International Journal of Economics and Financial Issues

ISSN: 2146-4138

available at http: www.econjournals.com

International Journal of Economics and Financial Issues, 2017, 7(3), 609-612.

Structure and Intensity Based Approach in Credit Risk Models: A Literature Review

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ABSTRACT

Credit risk modeling has been a subject of considerable research interest for finance and statistical researchers. The quantification of credit risk by assigning measurable and comparable numbers to the likelihood of default or spread risk is a major frontier in modern finance. In this paper we provide a literature review of credit risk models including both structural and intensity based approaches. Our focus is placed on probability of default and hazard rate of time to default.

Keywords: Credit Risks, Defaults, Weiner Process, Volatility
JEL Classification: C01, C02,C87

1. INTRODUCTION TO CREDIT RISK MODELLING

Credit risk is pivotal area in banking and is a concern to a variety of stake holders namely institutions, customers, and stakeholders. The predominant factor to credit risk is the default event where the debtor is unable to meet his legal obligation according to the debt contract. The examples of defaults include bond default, corporate bankruptcy, the credit card charge off and the mortgage foreclosure. The other forms of credit risk include repayment delinquency in retail loans, the loss severity upon default events and unexpected change in credit rating.

An extensive literature has been fostered by both academics in finance and practitioners in industry. There are two parallel worlds based upon a simple dichotomous rule of data availability. Firstly, the direct measurements of credit performance and secondly the prices observed from credit market. The two worlds of credit risk can be characterized by default probability, one being actual and other being implied. The former corresponds to direct observation of defaults, also known as physical default probability in finance. Credit ratings and scores represent the credit worthiness of individual corporations and consumers.The final evaluations are based on expected probability as well as judgment by rating or scoring specialist. The latter refers to risk neutral default probability implied from credit market data e.g. corporate bond yields.

When compared to the academics of implied default probability, academic literature based on actual default is much smaller which we believe is largely due to limited access of an academic researcher to proprietary internal data of historical defaults. Alen, Odd , Borgan (2008) show how counting integrals fit well with censored data using standard analyses such as Kaplan Meier plots and Cox regression in the book Survival and event History analysis. Berlin M and Mester I J (2004) argues stochastic models arem more reliable in retail credit risk measurement . Bielecki, T. R. and Rutkowski, M. (2004) formulates consistency conditions that tie together credit spread and recovery rates in order to construct a risk neutral probablity. Christian Bluhm and Ludger Overbeck (2007) discuss advanced mathematical models in portflio credit in the book ‘Structured Credit Portforlio Analysis, Baskets and CDOs” they also use calculations to discuss the business applications. D.R Cox (1972) obtained a conditional liklihood leading to inferences about unknown regression coefficients. Duffie and Singleton (2003) provide comprehensive treatment of pricing derivative and critical assement of alternative approaches to credit risk modelling. Due et al., (2007) offers an econometric method for estimating term structure of corporate defualt probablitites over multiple future periods connditioanl on firm specific and macroeconomic co-variates. Breeden (2007) describes an approach to analyzing these multiple time series as a single set such that the underlying lifecycles and calendar-based shocks may be measured.

Apart from this mathematical models, paramentric regression models, statistical models were used by experts like Abramovich and Steinberg (1996), Abramowitz and Stegun (1972), Andersen et al. (1993), Andersen and Gill (1982), Arjas (1986), Breslow (1972), Cressie (1993), Donoho and Johnstone (1994), Duchesne and Lawless (2000), Efron (2002), Esper et al. (2002).

Cox et al. (1985) framed a theory of term structure in interest rates, Das et al. (2007),determines the correlation of frequency in defults based on intensity of defult determinants, Deng et al. (2000), presents a unified model of cmpeting risks of mortgage termination by prepayment and defult considering two hazards as dependent competing risks. Chen (2007) interprets improvement in liquidity of bond reduces bond yields and illiquid bonds have more yield. In this paper we provide literature review on structural and intensity based approaches with primary focus on probability of default and hazard rate of time to default.

2. STRUCTURAL APPROACH MODELS

In credit risk modeling, structural approach is also known as firm value approach, since a firm's inability to meet contractual debt is assumed to be determined by its asset value. It was impaired by the 1970's Black Shoes, Merton Methodology for financial option pricing. The two classic structural models are (Merton, 1974) and the first passage time model (Black and Cox, 1975).

The Merton model assumes that default event occurs at a maturity date of debt if the asset value is less than the debt level. Let D, be the debt level with maturity date T, and let V (t) be the latent asset value following a geometric Brownian motion.

$$dV(t) = \mu V(t)\,dt + \sigma V(t)\,dW(t) \tag{1}$$

With drift μ, volatility σ and the standard wiener process (t), we recall that EW (t) = 0, EW (t) W(s) = min (t, s). Given the initial asset value V (0) >D by Ito's Lemma,

$$\frac{V(t)}{V(0)} = \exp\{(\mu \frac{1}{2}\sigma^2)\,t + \sigma W_t\} \sim \text{Log normal}\,((\mu - \frac{1}{2}\sigma^2))\,t,\ \sigma 2t) \tag{2}$$

Form which may evaluate the default probability P (V (T) ≤ D).

The notion of distance to default facilitates the computation of conditional default probability. Given the sample path of asset values up to t, one may first estimate the unknown parameters in (2.1) by maximum likelihood method. According to Due and Singleton (2003) distance to default is defined as X (t) by the number of standard deviations such that log V_t exceeds log D i.e.,

$$X(t) = (\log V(t) - \log D)/\sigma \tag{3}$$

Cleary, X (t) is a drifted Weiner process of the form

$$X(t) = c + bt + W(t),\ t \geq 0 \tag{4}$$

With $b = \mu \frac{-\sigma^2/2}{2}$ and $c = \frac{\log(0) - \log D}{\sigma}$

Then it is easy to verify that conditional probability of default at maturity date T is

$$P(V(T) \leq D \mid V(t) > D) = P(X(t) > 0) = \phi\,\frac{(X(t) + b(T-t)}{\sqrt{T-t}} \tag{5}$$

Where ϕ (.) is the cumulative normal distribution function

The first-passage-time model by Black and Cox (1975) extends the Merton model so that the default event could occur as soon as the asset value reaches a pre-specified debt barrier. By (2.5) and (2.6), V (t) hits the debt barrier once the distance-to-default process X (t) hits zero. Given the initial distance-to-default c ≡X (0) > 0, consider the first passage time,

$$\daleth = \inf\{\, t \geq 0 : X(t) < 0\}, \tag{6}$$

Where inf θ = ∞ as usual. It is well known that ך follows the inverse Gaussian distribution (Schrodinger, 1915; Tweedie, 1957; Chhikara and Folks, 1989) with the density,

$$f(t) = \frac{c}{\sqrt{2\pi}} t^{-3/2} exp\{-\frac{(c+bt)^2}{2t}\}, t \geq 0 \tag{7}$$

There are also other types of parameterization in Marshall and Olkin (2007). The survival function S (t) is defined by P (ך>t) for any t ≥ 0 and is given by,

$$S(t) = \phi(\frac{c+bt}{\sqrt{t}}) - e^{-2bc}\phi(-\frac{(c+bt)}{\sqrt{t}}) \tag{8}$$

The hazard rate or conditional default rate is defined by instantaneous rate of default conditional on survivorship.

$$\lambda(t) = \lim_{\Delta t \downarrow 0} \frac{1}{\Delta t} P(t \pounds\ 7 < t + \Delta t \mid 7\,{}^3 t = \frac{f(t)}{s(t)} \tag{9}$$

Using the inverse Guassian density and survival functions we obtain the form of first passage time hazard rate:

$$\lambda(t;c,b) = \frac{\frac{c}{\sqrt{2\pi t^3}} \exp\{-\frac{(c+bt)^2}{2t}}{\phi(\frac{c+bt}{\sqrt{t}})\, e^{-2bc}\phi(\frac{-c+bt}{\sqrt{t}})}\ c > 0 \tag{10}$$

This is one of the most important forms of hazard function in structural approach to credit risk modeling. Asymptotic theory for Cox model with staggered entry was developed by Bilias et al. (1997). Modern developments of structural models based on Merton and Black-Cox models can be referred to Bielecki and Rutkowski (2004).

3. INTENSITY BASED APPROACH

The intensity-based approach is also called the reduced-form approach, proposed independently by Jarrow and Turnbull (1995) and Madan and Unal (1998). Many follow-up papers can be found in Lando (2004), Bielecki and Rutkowski (2004) and references therein. Unlike the structural approach that assumes the default to be completely determined by the asset value subject to a barrier, the default event in the reduced-form approach is governed by an externally specified intensity process that may or may not be related to the asset value. The default is treated as an unexpected event that comes "by surprise". This is a practically appealing feature, since in the real world the default event (e.g., Year, 2001 bankruptcy of Enron Corporation) often all of a sudden happening without announcement.The default intensity corresponds to the hazard rate λ (t) = f (t)/S (t) defined in (2.9) and its roots in statistical reliability and survival analysis of time to failure. When S (t) is absolutely continuous with f (t) = d (1-S (t))/dt we have that,

$$\lambda(t) = (\frac{-dS(t)}{S(t)dt} = -\frac{(d[logS(t)])}{dt}, S(t) = exp\{-\int_0^t \lambda(s)ds\} \quad t \geq 0$$

In survival analysis, λ (t) is usually assumed to be deterministic function in time. In credit risk modeling λ (t) is often treated as stochastic. Thus, the default time ד is doubly stochastic. Lando (1998) adopted the term "doubly stochastic poison process" (or Cox process) that refers to a counting process with possibly recurrent events. What matters in modeling default is only the first jump of the counting process with possibly recurrent events, in which case the default intensity is equivalent to the hazard rate. In finance, the intensity-based models are mostly the term-structure models borrowed from the literature of interest-rate modeling. Below is an incomplete list:

Vasicek: $d\lambda(t) = \kappa(\theta-\lambda(t))\,dt+\sigma dW_t$

Cox-ıngersoll-roll: $d\lambda(t) = \kappa(\theta-\lambda(t))\,dt+\sigma\sqrt{\lambda}(t)\,dWt$ (11)

Affine jump: $d\lambda(t) = \mu\lambda(t))\,dt+\sigma(\lambda(t))\,dWt+dJ_t$

$1\sigma(\lambda(t)) = \sqrt{\sigma_0^2+\sigma_1^2(\lambda(t))}$

The term-structure models provide straightforward ways to simulate the future default intensity for the purpose of predicting the conditional default probability. However, they are ad hoc models lacking fundamental interpretation of the default event. The choices (11) are popular because they could yield closed-form pricing.The choices (11) are popular because theycould yield closed-form pricingformulas for credit risk modelling, while the real-world default intensities deserve more flexible and meaningful forms. For instance, the intensity models in (11) cannot be used to model the endogenous shapes of first-passage-time hazards. The dependence of default intensity on state variables z (t) (e.g., macroeconomic covariates) is usually treated through a multivariate term-structure model for the joint of (λ (t), z (t)). This approach essentially presumes a linear dependence in the diffusion components, e.g., by correlated Wiener processes.

4. CONCLUSION

Credit risk is the risk associated to claims that have a positive probability of default. Credit risk is important because investors are not guaranteed to get the expected return on their investments, but instead they can lose their invested capital if default occurs. In this thesis we studied the modeling of credit risk.

In this article we have studied the structure and intensity based models in credit risk.The structural based models have been proved empherically in many litratures compared to the intensity based models. The general approach of the intensity based models is to model the default event τ as the first jump of a Poisson process. Aalen and Gjessing (2001), used Survival analysis in the medical context and focused on the concepts of survival function and hazard rate, the latter of these beingthe basis both for the Cox regression model and of the counting process approach. The survival probability of τ takes the form of a discount factor, and the default intensity plays the same role as an interest rate process. This default intensity can be modeled as a constant.

In practice, the effect of state variable on the default intensity can be non-linear in many other ways. The intensity-based approach also includes the duration models for econometric analysis of actual historical defaults. They correspond to the classical survival analysis in statistics, which opens another door for approaching credit risk.

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