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## **An Overview of Geometry Curriculum and Its Understanding by Albanian Students**

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**Abstract:** In recent years, the Mathematics curriculum of pre-university education in Albania has undergone several reforms, which reflected changes in teaching and learning methods, but also in the reduction or expansion of various teaching topics. As a result, geometric topics have been the subject of these implemented reforms. Mainly, it is focused on geometric transformations, polygons, circles, geometric bodies, and two-dimensional vectors. Meanwhile, the curriculum of Geometry in programs such as Mathematics, Engineering, and Physics has not been revised by this reform. This paper aims to investigate the situation and to give ideas on methods and ways that have the potential to transform and reduce this gap that has been created in the treatment of Geometry in pre-university and university education.

**Keywords:** Geometry curriculum, Teaching method, Conceptual error, Procedural error

### **Introduction**

Geometry plays an important role in teaching Mathematics. It begins in preschool education and continues until university education. The teaching of Geometry helps students to have confidence in their mathematical abilities, to be good problem solvers, and to communicate and reason mathematically correctly (Ramlan 2016) for the fact that it is the best tool for the development of logical thinking. Geometry helps to understand concepts and solve problems in Algebra, Analysis, Probability, and Statistics and not only but also Physics, Chemistry, Informatics, etc. According to Bassarear (2012), Geometry is the subject that studies shapes, their relationships, and their properties. Recent studies have confirmed that students have difficulties understanding the concepts of Geometry (Ada & Kurtulus, 2010). These results are also identified in Albanian students, who have been under a new curriculum of Mathematics since 2010. The new curriculum of Mathematics contains fewer themes of Geometry in comparison with the old one. It is noticed that the study of ellipse, hyperbola, and parabola as a geometric set is absent. In addition, the student does not know lines and planes in 3D Geometry. Also, the concept of the vectorial product of two vectors is not given.

Due to these changes, it is seen that many students of Bachelor of Mathematics, Informatics, and Economics are faced with difficulties in learning several concepts of Geometry. Consequently, in this paper, we have identified the problems that students have in the implementation of the new curricula on geometric concepts.

The objectives of this study are:

1. The identification of difficulties and misconceptions of students related to geometric conception.
2. Analyzing the collected data by classifying them.
3. Giving ideas to overcome the gap between university programs and pre-university education.

Table 1. Themes related to geometry in the tenth grade of the new curriculum of mathematics are given in the table below (Fearnley et al., 2016)

10-th GRADE		
Themes	Concepts	Hours
Angles and polygons	Angles and lines	2 hours
	Triangles and quadrilaterals	
	Congruences and similarity	
2D Geometry	Angles of polygons	2 hours
	The measure of length and angles	
The Circle and geometric position	The syprine of 2D shapes	2 hours
	Circle	
	Circle Theorems	

Table 2. In the 11-th grade, themes, and concepts (Fearnley et al. 2016) are presented in

11-th GRADE		
Themes	Concepts	Hours
3 D Geometry	Geometric bodies	2 hours
	Volume of prisms	
	Total surface and volume of geometric bodies	
Graphs	The equation of a line	2 hours
	The graph of a parabola	

Table 3. Themes of geometry in 12-ve grade (Jefferson, 2019), are given in.

12-ve GRADE		
Themes	Concepts	Hours
Vectors	Definitions and properties	2 hours
	Coordinates of a vector	

## Method

To achieve the goal of this study, a test was conducted. The target population was 12th-grade high school students and first-grade students of programs Bachelor in Mathematics, Informatics, and Economics. The participants of the champion were required to write their solutions in the test paper. The test was created based on the geometric knowledge that a student should have by his curriculum according to the method (Bassarar 2012). The duration of the test was 45 minutes. The methodology of this study consisted of collecting and analyzing data through descriptive statistics. Students' errors were categorized according to the classification of (Luneta 2015).

## Results and Discussion

The test was completed by 192 of 12th-grade high school students and 87 university students. The test consisted of 5 questions. The two first questions were given to identify the misconceptions about 2D shapes and their relationship with 3D shapes. The purpose of the third question was to survey errors in location determination, reasoning, and geometric modeling for problem-solving. Question 4 intended to study the knowledge of students on vectors (part of coordinative geometry). The last question, Question 5 examined students' understanding of geometric transformations. Below, there are given the questions of the test and the results.

**Question 1.**  $ABCD$  is a rectangle with length  $AB = 6\text{cm}$ , and diagonal  $AC = 10\text{cm}$ .

- Find the perimeter and surface of  $ABCD$ .
- Label and show elements of the obtained geometric body by the rotation of the rectangular around its length  $AB$ .

The answers' results in percent are given in the following diagram:

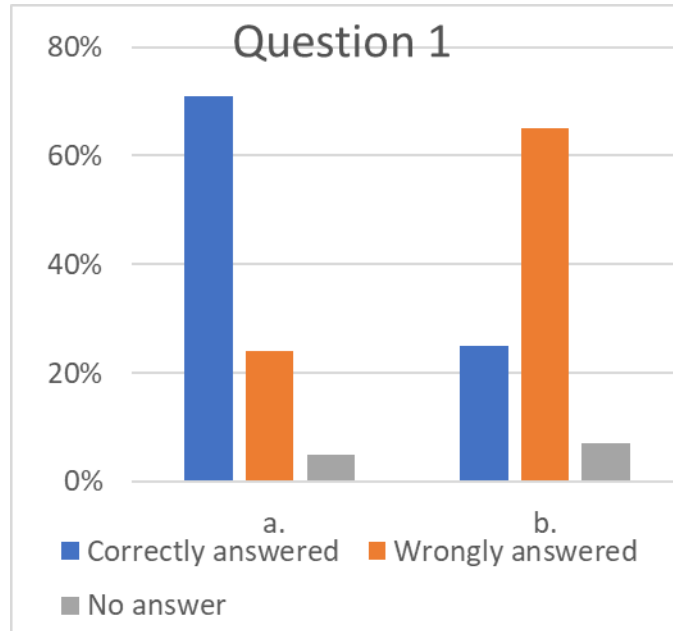


Diagram 1. Question 1 results

Question a. was solved correctly by 71% of students, 5% of them left the test blank and 24% solved it incorrectly or partially incorrectly. Question b. was solved correctly by 27% of students, 8% of them did not touch it at all, and 65% solved it incorrectly. In total, 25% of students solved the first exercise correctly, 5% of them did not touch it at all and 70% solved it incorrectly. 13% of students who solved incorrectly, Question 1, was made conceptual mistakes. They did not know the concepts of rotation, and cylinder. The others made procedural mistakes by applying Pythagoras Theorem incorrectly etc.

**Question 2.** The cuboid with dimensions 4 cm, 5 cm, and 8 cm is given.

- List all pairs of parallel sides of the cuboid.
- Sketch three openings of this cuboid.
- Find the total surface area and volume of the cuboid.

The students' results are shown in the following diagram.

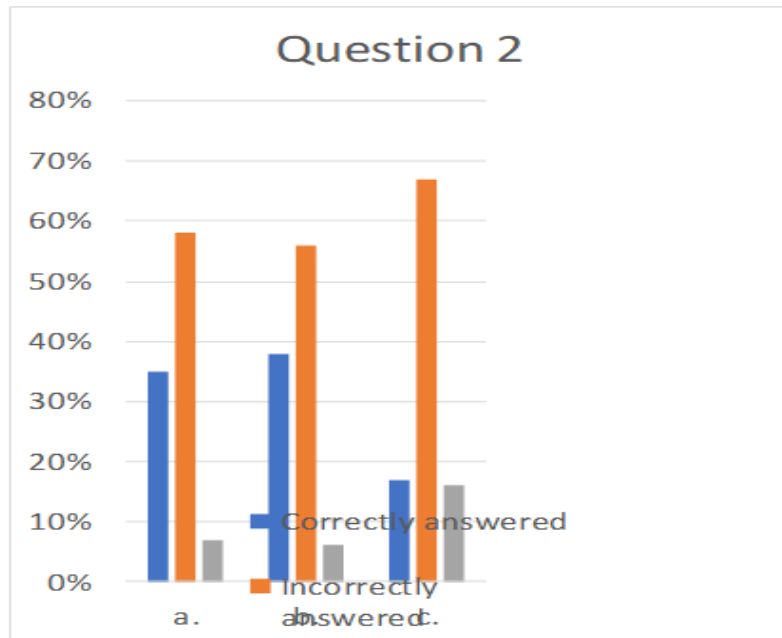


Diagram 2. Question 2 results

35% of the participants solved question a. correctly; 58% incorrectly and 7% left it blank. 38% of students solved it correctly; 56% incorrectly and 6% did not solve it at all.

Question c, 17% of them solved correctly; 67% incorrectly, and 16% left it blank. In total, 17% of the test participants solved the second exercise correctly, 5% of them left it blank and 72% of them solved the second exercise incorrectly. Conceptual errors made by students were 13%. Most of them do not know the concepts of the lateral surface and the total surface of the cuboid. 54% of students' errors were procedural (participants either opened the cuboid incorrectly or could not indicate 3 openings of it; also, they did not show all pairs of parallel sides).

**Question 3.** A ship sails from port  $O$  on a course of  $30^\circ$  for a distance of 50 km to reach port  $A$ . Another ship departs from port  $O$  on a course of  $200^\circ$  at a distance of 120 km to reach port  $B$ .

- Represent the data in the rectangular coordinate system  $Oxy$ .
- Find the course from port  $A$  to port  $B$ .
- Find the distance between  $A$  and  $B$ .

Below, there are given results of students in percent.

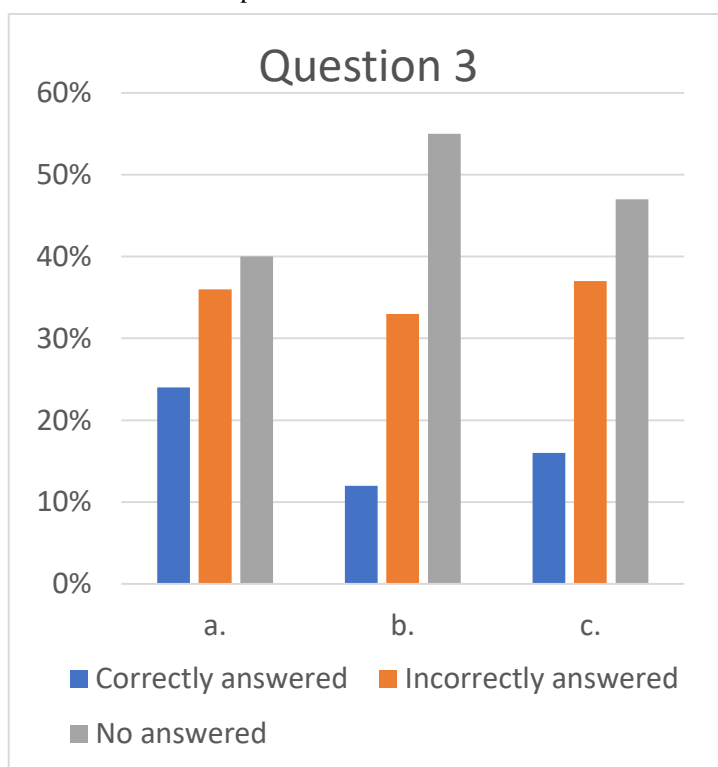


Diagram 3. Question 3 results

24% of participants gave a correct solution for question a.; 36% of them answered incorrectly and the other part of students did not give any answer. 12% of students answered correctly to question b.; 33% of them answered incorrectly and 55% did not answer.

Question c., there were answered correctly by 16% of students; 37% incorrectly and 47% left the test blank.

19% of mistakes were conceptual (The students made mistakes, especially in the course' concept, they chose an oriented direction, the axe of  $ox$ ). 27% of mistakes were procedural (the participants used inappropriate theorems to find the required elements, for example, they used Pythagoras's Theorem instead of the Theorem of cosine).

**Question 4.** Points  $A$ ,  $B$ , and  $C$  have a vectorial radius  $2\vec{i} + \vec{j}$ ,  $-\vec{i} + 3\vec{j}$ ,  $\vec{i} - \vec{j}$  respectively.

- Draw the points  $A$ ,  $B$  and  $C$  in a cartesian plane  $oxy$  and find the coordinates of vectors  $\vec{AB}$ ,  $\vec{BC}$  and  $\vec{CA}$ .
- Evaluate the surface of triangle  $ABC$ .

Diagram 4 shows the results of students in percent.

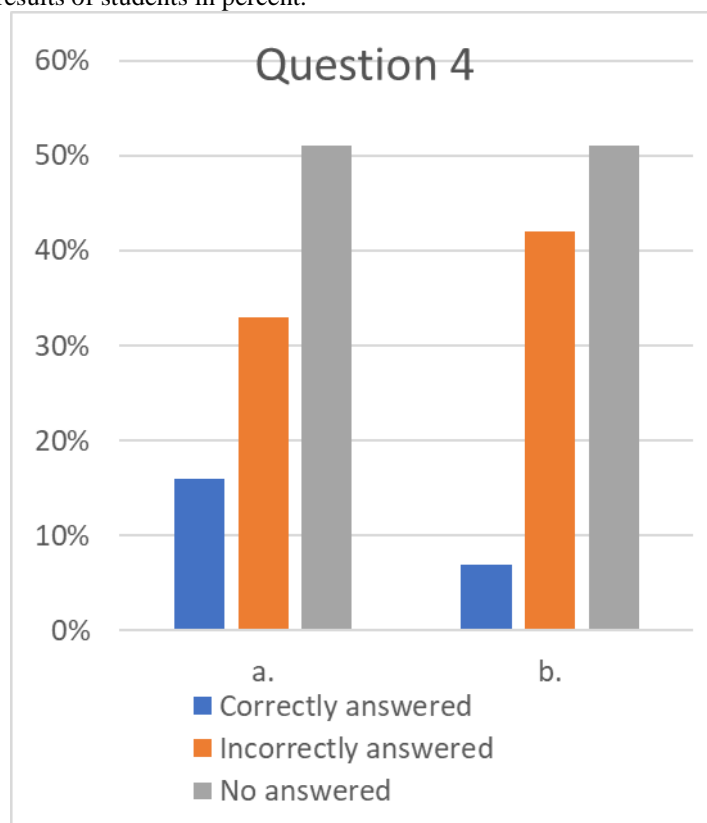


Diagram 4. Question 4 results

16% of participants gave correct answers for question a.; 33% of them answered wrongly or partially wrong and 51% of students left the test question a. blank.

Question b., 7% of students solved it correctly; 42% of them incorrectly, and 51% of participants did not answer.

28% of errors made in Question 4 were conceptual (Students did not know how to represent a vector as a linear combination of unit vectors  $\vec{i}$ ,  $\vec{j}$ ; and they wrote the length of vectors as a vector). 12% of mistakes were

procedural. (The participants used height to find the surface of the triangle, and they took it as a bisector of a triangle.)

**Question 5.** Let  $ABC$  be a triangle where  $A(-1,2)$ ,  $B(2, 3)$ ,  $C(5,0)$ .

- Draw the triangle  $A'B'C'$  as an image of triangle  $ABC$  during a rotation with center  $O$  and angle  $90^\circ$  on anticlockwise orientation.
- Draw the image  $A''B''C''$  of triangle  $A'B'C'$  in a symmetry respect to line  $y = x$ .
- Describe the geometric transformation that transform triangle  $ABC$  to triangle  $A''B''C''$ .
- Show the relationship between elements of triangles  $ABC$ ,  $A'B'C'$  dhe  $A''B''C''$ ?

The results of students' answers are given below:

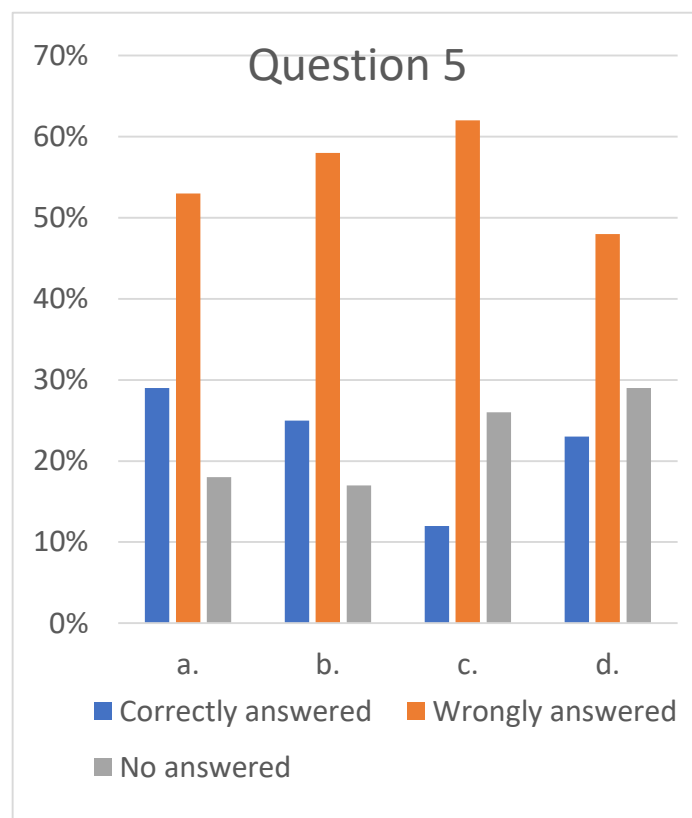


Diagram 5. Question 5

29% of participants answered correctly; 53% of students solved the question a. Incorrectly, and the others left it blank. Question b, 25% of students sketched correctly the image of triangle  $A'B'C'$  during the symmetry with line  $y = x$ ; 58% of them drew it wrongly and 17% did not give any answer. The results for question c. showed that there were a few students that solved it correctly, only 12% of participants; 62% of them gave wrong or partially wrong solutions. For question d., 23% of participants gave the right answer; meanwhile, 48% of them left the question blank. 52% of mistakes in Question 5 were conceptual (mainly, the participants did not have clear concepts of rotation and line symmetry. Most of them did not determine the geometric transformation that is used when preimage and image were given.

The other mistakes were procedural. The students used the wrong properties to show the relationship between elements of preimage and image during a geometric transformation. They had difficulties to determine the composed transformation.

## Conclusions

In this paper, there are shown difficulties of students in geometric concepts related to the Albanian new curriculum of Mathematics. The study shows that high school students struggle with perception of geometric concepts. Furthermore, the students, manifest problems with the procedure in solving a geometric exercise. There exists a handicap between learning geometric concepts and using them in several geometric situations. As a result, abstraction, the third Van Hiele level is not understood by the students. It is noticed that the right or wrong solutions of exercises are not explained with words, consequently, there exists a lack of geometric expressions on the part of students.

## Recommendations

According to this study, students need to avoid mistakes in perceiving geometric concepts and solving geometric problems. We recommend:

1. The implementation of Van Hiele's levels at least up to the level of abstraction (Van Hiele 1986) in explaining geometric concepts and solving geometric problems by the teacher.
2. Using the learning of geometric conceptions and properties intertwined with technology.
3. Ask the student for an explanation of the solution of an exercise.

## Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPESS Journal belongs to the author.

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## References

- Ada, T. & Kurtulus, A. (2010). Students' misconceptions and errors in transformation geometry. *International Journal of Mathematics Education in Science and Technology*, 41(7), 901-909.
- Bassarear, T. (2012). *Mathematics for elementary school teachers*. (5th ed.). London: Brooks/Cole.
- Fearnley, S., Haighton, J., Lomax, S., Mullarkey, P., Nicholson, J., & Nixon, M.(2016.) *Matematika 10-11, Pjesa 1*. Botime Pegi.
- Fearnley, F., Haighton, J., Lomax, S., Mullarkey, P., Nicholson, J. & Nixon, M.(2016) *Matematika 10-11, Pjesa 2*. Botime Pegi.
- Jefferson, B. (2019). *Matematika 12*, Botime Pegi.
- Luneta, K., & Makonye J.P. (2011). Undergraduate students' preferences of knowledge to solve particle mechanics problems. *Journal for Science and Mathematics Education for Southeast Asia*, 34(2), 237–261.
- Ramlan, A. M. (2016). The effect of van Hiele Learning model toward geometric reasoning ability based on the self-efficacy of senior high school students. *Journal of Mathematics Education* 1(2) 63-72.
- Van Hiele, P. M.(1986). *Structure and insight. A theory of mathematics education*, Academic Press Inc.

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