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A Novel Method for Assessing the Weight Coefficients of Criteria within the Framework of Multi-Criteria Decision-Making: Measurement Relying on the Impacts of an Exponential Curve Function (MIEXCF)

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Keywords

MCDM

Exponential Function

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MIEXCF

Abstract

In the realm of multi-criteria decision making (MCDM), this study introduces the Measurement Relying on the Impacts of an Exponential Curve Function (MIEXCF) as a novel approach for objectively determining criteria weight coefficients. Utilizing exponential curve interactions among criteria, MIEXCF is designed to enrich the MCDM literature. The dataset comprises criterion values extracted from Global Innovation Index (GII) evaluations for 19 G20 members. Results demonstrate the efficacy of MIEXCF in objectively deriving criteria weights for diverse nations. Comparative analyses with other methods (ENTROPY, CRITIC, SD, SVP, LOPCOW, MEREC) further validate MIEXCF's credibility, reliability, and stability. Notably, the simulation analysis indicates MIEXCF's effectiveness in discerning criteria weights and stability across scenarios. In conclusion, MIEXCF stands out as a robust objective criterion weighting technique, offering substantial contributions to exponential functions and the broader MCDM literature.

Cite

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1. INTRODUCTION

Multi-criteria decision making (MCDM) is a field utilized in intricate decision-making processes and mathematical modelling, where a multitude of factors are considered. This method aims to assess and rank alternatives based on the preferences and priorities of decision-makers. To achieve this, it is imperative to ascertain the weights of the criteria that reflect the preferences of the decision-makers (Bircan, 2020).

In the extensive landscape of (MCDM) literature, one encounters a plethora of techniques (such as ENTROPY, CRITIC, SD, SVP, MEREC, LOPCOW) designed to ascertain weight coefficients (Keleş, 2023). Within this body of literature, the objective weights assigned to criteria exhibit two fundamental characteristics. The first pertains to the degree of contrast in the performance of decision alternatives concerning each criterion, encapsulating the disparity between maximum and minimum values across the criteria. The second characteristic relates to the distinctiveness or conflict among these criteria. By elucidating and leveraging these inherent characteristics embedded in the data defining the multi-criteria problem, decision-makers can derive valuable insights to inform their decision-making process (Ecer, 2020). Within this framework, criteria that exhibit a heightened degree of interdependence can manifest a more pronounced distinctiveness when compared to other criteria in terms of their discriminative capacity. Beyond these established methods, such as MIEXCF (Measurement Relying on the Impacts of an Exponential Curve Function), this study introduces a novel approach for calculating objective weight coefficients for variables based on exponential curve

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functions. This method facilitates the analysis and modelling of variables in light of their exponential curve relationships. Consequently, the research centers its attention on exploring the analytical and modeling potential inherent in exponential curve, as these functions have a well-documented history of utility in diverse domains and have proven effective in addressing various problem-solving challenges (Chakrabarty & Rahman, 2007).

The primary objective of composing this article is to introduce an alternative and objective approach for determining weighting coefficients of criteria within the domain of MCDM methods, with a foundation in exponential curve measurement. It has come to our attention that the existing literature contains methods that rely solely on exponential curve calculations and are marked by limitations. Therefore, this study puts forth a novel method. The core aim of this research is to establish a framework that effectively captures the impact values among criteria through exponential curve functions, transforms these impact values into criteria weighting coefficients, and thereby produces dependable outcomes. The method outlined in this article is expected to more precisely reflect the intensity of the interplay among criteria, facilitating the calculation of criteria weighting coefficients. Consequently, this study is perceived as a valuable addition to the existing body of literature concerning methods for computing criteria weighting coefficients, offering a fresh perspective to the field.

Within this context, the research is two goals. First, it aims to introduce a novel method for computing the weight coefficients of criteria concerning decision alternatives within the realm of MCDM. The second objective is to promote the utilization of exponential curve functions and enhance awareness of their inherent potential, given their significant role in addressing and dissecting complex problems. To achieve these goals, the research literature elucidates both objective weighting methods and the attributes of exponential curve functions. In this context, the method section of the study elucidates objective criteria weight coefficients, the exponential curve function, relationship between calculating Criterion Weights and exponential curve function, data set and description of the proposed method (MIEXCF). According to the proposed method, the weights of criteria for the 19 countries in G20 group, demonstrating validity and reliability, are measured and ranked using the Logistic Performance Index data for these countries. Secondly, sensitivity analyses are conducted to assess the method's sensibility. Subsequently, thirdly, comparative analyses are performed to gauge the method's credibility and reliability levels. Finally, simulation analyses are provided to deconstruct the criteria weights and establish the stability level of the method.

2. MATERIAL AND METHOD

2.1. Methods for Calculating Criterion Weights in the Scope of MCDM

The process of choosing among different alternatives is a crucial aspect of many decision-making procedures. However, in such situations, it often happens that each alternative exhibits varying performances on different criteria. Therefore, accurately determining the relative importance of these criteria is essential for effectively comparing the performance of decision alternatives and ultimately selecting the most suitable one (Saaty, 2008). This is because, traditionally, the significance of criteria is established by assigning weight coefficients in MCDM problems (Ecer, 2020).

Subjective weight coefficients are primarily derived from the personal experiences and evaluations of decision-makers, making them inherently dependent on individual opinions. Consequently, these values tend to vary among different individuals (Baş, 2021). These weight coefficients are typically determined based on expert judgments. However, relying solely on subjective assessments by experts can introduce errors and biases into the decision-making process. In contrast, objective methodologies do not account for decision-makers' inconsistencies and uncertainties. Instead, they leverage mathematical models and the information within the decision matrix to compute the criteria weights. In essence, objective weighting techniques consider the underlying data structure in the evaluation process (Paksoy, 2017; Arslan, 2020; Demir et al., 2021).

Within the realm of MCDM literature, one encounters a variety of objective weighting methods, including CRITIC (Criteria Importance Through Inter Criteria Correlation), ENTROPY, CILOS (Criterion Impact Loss), IDOCRIW (Integrated Determination of Objective Criteria Weights), SVP (Statistical Variance Procedure), SD (Standard Deviation), MEREC (Method Based On Removal Effects of Criteria), LOPCOW

(Logarithmic Percentage Change-driven Objective Weighting) and, SECA (Simultaneous Evaluation of Criteria and Alternatives) (Ecer, 2020). The CRITIC method, in particular, operates on the principle of utilizing information inherent in a system. Accordingly, the more disorder or distinctiveness a criterion exhibits compared to others, the greater its importance becomes. Thus, the CRITIC method relies on the interrelationships among criteria. This technique involves scrutinizing the correlations between criteria to pinpoint any inconsistencies among them. Subsequently, these contradictions related to the criteria are weighted using the standard deviation, thus facilitating the determination of criterion weight coefficient values. The CRITIC method commences with the creation of a decision matrix. Next, the normalized values of this matrix are computed. By examining the correlations among the criteria based on these normalized values, the criteria weights can be quantified (Diakoulaki et al., 1995).

The ENTROPY method proves to be a valuable tool in the decision-making process. In this approach, after constructing the decision matrix, the standard values of the decision matrix and the ENTROPY measure of the criteria are employed to ascertain the ENTROPY-based criterion weights (Ayçin, 2019).

Within the CILOS method, the relative importance of criteria hinges on the degree of impact deviation of other criteria from their respective ideal maximum and minimum values. In essence, if a criterion exhibits a lower impact deviation, its weight coefficient is correspondingly increased. The methodology involves a step-by-step process, including the calculation of the decision matrix, normalization, square matrix, and the weight system matrix values. Subsequently, a system of linear equations is solved to determine the weight coefficients for the criteria (Zavadskas & Podvezko, 2016; Sel, 2020).

The IDOCRIW method is a fusion of both the ENTROPY and CILOS methods. This approach centers around the determination of the relative impact of a missing index. Initially, the weight coefficients for the criteria are ascertained through the ENTROPY and CILOS methods, utilizing the decision matrix values. Subsequently, the ENTROPY and CILOS weights are integrated to yield the IDOCRIW weights (Zavadskas & Podvezko, 2016).

SVP, as a target weighting method, is designed to provide objective weights for the computation of criterion weights or their relative importance levels (Nasser et al., 2019). This method quantifies the weight values assigned to criteria objectively, ensuring that they are not influenced by expert opinions or subjective evaluations. The SVP method's approach to calculating criterion weights is rooted in the variance metrics associated with these criteria (Gülençer & Türkoğlu, 2020) After determining the variance values for each criterion, the weights for individual criteria are computed by dividing the variance value of each criterion by the total variance value encompassing all criteria. In essence, the SVP method serves as an objective approach for determining weights, enabling the computation of criterion weights or their significance levels based on variance values linked to the criteria (Odu, 2019).

The SD method relies on assessing the distance of criteria values from the arithmetic mean of these criteria. To apply this method, the initial step involves normalizing the decision matrix using the values contained within it. Subsequently, the standard deviation values for each criterion are computed, serving as a basis for determining the criteria weights (Uludağ & Doğan, 2021). In the case of the SVP method, criterion weights are determined by calculating the variances of the criteria using the values from the decision matrix (Demir et al., 2021).

Within the MEREC method, much like other weighting methodologies, the process commences with obtaining the decision matrix and its normalized counterpart. Following this, the total performance values of the decision alternatives are computed using a structure based on natural logarithms. Subsequently, by considering the value of each decision alternative, adjustments in the performance values of the other decision alternatives are recalculated based on the natural logarithm. Towards the conclusion of this method, the weight values for the criteria are determined through the calculation of the removal effect on each criterion, specifically the sum of absolute deviations. Furthermore, in this method, as the impact of criteria on decision alternatives grows, the weight coefficients of the criteria also increase (Keshavarz-Ghorabaee et al., 2021).

The LOPCOW method involves the integration of data from various dimensions to derive appropriate or ideal weights. Additionally, this approach aims to narrow the gaps between the most significant and least significant criteria. Moreover, LOPCOW takes into consideration the relationships between criteria. In this method, the initial step involves preparing the decision matrix, followed by normalizing the values within that matrix. Subsequently, the average square value, expressed as a percentage of the criterion's standard deviation, is calculated to mitigate the variations caused by the data's magnitude, ultimately determining the weight coefficients for the criteria (Ecer & Pamucar, 2022).

The SECA method offers a means to assess both the performance of decision alternatives and the weight coefficients of criteria concerning these alternatives. In this approach, the values within the decision matrix are standardized. Subsequently, disagreement degrees and standardization values are computed using the standard deviation. This data serves as the basis for calculating the weights of the criteria, achieved by solving a multi-objective linear model through model optimization (Keshavarz-Ghorabaee et al., 2018).

The DEMATEL method can be used to reveal the interaction between criteria, as well as to subjectively determine the weights of criteria based on their relationships. To do this, the effects of criteria on each other are determined using subjective evaluations, where 0 represents no effect and 4 represents a very high effect. This information is used to create a direct relationship matrix, which is then used to calculate the standard relationship matrix, total relationship matrix, relationship diagram, threshold value, and finally, the weight coefficients of the criteria (Fontela & Gobus, 1976). Altıntaş (2021), emphasized that the effects of criteria on each other can also be determined using the Somers' d correlation coefficient, which allows for the objective calculation of the weight coefficients of the criteria. This eliminates the need for subjective evaluations in the direct relationship matrix. Upon a review of the literature, it becomes evident that numerous studies have utilized objective weighting and relationship-oriented DEMATEL methods. The ongoing research pertaining to the discussed weighting is outlined in Table 1.

Table 1. *Present Studies Concerning Criteria Weighting*

Author(s)	Methods	Research Theme
Alrababah & Gan, 2023	CRITIC based VIKOR	The Impact of the Hybrid CRITIC–VIKOR Method on Product Aspect Ranking in Customer Reviews
Wang et al., 2023	ENTROPY based MARCOS	Sustainable Evaluation of Major Third-Party Logistics providers
Ali et al., 2022	SAW-AHP, SAW-CILOS, SAW-AHP-CILOS, TOPSIS-AHP, TOPSIS-CILOS, TOPSIS-AHP-CILOS, CoCoSo-AHP, CoCoSo-CILOS, CoCoSo-AHP-CILOS, MARCOS-AHP, MARCOS-CILOS, and MARCOS-AHP-CILOS	Lessons learned from the COVID-19 pandemic in planning the future energy systems of developing countries
Ayan et al., 2023	CILOS, IDOCRIW, FUCOM, LBWA, SAPEVO-M, and MEREC	A Comprehensive Review of the Novel Weighting Methods
Vavrek, 2019	CRITIC, SD,SVP, MR based TOPSIS	Evaluation of the Impact of Selected Weighting Methods on the Results of the TOPSIS Technique
Mukhametzyanov, 2021	Entropy, CRITIC, SD	specific character of objective methods for determining weights of criteria
Sümerli Sarigül et al., 2023	MEREC based MARCOS and COCOSO	evaluating airport service quality
Ersoy, 2023	LOPCOW based RSMVC	Performance Measurement in the BIST Retail and Trade Sector
Rasmussen et al., 2023	F-AHP based F-TOPSIS and SECA	Supplier selection for aerospace & defense industry
Göncü & Çetin, 2022	Dematel and ANP Method	Supplier Selection Criteria in Healthcare Enterprises

2.2. Exponential Curve Function

The Exponential Curve function is based on the exponential function. A function written in the form of $f(x) = b^x$, where $b \in R^+ - \{1\}$ and $x \in R^+$, is referred to as an exponential function. Here, $''b''$ is called the base, and $''x''$ is referred to as the exponent. The domain of exponential functions is the set of real numbers, while the range consists of positive numbers (Önalan, 2010; Balaban, 2015; Ertik et al., 2015; Eroğlu, 2017). If $0 < b < 1$, the function $y = b^x$ exhibits a decreasing characteristic, approaching the $''y''$ axis for increasing values of the base "b". If $b > 1$, the function $y = b^x$ is an increasing function, and again, it approaches the $''y''$ axis for growing values of the base $''b''$ (Kartal et al., 2014; Bernett et al., 2015; Kuruüzüm & İpekçi, 2015; Pekkaya, 2016). In a function written as $y = b^x$, if the irrational number $''e \approx 2{,}718281829 \ldots\ldots''$is used instead of the base $''b''$, the function becomes $y = e^x$ (Kuruüzüm & İpekçi, 2015).

The Exponential Curve function fundamentally arises from the composition of the $''e^{x''}$ ' function with any constant value, forming the function $y = a.e^{bx}$. The logarithmic transformation of this function can be expressed as $In(y) = In(a) + bx$ (Chakrabarty & Rahman, 2007; IBM, 2013).

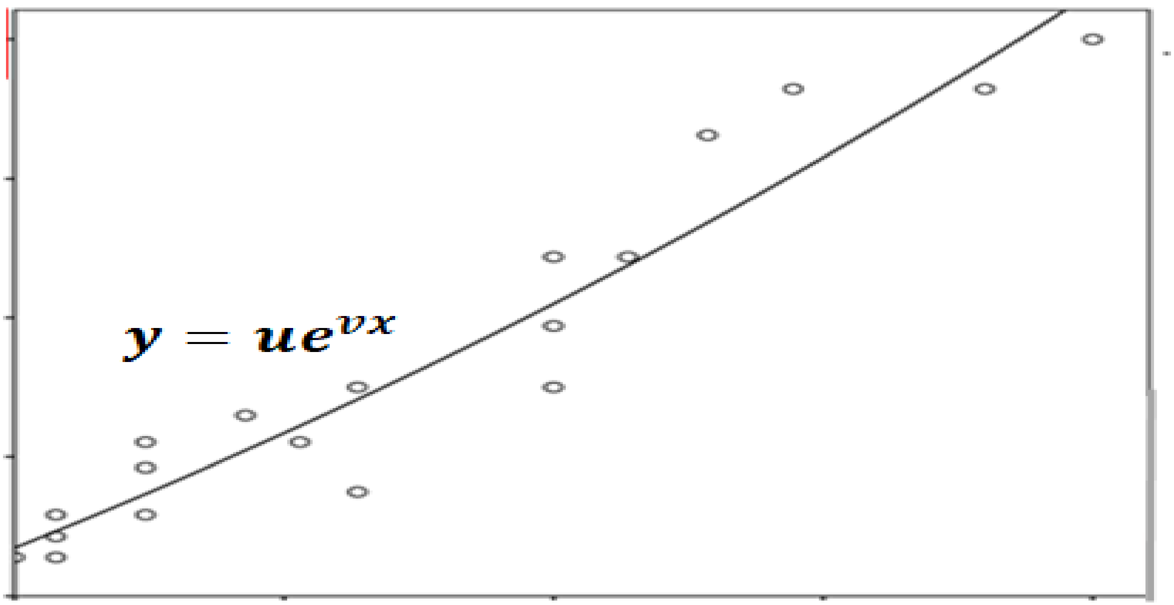

Figure 1. *Exponential Curve Function (Note: Figure 1 was obtained using the SPSS 23 program)*

The given expression $y = u.e^{vx}$models the relationship between the constants "u" and "v" and the variable "x" in an exponential function. "u" represents the initial value of the function (the y-intercept) and "v" controls the rate of growth. As the value of "x" increases, the value of "y" changes exponentially, increasing if "v" is positive and decreasing if "v" is negative. Positive "v" indicates growth with increasing "x", while negative "v" indicates a decrease in "y" with increasing "x" (Thomas, 1991; Chakrabarty & Rahman, 2007).

The exponential curve function provides significant advantages in mathematical modeling. Firstly, due to the function's continuity and differentiability, it possesses a suitable structure for modeling physical systems. Secondly, the function's monotonic property facilitates the derivation, integration, and other operations, simplifying predictions of the function's behavior. This enhances the predictability of the function's behavior. Thirdly, the function's limits allow for easy estimation of how the function behaves at extreme points. Fourthly, the logarithmic transformation of the function results in a symmetric curve, making it easier to understand relationships between variables visually. Lastly, the logarithmic transformation can lead to a linear form, simplifying the function's structure and overcoming complexity (Shparlinski & Konyagin, 1999; Lin, 2014; Kahn, 2015; Joujan, 2018).

Upon reviewing the literature, it has been observed that many studies have been conducted utilizing the exponential curve function (Natalija, 2021). In their work, Landsberg (1977) explained that the exponential curve function is widely applicable in biological research and is an appropriate function for such studies.

Chakrabarty and Rahman (2007) utilized the exponential curve function to estimate and project the overall population in India. Weon & Je (2014) developed a versatile survival distribution based on the stretched exponential curve function, incorporating an age-dependent shaping exponent. Alfaro et al. (2020) employed the exponential curve function to forecast real-time returns of U.S.A stocks by considering unforeseen alterations in the progression of COVID-19 infections. Fosu and Edunyah (2020) employed the exponential curve function to estimate the effectiveness of measures developed against the COVID-19 pandemic in developing and impoverished countries. Hamill et al. (2005) utilized the exponential curve function in the development of a methodology that facilitates the creation of information security strategies and applies measures to assess them. Jones (2023) elucidated that the utilization of the exponential curve function is instrumental in describing the exponential growth resulting from combinatorially chosen samples extracted from standard thin-tailed distributions defined mathematically. Murillo-Escobar et al. (2023) introduced a methodology developed within the realm of encryption algorithms for clinical signals. This methodology enhances the randomness of five specifically chosen chaotic maps by incorporating trigonometric functions (sine, cosine, and tangent) and exponential curve functions. Wood (2023) has developed a new model using structural equation modeling software that allows for the combination of logistic and exponential curve functions.

2.3. Calculating Weight Coefficients for Criteria in the Context of MCDM with the Exponential Function (Theoretical Background)

In determining the weights of criteria, the distinctiveness and conflict among criteria bring out the nature of the criteria (Ecer, 2020). Accordingly, in the DEMATEL literature, criteria with higher impact values in interactional models lead to greater prioritization and significance compared to other criteria (Fontela & Gobus, 1976; Akın, 2017). Similarly, in structural equation modeling, especially in non-recursive models, when one criterion influences another criterion to a greater extent in absolute terms, the criterion causing a greater impact in the relationship between these two criteria is explained to contribute more to the relational structure and is considered more important in that relationship. On the other hand, in recursive models, if one criterion has a greater impact value in absolute terms on other criteria, it is stated to be the most important criterion (Bayram, 2010; Çelik & Yılmaz, 2013; Meydan & Şeşen, 2015; Özdamar, 2016; Civelek, 2018; Gürbüz, 2019; Kline, 2019).

Another attribute to consider in assessing the criteria is their potential for mutual influence, as quantified by quantitative outcomes. If one criterion has a modest positive impact on another, it opens up opportunities to develop strategies to enhance the influenced criterion by leveraging the influencing one. Conversely, when a positive influence of one criterion hinders the progress of another, strategies can be devised to reduce or mitigate the influence of the influencing criterion on the influenced one. Consequently, it becomes possible to formulate strategies, policies, and recommendations for advancing the criteria based on the interdependencies among them within a given context. In this context, exponential curve functions can be utilized to calculate the weight coefficients of criteria. This is due to the fact that, with exponential curve functions, the values of criteria influencing one another can be determined as dependent and independent variables (Karagöz, 2017).

Another crucial advantage of the exponential curve function is its capability to perform regression analysis between two variables. Accordingly, exponential curve equations describing the relationship between two variables based on the data of these variables can be established through regression analysis (curve estimation) using SPSS. This enables the calculation of the quantitative impact between two variables (IBM, 2013; Karagöz, 2017). In comparison to many functions in the literature, the exponential curve function allows for a more precise and controlled determination of how much the dependent variable changes with the smallest change in the independent variable (Lin, 2014). Therefore, this situation provides an opportunity for accurate and reliable modeling in the relationship or interaction structure between two variables (Chakrabarty & Rahman, 2007).

The derivative $f'(x)$ is referred to as the antiderivative or indefinite integral of the function $f(x)$. $f'(x)$ equals can be expressed as $f'(x)dx = df(x)$. This expression is commonly represented using the infinite and continuous summation symbol $\int$. From this equation, we can derive the equation $\int f'(x)dx = f(x)$. Hence, the function to be integrated is $f'(x)$. Moreover, $\int f(x)dx = F(x) + C$, where $\int_r^p f(x)dx = F(p) - F(r)$

represents the definite integral. In this context, $''r''$ denotes the lower limit of the integral, while $''p''$ signifies the upper limit (Kartal et al., 2014). Therefore, after establishing the exponential curve relationships between the criteria using the exponential curve, one can measure the extent to which the variation of the independent variable $''x''$ between the limits $''p''$ and $''r''$ impacts or alters the dependent variable $''y''$ through definite integration (Kuruüzüm & İpekçi Çetin, 2015). The visual representation of this situation is illustrated in Figure 2 (IBM, 2013).

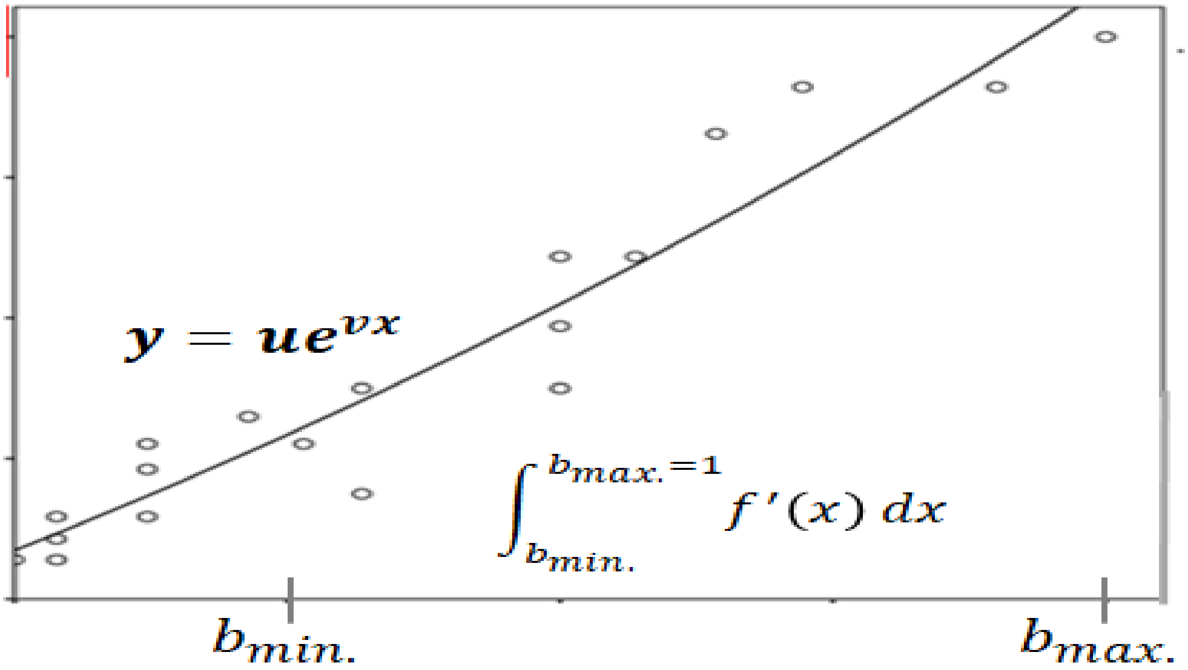

Figure 2. *Exponential Curve Function and Its Impact on Criterion* (p) *with Criterion* (r) *Model*
(Note: Figure 2 was obtained using the SPSS 23 program)

After creating a exponential curve function between the values of two criteria (p, r) in the decision matrix based on decision alternatives, the impact of one criterion (p) on the other critera (r), or how changes in the independent variable (r) criterion affect the dependent variable criterion, can be calculated by determining the maximum and minimum values of the criterion taken as the independent variable in the decision matrix. In any decision matrix containing cost or benefit-oriented criteria with no zeros or negative values, the maximum value of the independent criterion is 1 when the criteria are normalized between 0 and 1. If any value in any decision matrix is negative or zero, the exponential curve interaction between the criteria does not occur (Kartal et al., 2014). Because the exponential curve function is logarithmically transformed, the relationship between the criteria remains undefined (Shparlinski & Konyagin, 1999; Lin, 2014). Accordingly, $f(x) = ue^{v.x}$ where $f(x)$'s derivative is determined as $f(x)' = uve^{ux}$. Subsequently, the impact of the p criterion on the r criterion can be calculated with Equation 1.

$$\int_{r_{min.}}^{p_{max.}=1} f(x)'dx = F(x)]_{r_{min.}}^{p_{max.}} = F(Pmax) - f(rmin) \quad (1)$$

2.4. Data Set and Analysis of the Study

The dataset for the research consists of the Logistic Performance Index (LPI) criteria for the year 2022 for 19 countries in the G20 group. The reason for selecting this dataset in the study is to determine the discriminative power of the proposed model criteria among countries, considering the significant differences in values within this dataset. In this regard, abbreviations for this dataset are explained in Table 2 for convenience in the research.

Table 2. *LPI Criteria Abbreviations (Arvis et al., 2023)*

LPI Criteria	Criteria Abbreviations
Customs	LPI1
Infrastructure	LPI2
International Shipments	LPI3
Logistics Competence and Quality	LPI4
Timeliness Score	LPI5
Tracking and Tracing	LPI6

2.4. Proposed Method: Measurement Relying on the Impacts of an Exponential Curve Function (MIEXCF)

The fundamental logic of the proposed method is based on the exponential curve effect of criteria on each other. Regarding this, the method for calculating the weight coefficients of criteria according to the MIEXCF method is explained below.

Altıntaş (2023) established the mathematical model based on cubic effects among the criteria to determine the weights of criteria. Within this scope, in this study, Altıntaş's (2023) modeling approach was utilized in the following steps: in the third step, functions were formulated; in the fourth step, impact values among the criteria were calculated based on these functions; in the fifth step, total impact values of each criterion according to the function structures were computed; and finally, in the sixth step, Altıntaş's (2023) logic was employed in determining the criterion weights.

Step 1: Obtaining the Decision Matrix

i: $1, 2, 3 \dots n$: where *n* represents the number of decision alternatives

j: $1, 2, 3 \dots m$: where *m* represents the number of criteria

D: Decision matrix

ECF: Criterion

d_{ij}: The decision matrix is constructed according to Equation 2, where "ij" represents the $i-th$ decision alternative on the $j-th$ criterion.

$$D = [d_{ij}]_{nxm} = \begin{bmatrix} ECF_1 & ECF_2 & \dots & ECF_m \\ x_{11} & x_{12} & & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \tag{2}$$

Step 2: Normalization of Decision Matrix (d_{ij}^*)

The normalization of the decision matrix is conducted through the utilization of the subsequent equation. Benefit criteria undergo normalization using Equation 3, whereas cost criteria are subjected to normalization employing Equation 4.

$$d_{ij}^* = \frac{min.\, d_{ij}}{d_{ij}} \tag{3}$$

$$d_{ij}^* = \frac{d_{ij}}{max.\, d_{ij}} \tag{4}$$

Step 3: Generation of Exponential Curve Functions

Based on the number of criteria, m, exponential curve function $\left(f(x) = y = ae^{bx}\right)$ are generated for the variables up to a quantity of using SPSS assistance (Regression-CURVE ESTIMATION), considering the exponential curve relationship between them.

$$(1)\ f(ECF_1) = ECF_2, f(ECF_1) = ECF_3, \dots \dots\ f(ECF_1) = ECF_m \tag{5}$$

$$(2)\ f(ECF_2) = ECF_1, f(ECF_2) = ECF_3, \dots \dots\ f(ECF_2) = ECF_m \tag{6}$$

$$(3)\ f(ECF_3) = ECF_1, f(ECF_3) = ECF_2, \dots \dots\ f(ECF_3) = ECF_m \tag{7}$$

$$\vdots \quad \vdots \quad \vdots \qquad \vdots \quad \vdots \quad \vdots \qquad \vdots \quad \vdots \quad \vdots$$

$$(m)\ f(ECF_m) = ECF_1, f(ECF_m) = ECF_2, \dots \dots\ f(ECF_m) = ECF_{m-1} \tag{8}$$

Step 4: Calculation of Exponential Curve Impact Value between Criteria

In this stage, we assess the influence or alteration of a dependent variable (one criterion) by an independent variable (another criterion) within the scope of its minimum and maximum values, achieved through the application of definite integral calculations. In this context, 't' represents the exponential curve impact value of one criterion on the other. It is important to ensure the absolute value of the impact values after the integral calculation.

$$(1)\ f(ECF_1) = ECF_2\ , \int_{ECF_{1min.}}^{ECF_{1max.}} (f'(ECF_1))\, dx = \left|t_{ECF_1 \to ECF_2}\right| \tag{9}$$

$$(2)\ f(ECF_1) = ECF_3\ , \int_{ECF_{1min.}}^{ECF_{1max.}} (f'(ECF_2))\, dx = \left|t_{ECF_1 \to ECF_3}\right| \tag{10}$$

$$(3)\ f(ECF_1) = ECF_4\ , \int_{ECF_{1min.}}^{ECF_{1max.}} (f'(C_1))\, dx = \left|t_{ECF_1 \to ECF_4}\right| \tag{11}$$

$$\vdots \quad \vdots \quad \vdots \qquad \vdots \quad \vdots \quad \vdots \qquad \vdots \quad \vdots \quad \vdots$$

$$\left(\frac{m!}{(m-2)!}\right) f(ECF_m) = ECF_{m-1}\ , \int_{ECF_{mmin.}}^{ECF_{mmax.}} (f'(ECF_m))\, dx = \left|t_{ECF_m \to ECF_{m-1}}\right| \tag{12}$$

Step 5: Calculation of the Total Exponential Curve Impact Values of Each Criterion (S_{ECF})

During this stage, we aggregate the exponential curve impact values of one criterion on the remaining criteria to quantify the comprehensive exponential curve impact value of that criterion on the others.

$$(1)\ for\ ECF_1\ \left|t_{ECF_1\rightarrow ECF_2}\right| + \left|t_{ECF_1\rightarrow ECF_3}\right| \dots\dots + \left|t_{ECF_1\rightarrow ECF_m}\right| = \left(\sum_{j=1}^{m-1}\left|t_{ECF_1\rightarrow ECF_{j+1}}\right|\right) = S_{ECF_1} \tag{13}$$

$$(2)\ for\ ECF_2\ \left|t_{ECF_2\rightarrow ECF_1}\right| + \left|t_{ECF_2\rightarrow ECF_3}\right| \dots\dots + \left|t_{ECF_2\rightarrow ECF_m}\right| = \left(\sum_{j=0, j\neq 1}^{m-1}\left|t_{ECF_2\rightarrow ECF_{j+1}}\right|\right) = S_{ECF_2} \tag{14}$$

$$(3)\ for\ ECF_3\ \left|t_{ECF_3\rightarrow ECF_1}\right| + \left|t_{ECF_3\rightarrow ECF_2}\right| \dots\dots + \left|t_{ECF_3\rightarrow ECF_m}\right| = \left(\sum_{j=0, j\neq 2}^{m-1}\left|t_{ECF_3\rightarrow ECF_{j+1}}\right|\right) = S_{ECF_3} \tag{15}$$

$$\vdots \quad \vdots \quad \vdots \qquad \vdots \quad \vdots \quad \vdots \qquad \vdots \quad \vdots \quad \vdots$$

$$(m)\ for\ ECF_m\ \left|t_{ECF_m\rightarrow ECF_1}\right| + \left|t_{ECF_m\rightarrow ECF_2}\right| \dots\dots + \left|t_{ECF_m\rightarrow ECF_{m-1}}\right| = \left(\sum_{j=1}^{m-1}\left|t_{ECF_m\rightarrow ECF_{j+1}}\right|\right) = S_{ECF_m} \tag{16}$$

Step 6: Determination of Criterion Weight Values (w_j)

In this stage, the collective exponential curve impact value of each criterion on the remaining criteria is divided by the sum of the collective exponential curve impact values of all criteria. This division enables the computation of the weight coefficient for each criterion.

$$w_j = \frac{S_{ECF_j}}{\sum_{j=1}^{m} S_{ECF_j}} \tag{17}$$

The proposed method has numerous advantages. The first of these is the non-linear nature of the method. This is because the exponential curve function, being non-linear, is more successful in accurately predicting relationships between variables than linear methods. This situation ensures a more accurate resolution of relationships between variables compared to linear models (Chakrabarty & Rahman, 2007). Therefore, since the MIEXCF method is based on the exponential curve function, accurate results can be obtained in determining the relationships between criteria and, consequently, the weights of criteria. However, in other objective criterion weighting methods (ENTROPY, CRITIC, SD, SVP, and MEREC), the determination of criterion weights is not based on a non-linear linear structure. Therefore, these methods cannot benefit from the advantages provided by a non-linear structure.

Secondly, in the MIEXCF method, the calculation of the mutual influence value of variables is conducted through the integral method. The integral precisely and accurately demonstrates the total change or impact of the dependent variable in the variation of the independent variable within the boundary values of the independent variable. Therefore, this circumstance elucidates the original effect of the independent variable on the dependent variable without requiring a process (Bernett et al., 2015). In this context, within the scope of the MIEXCF method, the weights of criteria can be calculated more sensitively and realistically. In contrast, in other objective criterion weighting methods, the computation of criterion weights occurs as a result of highly transformative processes involving the values of the decision matrix.

Finally, the third aspect is the opportunity provided by the MIEXCF method for enhancing criteria. In the interactive analysis of variables, the variable or variables with the highest impact value can promote the development of other variables by influencing them (Karagöz, 2017). Because of the metric structure of the relationship between criteria, within the scope of the MIEXCF method, the criterion with the highest weight coefficient can be evaluated to determine its impact on the criteria that need improvement (those with the least impact) or to assess whether the criteria with lower impact or requiring improvement need to enhance their influence on other criteria. In contrast, in other objective weighting methods, since the weight values of criteria are not determined based on their influence on each other, opportunities to enhance criteria remain limited.

The MIEXCF method possesses both advantages and also drawbacks. One of its disadvantage is the complexity of calculations required for determining criteria weight coefficients, particularly compared to many objective weighting methods in the literature. As the number of criteria increases, the calculations become more intricate due to the numerous interaction values between criteria. However, these calculations can be easily performed using the MATLAB program or the Python programming language. Another drawback is the reliance on SPSS or other statistical software programs to identify exponential relationships between criteria. Without SPSS, the calculation of weight coefficients becomes more complex and time-consuming. However, the exponential relationship between the criteria can be obtained by transferring the formulas to the EXCEL program within the scope of regression analysis. As a result, the calculation operations can be simplified. A third disadvantage arises when there is no theoretical cause-and-effect relationship between criteria, limiting opportunities for criteria improvement. The aforementioned disadvantage is also present in the CRITIC, MEREC and DEMATEL methods. This is because the CRITIC method determines the weight coefficients of the criteria based on the level of Pearson correlation between the criteria. On the other hand, the logic of calculating the weight values of the criteria in the MEREC method is based on the effect of the criteria on the decision alternatives. In addition to these, the weights of the criteria in the DEMATEL method depend on the extent to which they impact each other and are affected by each other. Therefore, in the determination of the weight coefficients of the criteria, the cause-and-effect relationship between the criteria in the CRITIC and DEMATEL methods and the cause-and-effect relationship between the criteria and the decision alternatives in the MEREC method are not taken into account in a theoretical sense. In other words, in the CRITIC and DEMATEL methods, there is no a theoretical necessity for a cause-and-effect relationship between the criteria and MEREC, there is a no theoretical necessity for a cause-and-effect relationship between the criteria and the decision alternatives. Finally, the fourth drawback involves the need for transformation using Z-scores when values in the decision matrix are negative or 0 to ensure positive and non-zero values. This challenge is also present in the ENTROPY and MEREC methods, both relying on logarithmic measurements.

3. RESULTS AND DISCUSSION (THE CASE STUDY)

3.1. Computational Analyses

Considering the data set of the research, the weight coefficients of criteria for the 19 countries in the G20 group in 2022 were calculated using the LPI criterion data with the MIEXCF method. In this regard, in the first step of the method, a decision matrix was created with the help of Equation 2 and is presented in Table 3.

In the second step of the method, as all LPI criteria are beneficial, the normalized decision matrix was calculated using Equation 3 based on the decision matrix values described in Table 3 and is presented in Table 4.

In the third step of the model, with the presence of six components, we employ the functional equation expressed as to demonstrate the exponential relationships among these components.

To capture the interactions among the components effectively, we established 30 exponential curve function equations, denoted as Equations 5, 6, 7, and 8, using CURVE analysis (regression models) through SPSS. These respective functions are detailed in Table 5.

In the fourth phase of the approach, we calculated exponential curve influence factors among the criteria using equations 9, 10, 11, and 12. The process for determining the impact values between criteria are explained in the following sections.

$$\boldsymbol{f}(\boldsymbol{LPI1})$$

f(LPI1)=LPI2

$$\text{deriv}(0.305\boldsymbol{e}^{1.22x},x) = \frac{3721\boldsymbol{e}^{\frac{61x}{50}}}{10000}$$

$$\int_{0.6}^{1} \frac{3721\boldsymbol{e}^{\frac{61x}{50}}}{10000}\,dx = \frac{61\boldsymbol{e}^{\frac{61}{50}} - 61\boldsymbol{e}^{\frac{183}{250}}}{200} = 0{,}399$$

f(LPI1)=LPI3

$$\text{deriv}(0.381\boldsymbol{e}^{0.827x},x) = \frac{315087\boldsymbol{e}^{\frac{827x}{1000}}}{1000000}$$

$$\int_{0.6}^{1} \frac{315087\boldsymbol{e}^{\frac{827x}{1000}}}{1000000}\,dx = \frac{381\boldsymbol{e}^{\frac{827}{1000}} - 381\boldsymbol{e}^{\frac{2481}{5000}}}{1000} = 0{,}245$$

f(LPI1)=LPI4

$$\text{deriv}(0.32\boldsymbol{e}^{1.112x},x) = \frac{1112\boldsymbol{e}^{\frac{139x}{125}}}{3125}$$

$$\int_{0.6}^{1} \frac{1112\boldsymbol{e}^{\frac{139x}{125}}}{3125}\,dx = \frac{8\boldsymbol{e}^{\frac{139}{125}} - 8\boldsymbol{e}^{\frac{417}{625}}}{25} = 0{,}349$$

f(LPI1)=LPI5

$$\text{deriv}(0.477\boldsymbol{e}^{0.688x},x) = \frac{20511\boldsymbol{e}^{\frac{86x}{125}}}{62500}$$

$$\int_{0.6}^{1} \frac{20511\boldsymbol{e}^{\frac{86x}{125}}}{62500}\,dx = \frac{477\boldsymbol{e}^{\frac{86}{125}} - 477\boldsymbol{e}^{\frac{258}{625}}}{1000} = 0{,}228$$

f(LPI1)=LPI6

$$\text{deriv}(0.303\boldsymbol{e}^{1.106x},x) = \frac{167559\boldsymbol{e}^{\frac{553x}{500}}}{500000}$$

$$\int_{0.6}^{1} \frac{167559\boldsymbol{e}^{\frac{553x}{500}}}{500000}dx = \frac{303\boldsymbol{e}^{\frac{553}{500}} - 303\boldsymbol{e}^{\frac{1659}{2500}}}{1000} = 0{,}327$$

$$\boldsymbol{f(LPI2)}$$

f(LPI2)=LPI1

$$\text{deriv}(0.287\boldsymbol{e}^{1.229x},x) = \frac{352723\boldsymbol{e}^{\frac{1229x}{1000}}}{1000000}$$

$$\int_{0.628}^{1} \frac{352723\boldsymbol{e}^{\frac{1229x}{1000}}}{1000000}dx = \frac{287\boldsymbol{e}^{\frac{1229}{1000}} - 287\boldsymbol{e}^{\frac{192953}{250000}}}{1000} = 0{,}360$$

f(LPI2)=LPI3

$$\text{deriv}(0.377\boldsymbol{e}^{0.821x},x) = \frac{309517\boldsymbol{e}^{\frac{821x}{1000}}}{1000000}$$

$$\int_{0.628}^{1} \frac{309517\boldsymbol{e}^{\frac{821x}{1000}}}{1000000}dx = \frac{377\boldsymbol{e}^{\frac{821}{1000}} - 377\boldsymbol{e}^{\frac{128897}{250000}}}{1000} = 0{,}226$$

f(LPI2)=LPI4

$$\text{deriv}(0.311\boldsymbol{e}^{1.122x},x) = \frac{174471\boldsymbol{e}^{\frac{561x}{500}}}{500000}$$

$$\int_{0.628}^{1} \frac{174471\boldsymbol{e}^{\frac{561x}{500}}}{500000}dx = \frac{311\boldsymbol{e}^{\frac{561}{500}} - 311\boldsymbol{e}^{\frac{88077}{125000}}}{1000} = 0{,}326$$

f(LPI2)=LPI5

$$\text{deriv}(0.474\boldsymbol{e}^{0.678x},x) = \frac{80343\boldsymbol{e}^{\frac{339x}{500}}}{250000}$$

$$\int_{0.628}^{1} \frac{80343\boldsymbol{e}^{\frac{339x}{500}}}{250000}dx = \frac{237\boldsymbol{e}^{\frac{339}{500}} - 237\boldsymbol{e}^{\frac{53223}{125000}}}{500} = 0{,}208$$

f(LPI2)=LPI6

$$\text{deriv}(0.298\boldsymbol{e}^{1.698x},x) = \frac{126501\boldsymbol{e}^{\frac{849x}{500}}}{250000}$$

$$\int_{0.628}^{1} \frac{126501\boldsymbol{e}^{\frac{849x}{500}}}{250000}dx = \frac{149\boldsymbol{e}^{\frac{849}{500}} - 149\boldsymbol{e}^{\frac{133293}{125000}}}{500} = 0{,}762$$

$$\boldsymbol{f}(\boldsymbol{LPI3})$$

f(LPI3)=LPI1

$$\text{deriv}(0.301\boldsymbol{e}^{1.257x},x) = \frac{378357\boldsymbol{e}^{\frac{1257x}{1000}}}{1000000}$$

$$\int_{0.622}^{1} \frac{378357\boldsymbol{e}^{\frac{1257x}{1000}}}{1000000}dx = \frac{301\boldsymbol{e}^{\frac{1257}{1000}} - 301\boldsymbol{e}^{\frac{390927}{500000}}}{1000} = 0{,}400$$

f(LPI3)=LPI2

$$\text{deriv}(0.313\boldsymbol{e}^{1.239x},x) = \frac{387807\boldsymbol{e}^{\frac{1239x}{1000}}}{1000000}$$

$$\int_{0.622}^{1} \frac{387807\boldsymbol{e}^{\frac{1239x}{1000}}}{1000000}dx = \frac{313\boldsymbol{e}^{\frac{1239}{1000}} - 313\boldsymbol{e}^{\frac{385329}{500000}}}{1000} = 0{,}404$$

f(LPI3)=LPI4

$$\text{deriv}(0.297\boldsymbol{e}^{1.269x},x) = \frac{376893\boldsymbol{e}^{\frac{1269x}{1000}}}{1000000}$$

$$\int_{0.622}^{1} \frac{376893\boldsymbol{e}^{\frac{1269x}{1000}}}{1000000}dx = \frac{297\boldsymbol{e}^{\frac{1269}{1000}} - 297\boldsymbol{e}^{\frac{394659}{500000}}}{1000} = 0{,}403$$

f(LPI3)=LPI5

$$\text{deriv}(0.424\boldsymbol{e}^{0.886x},x) = \frac{23479\boldsymbol{e}^{\frac{443x}{500}}}{62500}$$

$$\int_{0.622}^{1} \frac{23479\boldsymbol{e}^{\frac{443x}{500}}}{62500}dx = \frac{53\boldsymbol{e}^{\frac{443}{500}} - 53\boldsymbol{e}^{\frac{137773}{250000}}}{125} = 0{,}293$$

f(LPI3)=LPI6

$$\mathrm{deriv}(0.28\boldsymbol{e}^{1.268x}, x) = \frac{2219\boldsymbol{e}^{\frac{317x}{250}}}{6250}$$

$$\int_{0.622}^{1} \frac{2219\boldsymbol{e}^{\frac{317x}{250}}}{6250} dx = \frac{7\boldsymbol{e}^{\frac{317}{250}} - 7\boldsymbol{e}^{\frac{98587}{125000}}}{25} = 0{,}379$$

$\boldsymbol{f(LPI4)}$

f(LPI4)=LPI1

$$\mathrm{deriv}(0.283\boldsymbol{e}^{1.286x}, x) = \frac{181969\boldsymbol{e}^{\frac{643x}{500}}}{500000}$$

$$\int_{0.619}^{1} \frac{181969\boldsymbol{e}^{\frac{643x}{500}}}{500000} dx = \frac{283\boldsymbol{e}^{\frac{643}{500}} - 283\boldsymbol{e}^{\frac{398017}{500000}}}{1000} = 0{,}397$$

f(LPI4)=LPI2

$$\mathrm{deriv}(0.277\boldsymbol{e}^{1.229x}, x) = \frac{340433\boldsymbol{e}^{\frac{1229x}{1000}}}{1000000}$$

$$\int_{0.619}^{1} \frac{340433\boldsymbol{e}^{\frac{1229x}{1000}}}{1000000} dx = \frac{277\boldsymbol{e}^{\frac{1229}{1000}} - 277\boldsymbol{e}^{\frac{760751}{1000000}}}{1000} = 0{,}354$$

f(LPI4)=LPI3

$$\mathrm{deriv}(0.345\boldsymbol{e}^{0.968x}, x) = \frac{8349\boldsymbol{e}^{\frac{121x}{125}}}{25000}$$

$$\int_{0.968}^{1} \frac{8349\boldsymbol{e}^{\frac{121x}{125}}}{25000} dx = \frac{69\boldsymbol{e}^{\frac{121}{125}} - 69\boldsymbol{e}^{\frac{14641}{15625}}}{200} = 0{,}028$$

f(LPI4)=LPI5

$$\mathrm{deriv}(0.447\boldsymbol{e}^{0.78x}, x) = \frac{17433\boldsymbol{e}^{\frac{39x}{50}}}{50000}$$

$$\int_{0.619}^{1} \frac{17433e^{\frac{39x}{50}}}{50000}dx = \frac{447e^{\frac{39}{50}} - 447e^{\frac{24141}{50000}}}{1000} = 0{,}251$$

f(LPI4)=LPI6

$$\text{deriv}(0.277e^{1.229x},x) = \frac{340433e^{\frac{1229x}{1000}}}{1000000}$$

$$\int_{0.619}^{1} \frac{340433e^{\frac{1229x}{1000}}}{1000000}dx = \frac{277e^{\frac{1229}{1000}} - 277e^{\frac{760751}{1000000}}}{1000} = 0{,}354$$

$f(LPI5)$

f(LPI5)=LPI1

$$\text{deriv}(0.191e^{1.687x},x) = \frac{322217e^{\frac{1687x}{1000}}}{1000000}$$

$$\int_{0.707}^{1} \frac{322217e^{\frac{1687x}{1000}}}{1000000}dx = \frac{191e^{\frac{1687}{1000}} - 191e^{\frac{1192709}{1000000}}}{1000} = 0{,}402$$

f(LPI5)=LPI2

$$\text{deriv}(0.203e^{1.646x},x) = \frac{167069e^{\frac{823x}{500}}}{500000}$$

$$\int_{0.707}^{1} \frac{167069e^{\frac{823x}{500}}}{500000}dx = \frac{203e^{\frac{823}{500}} - 203e^{\frac{581861}{500000}}}{1000} = 0{,}403$$

f(LPI5)=LPI3

$$\text{deriv}(0.228e^{1.411x},x) = \frac{80427e^{\frac{1411x}{1000}}}{250000}$$

$$\int_{0.707}^{1} \frac{80427e^{\frac{1411x}{1000}}}{250000}dx = \frac{57e^{\frac{1411}{1000}} - 57e^{\frac{997577}{1000000}}}{250} = 0{,}317$$

f(LPI5)=LPI4

$$\text{deriv}(0.198e^{1.64x},x)$$

$$\int_{0.707}^{1} \frac{4059\boldsymbol{e}^{\frac{41x}{25}}}{12500}dx = \frac{99\boldsymbol{e}^{\frac{41}{25}} - 99\boldsymbol{e}^{\frac{28987}{25000}}}{500} = 0{,}389$$

f(LPI5)=LPI6

$$\text{deriv}(0.193\boldsymbol{e}^{1.592x},x) = \frac{38407\boldsymbol{e}^{\frac{199x}{125}}}{125000}$$

$$\int_{0.707}^{1} \frac{38407\boldsymbol{e}^{\frac{199x}{125}}}{125000}dx = \frac{193\boldsymbol{e}^{\frac{199}{125}} - 193\boldsymbol{e}^{\frac{140693}{125000}}}{1000} = 0{,}354$$

$$\boldsymbol{f}(\boldsymbol{LPI6})$$

f(LPI6)=LPI1

$$\text{deriv}(0.292\boldsymbol{e}^{1.323x},x) = \frac{96579\boldsymbol{e}^{\frac{1323x}{1000}}}{250000}$$

$$\int_{0.595}^{1} \frac{96579\boldsymbol{e}^{\frac{1323x}{1000}}}{250000}dx = \frac{73\boldsymbol{e}^{\frac{1323}{1000}} - 73\boldsymbol{e}^{\frac{157437}{200000}}}{250} = 0{,}455$$

f(LPI6)=LPI2

$$\text{deriv}(0.305\boldsymbol{e}^{1.302x},x) = \frac{39711\boldsymbol{e}^{\frac{651x}{500}}}{100000}$$

$$\int_{0.595}^{1} \frac{39711\boldsymbol{e}^{\frac{651x}{500}}}{100000}dx = \frac{61\boldsymbol{e}^{\frac{651}{500}} - 61\boldsymbol{e}^{\frac{77469}{100000}}}{200} = 0{,}460$$

f(LPI6)=LPI3

$$\text{deriv}(0.351\boldsymbol{e}^{1.001x},x) = \frac{351351\boldsymbol{e}^{\frac{1001x}{1000}}}{1000000}$$

$$\int_{0.595}^{1} \frac{351351\boldsymbol{e}^{\frac{1001x}{1000}}}{1000000}dx = \frac{351\boldsymbol{e}^{\frac{1001}{1000}} - 351\boldsymbol{e}^{\frac{119119}{200000}}}{1000} = 0{,}318$$

f(LPI6)=LPI4

$$\text{deriv}(0.303\boldsymbol{e}^{1.271x},x) = \frac{385113\boldsymbol{e}^{\frac{1271x}{1000}}}{1000000}$$

$$\int_{0.595}^{1} \frac{385113\boldsymbol{e}^{\frac{1271x}{1000}}}{1000000} dx = \frac{303\boldsymbol{e}^{\frac{1271}{1000}} - 303\boldsymbol{e}^{\frac{151249}{200000}}}{1000} = 0{,}435$$

f(LPI6)=LPI5

$$\text{deriv}(0.46\boldsymbol{e}^{0.788x},x) = \frac{4531\boldsymbol{e}^{\frac{197x}{250}}}{12500}$$

$$\int_{0.595}^{1} \frac{4531\boldsymbol{e}^{\frac{197x}{250}}}{12500} dx = \frac{23\boldsymbol{e}^{\frac{197}{250}} - 23\boldsymbol{e}^{\frac{23443}{50000}}}{50} = 0{,}276$$

In the fifth stage of the process, we computed the cumulative exponential curve impact values for each criterion using formulas 13, 14, 15, and 16. These calculated values are presented in Table 6.

Table 3. *Decision Matrix*

Economies	LPI1	LPI2	LPI3	LPI4	LPI5	LPI6
Argentina	2.7	2.8	2.7	2.7	3.1	2.9
Australia	3.7	4.1	3.1	3.9	3.6	4.1
Brazil	2.9	3.2	2.9	3.3	3.2	3.5
Canada	4	4.3	3.6	4.2	4.1	4.1
China	3.3	4	3.6	3.8	3.7	3.8
France	3.7	3.8	3.7	3.8	4.1	4
Germany	3.9	4.3	3.7	4.2	4.1	4.2
India	3	3.2	3.5	3.5	3.6	3.4
Indonesia	2.8	2.9	3	2.9	3.3	3
Italy	3.4	3.8	3.4	3.8	3.9	3.9
Japan	3.9	4.2	3.3	4.1	4	4
Korea, Rep.	3.9	4.1	3.4	3.8	3.8	3.8
Mexico	2.5	2.8	2.8	3	3.5	3.1
Russia Fed.	2.4	2.7	2.3	2.6	2.9	2.5
Saudi Arabia	3	3.6	3.3	3.3	3.6	3.5
South Africa	3.3	3.6	3.6	3.8	3.8	3.8
Türkiye	3	3.4	3.4	3.5	3.6	3.5
United King.	3.5	3.7	3.5	3.7	3.7	4
USA	3.7	3.9	3.4	3.9	3.8	4.2

Table 4. *Normalized Matrix*

Countries	LPI1	LPI2	LPI3	LPI4	LPI5	LPI6
Argentina	0.889	0.964	0.852	0.963	0.936	0.862
Australia	0.649	0.659	0.742	0.667	0.806	0.610
Brazil	0.828	0.844	0.793	0.788	0.906	0.714
Canada	0.600	0.628	0.639	0.619	0.707	0.610
China	0.727	0.675	0.639	0.684	0.784	0.658
France	0.649	0.711	0.622	0.684	0.707	0.625
Germany	0.615	0.628	0.622	0.619	0.707	0.595
India	0.800	0.844	0.657	0.743	0.806	0.735
Indonesia	0.857	0.931	0.767	0.897	0.879	0.833
Italy	0.706	0.711	0.677	0.684	0.744	0.641
Japan	0.615	0.643	0.697	0.634	0.725	0.625
Korea, Rep.	0.615	0.659	0.677	0.684	0.763	0.658
Mexico	0.960	0.964	0.821	0.867	0.829	0.807
Russia Fed.	1.000	1.000	1.000	1.000	1.000	1.000
Saudi Arab.	0.800	0.750	0.697	0.788	0.806	0.714
South Afr.	0.727	0.750	0.639	0.684	0.763	0.658
Türkiye	0.800	0.794	0.677	0.743	0.806	0.714
United Kin.	0.686	0.730	0.657	0.703	0.784	0.625
USA	0.649	0.692	0.677	0.667	0.763	0.595

Furthermore, in Equation 17, we calculate weight coefficients that represent the significance levels of each criterion. These coefficients measure the relative importance of the criteria within the scope of the analysis. The resulting values are presented in Table 7.

After a comprehensive analysis of Table 4, we have organized the significance attributed to the various components of the LPI (Exponential Impact) as follows: LPI6 carries the highest weight coefficient, followed by LPI2, LPI3, LPI5, LPI1 and, finally, LPI4. This sequence clarifies the differing levels of importance assigned to each constituent within the LPI framework.

3.2. Sensibility Analysis

In the context of this research, we conducted an evaluation of the MIEXCF method to assess its methodological sensitivity. Sensitivity analysis, within the framework of MCDM, involves applying various criteria weighting methods to the same dataset, enabling a comparison of the resulting values and rankings. To ensure the sensitivity of the weight coefficient calculation method, the weight rankings of the criteria identified using the method chosen for sensitivity analysis are expected to differ from the weight coefficient rankings obtained with other methods (Gigovic et al., 2016).

Following this approach, for the purpose of sensitivity analysis, we calculated and organized the weighting coefficients associated with the components of the LPI using well-established objective weighting techniques that are commonly found in scholarly literature. Some notable examples of these techniques include ENTROPY, CRITIC, SD (Standard Deviation), SVP (Statistical Variance Procedure), MEREC, and LOPCOW. The corresponding numerical results have been thoroughly documented in Table 8.

Table 5. *Exponential Curve functions Derived from the Correlation Among the Criteria*

IDC	DC	Function
LPI1→	LPI2	y=0.305$e^{1.220x}$
	LPI3	y=0.381$e^{0.827x}$
	LPI4	y=0.320$e^{1.112x}$
	LPI5	y=0.477$e^{0.688x}$
	LPI6	y=0.303$e^{1.106x}$
LPI2→	LPI1	y=0.287$e^{1.129x}$
	LPI3	y=0.377$e^{0.821x}$
	LPI4	y=0.311$e^{1.122x}$
	LPI5	y=0.474$e^{0.678x}$
	LPI6	y=0.298$e^{1.098x}$
LPI3→	LPI1	y=0.297$e^{1.269x}$
	LPI2	y=0.301$e^{1.257x}$
	LPI4	y=0.313$e^{1.239x}$
	LPI5	y=0.424$e^{0.886x}$
	LPI6	y=0.280$e^{1.268x}$
LPI4→	LPI1	y=0.283$e^{1.286x}$
	LPI2	y=0.291$e^{1.288x}$
	LPI3	y=0.345$e^{0.968x}$
	LPI5	y=0.447$e^{0.780x}$
	LPI6	y=0.277$e^{1.229x}$
LPI5→	LPI1	y=0.191$e^{1.687x}$
	LPI2	y=0.203$e^{1.646x}$
	LPI3	y=0.228$e^{1.411x}$
	LPI4	y=0.198$e^{1.640x}$
	LPI6	y=0.193$e^{1.592x}$
LPI6→	LPI1	y=0.292$e^{1.323x}$
	LPI2	y=0.305$e^{1.302x}$
	LPI3	y=0.351$e^{1.001x}$
	LPI4	y=0.303$e^{1.271x}$
	LPI5	y=0.450$e^{0.788x}$
IDC: Independent Criteria, DC: Dependent Criteria		

Table 6. *The Total Exponential Curve Impact Values of LPI Components on Each Other*

Independent Component	Dependent Criteria	Effect
LPI1→	LPI2	0.399
	LPI3	0.245
	LPI4	0.349
	LPI5	0.228
	LPI6	0.327
	Total	1.548
LPI2→	LPI1	0.36
	LPI3	0.226
	LPI4	0.326
	LPI5	0.208
	LPI6	0.762
	Total	1.882
LPI3→	LPI1	0.4
	LPI2	0.404
	LPI4	0.403
	LPI5	0.293
	LPI6	0.379
	Total	1.879
LPI4→	LPI1	0.397
	LPI2	0.354
	LPI3	0.028
	LPI5	0.251
	LPI6	0.354
	Total	1.384
LPI5→	LPI1	0.402
	LPI2	0.403
	LPI3	0.317
	LPI4	0.389
	LPI6	0.354
	Total	1.865
LPI6→	LPI1	0.455
	LPI2	0.46
	LPI3	0.318
	LPI4	0,435
	LPI5	0,276
	Total	1,944

When Tables 6 and 7 are compared side by side, it becomes clear that the prioritization of criteria weighting coefficients for the Global Logistic Performance Index (LPI) differs when calculated using the MIEXCF method compared to other approaches. This demonstrates the sensitivity of the MIEXCF method.

3.3. Comparative Analysis

The comparative analysis assesses the relationships and positions of the proposed method against other objective weight coefficient calculation methods. The proposed method should be credible, reliable, and consistent with other methods, while also demonstrating a positive and significant correlation with different weight coefficient methods (Keshavarz-Ghorabaee et al., 2021). Based on the data presented in Table 7, the positions of the methods are illustrated in Figures 3, 4 and 5.

Table 7. *Weighting Coefficients* (w) *of the LPI Criteria*

LPI Criteria	Total Effects	$(\boldsymbol{w})$	Ranking
LPI1	1.548	0.1474	5
LPI2	1.882	0.1792	2
LPI3	1.879	0.1789	3
LPI4	1.384	0.1318	6
LPI5	1.865	0.1776	4
LPI6	1.944	0.1851	1
Total	10,502	------	-----

Table 8. *Results from Alternative Methods of Calculating Objective Weighting Coefficients*

LPI Criteria	**ENTROPY**		**CRITIC**		**SD**	
	Score	**Rank**	**Score**	**Rank**	**Score**	**Rank**
LPI1	0.223513	1	0.157853	3	0.196183	1
LPI2	0.210311	2	0.130547	4	0.189777	2
LPI3	0.131934	5	0.295611	1	0.149215	5
LPI4	0.177685	3	0.088489	6	0.173718	3
LPI5	0.083043	6	0.204207	2	0.120169	6
LPI6	0.173513	4	0.123294	5	0.170939	4
LPI Criteria	SVP		LOPCOW		MEREC	
	Score	Rank	Score	Rank	Score	Rank
LPI1	0.200922	2	0.15136	5	0.147113	5
LPI2	0.224147	1	0.146976	6	0.131193	6
LPI3	0.113687	5	0.187393	1	0.180158	3
LPI4	0.183917	4	0.162869	4	0.151631	4
LPI5	0.091382	6	0.172376	3	0.190409	2
LPI6	0.185945	3	0.179025	2	0.199497	1

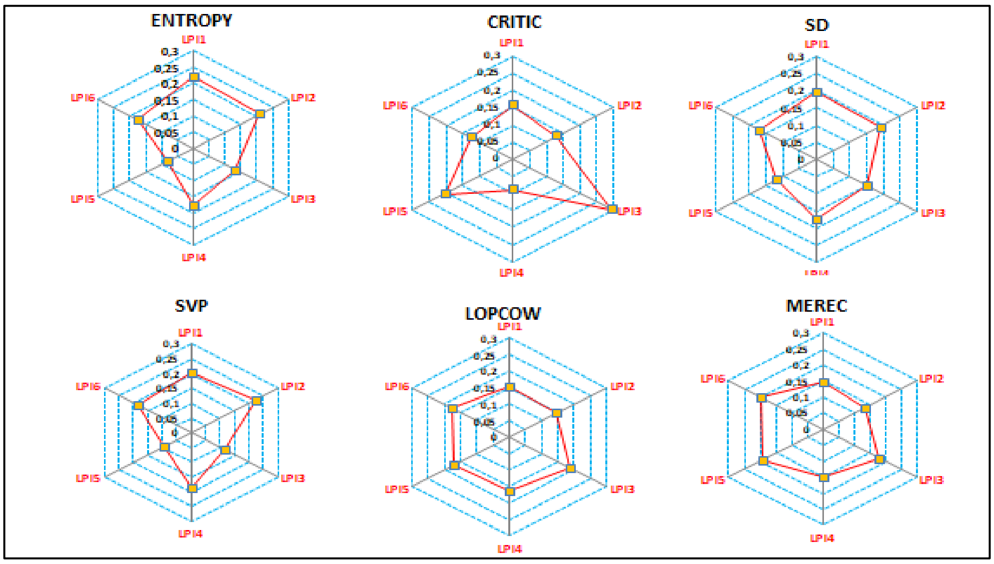

Figure 3. *Positions of the ENTROPY, CRITIC, SD, SVP, LOPCOW, and MEREC Methods (Note: The axises is graduated in increments of 0.30, 0.25, 0.20, 0.15, 0.10, 0.05, and 0)*

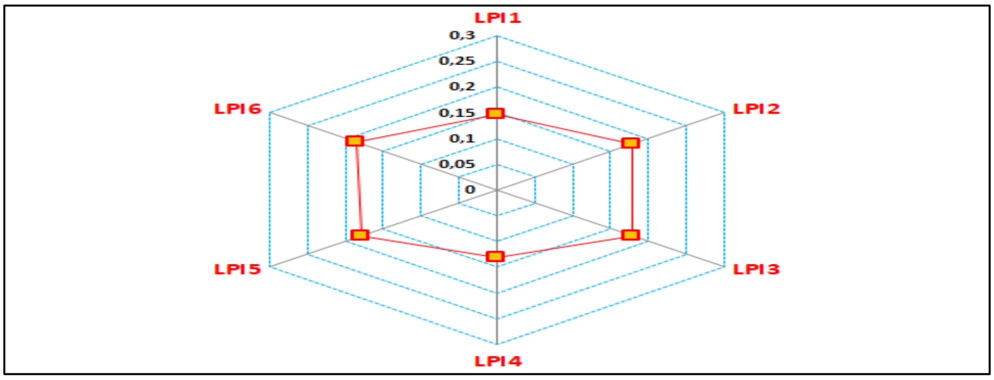

Figure 4. *Positions of the MIEXCF Methods (Note: The axis is graduated in increments of 0.30, 0.25, 0.20, 0.15, 0.10, 0.05, and 0)*

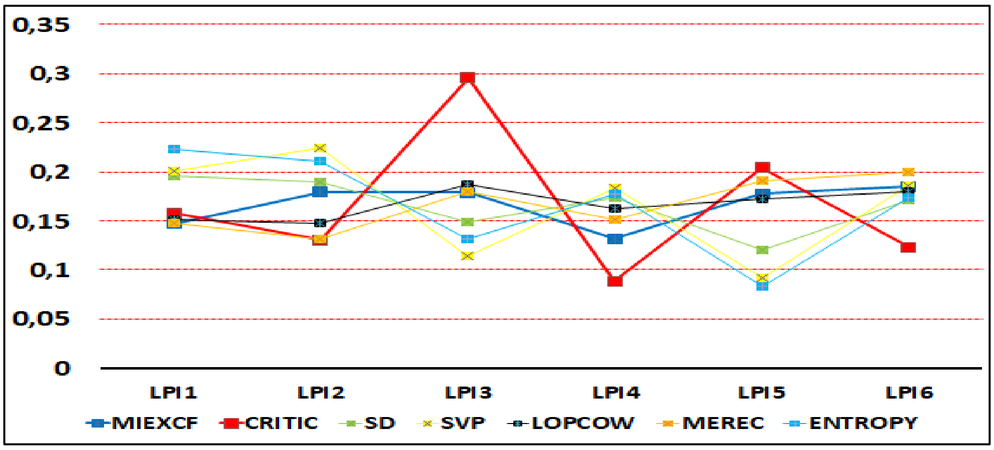

Figure 5. *Positions of the ENTROPY, CRITIC, SD, SVP, LOPCOW, MEREC, and MIEXCF Methods*

According to Figure 3, 4 and 5 the point locations of the MIEXCF method exhibit a higher degree of proportional similarity to the MEREC method compared to other methods. Moreover, in Figure 3, the differences between the MIEXCF method and the MEREC points are less pronounced than the differences between the MIEXCF method and the points associated with other methods. Based on this data, it can be concluded that the relationship between the MIEXCF method and the MEREC method is positive, and significant, The correlation values of the MIEXCF method with other methods are presented in Table 8.

Table 8. *Pearson Correlation Values of the MIEXCF Method with Other Methods*

r	**CRITIC**	**SD**	**SVP**
MIEXCF	0.452*	-0.384	-0.293
r	**LOPCOW**	**MEREC**	**ENTROPY**
MIEXCF	0.422*	0.504*	-0.381
p*<.05			

Keshavarz-Ghorabaee et al. (2021), referring to Walters' (2009) study during the measurement of the Pearson correlation between the MEREC method and other methods (SD, ENTROPY, and CRITIC), stated that a positive significant relationship in the range of 0.400-0.600 indicates a moderate level of relationship between variables, and if it exceeds 0.600, the relationship is considered significant. In this context, according to Table 8, it is observed that the MIEXCF method has a significant, positive, and moderate-level relationship with the CRITIC, MEREC, and LOPCOW methods. Although the correlation values of the MIEXCF method with the CRITIC and LOPCOW methods are moderate, significant, and positive, these correlation values are not too far from the 0.600 correlation value. Therefore, based on these results, it can be inferred that the MIEXCF method is close to the credibility and reliability status.

3.4. Simulation Analysis

To conduct the simulation analysis, various scenarios are generated by assigning different values to decision matrices. To ensure the stability of results obtained using the proposed method, it is expected that the proposed method will exhibit differences from other methods as the number of scenarios increases. In the second step, the average of the variance values determined by the proposed method across the scenarios should be greater than one or more of the other objective weighting methods. This indicates that the proposed method is relatively effective in distinguishing the criteria weights. Lastly, the uniformity of variances in criterion weights across the methods within the scenarios must be established (Keshavarz-Ghorabaee et al., 2021). In the simulation analysis, the correlation values of the MIEXCF method with other methods were calculated based on the initial 10 scenarios and are presented in Table 9.

Table 9. *Pearson Correlation Values of the MIEXCF Method among Other Methods Within the Scope of Scenarios*

Group	**Scenarios**	**ENTROPY**	**CRITIC**	**SD**	**SVP**	**LOPCOW**	**MEREC**
First group	1. Scenario	-0.39	0.472*	-0.365	-0.305	0.445*	0.535*
	2. Scenario	-0.41	0.460*	-0.37	-0.315	0.436*	0.510*
	3. Scenario	-0.4	0.440*	-0.365	-0.285	0.438*	0.520*
Group	Scenarios	**ENTROPY**	**CRITIC**	**SD**	**SVP**	**LOPCOW**	**MEREC**
Second group	4. Scenario	-0.39	0.430*	-0.365	-0.315	0.448*	0,522*
	5. Scenario	-0.395	0.440*	-0.38	-0.335	0.420*	0,480*
	6. Scenario	-0.344	0.433*	-0.34	-0.285	0.435*	0,465*
	7. Scenario	-0.335	0.425*	-0.35	-0.29	0.405*	0,490*
	8. Scenario	-0.32	0.440*	-0.345	-0.27	0.410*	0,480*
	9. Scenario	-0.315	0.435*	-0.333	-0.225	0.390*	0.505*
	10. Scenario	-0.3	0.403*	-0.34	-0.24	0.382*	0.497*
Mean		-0,36	0.438	-0.355	-0.287	0.421	0.500
p*<.05							

As the number of scenarios increases, the criterion weights differ from each other based on the methods, as shown in Table 9. Notably, the MIEXCF method with CRITIC, LOPCOW and MEREC exhibit positive and significant relationships across all scenarios. This indicates a consistent pattern of correlation between the

MIEXCF and MEREC, CRITIC methods. The data presented in Table 9 were divided into two groups based on the values of the criterion weights. The correlation values between the two groups are shown in Figure 6.

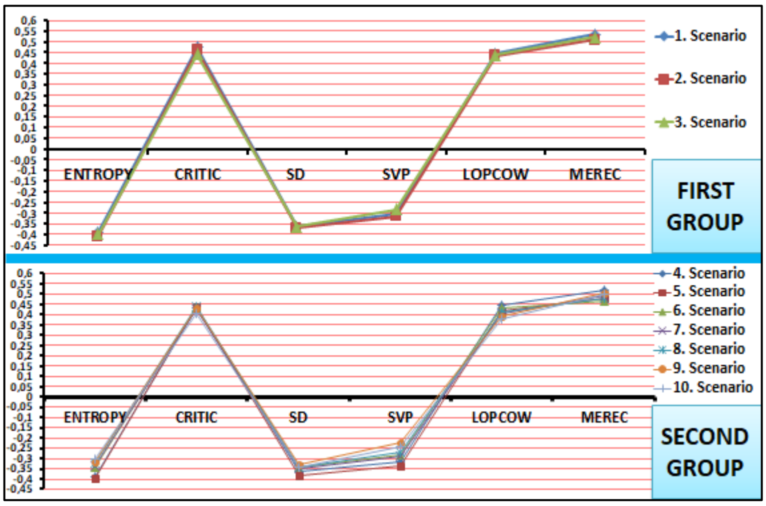

Figure 6. *The Correlation Status of MIEXCF Method with Other Approaches within Various Scenarios*

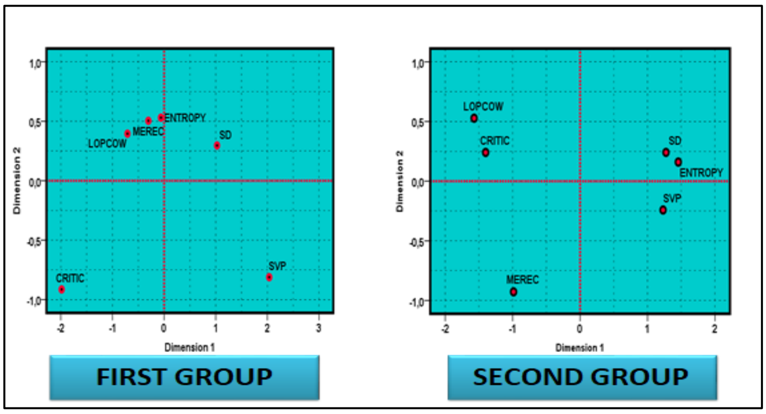

Figure 7. *The Discriminant Analysis of Correlation Status between MIEXCF Method and Other Methodologies across Different Scenarios*

Upon examination of Figure 7, it is observed that within the second group, the MIEXCF method exhibits variations in correlation values with other weight coefficient calculation methods compared to the first group, demonstrating a more pronounced dispersion in the space. Consequently, it has been observed that the distinctive features of the methods become increasingly prominent with the expansion of scenarios, resulting in a greater discernible differences between the methods. During the simulation analysis, the variance values of the methods were computed across different scenarios, and the resultant values are detailed in Table 10.

According to Table 10, it is observed that the average variance values of the MIEXCF method across scenarios are higher compared to the variance values of the ENTROPY, CRITIC, SD, and SVP methods. Conversely, these values are lower than those of the LOPCOW and MEREC methods. Hence, it can be assessed that the MIEXCF method exhibits a relatively enhanced capability in discerning criteria weights, as indicated by its higher average variance value compared to the ENTROPY, CRITIC, SD, and SVP methods. In the continuation of the simulation analysis, the homogeneity of variances in the criterion weights of the EXEBM method was examined through ADM (ANOM for variances with Levene) analysis across different scenarios. This analytical approach provides a graphical representation to assess the uniformity of variances. The graphical depiction comprises three variables: the general average ADM serves as the center line, along with the upper decision limits (UDL) and lower decision limits (LDL). If the standard deviation of a group (cluster) exceeds the decision limits, it indicates a significant difference from the general average ADM, signifying heterogeneity in variances. Conversely, if the standard deviations of all clusters fall within the LDL and UDL, it confirms the homogeneity of variances (Keshavarz-Ghorabaee et al., 2021). The visual representation of the ADM analysis is presented in Figure 8.

Table 10. *Variability in Methodologies across Scenarios*

Scenario	MIEXCF	ENTROPY	CRITIC	SD	SVP	LOPCOW	MEREC
1. Scenario	0.000279	0.000297	0.000280	0.000259	0.000252	0.000313	0.000328
2. Scenario	0.000297	0.000279	0.000291	0.000268	0.000259	0.000315	0.000290
3. Scenario	0.000302	0.000288	0.000282	0.000261	0.000254	0.000278	0.000327
4. Scenario	0.000285	0.000295	0.000288	0.000268	0.000256	0.000314	0.000304
5. Scenario	0.000290	0.000280	0.000297	0.000261	0.000257	0.000285	0.000326
6. Scenario	0.000287	0.000289	0.000283	0.000260	0.000258	0.000312	0.000315
7. Scenario	0.000301	0.000297	0.000286	0.000265	0.000255	0.000313	0.000285
8. Scenario	0.000295	0.000283	0.000294	0.000270	0.000260	0.000280	0.000310
9. Scenario	0.000289	0.000291	0.000290	0.000262	0.000257	0.000279	0.000321
10. Scenario	0.000297	0.000292	0.000289	0.000269	0.000259	0.000287	0.000297
Mean	0.000292	0.000282	0.000283	0.000264	0.000257	0.000315	0.000324

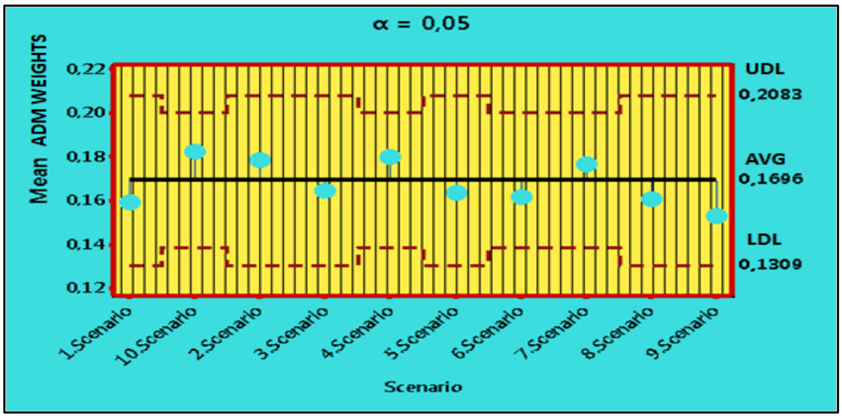

Figure 8. *ADM Visual*

As depicted in Figure 8, the ADM values calculated for each scenario are situated below the UDL values and above the LDL values. Consequently, the variances in the identified weights for each scenario exhibit homogeneity. This determination was further confirmed through the Levene Test. The fundamental statistics for the Levene Test are outlined in Table 11.

Table 11. *Levene Test*

Levene Statistic	df1	df2	Sig.
0.426	2	10	0.212
p**<.05			

Based on the findings from Table 11, the p-value (p=0.212) surpasses the significance threshold of 0.05, affirming the homogeneity of variances in criterion weights across scenarios. Overall, the outcomes of the simulation analysis suggest the robustness and stability of the MIEXCF method.

4. CONCLUSION

Multi-criteria decision-making is a widely used approach for complex problems, involving the consideration of various criteria. Assigning weights to these criteria is crucial as their importance may vary, ensuring an unbiased decision-making process. Researchers have developed diverse methods for computing weight coefficients, contributing to the field of MCDM. In this study, we propose a Measurement Relying on the Impacts of an Exponential Curve Function (MIEXCF) as a novel approach for determining criterion weights.

The fundamental principle of the MIEXCF method lies in establishing exponential curve effects among criteria. Leveraging the benefits of exponential curve functions and the proposed method, effects among criteria can be computed. The criterion with the highest cumulative effect is then deemed the most significant. This criterion holds the potential to influence others, contributing to their development through associated activities or measures. Furthermore, a system can be devised to determine decision alternative strategies based on the most crucial criterion(s).

The study utilized 2023 Logistics Performance Index (LPI) data for 19 G20 countries. Initially, the MIEXCF method calculated weight coefficients for LPI components. A sensitivity analysis compared these with other methods (ENTROPY, CRITIC, SD, SVP, MEREC, and LOPCOW). Results revealed differing weight coefficient rankings for LPI criteria between the MIEXCF method and others. This underscores the proposed method's sensitivity.

The study's second approach involved a comparative analysis of the MIEXCF method, examining its similarity to other objective weight methods. Results showed a positive, significant, and moderate correlation with the MEREC method, and a positive, significant, and low correlation with the LOPCOW and CRITIC methods. Generally, the MIEXCF method was observed to have limited similarity with other criterion weighting methods. These findings led to the conclusion that the MIEXCF method is both credible and reliable.

In a simulation analysis, ten distinct Logistics Performance Index decision matrices were generated using the MIEXCF method and other weighting techniques. These matrices were divided into two groups: one with three scenarios and another with seven scenarios. Correlation values between the MIEXCF method and other weighting methods decreased as the number of scenarios increased, highlighting the unique features of the MIEXCF method. Variance analysis revealed that the MIEXCF method effectively distinguishes criteria weights, with higher average variance compared to alternative methods such as ENTROPY, CRITIC, SD and SVP. Homogeneity tests confirmed consistent variances within MIEXCF scenarios, indicating reliability. The Levene test showed no significant variance differences between the MIEXCF method and others.

According to the findings, it has been concluded that the MIEXCF method is a sensitive, stable and close to, credible, and reliable state. The study aims to demonstrate the feasibility of using the MIEXCF method to quantify criterion weights in MCDM literature, providing an objective tool for assessing decision option effectiveness. The outcomes have significant implications for scholars and decision-makers, anticipating increased attention to exponential curve functions in mathematical modeling. The MIEXCF method proves effective for decision-makers dealing with complex tasks, especially in performance evaluation. Simulation analysis data confirms the method's stability and robustness.

In future studies, the calculation of criterion weighting coefficients and the assessment of relationships among criteria can be expanded beyond exponential functions to include other functions like sigmoid, quadratic, cubic, linear, inverse, and so forth. Additionally, research endeavors can explore computing criterion weighting coefficients by considering not only the influence of criteria but also the values of criterion interdependence. The goal is to identify the criteria contributing to the intensity of relationships between two criteria.

CONFLICT OF INTEREST

The author declares no conflict of interest.

REFERENCES

Akın, N. G. (2017). İşletme bölümü öğrencilerinin meslek seçimini etkileyen faktörlerin bulanık dematel yöntemi ile değerlendirilmesi. *Uluslararası Yönetim İktisat ve İşletme Dergisi*, 13(4), 873-890. http://www.doi.org/10.17130/ijmeb.2017433413.

Alfaro, L., Chari, A., Greenland, A. N., & Schott, P. K. (2020). Aggregate and firm-level stock returns during pandemics, in real time. *NBER Working Paper* (26950), 1-31.

Ali, T., Aghaloo, K., Chiu, Y.-R., & Ahmad, M. (2022). Lessons learned from the COVID-19 pandemic in planning the future energy systems of developing countries using an integrated MCDM approach in the off-grid areas of Bangladesh. *Renewable Energy*, 189, 26-38. https://www.doi.org/10.1016/j.renene.2022.02.099.

Alrababah, S., & Gan, K. H. (2023). Effects of the hybrid CRITIC–VIKOR method on product aspect ranking in customer reviews. *Appl. Sci*., 13, 1-14. https://www.doi.org/10.3390/app13169176.

Altıntaş, F. F. (2021). Sağlık güvenliği bileşenleri bileşenleri arasındaki ilişkilerin analizi: Somer's d temelli DEMATEL yöntemi ile bir uygulama. *Eurasian Academy of Sciences Social Sciences Journal*, 36, 48-63. https://www.doi.org/10.17740/eas.soc.2021.V36-04.

Altıntaş, F. F. (2023). A novel approach to measuring criterion weights in multiple criteria decision making: cubic effect-based measurement (CEBM). *Nicel Bilimler Dergisi*, 5(2), 151-195. https://www.doi.org/10.51541/nicel.1349382.

Arslan, R. (2020). Critic yöntemi. In: H. Bircan (Eds.), *Çok Kriterli Karar Verme Problemlerinde Kriter Ağırlıklandırma Yöntemleri* (pp. 120-122). Nobel Yayıncılık.

Arvis, J.-F., Ojala, L., Shepherd, B., Ulybina, D., & Wiederer, C. (2023). *Connecting to compete trade logistics in the global economy*. International Bank for Reconstruction and Development/The World Bank.

Ayan, B., Abacıoğlu, S., & Basilio, M. P. (2023). A comprehensive review of the novel weighting methods for multi-criteria decision-making. *Information*, 14(5), 1-28. https://www.doi.org/10.3390/info14050285.

Ayçin, E. (2019). *Çok Kriterli Karar Verme*. Ankara: Nobel Yayın.

Balaban, E. (2015). *Temel Matematik ve İşletme Uygulamaları*. İstanbul: Türkmen Kitapevi.

Baş, F. (2021). *Çok Kriterli Karar Verme Yöntemlerinde Kriter Ağırlıklarının Belirlenmesi*. Ankara: Nobel Bilimsel.

Bayram, N. (2010). *Yapısal Eşitlik Modellemesine Giriş: Amos Uygulamaları*. Bursa: Ezgi Kitapevi.

Bernett, M. A., Ziegler, M., & Byleen, K. E. (2015). *Calculus for business, economics, life sciences and social sciences*. Pearson.

Bircan, H. (2020). *Çok Kriterli Karar Verme Problemlerinde Kriter Ağırlıklandırma Yöntemleri*. Ankara: Nobel Yayıncılık.

Çelik, E. H., & Yılmaz, V. (2013). *Lisrel 9.1 ile Yapısal Eşitlik Modellemesi*. Ankara: Anı Yayıncılık.

Chakrabarty, D., & Rahman, A. (2007). Exponential curve: Estimation-using the just preceding observation in fitted curve. *Int.J.Agricult.Stat.Sci*, 3(2), 381-186.

Civelek, M. E. (2018). *Yapısal Eşitlik Modellemesi Metodolojisi*. İstanbul: Beta Yayımları.

Demir, G., Özyalçın, T., & Bircan, H. (2021). *Çok Kriterli Karar Verme Yöntemleri ve ÇKKV Yazılımı ile Problem Çözümü*. Ankara: Nobel.

Diakoulaki, D., Mavrotas, G., & Papayannakis , L. (1995). Determining objective weights in multiple criteria problems: The Critic method. *Computers & Operations Research*, 22(7), 763-770.

Ecer, F. (2020). *Çok Kriterli Karar Verme*. Ankara: Seçkin Yayıncılık.

Ecer, F., & Pamucar, D. (2022). A novel LOPCOW-DOBI multi-criteria sustainability performance assessment methodology: An application in developing country banking sector. *Omega*, 1-35. https://www.www.doi.org/10.1016/j.omega.2022.102690.

Eroğlu, E. (2017). *İşletme, Iktisat Ve Sosyal Bilimler Için Matematik*. Bursa: Dora Yayın.

Ersoy, N. (2023). BIST perakende ticaret sektöründe LOPCOW-RSMVC modeli ile performans ölçümü. *Sosyoekonomi*, 31(57), 419-436.

Ertik, H., Şendur, A., & Tulga, İ. (2015). *Matematik İşletme İktisat ve Ekonomi Uygulamaları*. Çanakkale: Paradigma Akademi Yayınları.

Fontela, E., & Gobus, A. (1976). *The DEMATEL observer*. Battelle Geneva Research Center.

Fosu, G. O., & Edunyah, G. (2020). Flattening the exponential growth curve of covid-19 in Ghana and other developing countries; Divine ıntervention is a necessity. *SSRN*, 1-13. http://www.www.doi.org/10.2139/ssrn.3565147.

Gigovic, L., Pamucar, D., Bajic, Z., & Milicevic, M. (2016). The combination of expert judgment and GIS-MAIRCA analysis for the selection of sites for ammunition depots. *Sustainability*, 8, 1-30. https://www.doi.org/10.3390/su8040372.

Göncü, K. K., & Çetin, O. (2022). A Decision model for supplier selection criteria in healthcare enterprises with dematel ANP method. *Sustainability*, 14, 1-16. https://www.doi.org/10.3390/su142113912.

Gülençer, İ., & Türkoğlu, S. P. (2020). Gelişmekte olan Asya ve Avrupa ülkelerinin finansal gelişmişlik performansının İstatistiksel Varyans Prosedürü temelli OCRA yöntemiyle analizi. *Üçüncü Sektör Sosyal Ekonomi Dergisi*, 55(2), 1330-1344. https://www.doi.org/10.15659/3.sektor-sosyal-ekonomi.

Gürbüz, S. (2019). *AMOS ile Yapısal Eşitlik Modellemesi*. Ankara: Seçkin Yayınevi.

Hamill, J. T., Deckro, R. F., & Kloeber, J. M. (2005). Evaluating Information Assurance Strategies. *Decision Support Systems*, 39(3), 463-484.

IBM. (2013). *SPSS tutorial*. IBM.

Jones, C. I. (2023). Recipes and Economic Growth: A combinatorial march down an exponential tail. *Journal of Applied Econometrics*, 38(55), 767–785. https://www.doi.org/10.1086/723631.

Joujan, A. (2018). *Summit Math Series: Algebra 2: Book 7: Exponential functions*. XanEdu Publishing Inc.

Kahn, D. S. (2015). *Attacking problems in logarithms and exponential functions*. Dover Publications.

Karagöz, Y. (2017). *SPSS ve AMOS 23 Uygulamalı İstatistiksel Analizler*. Ankara: Nobel Akademik Yayıncılık.

Kartal, M., Karagöz, Y., & Kartal, Z. (2014). *Temel Matematik*. Ankara: Nobel Yayın.

Keleş, N. (2023). *Uygulamalarla Klasik ve Güncel Karar Verme Yöntemleri*. Ankara: Nobel.

Keshavarz-Ghorabaee, M., Amiri, M., Zavadskas, E. K., Turskis, Z., & Antucheviciene, J. (2021). Determination of objectiveweights using a new method based on the removal effects of criteria (MEREC). *Symmetry*, 13, 1-20. https://www.doi.org/10.3390/sym13040525.

Keshavarz-Ghorabaee, M., Amiri, M., Zavadskas, E. K., Turskis, Z., & Antucheviciene, J. (2018). Simultaneous evaluation of criteria and alternatives (SECA) for multi-criteria decision-making. *Informatica*, 29(2), 265–280. https://www.doi.org/10.15388/Informatica.2018.167.

Kline, R. B. (2019). *Yapısal Eşitlik Modellemesinin İlkeleri ve Uygulaması*, Translate: Şen, S. İstanbul: Nobel Yayınları.

Kuruüzüm, A., & İpekçi Çetin, E. (2015). *İşletme ve Ekonomi Öğrencileri Için Uygulamalı Matematik*. Ankara: Gazi Kitapevi.

Landsberg, J. J. (1977). Some useful equations for biological studies. *Expl Agric.*, 13, 273-286.

Lin, C.-Y. (2014). *An Exponential function approach to parabolic equations*. World Scientific Publishing Company.

Meydan, C. H., & Şeşen, H. (2015). *Yapısal Eşitlik Modellemesi AMOS Uygulamaları*. Ankara: Detay Yayıncılık.

Mukhametzyanov, I. Z. (2021). Specific character of objective methods for determining weights of criteria in MCDM problems: Entropy, CRITIC, SD. *Decision Making: Applications in Management and Engineering* 4(2), 76-105. https://www.doi.org/10.31181/dmame210402076i.

Murillo-Escobar, M. A., Quintana-Ibarra, J. A., & Cruz-Hernández, C. (2023). Biosignal encryption algorithm based on Ushio chaotic map for e-health. *Multimed Tools Appl*(82), 23373–23399. https://www.doi.org/10.1007/s11042-022-14092-4.

Nasser, A. A., Alkhulaidi, A. A., Ali, M. N., Hankal, M., & Al-olofe, M. (2019). A weighted euclidean distance - statistical variance procedure based approach for ımproving the healthcare decision making system in Yemen. *Indian Journal of Science and Technology*, 12(3), https://www.doi.org/1-15. 10.17485/ijst/2019/v12i3/140661.

Natalija, B. (2021). Exponential functions through a real-world context. *Open Schools Journal for Open Science*, 3, 1-10. https://www.doi.org/10.12681/osj.24891.

Odu, G. 0. (2019). Weighting methods for multi-criteria decision making technique. *J. Appl. Sci. Environ. Manage*, 23(8), 1449-1457. https://dx.www.doi.org/10.4314/jasem.v23i8.7.

Önalan, Ö. (2010). *İşletme Matematiği*. İstanbul: Avcıol Basım Yayım.

Özdamar, K. (2016). *Eğitim, Sağlık ve Davranış Bilimlerinde Ölçek Geliştirme ve Yapısal Eşitlik Modellemesi*. Eskişehir: Nisan Kitapevi.

Paksoy, S. (2017). *Çok Kriterli Karar Vermede Güncel Yaklaşımlar*. Adana: Karahan Kitapevi.

Pekkaya, M. (2016). *İşletme ve Iktisat Için Genel Matematik ve Matematiksel Yöntemler*. Bursa: Ekin Yayınevi.

Rasmussen, A., Sabic, H., Saha, S., & Nielsen, I. E. (2023). Supplier selection for aerospace & defense industry through MCDM methods. *Cleaner Engineering and Technology*, 12, 1-12. https://www.doi.org/10.1016/j.clet.2022.100590.Saaty, T. L. (2008). Decision making with the analytic hierarchy process. *International Journal of Services Sciences*, 1(1), 83-98.

Sel, A. (2020). IDOCRIW Yöntemi, In: H. Bircan (Eds.), *Çok Kriterli Karar Verme Problemlerinde Kriter Ağirliklandirma Yöntemleri* (pp. 37-50). Nobel Akdemik Yayıncılık.

Shparlinski, I., & Konyagin, S. (1999). *Character sums with exponential functions and their applications*. Cambridge University Press.

Sümerli Sarigül, S., Ünlü, M., & Yaşar, E. (2023). A new MCDM approach in evaluating airport service quality: MEREC-Based MARCOS and CoCoSo Methods. *Uluslararası Yönetim Akademisi Dergisi*, 6(1), 90-108. https://www.doi.org/10.33712/mana.1250335.

Thomas, C. (1991). *Introduction to exponents and logarithms*. University of Sydney.

Uludağ, A. S., & Doğan, H. (2021). *Üretim Yönetiminde Çok Kriterli Karar Verme*. Ankara: Nobel.

Vavrek, R. (2019). Evaluation of the ımpact of selected weighting methods on the results of the TOPSIS technique. *International Journal of Information Technology & Decision Making*, 18(6), 1821–1843. https://www.doi.org/10.1142/S021962201950041X.

Walters, S. J. (2009). *Quality of life outcomes in clinical trials and health-care evaluation: A practical guide to analysis and interpretation*. Wiley.

Wang, C.-N., Nguyen, N.-A.-T., & Dang, T.-T. (2023). Sustainable evaluation of major third-party logistics providers: A framework of an MCDM-Based Entropy objective weighting method. *Mathematics*, 11, 1-27. https://www.doi.org/10.1142/S021962201950041X.

Weon, B. M., & Je, J. H. (2014). Plasticity and rectangularity in survival curves. *Scientific Reports*, 1(104), 1-5.

Wood, P. K. (2023). Combined logistic and confined exponential growth models. *A Multidisciplinary Journal*, 1-10. http://www.www.doi.org/10.1080/10705511.2023.2220918.

Zavadskas, E. K., & Podvezko, V. (2016). Integrated determination of objective criteria wights in MCMD. *International Journal of Information Technology & Decision Making*, 15(2), 267-283. https://www.doi.org/10.1142/S0219622016500036.