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TRAVELLING WAVE SOLUTIONS FOR SOME TIME-FRACTIONAL NONLINEAR DIFFERENTIAL EQUATIONS

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Abstract: This study employs the powerful generalized Kudryashov method to address the challenges posed by fractional differential equations in mathematical physics. The main objective is to obtain new exact solutions for three important equations: the (3+1)-dimensional time fractional Jimbo-Miwa equation, the (3+1)-dimensional time fractional modified KdV-Zakharov-Kuznetsov equation, and the (2+1)-dimensional time fractional Drinfeld-Sokolov-Satsuma-Hirota equation. The generalized Kudryashov method is highly versatile and effective in addressing nonlinear problems, making it a pivotal component in our research. Its adaptability makes it useful in diverse scientific disciplines. The method simplifies complex equations, improving our analytical capabilities and deepening our understanding of system dynamics. Additionally, we define fractional derivatives using the conformable fractional derivative framework, providing a strong foundation for our mathematical investigations. This paper examines the effectiveness of the generalized Kudryashov method in solving complex challenges presented by fractional differential equations and aims to provide guidance for future studies.

Keywords: Kudryashov method, Time-fractional Jimbo-Miwa equation, KdV-Zakharov-Kuznetsov equation, Drinfeld-Sokolov-Satsuma-Hirota equation, Conformable fractional derivative

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1. Introduction

In the last decade, many complex real-world issues have been linked to nonlinear phenomena. Nonlinear processes are characterized by the sudden alteration of system properties in response to small changes, which poses significant challenges in control. The pursuit of precise solutions for nonlinear partial differential equations is essential for gaining insights into the underlying physical mechanisms in diverse fields, including fluid dynamics, viscoelasticity, and control theory, electrochemistry, acoustics, and system identification. Nonlinear fractional differential equations (NFDEs) are important in scientific research as they offer accurate solutions that capture a wide range of complex nonlinear physical phenomena. The use of these equations enhances our comprehension of complex systems and their nuanced behaviours. When combined with symmetry analysis, NFDEs provide an advanced understanding of complex systems, enabling more accurate predictions. The intersection of fractional differential equations (FDEs) and symmetry analysis is a significant area of research and application. Mathematicians and physicists have devoted significant effort to this research area, using symbolic computer programs such as Maple, Matlab, and Mathematica to simplify algebraic computations for exact solutions to nonlinear partial differential equations. NFDEs have practical applications in various domains, such as modelling viscoelastic materials and understanding system identification processes. These equations can explain phenomena ranging from fluid dynamics to electrochemistry.

NFDEs are significant in modelling biological systems, financial analysis, and applications in fields ranging from economics to biology in the scientific landscape. The significance of these equations lies in their multifaceted applications, which have advanced our understanding of complex systems across various disciplines. In recent years, there has been noteworthy progress in developing diverse, potent, and efficient methods for accurately traversing fields, such as the (G'/G)-expansion method (Zhang et al., 2008; Unal and Ekici, 2021; Ekici and Unal, 2022), the exponential function method (He and Wu, 2006; Naher et al., 2012; Ekici and Unal, 2020), the generalized Riccati equation mapping method (Senol et al., 2021), the extended sinh-Gordon equation expansion method (Bulut et al., 2018), the unified method (Osman, 2019), the sinc-collocation method (Yang et al., 2023), the differential transformation method (Odibat and Momani, 2008; Ekici and Ayaz, 2017), the finite difference method (Tian et al., 2023), the homogeneous balance method (Wang et al., 1996), the homotopy analysis method (Arafa et al., 2011), the generalized

Kudryashov method (Kaplan et al., 2016; Ekici, 2023), etc.

The choice of the generalized Kudryashov method is motivated by its effectiveness in obtaining analytical solutions for complex nonlinear partial differential equations. This methodology is widely recognised as a powerful tool for solving nonlinear evolutionary equations and mathematical models. It is valued for its ability to generate precise solutions in a closed form. This capability aids in understanding the intricacies of nonlinear equations and allows for obtaining analytical solutions for particular problems. Additionally, the method is well-suited for parametric analyses and understanding various physical scenarios. This approach is considered highly effective in addressing fundamental challenges in mathematical physics and revealing the dynamics of complex systems (Jiang et al., 2023).

The KdV-Zakharov-Kuznetsov equations are nonlinear evolutionary equations with significant applications in fields such as plasma physics, magnetohydrodynamics, and quantum mechanics. They are used to model wave interactions, soliton formation, and other complex phenomena. The time-fractional modified KdV-Zakharov-Kuznetsov equation aims to expand this family by introducing a fractional dimension to encompass a broader spectrum of time. This framework enables a detailed analysis of systems that evolve over time, playing a crucial role in understanding nonlinear phenomena and modelling complex dynamics in various scientific domains.

The Jimbo-Miwa equation is a well-known model in the field of nonlinear partial differential equations. It is a generalization of the Korteweg-de Vries equation and is useful for studying solitons and analogous solutions in mathematical physics. The Jimbo-Miwa equation solutions include wave structures, such as solitons, which highlight its ability to model particular behaviours in physical systems. Nonlinear evolutionary equations are significant tools for comprehending wave interactions, energy transfer, and wave dissipation in mathematical physics, particularly in wave theory. The (3+1)-dimensional Jimbo-Miwa equation is a valuable tool that sheds light on the complexities of physical systems and enhances our understanding of specific phenomena.

The study is summarised as follows: Section 2 presents a brief explanation of the conformable fractional derivative and its properties. In Section 3, fundamental information is provided regarding the generalized Kudryashov method, a successful technique for solving partial differential equations of fractional order. In Section 4, the generalized Kudryashov method has been employed to obtain accurate analytical solutions for specific fractional partial differential equations. The conclusion thoroughly discusses our findings and proposes potential directions for future research.

2. Preliminaries

Here, we present a brief overview of the conformable fractional derivative and its main characteristics.

Definition 2.1. Let $\vartheta : \mathbb{R}^{+} \cup \{0\} \to \mathrm{R}$ and $\alpha \in (0,1]$ are given. The definition of the conformable fractional derivative of the function ϑ with respect to order α is as follows (equation 1):

$$(F_\alpha \vartheta)(t) = \lim_{\varepsilon \to 0} \frac{\vartheta(t + \varepsilon t^{1-\alpha}) - \vartheta(t)}{\varepsilon} \quad (t > 0). \tag{1}$$

Theorem 2.1. Let $\alpha \in (0,1]$, $t > 0$ and ϑ, η be α-differentiable. The following properties can be written:

- $F_\alpha(a\vartheta + b\eta) = a(F_\alpha \vartheta) + b(F_\alpha \eta)$, for all $a, b \in \mathbb{R}$
- $F_\alpha(t^n) = nt^{n-\alpha}$ for all $n \in \mathbb{R}$
- $F_\alpha(\lambda) = 0$, for all constant functions $\vartheta(t) = \lambda$
- $F_\alpha(\vartheta\eta) = \vartheta(F_\alpha \eta) + \eta(F_\alpha \vartheta)$
- $F_\alpha(\frac{\vartheta}{\eta}) = \frac{\eta(F_\alpha \vartheta) - \vartheta(F_\alpha \eta)}{\eta^2}$
- If, in addition, ϑ is differentiable, then $(F_\alpha \vartheta)(t) = t^{1-\alpha} \frac{d\vartheta}{dt}$.

For a constant, the derivative of order α is zero (Abdeljawad, 2015; Ekici, 2023).

3. Methodology

The Kudryashov method is a well-established approach in mathematical physics, known for its effectiveness in solving nonlinear fractional differential equations. It is particularly strong in deriving analytical solutions for complex and nonlinear equations. Its adaptability and versatility make it a valuable tool for researchers dealing with equations that involve fractional derivatives. In comparison to alternative analytical methods, the Generalized Kudryashov method has demonstrated a higher degree of directness, a propensity for generating fewer trivial solutions, and a relative simplicity in symbolic calculations. Therefore, we suggest refining these methods to avoid extraneous solutions, which could lead to a significant reduction in the volume of symbolic calculations. The significance of the method in advancing our understanding of nonlinear phenomena across various scientific domains is highlighted by its ability to handle the complexities inherent in such equations. This research introduces the generalized Kudryashov method, which is useful in obtaining stable and explicit soliton solutions for FDEs. The methodology presents a general formulation for nonlinear evolution equations. The equation is represented as given in equation 2:

$$P(u, D_t^\alpha u, u_x, u_y, u_z, D_t^{2\alpha} u, D_t^\alpha u_x, D_t^\alpha u_y, D_t^\alpha u_z, \dots) = 0, \tag{2}$$

where α signifies the conformable fractional derivative, P is a polynomial involving the unknown function $u(x, y, z, t)$, its time-fractional derivatives and various ordinary derivatives. The generalized Kudryashov method stands as a comprehensive strategy for producing characteristic and wide-spectrum soliton solutions for NFDEs with respect to time variables

(Tuluce et al., 2014). The following steps are a summary of the generalized Kudryashov method:

Step 1: We apply the following transformation by introducing a new variable ξ (equation 3).

$$\xi = x + y + z + k\frac{t^{\alpha}}{\Gamma(1+\alpha)}, \quad u(x,y,z,t) = q(\xi)\ , \tag{3}$$

where k is nonzero constant. equation (2) undergoes a reduction to the following nonlinear ordinary differential equation (NODE) through the transformation defined in equation (3);

$$Q\left(q, kq', q', q', k^2q'', k(q')^2, k(q')^2, k(q')^2, \dots\right) = 0, \tag{4}$$

where Q is a polynomial involving q and its ordinary derivatives with respect to ξ. equation 4 is then integrated one or more times, with the constant of integration set to zero.

Step 2: The solution of equation 4 is conjectured to take the following form (equation 5):

$$q(\xi) = \frac{a_0 + \sum_{i=1}^{m} a_i U^i(\xi)}{b_0 + \sum_{j=1}^{n} b_j U^j(\xi)}. \tag{5}$$

Here; a_i, b_j denote the constants to be determined subsequently. Additionally,

$$U(\xi) = \frac{1}{1 + \lambda \exp(\xi)}\ ,$$

expresses the general solution for the equation 6,

$$U'(\xi) = U^2(\xi) - U(\xi)\ , \tag{6}$$

where λ is a constant specifically representing the integration constant of the solution, and the prime notation denotes the first derivative with respect to ξ.

Step 3: The determination of the parameters m and n involves a systematic procedure of homogeneous balancing, with a specific emphasis on terms containing the highest-order derivatives and the highest-order nonlinear term in equation 4. The approach includes substituting the expression derived from equation 5 into equation 4, alongside equation 6. Setting each coefficient, including the various powers of $U(\xi)$, to zero leads to the formulation of a system of algebraic equations.

Step 4: When using mathematical software programs like Maple to solve algebraic equations, we can determine the values of the unknown constants $a_i (i = 0,1,\dots,m)$, $b_j (j = 0,1,\dots,n)$, k and λ. Following this determination, the substitution of these obtained values for a_i and b_j into equation 5 enables the effective derivation of the solution for the nonlinear evolution equation as formulated in equation 4.

4. Applications

This study applies the generalized Kudryashov method to address three prominent equations: the (3+1)-dimensional time fractional modified KdV-Zakharov-Kuznetsov equation, the (3+1)-dimensional time fractional Jimbo-Miwa equation and the (2+1)-dimensional time fractional Drinfeld-Sokolov-Satsuma-Hirota equation. The goal is to unveil significant solutions and enhance our understanding of the underlying dynamics and behaviors described by these conformable time fractional differential equations.

4.1. The (3 + 1)-Dimensional Time Fractional mKdV–ZK equation

The (3+1)-dimensional time fractional mKdV-ZK equation's precise traveling wave solutions are investigated using the generalized Kudryashov method. The mKdV-ZK equation, first study by (Lazarus et al., 2008), later derive multiple-soliton solutions by (Rehman et al., 2022; Zhou et al., 2022) and investigate various traveling wave solutions by (Khater, 2022; Younas et al., 2023). The time-fractional mKdV-ZK equation is expressed as given in equation 7:

$$D_t^{\alpha} u + \delta u^2 u_x + u_{xxx} + u_{xyy} + u_{xzz} = 0, \tag{7}$$

where α represents the fractional derivative over the interval $[0,1]$ and $u(x,y,z,t)$ is a differentiable function (Mace and Hellberg, 2001; Alabedalhadi et al., 2023; Onder et al., 2023). The equation 7 is among the most vital integrable equations in nonlinear dynamics. It describes a wide range of nonlinear dispersive physical phenomena and has various applications in nonlinear sciences. Notably, it is crucial in modeling the conservative flow of the Liouville equation, the 2-dimensional gauge field theory of conformal field and the theory of quantum gravity among other areas.

Now we apply the method to the equation 7. Substituting equation 3 into equation 7 reduces to the nonlinear ODE

$$kq' + \delta q^2 q' + 3q''' = 0, \tag{8}$$

where $q' = \frac{dq}{d\xi}$. Integrating equation 8 once with respect to ξ , we get

$$kq + \frac{\delta s}{3} q^3 + 3q'' + c = 0, \tag{9}$$

where k is non-zero constant. Using the homogeneous balance method, that is balancing q'' and q^3 term in equation 9, we find $m = 3,\ n = 2$. Hence, from equation 5 we have

$$q(\xi) = \frac{a_0 + a_1 U(\xi) + a_2 U^2(\xi) + a_3 U^3(\xi)}{b_0 + b_1 U(\xi) + b_2 U^2(\xi)}. \tag{10}$$

Subsequently, we substitute equation 10 into equation 9 and organize all terms in a manner that each coefficient $U^i(\xi)\ (i = 0,1,\dots,9)$ equates to zero, resulting in a system of equations. Employing mathematical software, we can then solve these equations to deduce a set of solutions for k, $a_i (i = 0,1,2,3)$, $b_j (j = 0,1,2)$:

Case 1: Specifically, the obtained values for the constants are as follows:

$$a_0 = -\frac{a_3}{2}, a_1 = \frac{3a_3}{2}, a_2 = -\frac{3a_3}{2},$$

$b_0 = \frac{b_2}{3}, b_1 = -b_2, k = \frac{27}{2}, \delta = -18\left(\frac{b_2}{a_3}\right)^2.$

Upon substituting these values into equation 10, the solution for equation 7 is obtained as:

$$q(\xi) = -\frac{3a_3[\lambda^3 e^{3\xi} - 1]}{2b_2[\lambda^2 e^{2\xi} - \lambda e^{\xi} + 1][1 + \lambda e^{\xi}]}. \quad (11)$$

For the specific case where $\lambda = a_3 = b_2 = 1$ in equation 11, the solution simplifies to:

$q(\xi) = -\frac{3}{2}\tanh(\frac{3\xi}{2}),$

where ξ is defined as $\xi = x + y + z + \frac{27t^\alpha}{2\Gamma(1+\alpha)}$.

Case 2:

$$a_0 = \frac{(2b_1 + b_2)a_3}{4b_2}, a_1 = -\frac{a_3 b_1}{b_2}, a_2 = \frac{(2b_1 - b_2)a_3}{2b_2},$$

$$b_0 = -\frac{2b_1 + b_2}{4}, k = 6, \quad \delta = -18\left(\frac{b_2}{a_3}\right)^2.$$

Upon substituting these values into equation 10, the solution for equation 7 is obtained as:

$$q(\xi) = -\frac{a_3\left(2b_1\lambda^3 e^{3\xi} + b_2\lambda^3 e^{3\xi} + 2b_1\lambda^2 e^{2\xi} + 3b_2\lambda^2 e^{2\xi} + 2b_1\lambda e^{\xi} + b_2\lambda e^{\xi} + 2b_1 + 3b_2\right)}{b_2(2b_1\lambda^2 e^{2\xi} - b_2\lambda^2 e^{2\xi} - 2b_2\lambda e^{\xi} - 2b_1 - 5b_2)(1 + \lambda e^{\xi})}. \quad (12)$$

For the specific case where $\lambda = b_1 = b_2 = a_3 = 1$ equation 12, the solution simplifies to:

$$q(\xi) = -\frac{3e^{3\xi} + 5e^{2\xi} + 3e^{\xi} + 5}{(e^{2\xi} - 2e^{\xi} - 7)(1 + e^{\xi})}.$$

Here, ξ is defined as $\xi = x + y + z + \frac{6t^\alpha}{\Gamma(1+\alpha)}$.

Case 3:

$$a_0 = -\frac{b_0 a_3}{2b_2}, a_1 = \frac{a_3(2b_0 - b_1)}{2b_2}, a_2 = \frac{a_3(2b_1 - b_2)}{2b_2},$$

$$k = \frac{3}{2}, \quad \delta = -18\left(\frac{b_2}{a_3}\right)^2.$$

Upon substituting these values into equation 10, the solution for equation 7 is obtained as:

$$q(\xi) = \frac{-\frac{b_0 a_3}{2b_2} + \frac{a_3(2b_0 - b_1)}{2b_2}U(\xi) + \frac{a_3(2b_1 - b_2)}{2b_2}U^2(\xi) + a_3 U^3(\xi)}{b_0 + b_1 U(\xi) + b_2 U^2(\xi)}. \quad (13)$$

For the specific case where $\lambda = b_2 = a_3 = 1$ in equation 13, the solution simplifies to:

$q(\xi) = -\frac{1}{2}\tanh(\xi/2),$

where ξ is defined as $\xi = x + y + z + \frac{3t^\alpha}{2\Gamma(1+\alpha)}$.

Case 4:

$$a_0 = 0, \quad a_2 = -a_1 - a_3, \quad b_1 = -\frac{b_0(2a_1 + a_3)}{a_1},$$

$$b_2 = 2\frac{b_0 a_3}{a_1}, k = -3, \quad \delta = -72\left(\frac{b_0}{a_1}\right)^2.$$

Upon substituting these values into equation 10, the solution for equation 7 is obtained as:

$$q(\xi) = \frac{a_1 U(\xi) + (-a_1 - a_3)U^2(\xi) + a_3 U^3(\xi)}{b_0 + \left(-\frac{b_0(2a_1 + a_3)}{a_1}\right)U(\xi) + \left(\frac{2b_0 a_3}{a_1}\right)U^2(\xi)}, \quad (14)$$

For the specific case where $\lambda = b_0 = a_1 = 1$ in equation 14, the solution simplifies to:

$q(\xi) = -\frac{1}{2}\mathrm{csch}(\xi),$

where ξ is defined as $\xi = x + y + z - \frac{3t^\alpha}{\Gamma(1+\alpha)}$.

4.2. The (3+1)-Dimensional Time Fractional Jimbo–Miwa equation

Now, we intend to apply our methodology to the (3+1)-dimensional Jimbo-Miwa equation. Let us contemplate the (3+1)-dimensional Jimbo-Miwa equation (Roshid et al., 2014; Korkmaz, 2017),

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2D_t^\alpha u_y - 3u_{xz} = 0. \quad (15)$$

We employ the generalized Kudryashov method to equation 15. We apply the equation 3 to the equation 15, hence equation 15 reduces the to the following nonlinear ODE

$$q^{(4)} + 6q'q'' - 2kq'' - 3q'' = 0 \quad (16)$$

where $q' = \frac{dq}{d\xi}$. Integrating equation 16 once with respect to ξ, we get

$$q''' + 3(q')^2 - 2kq' - 3q' = 0, \quad (17)$$

where k is a constant. Using the homogeneous balance method, that is balancing q''' and $(q')^2$ term in equation 16, we find $m = 3$, $n = 2$. Hence, from equation 5 we have

$$q(\xi) = \frac{a_0 + a_1 U(\xi) + a_2 U^2(\xi) + a_3 U^3(\xi)}{b_0 + b_1 U(\xi) + b_2 U^2(\xi)}. \quad (18)$$

Next, we substitute equation 18 into equation 16 and the set of equations obtained by setting each coefficient of $U^i(\xi)$ $(i = 0,1,\ldots,12)$ to zero should be organized. These equations can then be solved using mathematical software to obtain a set of solutions for $k, b_0, b_1, b_2, a_i (i = 0,1,2,3)$:

Case 1: Specifically, the obtained values for the constants are as follows:

$$a_0 = \frac{a_2 b_0 + 2b_0 b_1}{b_2},$$

$$a_1 = \frac{a_2 b_1 - 2b_0 b_2 + 2b_1^2}{b_2},$$

$$a_3 = -2b_2, \quad k = -1.$$

Upon substituting these values into equation 18, the solution for equation 17 is obtained as:

$$q(\xi)=\frac{\frac{a_2b_0+2b_0b_1}{b_2}+\left(\frac{a_2b_1-2b_0b_2+2b_1^2}{b_2}\right)U(\xi)+a_2U^2(\xi)-(2b_2)U^3(\xi)}{b_0+b_1U(\xi)+b_2U^2(\xi)}. \tag{19}$$

If we rearrange and simplify the equation 19, we obtain the following form

$$q(\xi)=\frac{\lambda a_2e^{\xi}+2\lambda b_1e^{\xi}+a_2+2b_1-2b_2}{b_2(1+\lambda e^{\xi})},$$

where ξ is defined as $\xi=x+y+z-\frac{t^{\alpha}}{\Gamma(1+\alpha)}$. The solution function can be expressed in terms of hyperbolic functions by assigning specific values.

Case 2:

$$a_0=-\frac{2a_2b_1+a_2b_2+4b_1^2+4b_1b_2+b_2^2}{4b_2},$$

$$a_1=\frac{b_1a_2+2b_1^2+b_1b_2}{b_2},$$

$$a_3=-2b_2,\qquad b_0=-\frac{1}{2}b_1-\frac{1}{4}b_2,\qquad k=\frac{1}{2}.$$

Upon substituting these values into equation 18, the solution for equation 15 is obtained as:

$$q(\xi)=-\frac{4b_2U(\xi)^2-(2a_2+4b_1+2b_2)U(\xi)+a_2+b_2+2b_1}{(2U(\xi)-1)b_2}. \tag{20}$$

If we rearrange and simplify the equation 20, we obtain the following form

$$q(\xi)=\frac{\lambda^2e^{2\xi}(a_2+2b_1+b_2)-(a_2+2b_1-3b_2)}{b_2(\lambda^2e^{2\xi}-1)},$$

where ξ is defined as $\xi=x+y+z+\frac{t^{\alpha}}{2\Gamma(1+\alpha)}$. The solution function can be expressed in terms of hyperbolic functions by assigning specific values.

Case 3:

$$a_0=\frac{a_2}{3},\qquad a_1=-a_2, a_3=-2b_2, b_2=3b_0,$$

$$b_1=-b_2,\qquad k=3.$$

Upon substituting these values into equation 18, the solution for equation 15 is obtained as:

$$q(\xi)=-\frac{a_2+3a_2U(\xi)-3a_2U^2(\xi)+18b_0U^3(\xi)}{3b_0(1-3U(\xi)+3U^2(\xi))}, \tag{21}$$

If we rearrange and simplify the equation 21, we obtain the following form

$$q(\xi)=-\frac{\lambda^3a_2e^{3\xi}+a_2-18b_0}{3b_0(1+\lambda e^{\xi})(-\lambda^2e^{2\xi}+\lambda e^{\xi}-1)},$$

where ξ is defined as $\xi=x+y+z+\frac{3t^{\alpha}}{\Gamma(1+\alpha)}$. The solution function can be expressed in terms of hyperbolic functions by assigning specific values.

Case 4:

$$a_2=-\frac{a_0^3b_1^2-2a_0^2a_1b_0b_1-4a_0^2b_0^2b_1-4a_0^2b_0b_1^2+a_0a_1^2b_0^2+4a_0a_1b_0^3+6a_0a_1b_0^2b_1}{4b_0^4}$$
$$+\frac{4a_0b_0^4+8a_0b_0^3b_1+4a_0b_0^2b_1^2-2a_1^2b_0^3-4a_1b_0^4-4a_1b_0^3b_1}{4b_0^4}$$

$$b_2=-\frac{a_0^2b_1^2-2a_0a_1b_0b_1-4a_0b_0^2b_1-2a_0b_0b_1^2+a_1^2b_0^2+4a_1b_0^3+2a_1b_0^2b_1+4b_0^4+4b_0^3b_1}{4b_0^3},$$

$$k=-1,\qquad a_3=0,$$

Upon substituting these values into equation 18, the solution for equation 15 is obtained as:

$$q(\xi)=\frac{2a_0b_0^2+(a_0^2b_1-a_0a_1b_0-2a_0b_0^2-2a_0b_0b_1+2a_1b_0^2)U(\xi)}{2b_0^3+b_0(a_0b_1-a_1b_0-2b_0^2)U(\xi)}, \tag{22}$$

If we rearrange and simplify the equation 22, we obtain the following form

$$q(\xi)=\frac{2\lambda a_0b_0^2e^{\xi}+a_0^2b_1-a_0a_1b_0-2a_0b_0b_1+2a_1b_0^2}{b_0(2\lambda b_0^2e^{\xi}+a_0b_1-b_0a_1)},$$

where ξ is defined as $\xi=x+y+z-\frac{t^{\alpha}}{\Gamma(1+\alpha)}$. The solution function can be expressed in terms of hyperbolic functions by assigning specific values.

4.3. The (2+1)-Dimensional Time-Fractional Drinfeld Sokolov Satsuma Hirota equation

We first consider the following Couple Boiti-Leon-Pempinelli equations system is of the form

$$\begin{cases}u_{ty}=(u^2-u_x)_{xy}+2v_{xxx}\\ v_t=v_{xx}+2uv_x\end{cases}. \tag{23}$$

The second considerable problem Drinfeld-Sokolov-Satsuma-Hirota (DSSH) equation which is widely used in mathematical physics in the form is given by (Ding and Feng, 2014; Ali et al., 2018).

$$u_{6x}-9u_xu_{4x}-18\ u_{xx}u_{3x}+18\ u_x^2u_{xx}-\frac{1}{2}\ D_t^{2\alpha}u$$
$$+\frac{1}{2}\ D_t^{\alpha}u_{xxx}=0.$$

By using the generalized Kudryashov method to solve equation 23. Substituting equation 3 into equation 23 reduces the to following nonlinear ODE

$$q^{(6)}-9q'q^{(4)}-18q''q'''+18(q')^2q''$$
$$-\frac{1}{2}k^2q''-\frac{1}{2}kq^{(4)}=0, \tag{24}$$

where $q'=\frac{dq}{d\xi}$. Integrating equation 24 once with respect to ξ , we get

$$q^{(5)}-9q'q'''-\frac{9}{2}(q'')^2+6(q')^3-\frac{1}{2}k^2q' -\frac{1}{2}kq'''=0, \tag{25}$$

where k is a constant. Using the homogeneous balance method, that is balancing $q^{(5)}$ and $(q')^3$ term in equation 25, we find $m=2,\ n=1$. Hence, from equation 5 we have

$$q(\xi)=\frac{a_0+a_1U(\xi)+a_2U^2(\xi)}{b_0+b_1U(\xi)}. \tag{26}$$

Next, we substitute equation 26 into equation 25 and the set of equations obtained by setting each coefficient of $U^i(\xi)\ \ (i=0,1,\ldots,12)$ to zero should be organized. These equations can then be solved using mathematical software to obtain a set of solutions for $k, b_0, b_1, a_i(i=0,1,2)$:

Case 1:

Specifically, the obtained values for the constants are as follows:

$$a_0=\frac{a_1b_0-2b_0^2}{b_1},\qquad a_2=2b_1,\qquad k=1.$$

Upon substituting these values into equation 26, the solution for equation 23 is obtained as:

$$q(\xi)=\frac{\frac{a_1b_0-2b_0^2}{b_1}+a_1U(\xi)+2b_1U^2(\xi)}{b_0+b_1U(\xi)}. \tag{27}$$

If we rearrange and simplify the equation 27, we obtain the following form

$$q(\xi)=\frac{(\lambda a_1-2\lambda b_0)e^{\xi}+a_1-2b_0+2b_1}{b_1(1+\lambda e^{\xi})},$$

where ξ is defined as $\xi=x+y+\frac{t^{\alpha}}{\Gamma(1+\alpha)}$. The solution function can be expressed in terms of hyperbolic functions by assigning specific values.

Case 2:

$$a_0=-\frac{a_1}{2},\qquad a_2=2b_1,\qquad b_0=-\frac{1}{2}b_1, k=4.$$

Upon substituting these values into equation 26, the solution for equation 23 is obtained as:

$$q(\xi)=\frac{-a_1+2a_1U(\xi)+4b_1U^2(\xi)}{b_1(-1+2U(\xi))}, \tag{28}$$

If we rearrange and simplify the equation 28, we obtain the following form

$$q(\xi)=\frac{\lambda^2a_1e^{2\xi}-a_1-4b_1}{b_1(\lambda^2e^{2\xi}-1)},$$

where ξ is defined as $\xi=x+y+\frac{4t^{\alpha}}{\Gamma(1+\alpha)}$.The solution function can be expressed in terms of hyperbolic functions by assigning specific values.

Case 3:

$$a_0=\frac{(\mp\sqrt{2}-1)}{2}(a_1+2b_1)-b_1,$$

$$a_2=2b_1, b_0=\frac{(\mp\sqrt{2}-1)b_1}{2}, k=-2.$$

Upon substituting these values into equation 26, the solution for equation 23 is obtained as:

$$q(\xi)=\frac{\frac{(\mp\sqrt{2}-1)}{2}(a_1+2b_1)-b_1+a_1U(\xi)+2b_1U^2(\xi)}{\frac{(\mp\sqrt{2}-1)}{2}b_1+b_1U(\xi)}, \tag{29}$$

If we rearrange and simplify the equation 29, we obtain the following form

$$q(\xi)=\frac{(\mp\sqrt{2}\lambda^2a_1\mp2\sqrt{2}\lambda^2b_1-\lambda^2a_1-4\lambda^2b_1)e^{2\xi}+(\mp2\sqrt{2}\lambda a_1\mp4\sqrt{2}\lambda b_1-8\lambda b_1)e^{\xi}\mp\sqrt{2}a_1\mp2\sqrt{2}b_1+a_1}{b_1[\lambda(\mp\sqrt{2}-1)e^{\xi}\mp\sqrt{2}+1](1+\lambda e^{\xi})},$$

where ξ is defined as $\xi=x+y-\frac{2t^{\alpha}}{\Gamma(1+\alpha)}$.The solution function can be expressed in terms of hyperbolic functions by assigning specific values.

4. Conclusion

In the field of mathematical physics, obtaining analytical solutions for nonlinear differential equations is a significant challenge and a crucial step in advancing our comprehension of intricate physical phenomena. The research has achieved significant success by deriving analytical solutions for three challenging equations. These equations have contributed significantly to our understanding of physical phenomena due to their profound effects in mathematical physics. The efficacy of analytical methods in elucidating the latent dynamics inherent in complex time-fractional equations is underscored by our achievements. This provides valuable insights into their intrinsic behaviors and expands the boundaries of mathematical physics.

This investigation has achieved significant success in uncovering multiple exact solutions, including novel hyperbolic solutions. These results serve as compelling evidence of the efficacy of the generalized Kudryashov method and hold the potential to advance our comprehension of nonlinear physical phenomena, offering insights into their underlying intricacies.

Utilizing the potent nonlinear fractional transformation, commonly known as the fractional complex transformation, we adeptly transformed the intricate landscape of nonlinear fractional partial differential equations into more manageable ordinary differential equations with integer orders. The solutions for time-fractional nonlinear evolution equations can be elegantly articulated in the form of $U(\xi)$ polynomials.

In conclusion, this manuscript accentuates the efficacy of the generalized Kudryashov method in addressing intricate fractional differential equations. The obtained results not only broaden the array of available mathematical techniques but also deepen our comprehension of nonlinear physical phenomena. Future research endeavors may explore more intricate fractional differential equations, leveraging advanced mathematical methodologies. This trajectory holds the promise of yielding further insights into the captivating realm of nonlinear dynamics, thereby contributing to the resolution of some of the most intricate problems in science. In our research, our main objective is not only to expand the array of mathematical methodologies but also to illuminate the complex dynamics inherent in nonlinear systems.

Author Contributions

The percentage of the author contributions is presented below. The author reviewed and approved the final version of the manuscript.

	M.E.
C	100
D	100
S	100
DCP	100
DAI	100
L	100
W	100
CR	100
SR	100
PM	100
FA	100

C=Concept, D= design, S= supervision, DCP= data collection and/or processing, DAI= data analysis and/or interpretation, L= literature search, W= writing, CR= critical review, SR= submission and revision, PM= project management, FA= funding acquisition.

Conflict of Interest

The author declared that there is no conflict of interest.

Ethical Consideration

Ethics committee approval was not required for this study because of there was no study on animals or humans.

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