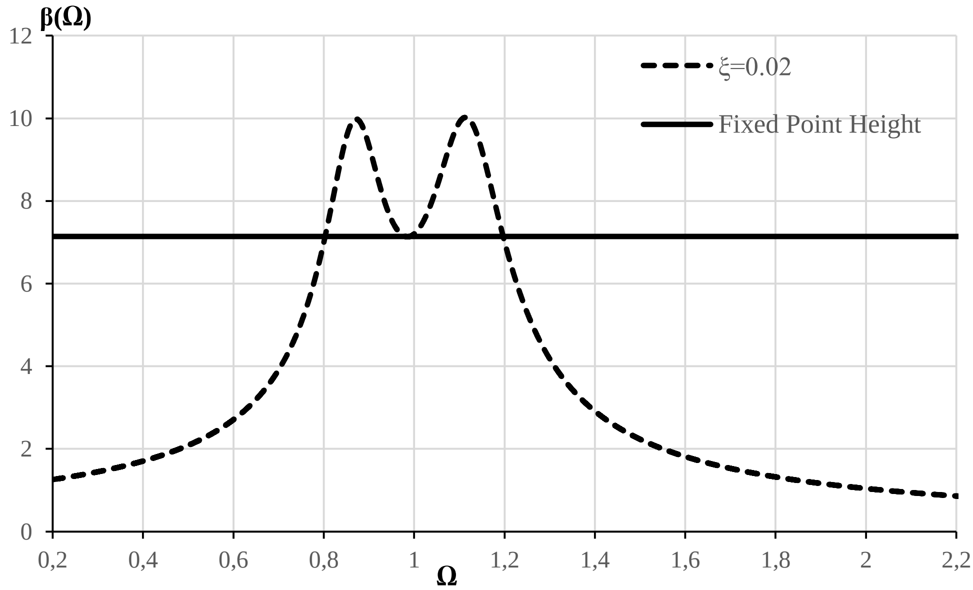
Figure 4. Change of optimum damping ratio with mass ratio considering optimum spring stiffness values

2.2.4. Evaluation of the V-TMD Performance with Optimum Parameters

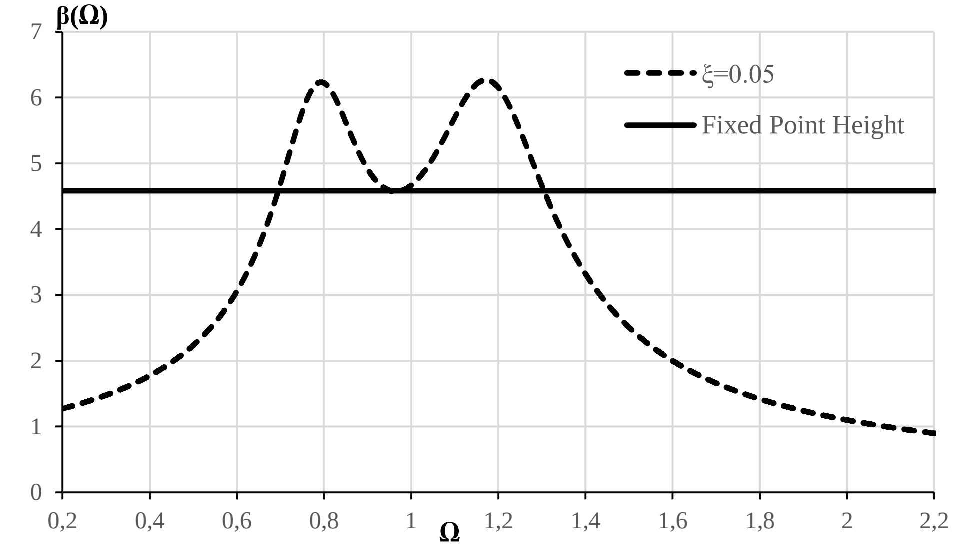
Using the optimum values, the DAF curve is plotted in Figure 5 for different mass ratios. Noticeably, an increase in mass ratio results in a decrease in the maximum DAF values. In figure 5, the continuous line corresponds to the height values of the fixed points. It is also seen that the symmetry, which we want to investigate, is obtained using the optimum damping ratios based on the proposed algorithm for relatively large mass ratios.



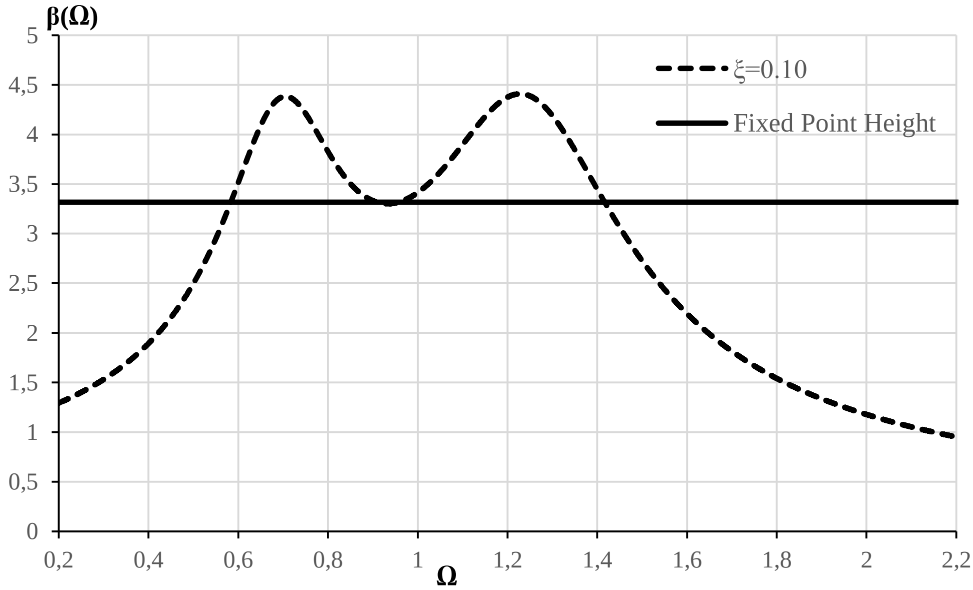
(a)



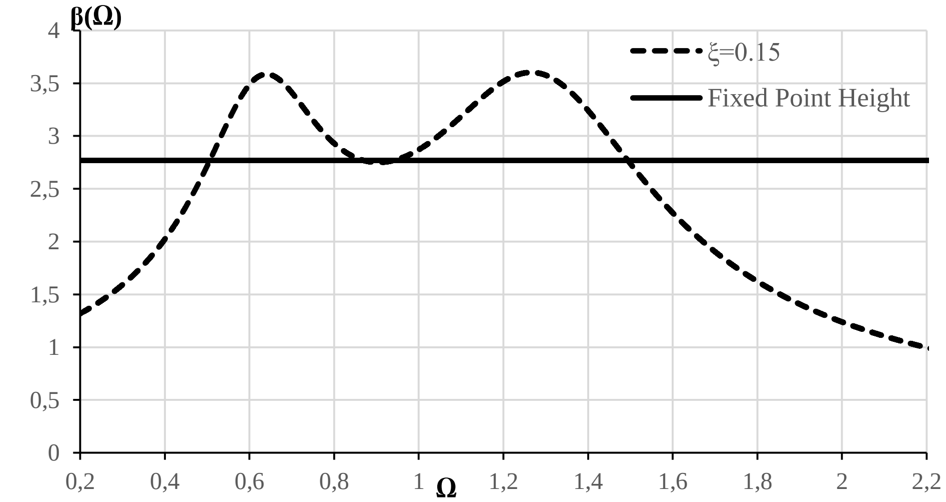
(b)


β(Ω)
ξ=0.05
Fixed Point Height
Ω

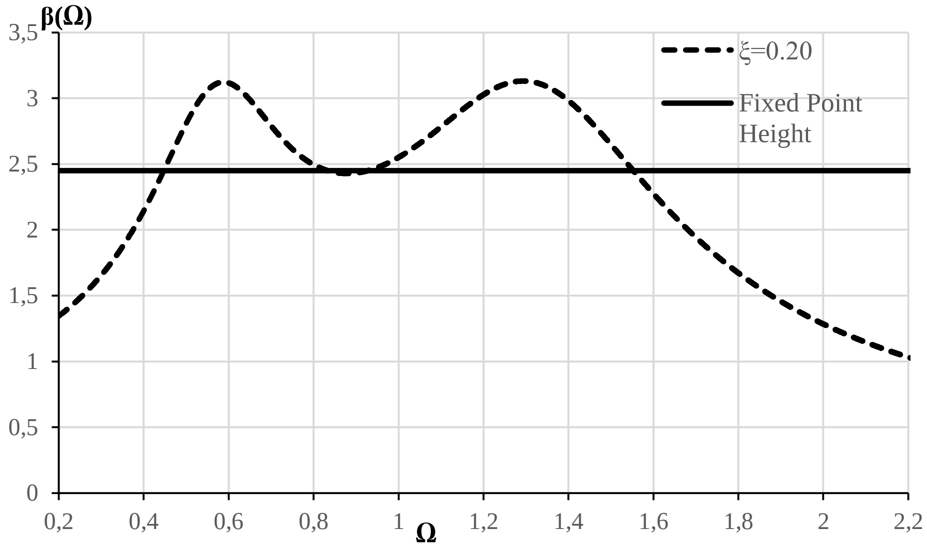
(c)


β(Ω)
ξ=0.10
Fixed Point Height
Ω

(d)


β(Ω)
ξ=0.15
Fixed Point Height
Ω

(e)



(f)

Figure 5. β(Ω) curves obtained using optimum parameters for different mass ratios (a) μ=0.01, (b) μ=0.02, (c) μ=0.05, (d) μ=0.10, (e) μ=0.15, and (f) μ=0.20

To demonstrate the efficiency of the recommended technique, the results of the V-TMD are compared to the available results in the literature and tabulated in Table 3. Additionally, the classical TMD results are given in order to make a comparison between the performance of TMD and V-TMD.

Table 3. Maximum DAFs for different mass ratios

Mass Ratio (μ)	Classical TMD Reference Study [3]	V-TMD Reference Study [15]	V-TMD Present Study
0.01	14.18	13.78	14.29
0.02	10.05	9.67	10.02
0.05	6.40	6.05	6.27
0.10	4.58	4.27	4.42
0.20	3.32	3.04	3.16

It is seen that the effectiveness of the V-TMD in diminishing the maximum DAF values is better than that of the classical TMD, as expected. Another observation from Table 3 is that the results of the present study agree well with the results of earlier research studies. Compared with the results of [15], highest error rate is nearly 4 % (when μ=0.20). This error is at an acceptable level, and symmetry disruption is inevitable for large mass ratios.

3. Conclusion

A Tuned mass damper (TMD) is a traditional passive vibration control device that is attached to a vibrating main structure. In this study, the characteristics of viscoelastically damped TMD, which is an alternative to classically damped TMD, are discussed. As a V-TMD, the Zener model is used. Some of the main points examined in this study are as follows:

- The fixed-point theory is generalized to obtain the optimal parameters of the V-TMD.
- A Zener-type TMD has three fixed points, unlike the traditional TMD with two fixed points.
- The optimal values of two stiffness parameters (k_a and k_2) are established considering the height of the DAF at three fixed points.
- An optimal damping ratio expression is developed considering the symmetry of the DAF with reference to the central fixed point.
- The change in the value of optimum stiffness parameters and damping ratio with reference to different mass ratios is also presented with representative figures and tables. It is observed that the present study results show accurate agreement with the results reported in the literature.
- Simple closed-form expressions are sufficiently accurate for practical use.
- The optimum damping ratio of the V-TMD increases with the increase in the mass ratio of the structure, as expected.
- The optimum stiffness ratio of the V-TMD increases with the increase in mass ratio of the structure, as expected.
- The authors recommend to use a viscoelastic damper model composed of multiple Maxwell and/or Voight elements for future research and searching to see if there will still be fixed points for this form of general viscoelastic model.

Contributions of the authors

The authors confirm contribution to the paper as follows: study conception and design: G. Tekin; data collection: M. A. Kösen; analysis and interpretation of results: G. Tekin and M. A. Kösen; draft manuscript preparation: G. Tekin and M. A. Kösen. All authors reviewed the results and approved the final version of the manuscript.

Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is in compliance with research and publication ethics.

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