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Some New Techniques of Computing Correlation Coefficient between q-Rung Orthopair Fuzzy Sets and their Applications in Multi-Criteria Decision-Making

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Highlights

- The paper presents two new techniques of q-rung orthopair fuzzy correlation coefficient.
- It discusses some theoretical properties of the new techniques.
- It explores the uses of the new techniques in medical diagnosis and employment procedures.
- It shows the advantages of the new techniques over the existing techniques.

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Abstract

The term q-rung orthopair fuzzy set is an essential variant of fuzzy set with the capacity of tackling fuzziness and imprecision in the decision-making process. A fundamental concept in the decision-making process is the idea of correlation coefficient because of its wide applications. The process of decision-making is complex due to imprecisions, and as such the idea of correlation coefficient has been investigated under q-rung orthopair fuzzy setting. Some authors have constructed some techniques of correlation coefficient under q-rung orthopair fuzzy sets with practical applications. However, these existing techniques are defectives with several drawbacks in terms of precision and alignment with the conditions of correlation coefficient. In this work, two new techniques for estimating correlation coefficient under q-rung orthopair fuzzy sets are presented and theoretically discussed. Moreover, we apply the new techniques of correlation coefficient under q-rung orthopair fuzzy sets in disease diagnosis and employment process by using simulated q-rung orthopair fuzzy data based on multi-criteria decision-making approach and recognition principle. Some comparative analyses are provided to ascertain the benefits of the new techniques of correlation coefficient under q-rung orthopair fuzzy sets over the obtainable techniques with regard to reliability and performance rating.

1. INTRODUCTION

The art of decision-making lies in identifying the option or alternative that most closely matches the selection criteria. The concept of fuzzy sets (FSs) [1] is appropriate to support reliable decision-making given the complexity of decision-making. However, FS is constrained in that it only recognizes the degree of membership (DM). Consequently, a great deal of FS variants have been introduced, including intuitionistic fuzzy set (IFS) [2], Pythagorean fuzzy set (PFS) [3, 4], Fermatean fuzzy set (FFS) [5, 6], and q-rung orthopair fuzzy set (q-ROFS) [7]. The FS variants were added to increase FS's modeling capability. To identify DM and degree of non-membership (DNM), IFSs used two values from the closed interval, $I = [0,1]$, such that their sum is less than or equal to one. A third parameter known as hesitation margin may also be present. IFS is used in numerous fields [8–11]. IFS is unable to simulate scenarios for which the sum of DM and DNM transcends one. In order to address such an issue, the concept of PFSs was presented [3, 4].

With the use of PFSs, numerous real-world issues have been solved [12–19]. Conversely, PFS is unable to simulate scenarios in which the aggregate of the squares for DM and DNM transcends 1. In order to manage

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this possible situation, the notion of FFSs was presented [5, 6]. It is distinguished by the characteristic that the summation of DM and DNM's squares can exceed one, while the sum of their cubes is almost one. Numerous FFS applications have been implemented using a variety of information measures [20–25]. The concept of FFSs cannot be used to reduce the imprecision in systems if the sum of the cubes of DM and DNM is greater than one. q-ROFSs [7] were introduced to address the shortcomings of IFSs, PFSs, and FFSs. Due to q-ROFS's exceptional qualities, numerous researchers have studied it in-depth and produced a vast amount of works, including multi-criteria decision-making (MCDM) [26–29], measure theory [30, 31], graph theory [32–34], multi-attribute decision-making (MADM) [35], and many more.

One of the most often used indices in decision-making, pattern recognition, data analysis, machine learning, and related fields is the idea correlation coefficient. Correlation analysis is a statistical technique for establishing the relationship between two numerically measured continuous variables. When a researcher wants to find out if there could be a relationship between two variables, they use this type of analysis. The correlation coefficient is a statistical approach of measuring how potently two variables are associated. Because of the imprecisions in data collection, the sense of correlation coefficient has been studied in FSs [36], and stretched in view of the three parameters of IFSs [37]. Due to the shortcomings of the methods in [38, 39], Huang and Guo [40] presented a strong method, but they did so by taking into account just two IFSs' parameters. From a statistical standpoint, Hung [41] examined intuitionistic fuzzy correlation coefficient. To increase accuracy, the method in [41] was altered in [42, 43]. Decision-making situations have been the subject of research and application of a few statistical methods for calculating intuitionistic fuzzy correlation coefficient based on variance and covariance [44–46].

Likewise, under PFSs, the theory of correlation measure has been explored. The Pythagorean fuzzy correlation measure was first used in real-world scenarios in [47]. An innovative correlation coefficient for PFSs was proposed by Thao [48] and used in pattern recognition. By considering DM, DNM, potency, and path of commitments of PFSs, Lin et al. [49] established some new directional correlation coefficients for PFSs. Certain unique correlation coefficients for PFSs with applications were introduced by Singh and Ganie [50]. Several Pythagorean fuzzy correlation coefficients have been studied from statistical viewpoints [51–53]. Owing to its significance, correlation coefficients have been studied within the context of q-ROFS. A method for computing correlation coefficient for q-ROFSs (CCq-ROFSs) was presented in [54] and used in practical decision-making contexts. Li et al. [55] presented some CCq-ROFSs methods along with a discussion of their uses in clustering. Furthermore, we found that the approach of CCq-ROFSs based on variance and covariance is not a suitable method for CCq-ROFSs [56]. Lastly, two CCq-ROFSs approaches were presented by Bashir et al. [57] after they expanded upon the correlation coefficient approaches in [37] and talked about their uses in clustering. However, we note certain shortcomings in the CCq-ROFSs methods covered in [54–57]. Contrary to expectations, the performances of the existing CCq-ROFSs techniques decline as q increases. Because the hesitation margin was disregarded when calculating the correlation coefficient, the CCq-ROFSs techniques in [54, 55] produce outputs that are unreliable due to exclusion error. Furthermore, some of the correlation coefficient's requirements are violated by the methods for obtaining CCq-ROFSs. Given all of these difficulties, it is essential to establish some new CCq-ROFSs methods that will address the difficulties and raise performance rating. The goals of this work are listed thus:

i. Evaluation of the available CCq-ROFSs methods [54–57] in order to draw attention to their shortcomings.
ii. Development of new CCq-ROFSs approaches, which can resolve the setbacks in the extant methodologies with enhanced performance assessment.
iii. Descriptions of the novel CCq-ROFSs techniques to demonstrate how well they align with correlation coefficient properties.
iv. Application of the new CCq-ROFSs approaches based on MCDM approach and recognition principle, in medical diagnosis and employment processes.
v. Comparative study using numerical samples to list the benefits of the innovative CCq-ROFSs methodologies over the current CCq-ROFSs methodologies.

The residual portions of the paper are systematized as follows: Section 2 dwells on the variant of FS; Section 3 summarizes some of the existing CCq-ROFSs techniques; Section 4 develops two new CCq-ROFSs techniques, characterizes their properties, and addresses medical diagnosis and employment processes based on the new CCq-ROFSs techniques using the MCDM approach and recognition principle; Section 5 analyses the new CCq-ROFSs techniques in comparison to the existing CCq-ROFSs techniques; and finally, Section 6 closes the paper with suggestions for further investigation.

2. PRELIMINARIES

This section reiterates certain variants of FS. We denote the finite non-empty set used in the paper by Q.

Definition 2.1 [2]. An IFS β in Q can be represented as:

$$\beta = \{\langle q_i, \mu_\beta(q_i), \nu_\beta(q_i)\rangle \mid q_i \in Q\}, \tag{2.1}$$

for $i = 1,2,\ldots,n$, where $\mu_\beta(q_i)$ denotes DM and $\nu_\beta(q_i)$ represents DNM of $q_i \in Q$ in β with $0 \leq \mu_\beta(q_i) + \nu_\beta(q_i) \leq 1$. The grade of indeterminacy of β is given by $\pi_\beta(q_i) = 1 - \mu_\beta(q_i) - \nu_\beta(q_i)$. For simplicity, Xu and Yager [34] used the notation (μ_β, ν_β) to embody the intuitionistic fuzzy number of β.

Definition 2.2 [4]. A PFS P in Q is given by

$$P = \{\langle q_i, \mu_P(q_i), \nu_P(q_i)\rangle \mid q_i \in Q\}, \tag{2.2}$$

for $i = 1,2,\ldots,n$, where $\mu_P: Q \to [0,1]$ denotes DM and $\nu_P: Q \to [0,1]$ denotes DNM of $q_i \in Q$ to the set P with

$$0 \leq \mu_P^2(q_i) + \nu_P^2(q_i) \leq 1.$$

The grade of indeterminacy of P is given by

$$\pi_p(q_i) = \sqrt{1 - \mu_P^2(q_i) - \nu_P^2(q_i)}.$$

For expediency, the Pythagorean fuzzy number (PFN) of PFS P is signified by (μ_P, ν_P).

Definition 2.3 [6]. A FFS represented by F in Q is a structure having the form

$$\mathrm{F} = \{\langle q_i, \mu_F(q_i), \nu_F(q_i)\rangle : q_i \in Q\}, \tag{2.3}$$

for $i = 1,2,\ldots,n$, where $\mu_F: Q \to [0,1]$ and $\nu_F: Q \to [0,1]$ symbolize DM and DNM, respectively, with the form

$$0 \leq \mu_\mathrm{F}^3(q_i) + \nu_\mathrm{F}^3(q_i) \leq 1, \forall q_i \in Q.$$

For any FFS F, the function $\pi_\mathrm{F}(q_i)$ defined by $\pi_\mathrm{F}(q_i) = \sqrt[3]{1 - \mu_\mathrm{F}^3(q_i) - \nu_\mathrm{F}^3(q_i)}$ is called the grade of hesitancy of q_i to F.

Definition 2.4 [7]. A q-ROFS represented by $\widetilde{\wp}$ in Q is defined by

$$\widetilde{\wp} = \{\langle q_i, \xi_{\widetilde{\wp}}(q_i), \eta_{\widetilde{\wp}}(q_i)\rangle \mid q_i \in Q\}, \tag{2.4}$$

for $i = 1,2,\ldots,n$, where the functions $\xi_{\widetilde{\wp}}(q_i), \eta_{\widetilde{\wp}}(q_i) \in [0,1]$ denote DM and DNM, respectively, of $q_i \in Q$ to the set $\widetilde{\wp}$ satisfying the property;

$$0 \leq \xi_{\widetilde{\wp}}^{r}(q_i) + \eta_{\widetilde{\wp}}^{r}(q_i) \leq 1, r/q \geq 1. \tag{2.5}$$

The degree of indeterminacy $\pi_{\widetilde{\wp}}(q_i)$ of q_i in $\widetilde{\wp}$ is given as:

$$\pi_{\widetilde{\wp}}(q_i) = \left[1 - \xi_{\widetilde{\wp}}^{r}(q_i) - \eta_{\widetilde{\wp}}^{r}(q_i)\right]^{\frac{1}{r}}. \tag{2.6}$$

For easiness, $\left(\xi_{\widetilde{\wp}}(q_i), \eta_{\widetilde{\wp}}(q_i)\right)$ is the q-ROF number (q-ROFN), and is signified by $\widetilde{\wp} = (\xi, \eta)$. Figure 1 is the graphical representation of q-ROFS.

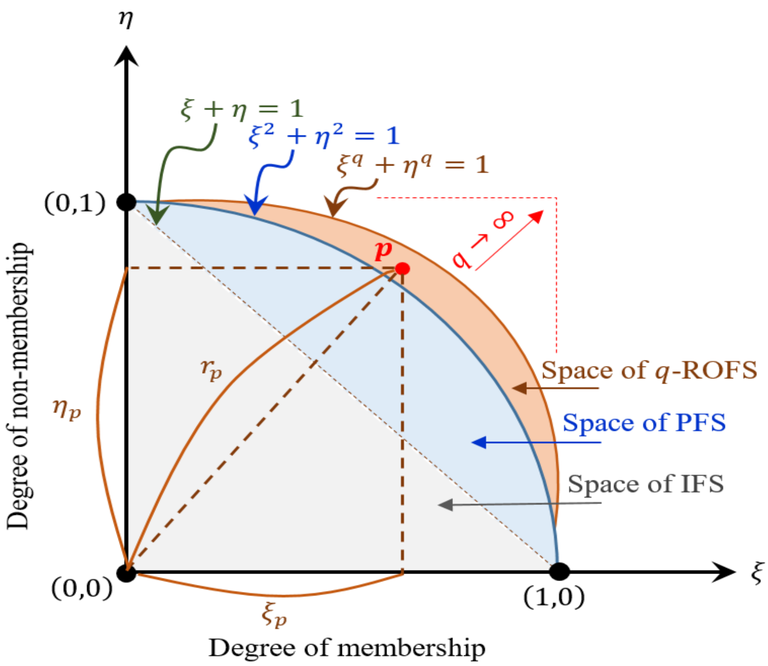

Figure 1. *Graphical representation of q-ROFS*

We present some operations on q-ROFNs, in forms of basic operations, ordering, and score and accuracy functions as defined in [35], as follows:

Definition 2.5. Suppose $\widetilde{\wp} = (\xi, \eta)$, $\widetilde{\wp}_1 = (\xi_1, \eta_1)$ and $\widetilde{\wp}_2 = (\xi_2, \eta_2)$ are q-ROFNs, and $\lambda > 0$, then the following are certain fundamental operations on q-ROFSs;

i. $\widetilde{\wp}_1 \oplus \widetilde{\wp}_2 = \left(\sqrt[r]{\xi_1{}^r + \xi_2{}^r - \xi_1{}^r \xi_2{}^r}, \eta_1\eta_2\right)$,
ii. $\lambda\widetilde{\wp} = \left(\sqrt[r]{1 - (1 - \xi^r)^\lambda}, \eta^\lambda\right)$,
iii. $\widetilde{\wp}_1 \otimes \widetilde{\wp}_2 = \left(\xi_1\xi_2, \sqrt[r]{\eta_1{}^r + \eta_2{}^r - \eta_1{}^r \eta_2{}^r}\right)$,
iv. $\widetilde{\wp}^\lambda = \left(\xi^\lambda, \sqrt[r]{1 - (1 - \eta^r)^\lambda}\right)$.

Definition 2.6. Assume $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ are q-ROFNs, then the ordering for the q-ROFNs are:

- For $S(\widetilde{\wp}_1) = S(\widetilde{\wp}_2)$;

i. $A(\widetilde{\wp}_1) > A(\widetilde{\wp}_2)$ if $\widetilde{\wp}_1 \succ \widetilde{\wp}_2$,

ii. $A(\widetilde{\wp}_1) = A(\widetilde{\wp}_2)$ for $\widetilde{\wp}_1 \approx \widetilde{\wp}_2$.

- $S(\widetilde{\wp}_1) > S(\widetilde{\wp}_2)$ if $\widetilde{\wp}_1 \succ \widetilde{\wp}_2$.

Definition 2.7. Assume $\widetilde{\wp} = (\xi, \eta)$ is a q-ROFN, then the score function, $S(\widetilde{\wp})$ of $\widetilde{\wp}$ is described as:

$$S(\widetilde{\wp}) = \frac{1}{2}(1 + \xi^q - \eta^q), \quad (2.7)$$

where $S(\widetilde{\wp}) \in [0, 1]$. In the same vein, the accuracy function, $A(\widetilde{\wp})$ of $\widetilde{\wp}$ is given by:

$$A(\widetilde{\wp}) = \xi^q + \eta^q. \quad (2.8)$$

3. CORRELATION COEFFICIENT UNDER q-ROFSs

Here, definitions of CCq-ROFSs under [0,1] and [−1,1] are represented. Suppose $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ are q-ROFSs in $Q = \{q_1, q_2, \ldots, q_n\}$, then we present two definitions of CCq-ROFSs.

Definition 3.1. The CCq-ROFSs $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ in [0,1] designated by $\rho(\widetilde{\wp}_1, \widetilde{\wp}_2)$, is a measuring function,

$\rho : \widetilde{\wp}_1 \times \widetilde{\wp}_2 \to [0,1]$ with the properties:

i. $\rho(\widetilde{\wp}_1, \widetilde{\wp}_2) = \rho(\widetilde{\wp}_2, \widetilde{\wp}_1)$,

ii. $\rho(\widetilde{\wp}_1, \widetilde{\wp}_2) \in [0,1]$,

iii. $\rho(\widetilde{\wp}_1, \widetilde{\wp}_2) = 1$ if and only if $\widetilde{\wp}_1 = \widetilde{\wp}_2$.

In the same way, the CCq-ROFSs $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ in [−1,1] designated by $\rho_*(\widetilde{\wp}_1, \widetilde{\wp}_2)$, is a measuring function, $\rho_* : \widetilde{\wp}_1 \times \widetilde{\wp}_2 \to [-1,1]$ with the properties:

i. $\rho_*(\widetilde{\wp}_1, \widetilde{\wp}_2) = \rho_*(\widetilde{\wp}_2, \widetilde{\wp}_1)$,

ii. $\rho_*(\widetilde{\wp}_1, \widetilde{\wp}_2) \in [-1,1]$,

iii. $\rho_*(\widetilde{\wp}_1, \widetilde{\wp}_2) = 1$ if and only if $\widetilde{\wp}_1 = \widetilde{\wp}_2$.

As $\rho(\widetilde{\wp}_1, \widetilde{\wp}_2)$, $\rho_*(\widetilde{\wp}_1, \widetilde{\wp}_2)$ tends to 1, it means a potent correlation exists between $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$. Also, as $\rho(\widetilde{\wp}_1, \widetilde{\wp}_2)$ tends to 0 or −1 for the case of $\rho_*(\widetilde{\wp}_1, \widetilde{\wp}_2)$, it means a very faint correlation exists between $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$. Whereas, $\rho(\widetilde{\wp}_1, \widetilde{\wp}_2), \rho_*(\widetilde{\wp}_1, \widetilde{\wp}_2) = 1$ points to a perfect positive correlation between $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$, and $\rho(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0$ or $\rho_*(\widetilde{\wp}_1, \widetilde{\wp}_2) = -1$ explains no correlation or perfect negative correlation between $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$.

3.1. Techniques of CCq-ROFSs

Here, we reiterate some existing techniques of computing CCq-ROFSs [54-57]. Suppose $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ are two arbitrary q-ROFSs in $Q = \{q_1, q_2, \ldots, q_n\}$, then the existing CC$q$-ROFSs between $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ are listed in what follows.

3.1.1. Du's technique

In [54], a technique for computing CCq-ROFSs was introduced, which modified the approach in [36, 37]. The technique is;

$$\rho_1(\tilde{\wp}_1,\tilde{\wp}_2)=\left(\frac{\left(\sum_{i=1}^{n}\left(\xi^{\,r}_{\tilde{\wp}_1}(q_i)\xi^{\,r}_{\tilde{\wp}_2}(q_i)+\eta^{r}_{\tilde{\wp}_1}(q_i)\eta^{r}_{\tilde{\wp}_2}(q_i)\right)\right)^2}{\sum_{i=1}^{n}\left(\xi^{\,2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)\right)\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_2}(q_i)+\eta^{2r}_{\tilde{\wp}_2}(q_i)\right)}\right)^{\frac{1}{2r}},\ \text{for } r\geq 1. \tag{3.1}$$

We observe that, this technique cannot reliably determine the correlation between the q-ROFSs as the hesitation margin is left out. In addition, the method fails a condition of correlation metric.

Example 3.1. Suppose $\tilde{\wp}_1=\{\langle q_1,0.1,0.2\rangle,\langle q_2,0.2,0.1\rangle,\langle q_3,0.29,0.0\rangle\}$ and $\tilde{\wp}_2=\{\langle q_1,0.1,0.3\rangle,\langle q_2,0.2,0.2\rangle,\langle q_3,0.29,0.1\rangle\}$ are q-ROFSs in $Q=\{q_1,q_2,q_3\}$. By applying the technique for $r=4$, we have $\rho_1(\tilde{\wp}_1,\tilde{\wp}_2)=1$ although the q-ROFSs are not equal.

3.1.2. Singh and Ganie's technique

An approach for computing CCq-ROFSs based on Pearson's correlation was introduced in [56], which is;

$$\rho_2(\tilde{\wp}_1,\tilde{\wp}_2)=\frac{1}{3}(\theta_1+\theta_2+\theta_3), \tag{3.2}$$

where

$$\theta_1=\frac{\sum_{i=1}^{n}\left[\left(\xi^{r}_{\tilde{\wp}_1}(q_i)-\left(\overline{\xi_{\tilde{\wp}_1}}\right)^r\right)\left(\xi^{r}_{\tilde{\wp}_2}(q_i)-\left(\overline{\xi_{\tilde{\wp}_2}}\right)^r\right)\right]}{\left[\sqrt{\sum_{i=1}^{n}\left(\xi^{r}_{\tilde{\wp}_1}(q_i)-\left(\overline{\xi_{\tilde{\wp}_1}}\right)^r\right)^2}\sqrt{\sum_{i=1}^{n}\left(\xi^{r}_{\tilde{\wp}_2}(q_i)-\left(\overline{\xi_{\tilde{\wp}_2}}\right)^r\right)^2}\right]},$$

$$\theta_2=\frac{\sum_{i=1}^{n}\left[\left(\eta^{r}_{\tilde{\wp}_1}(q_i)-\left(\overline{\eta_{\tilde{\wp}_1}}\right)^r\right)\left(\eta^{r}_{\tilde{\wp}_2}(q_i)-\left(\overline{\eta_{\tilde{\wp}_2}}\right)^r\right)\right]}{\left[\sqrt{\sum_{i=1}^{n}\left(\eta^{r}_{\tilde{\wp}_1}(q_i)-\left(\overline{\eta_{\tilde{\wp}_1}}\right)^r\right)^2}\sqrt{\sum_{i=1}^{n}\left(\eta^{r}_{\tilde{\wp}_2}(q_i)-\left(\overline{\eta_{\tilde{\wp}_2}}\right)^r\right)^2}\right]},$$

$$\theta_3=\frac{\sum_{i=1}^{n}\left[\left(\pi^{r}_{\tilde{\wp}_1}(q_i)-\left(\overline{\pi_{\tilde{\wp}_1}}\right)^r\right)\left(\pi^{r}_{\tilde{\wp}_2}(q_i)-\left(\overline{\pi_{\tilde{\wp}_2}}\right)^r\right)\right]}{\left[\sqrt{\sum_{i=1}^{n}\left(\pi^{r}_{\tilde{\wp}_1}(q_i)-\left(\overline{\pi_{\tilde{\wp}_1}}\right)^r\right)^2}\sqrt{\sum_{i=1}^{n}\left(\pi^{r}_{\tilde{\wp}_2}(q_i)-\left(\overline{\pi_{\tilde{\wp}_2}}\right)^r\right)^2}\right]},$$

$$\overline{\xi_{\tilde{\wp}_1}}=\frac{\sum_{i=1}^{n}\xi_{\tilde{\wp}_1}(q_i)}{n},\ \overline{\eta_{\tilde{\wp}_1}}=\frac{\sum_{i=1}^{n}\eta_{\tilde{\wp}_1}(q_i)}{n},\ \overline{\pi_{\tilde{\wp}_1}}=\frac{\sum_{i=1}^{n}\pi_{\tilde{\wp}_1}(q_i)}{n},$$

$$\overline{\xi_{\tilde{\wp}_2}}=\frac{\sum_{i=1}^{n}\xi_{\tilde{\wp}_2}(q_i)}{n},\ \overline{\eta_{\tilde{\wp}_2}}=\frac{\sum_{i=1}^{n}\eta_{\tilde{\wp}_2}(q_i)}{n},\ \overline{\pi_{\tilde{\wp}_2}}=\frac{\sum_{i=1}^{n}\pi_{\tilde{\wp}_2}(q_i)}{n},\ \text{for } r\geq 1.$$

We observe that, this technique fails a condition of correlation method. By applying the technique for $r=1$ to Example 3.1, we have $\rho_2(\tilde{\wp}_1,\tilde{\wp}_2)=1$ although the q-ROFSs are not equal.

3.1.3. Bashir *et al.*'s techniques

By adjusting the methods in [36, 37], Bashir et al. [57] presented new methods of CCq-ROFSs, which are;

$$\rho_3(\tilde{\wp}_1,\tilde{\wp}_2)=\frac{\sum_{i=1}^{n}\left(\xi^{\,r}_{\tilde{\wp}_1}(q_i)\xi^{\,r}_{\tilde{\wp}_2}(q_i)+\eta^{r}_{\tilde{\wp}_1}(q_i)\eta^{r}_{\tilde{\wp}_2}(q_i)\right)}{\max\left(\sum_{i=1}^{n}\left(\xi^{\,2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)\right),\ \sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_2}(q_i)+\eta^{2r}_{\tilde{\wp}_2}(q_i)\right)\right)}, \tag{3.3}$$

$$\rho_4(\widetilde{\wp}_1,\widetilde{\wp}_2)=\frac{\sum_{i=1}^{n}\left(\xi_{\widetilde{\wp}_1}^{r}(q_i)\xi_{\widetilde{\wp}_2}^{r}(q_i)+\eta_{\widetilde{\wp}_1}^{r}(q_i)\eta_{\widetilde{\wp}_2}^{r}(q_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\xi_{\widetilde{\wp}_1}^{2r}(q_i)+\eta_{\widetilde{\wp}_1}^{2r}(q_i)\right)\sum_{i=1}^{n}\left(\xi_{\widetilde{\wp}_2}^{2r}(q_i)+\eta_{\widetilde{\wp}_2}^{2r}(q_i)\right)}}, \quad (3.4)$$

for $r \geq 1$. It is noticed that, (3.1) equals (3.4) if $r = 1$. We observe that, these techniques omit the hesitation margin and also fail a condition of correlation method. By applying the technique for $r = 4$ to Example 3.1, we have $\rho_3(\widetilde{\wp}_1,\widetilde{\wp}_2)=\rho_4(\widetilde{\wp}_1,\widetilde{\wp}_2)=1$ although the q-ROFSs are not equal.

3.1.4. Li *et al.*'s techniques

In [55], two methods of working out CCq-ROFSs were established, which are reiterated as follows;

$$\rho_5(\widetilde{\wp}_1,\widetilde{\wp}_2)=(1-\lambda)\theta_\xi+\lambda\theta_\eta, \quad (3.5)$$

where

$$\theta_\xi=\frac{\sum_{i=1}^{n}\left(\left(\xi_{\widetilde{\wp}_1}^{r}(q_i)-\overline{\xi_{\widetilde{\wp}_1}}\right)\left(\xi_{\widetilde{\wp}_2}^{r}(q_i)-\overline{\xi_{\widetilde{\wp}_2}}\right)\right)}{\sqrt{\sum_{i=1}^{n}\left(\xi_{\widetilde{\wp}_1}^{r}(q_i)-\overline{\xi_{\widetilde{\wp}_1}}\right)^2\sum_{i=1}^{n}\left(\xi_{\widetilde{\wp}_2}^{r}(q_i)-\overline{\xi_{\widetilde{\wp}_2}}\right)^2}},$$

$$\theta_\eta=\frac{\sum_{i=1}^{n}\left(\left(\eta_{\widetilde{\wp}_1}^{r}(q_i)-\overline{\eta_{\widetilde{\wp}_1}}\right)\left(\eta_{\widetilde{\wp}_2}^{r}(q_i)-\overline{\eta_{\widetilde{\wp}_2}}\right)\right)}{\sqrt{\sum_{i=1}^{n}\left(\eta_{\widetilde{\wp}_1}^{r}(q_i)-\overline{\eta_{\widetilde{\wp}_1}}\right)^2\sum_{i=1}^{n}\left(\eta_{\widetilde{\wp}_2}^{r}(q_i)-\overline{\eta_{\widetilde{\wp}_2}}\right)^2}},$$

for

$$\overline{\xi_{\widetilde{\wp}_1}}=\frac{\sum_{i=1}^{n}\xi_{\widetilde{\wp}_1}(q_i)}{n}, \overline{\eta_{\widetilde{\wp}_1}}=\frac{\sum_{i=1}^{n}\eta_{\widetilde{\wp}_1}(q_i)}{n},$$

$$\overline{\xi_{\widetilde{\wp}_2}}=\frac{\sum_{i=1}^{n}\xi_{\widetilde{\wp}_2}(q_i)}{n}, \overline{\eta_{\widetilde{\wp}_2}}=\frac{\sum_{i=1}^{n}\eta_{\widetilde{\wp}_2}(q_i)}{n},$$

for $r \geq 1$ and $\lambda \in [0,1]$.

Likewise,

$$\rho_6(\widetilde{\wp}_1,\widetilde{\wp}_2)=(1-\lambda)\theta_\xi^*+\lambda\theta_\eta^*, \quad (3.6)$$

where

$$\theta_\xi^*=\frac{\sum_{i=1}^{n}\left(\left(\xi_{\widetilde{\wp}_1}^{r}(q_i)-\overline{\xi_{\widetilde{\wp}_1}}\right)\left(\xi_{\widetilde{\wp}_2}^{r}(q_i)-\overline{\xi_{\widetilde{\wp}_2}}\right)\right)}{max\left(\sum_{i=1}^{n}\left(\xi_{\widetilde{\wp}_1}^{r}(q_i)-\overline{\xi_{\widetilde{\wp}_1}}\right)^2,\ \sum_{i=1}^{n}\left(\xi_{\widetilde{\wp}_2}^{r}(q_i)-\overline{\xi_{\widetilde{\wp}_2}}\right)^2\right)},$$

$$\theta_\eta^*=\frac{\sum_{i=1}^{n}\left(\left(\eta_{\widetilde{\wp}_1}^{r}(q_i)-\overline{\eta_{\widetilde{\wp}_1}}\right)\left(\eta_{\widetilde{\wp}_2}^{r}(q_i)-\overline{\eta_{\widetilde{\wp}_2}}\right)\right)}{max\left(\sum_{i=1}^{n}\left(\eta_{\widetilde{\wp}_1}^{r}(q_i)-\overline{\eta_{\widetilde{\wp}_1}}\right)^2,\ \sum_{i=1}^{n}\left(\eta_{\widetilde{\wp}_2}^{r}(q_i)-\overline{\eta_{\widetilde{\wp}_2}}\right)^2\right)},$$

for

$$\overline{\xi_{\widetilde{\wp}_1}}=\frac{\sum_{i=1}^{n}\xi_{\widetilde{\wp}_1}(q_i)}{n}, \overline{\eta_{\widetilde{\wp}_1}}=\frac{\sum_{i=1}^{n}\eta_{\widetilde{\wp}_1}(q_i)}{n},$$

$$\overline{\xi_{\widetilde{\wp}_2}}=\frac{\sum_{i=1}^{n}\xi_{\widetilde{\wp}_2}(q_i)}{n}, \overline{\eta_{\widetilde{\wp}_2}}=\frac{\sum_{i=1}^{n}\eta_{\widetilde{\wp}_2}(q_i)}{n},$$

wherein $r \geq 1$ and $\lambda \in [0,1]$. We observe that, these techniques omit the hesitation margin and also fail a condition of correlation method. By applying the technique for $r = 1$ to Example 3.1, we have $\rho_5(\widetilde{\wp}_1, \widetilde{\wp}_2) = \rho_6(\widetilde{\wp}_1, \widetilde{\wp}_2) = 1$ although the q-ROFSs are not equal.

4. RESULTS

Having presented the existing techniques of computing of CCq-ROFSs, we introduce some new approaches of calculating CCq-ROFSs which improve the technique in [54, 57] in terms of reliability and accuracy.

4.1. Certain New Techniques of CCq-ROFSs

Let $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ be any two arbitrary q-ROFSs in $Q = \{q_1, q_2, \dots, q_n\}$, then the first new technique of CCq-ROFSs between $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ is as follows:

$$\tilde{\rho}_1(\widetilde{\wp}_1, \widetilde{\wp}_2) = \left. \begin{array}{l} \dfrac{\left(\sum_{i=1}^{n}\left(\xi^{r}_{\widetilde{\wp}_1}(q_i)\xi^{r}_{\widetilde{\wp}_2}(q_i) + \eta^{r}_{\widetilde{\wp}_1}(q_i)\eta^{r}_{\widetilde{\wp}_2}(q_i) + \pi^{r}_{\widetilde{\wp}_1}(q_i)\pi^{r}_{\widetilde{\wp}_2}(q_i)\right)\right)^{\frac{1}{r}}}{\sqrt{\left(\sum_{i=1}^{n}\left(\xi^{2r}_{\widetilde{\wp}_1}(q_i) + \eta^{2r}_{\widetilde{\wp}_1}(q_i) + \pi^{2r}_{\widetilde{\wp}_1}(q_i)\right)\right)^{\frac{1}{r}}\left(\sum_{i=1}^{n}\left(\xi^{2r}_{\widetilde{\wp}_2}(q_i) + \eta^{2r}_{\widetilde{\wp}_2}(q_i) + \pi^{2r}_{\widetilde{\wp}_2}(q_i)\right)\right)^{\frac{1}{r}}}} \\ = \left(\dfrac{\left(\sum_{i=1}^{n}\left(\xi^{r}_{\widetilde{\wp}_1}(q_i)\xi^{r}_{\widetilde{\wp}_2}(q_i) + \eta^{r}_{\widetilde{\wp}_1}(q_i)\eta^{r}_{\widetilde{\wp}_2}(q_i) + \pi^{r}_{\widetilde{\wp}_1}(q_i)\pi^{r}_{\widetilde{\wp}_2}(q_i)\right)\right)^{2}}{\sum_{i=1}^{n}\left(\xi^{2r}_{\widetilde{\wp}_1}(q_i) + \eta^{2r}_{\widetilde{\wp}_1}(q_i) + \pi^{2r}_{\widetilde{\wp}_1}(q_i)\right)\sum_{i=1}^{n}\left(\xi^{2r}_{\widetilde{\wp}_2}(q_i) + \eta^{2r}_{\widetilde{\wp}_2}(q_i) + \pi^{2r}_{\widetilde{\wp}_2}(q_i)\right)}\right)^{\frac{1}{2r}} \end{array} \right\} \quad (4.1)$$

for $r \geq 1$.

The second new technique of CCq-ROFSs between $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ is given as follows:

$$\tilde{\rho}_2(\widetilde{\wp}_1, \widetilde{\wp}_2) = \frac{\sum_{i=1}^{n}\left(\xi^{r}_{\widetilde{\wp}_1}(q_i)\xi^{r}_{\widetilde{\wp}_2}(q_i) + \eta^{r}_{\widetilde{\wp}_1}(q_i)\eta^{r}_{\widetilde{\wp}_2}(q_i) + \pi^{r}_{\widetilde{\wp}_1}(q_i)\pi^{r}_{\widetilde{\wp}_2}(q_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\xi^{2r}_{\widetilde{\wp}_1}(q_i) + \eta^{2r}_{\widetilde{\wp}_1}(q_i) + \pi^{2r}_{\widetilde{\wp}_1}(q_i)\right)\sum_{i=1}^{n}\left(\xi^{2r}_{\widetilde{\wp}_2}(q_i) + \eta^{2r}_{\widetilde{\wp}_2}(q_i) + \pi^{2r}_{\widetilde{\wp}_2}(q_i)\right)}}, \quad (4.2)$$

for $r \geq 1$. We observe that (4.1) is the extension of the technique in [54] by the addition of the hesitation margin of q-ROFSs to improve reliability by avoiding information loss. Also, (4.2) is the generalization of the approach in [37] in terms of q-ROFSs.

Remark 4.1. The correlation coefficients $\tilde{\rho}_1(\widetilde{\wp}_1, \widetilde{\wp}_2)$ and $\tilde{\rho}_2(\widetilde{\wp}_1, \widetilde{\wp}_2)$ for q-ROFSs $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ in Q are equal if $r = 1$.

To ascertain the validity of our new techniques of CCq-ROFSs numerically, we present examples of two close related q-ROFSs to test how suitable the new techniques will estimate their correlation coefficients.

Example 4.1. Let $\widetilde{\wp}_1$ and $\widetilde{\wp}_2$ be q-ROFSs in $Q = \{q_1, q_2\}$ defined by $\widetilde{\wp}_1 = \{\langle q_1, 0.4, 0.3\rangle, \langle q_2, 0.3, 0.2\rangle\}$ and $\widetilde{\wp}_2 = \{\langle q_1, 0.3, 0.2\rangle, \langle q_2, 0.2, 0.1\rangle\}$. By using the novel techniques, i.e., (4.1) and (4.2) for $r = 1$, we get $\tilde{\rho}_1(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.9974$ and $\tilde{\rho}_2(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.9947$. For $r = 2$, we have $\tilde{\rho}_1(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.9997$ and $\tilde{\rho}_2(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.9992$, and for $r = 3$, we have $\tilde{\rho}_1(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.99998$ and $\tilde{\rho}_2(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.9999$. Although these q-ROFSs are similar, the new techniques are able to give distinct results that satisfy the conditions of correlation coefficient.

By applying the new techniques, i.e., (4.1) and (4.2) for $r = 1$ to Example 3.1, we $\tilde{\rho}_1(\widetilde{\wp}_1, \widetilde{\wp}_2) = \tilde{\rho}_2(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.9183$, in agreement with Remark 4.1. For $r = 2$, we have $\tilde{\rho}_1(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.9911$ and $\tilde{\rho}_2(\widetilde{\wp}_1, \widetilde{\wp}_2) = 0.9822$. These results show that the new techniques are better than the techniques in [54, 55, 56, 57].

Now, we presents some properties of the new techniques as follow:

Proposition 4.1. *Suppose $\tilde{\wp}_1$ and $\tilde{\wp}_2$ are q-ROFSs in Q. Then*

i. $\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2) = \tilde{\rho}_1(\tilde{\wp}_2,\tilde{\wp}_1)$,

ii. $\tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2) = \tilde{\rho}_2(\tilde{\wp}_2,\tilde{\wp}_1)$.

Proof. By using the given hypothesis, we have

$$\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2) = \left(\frac{\left(\sum_{i=1}^{n}\left(\xi^{r}_{\tilde{\wp}_1}(q_i)\xi^{r}_{\tilde{\wp}_2}(q_i)+\eta^{r}_{\tilde{\wp}_1}(q_i)\eta^{r}_{\tilde{\wp}_2}(q_i)+\pi^{r}_{\tilde{\wp}_1}(q_i)\pi^{r}_{\tilde{\wp}_2}(q_i)\right)\right)^2}{\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_2}(q_i)+\eta^{2r}_{\tilde{\wp}_2}(q_i)+\pi^{2r}_{\tilde{\wp}_2}(q_i)\right)}\right)^{\frac{1}{2r}}$$

$$= \left(\frac{\left(\sum_{i=1}^{n}\left(\xi^{r}_{\tilde{\wp}_2}(q_i)\xi^{r}_{\tilde{\wp}_1}(q_i)+\eta^{r}_{\tilde{\wp}_2}(q_i)\eta^{r}_{\tilde{\wp}_1}(q_i)+\pi^{r}_{\tilde{\wp}_2}(q_i)\pi^{r}_{\tilde{\wp}_1}(q_i)\right)\right)^2}{\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_2}(q_i)+\eta^{2r}_{\tilde{\wp}_2}(q_i)+\pi^{2r}_{\tilde{\wp}_2}(q_i)\right)\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)}\right)^{\frac{1}{2r}}$$

$$=\tilde{\rho}_1(\tilde{\wp}_2,\tilde{\wp}_1).$$

The verification of (ii) is related. Hence, the new techniques of CCq-ROFSs are symmetric.

Proposition 4.2. *Suppose $\tilde{\wp}_1$ and $\tilde{\wp}_2$ are q-ROFSs in Q. Then $\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2) = 1 \Leftrightarrow \tilde{\wp}_1 = \tilde{\wp}_2$.* Similarly, $\tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2) = 1 \Leftrightarrow \tilde{\wp}_1 = \tilde{\wp}_2$.

Proof. Suppose $\tilde{\wp}_1 = \tilde{\wp}_2$, then we have

$$\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2) = \left(\frac{\left(\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)\right)^2}{\left(\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)\right)^2}\right)^{\frac{1}{2r}}$$

$$= \frac{\left(\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)\right)^{\frac{1}{r}}}{\left(\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)\right)^{\frac{1}{q}}},$$

$$= 1.$$

Conversely, if $\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2) = 1$, then it is clear that $\tilde{\wp}_1 = \tilde{\wp}_2$.
Now, we prove that $\tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2) = 1 \Leftrightarrow \tilde{\wp}_1 = \tilde{\wp}_2$. Assume that $\tilde{\wp}_1 = \tilde{\wp}_2$, then we have

$$\tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2) = \frac{\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)\sum_{i=1}^{n}\left(\xi^{2r}_{\tilde{\wp}_1}(q_i)+\eta^{2r}_{\tilde{\wp}_1}(q_i)+\pi^{2r}_{\tilde{\wp}_1}(q_i)\right)}}$$

$$= 1.$$

Conversely, suppose $\tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2) = 1$, then it is straightforward that $\tilde{\wp}_1$ and $\tilde{\wp}_2$ are equal.

Theorem 4.1. *Suppose $\tilde{\wp}_1$ and $\tilde{\wp}_2$ are q-ROFSs in X. Then, $0 \leq \tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2) \leq 1$ and $0 \leq \tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2) \leq 1$.*

Proof. Certainly, $\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2) \geq 0$ and $\tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2) \geq 0$. Now, we prove that $\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2) \leq 1$ and $\tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2) \leq 1$. Let us assume that

$$\sum_{i=1}^{n}\xi_{\tilde{\wp}_1}^{r}(q_i)=\mathrm{A},\ \sum_{i=1}^{n}\xi_{\tilde{\wp}_2}^{r}(q_i)=\mathrm{B},$$

$$\sum_{i=1}^{n}\eta_{\tilde{\wp}_1}^{r}(q_i)=\Gamma,\ \sum_{i=1}^{n}\eta_{\tilde{\wp}_2}^{r}(q_i)=\mathrm{E},$$

$$\sum_{i=1}^{n}\pi_{\tilde{\wp}_1}^{r}(q_i)=\mathrm{Z},\ \sum_{i=1}^{n}\pi_{\tilde{\wp}_2}^{r}(q_i)=\mathrm{H}.$$

Firstly, we show that $\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2)\leq 1$ as follows:

$$\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2)=\frac{\left(\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{r}(q_i)\xi_{\tilde{\wp}_2}^{r}(q_i)+\eta_{\tilde{\wp}_1}^{r}(q_i)\eta_{\tilde{\wp}_2}^{r}(q_i)+\pi_{\tilde{\wp}_1}^{r}(q_i)\pi_{\tilde{\wp}_2}^{r}(q_i)\right)\right)^{\frac{1}{r}}}{\left(\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{2r}(q_i)+\eta_{\tilde{\wp}_1}^{2r}(q_i)+\pi_{\tilde{\wp}_1}^{2r}(q_i)\right)\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_2}^{2r}(q_i)+\eta_{\tilde{\wp}_2}^{2r}(q_i)+\pi_{\tilde{\wp}_2}^{2r}(q_i)\right)\right)^{\frac{1}{2r}}}$$

$$=\left(\frac{\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{r}(q_i)\xi_{\tilde{\wp}_2}^{r}(q_i)+\eta_{\tilde{\wp}_1}^{r}(q_i)\eta_{\tilde{\wp}_2}^{r}(q_i)+\pi_{\tilde{\wp}_1}^{r}(q_i)\pi_{\tilde{\wp}_2}^{r}(q_i)\right)}{\left(\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{2r}(q_i)+\eta_{\tilde{\wp}_1}^{2r}(q_i)+\pi_{\tilde{\wp}_1}^{2r}(q_i)\right)\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_2}^{2r}(q_i)+\eta_{\tilde{\wp}_2}^{2r}(q_i)+\pi_{\tilde{\wp}_2}^{2r}(q_i)\right)\right)^{\frac{1}{2}}}\right)^{\frac{1}{r}}$$

$$=\left(\frac{\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{r}(q_i)\xi_{\tilde{\wp}_2}^{r}(q_i)\right)+\sum_{i=1}^{n}\left(\eta_{\tilde{\wp}_1}^{r}(q_i)\eta_{\tilde{\wp}_2}^{r}(q_i)\right)+\sum_{i=1}^{n}\left(\pi_{\tilde{\wp}_1}^{r}(q_i)\pi_{\tilde{\wp}_2}^{r}(q_i)\right)}{\left(\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{2r}(q_i)+\eta_{\tilde{\wp}_1}^{2r}(q_i)+\pi_{\tilde{\wp}_1}^{2r}(q_i)\right)\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_2}^{2r}(q_i)+\eta_{\tilde{\wp}_2}^{2r}(q_i)+\pi_{\tilde{\wp}_2}^{2r}(q_i)\right)\right)^{\frac{1}{2}}}\right)^{\frac{1}{r}}$$

$$=\left(\frac{\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH}}{\left((\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)\right)^{\frac{1}{2}}}\right)^{\frac{1}{r}}.$$

By raising bothsides to the power of $2r$, we get

$$\tilde{\rho}_1^{2r}(\tilde{\wp}_1,\tilde{\wp}_2)=\frac{(\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH})^2}{(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)},$$

and by substracting 1 from bothsides, we have

$$\tilde{\rho}_1^{2r}(\tilde{\wp}_1,\tilde{\wp}_2)-1=\frac{(\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH})^2}{(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)}-1$$

$$=\frac{(\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH})^2-(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)}{(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)}$$

$$=-\frac{\left((\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)-(\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH})^2\right)}{(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)}$$

$$\leq 0.$$

Thus, $\tilde{\rho}_1^{2r}(\tilde{\wp}_1,\tilde{\wp}_2)\leq 1$, and hence $\tilde{\rho}_1(\tilde{\wp}_1,\tilde{\wp}_2)\leq 1$ as desired.

Similarly, we have

$$\tilde{\rho}_2(\tilde{\wp}_1,\tilde{\wp}_2)=\frac{\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{2r}(q_i)+\eta_{\tilde{\wp}_1}^{2r}(q_i)+\pi_{\tilde{\wp}_1}^{2r}(q_i)\right)}{\left(\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{2r}(q_i)+\eta_{\tilde{\wp}_1}^{2r}(q_i)+\pi_{\tilde{\wp}_1}^{2r}(q_i)\right)\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{2r}(q_i)+\eta_{\tilde{\wp}_1}^{2r}(q_i)+\pi_{\tilde{\wp}_1}^{2r}(q_i)\right)\right)^{\frac{1}{2}}}$$

$$= \frac{\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{r}(q_i)\xi_{\tilde{\wp}_2}^{r}(q_i) + \eta_{\tilde{\wp}_1}^{r}(q_i)\eta_{\tilde{\wp}_2}^{r}(q_i) + \pi_{\tilde{\wp}_1}^{r}(q_i)\pi_{\tilde{\wp}_2}^{r}(q_i)\right)}{\left(\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{2r}(q_i) + \eta_{\tilde{\wp}_1}^{2r}(q_i) + \pi_{\tilde{\wp}_1}^{2r}(q_i)\right)\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_2}^{2r}(q_i) + \eta_{\tilde{\wp}_2}^{2r}(q_i) + \pi_{\tilde{\wp}_2}^{2r}(q_i)\right)\right)^{\frac{1}{2}}}$$

$$= \frac{\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{r}(q_i)\xi_{\tilde{\wp}_2}^{r}(q_i)\right)+\sum_{i=1}^{n}\left(\eta_{\tilde{\wp}_1}^{r}(q_i)\eta_{\tilde{\wp}_2}^{r}(q_i)\right)+\sum_{i=1}^{n}\left(\pi_{\tilde{\wp}_1}^{r}(q_i)\pi_{\tilde{\wp}_2}^{r}(q_i)\right)}{\left(\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_1}^{2r}(q_i)+ \eta_{\tilde{\wp}_1}^{2r}(q_i)+\pi_{\tilde{\wp}_1}^{2r}(q_i)\right)\sum_{i=1}^{n}\left(\xi_{\tilde{\wp}_2}^{2r}(q_i)+\eta_{\tilde{\wp}_2}^{2r}(q_i)+\pi_{\tilde{\wp}_2}^{2r}(q_i)\right)\right)^{\frac{1}{2}}}$$

$$= \frac{\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH}}{\left((\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)\right)^{\frac{1}{2}}}.$$

Hence,

$$\tilde{\rho}_2^2(\tilde{\wp}_1, \tilde{\wp}_2) = \frac{(\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH})^2}{(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)},$$

and thus we get

$$\begin{aligned}\tilde{\rho}_2^2(\tilde{\wp}_1, \tilde{\wp}_2) - 1 &= \frac{(\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH})^2}{(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)} - 1\\ &= \frac{(\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH})^2-(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)}{(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)}\\ &= -\frac{\left((\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)-(\mathrm{AB}+\Gamma\mathrm{E}+\mathrm{ZH})^2\right)}{(\mathrm{A}^2+\Gamma^2+\mathrm{Z}^2)(\mathrm{B}^2+\mathrm{E}^2+\mathrm{H}^2)}\\ &\leq 0.\end{aligned}$$

Therefore, $\tilde{\rho}_2^2(\tilde{\wp}_1, \tilde{\wp}_2) \leq 1$, and thus $\tilde{\rho}_2(\tilde{\wp}_1, \tilde{\wp}_2) \leq 1$ as desired.

4.2. Application of the New Methods of CCq-ROFSs

Here, we address the importance of the new CCq-ROFSs methods in the employment process and disease diagnosis via the MCDM approach and the recognition principle, respectively.

4.2.1. Medical evaluation

In medical diagnosis, which relies on pattern recognition principle to identify the ailment patients, the idea of correlation coefficient is crucial. A patient's medical history, physical examinations, and laboratory tests are all part of the intricate process of making a diagnosis. For a medical diagnosis, a more thorough understanding of a patient's conditions is crucial. The diagnostic process identifies the illness that corresponds with a patient's signs and symptoms, or what is causing the patient's illness. Sick people describe their conditions with symptoms (such as a fever, headache, stomachache, sore joints, etc.). Many times, certain symptoms are not specific at all. For example, patients with lung disease and patients with heart disease may present with the same symptom, namely chest pain. Fuzzy logic is a suitable concept in healthcare because of these complexities. However, q-rung orthopair fuzzy logic is the way to go in order to obtain a trustworthy diagnosis, particularly by using the CCq-ROFs approach.

Experimental illustration

Here, we present a medical diagnosis via knowledge-based data set from common medical information. Given that there is a q-rung orthopair fuzzy data (q-ROFD) of selected ailments such as typhoid fever, malaria fever, stomach problem, viral fever, and chest problem as presented in Table 1. From basic medical information with regards to some related symptoms represented by a set, $S = \{S_1, S_2, S_3, S_4, S_5\}$, where the elements of S represent temperature, headache, stomachache, cough, and chest ache, respectively.

Suppose five sick fellows approach a medical facility for medical diagnosis to decide their medical status, and the sick fellows are manifesting some indications of high temperature, stomachache, cough, headache, and chestache as seen in S. After a careful medical examination, the medical information of the patients are captured in q-ROFD, presented in Table 2. For easiness, we present the patients as a set represented by $P_i = \{P_1, P_2, P_3, P_4\}$, and the ailments as a set, $\breve{D}_j = \{\breve{D}_1, \breve{D}_2, \breve{D}_3, \breve{D}_4, \breve{D}_5\}$, where $\breve{D}_1$ is malaria fever, $\breve{D}_2$ is viral fever, $\breve{D}_3$ is typhoid fever, $\breve{D}_4$ is stomach problem, and $\breve{D}_5$ is chest problem, respectively.

Table 1. *q-ROF Data of Some Ailments*

Symptoms	$\breve{D}_1$	$\breve{D}_2$	$\breve{D}_3$	$\breve{D}_4$	$\breve{D}_5$
S_1	⟨0.7,0.2⟩	⟨0.6,0.0⟩	⟨0.5,0.3⟩	⟨0.1,0.8⟩	⟨0.1,0.7⟩
S_2	⟨0.7,0.1⟩	⟨0.5,0.3⟩	⟨0.6,0.2⟩	⟨0.3,0.5⟩	⟨0.1,0.8⟩
S_3	⟨0.1,0.8⟩	⟨0.2,0.7⟩	⟨0.2,0.7⟩	⟨0.9,0.0⟩	⟨0.2,0.7⟩
S_4	⟨0.8,0.1⟩	⟨0.5,0.3⟩	⟨0.3,0.6⟩	⟨0.1,0.7⟩	⟨0.6,0.3⟩
S_5	⟨0.1,0.8⟩	⟨0.2,0.7⟩	⟨0.1,0.8⟩	⟨0.2,0.7⟩	⟨0.9,0.0⟩

Table 2. *q-ROF Patients' Medical Information*

Patients	S_1	S_2	S_3	S_4	S_5
P_1	⟨0.8,0.1⟩	⟨0.7,0.2⟩	⟨0.1,0.8⟩	⟨0.7,0.2⟩	⟨0.2,0.7⟩
P_2	⟨0.0,0.9⟩	⟨0.5,0.4⟩	⟨0.7,0.1⟩	⟨0.2,0.7⟩	⟨0.2,0.8⟩
P_3	⟨0.9,0.0⟩	⟨0.8,0.1⟩	⟨0.0,0.7⟩	⟨0.2,0.6⟩	⟨0.1,0.5⟩
P_4	⟨0.6,0.1⟩	⟨0.5,0.3⟩	⟨0.4,0.5⟩	⟨0.7,0.1⟩	⟨0.4,0.5⟩

To obtain the diagnosis of the patients, the new techniques are deployed to calculate the correlation coefficient between the sick folks and the ailments using the steps below:

1. Find the hesitation margin of $\breve{D}_j$ and P_i for each $r/q = 1,2,\dots,10$.
2. Compute the correlation of $(P_i, \breve{D}_j)$, for $i = 1,2,3,4$ and $j = 1,2,3,4,5$ using the innovative techniques of CCq-ROFSs for each $r/q = 1,2,\dots,10$.
3. For each $r/q = 1,2,\dots,10$, the greatest correlation coefficient of $(P_i, \breve{D}_j)$ determines the sick folks' medical status.

Diagnostic process via CCq-ROFSs

After following the given steps, the results of the correlation coefficient concerning each sick folks and each ailments using our first approach i.e., (4.1), are presented in Tables 3-6 and Figure 2.

Table 3. *Correlation Coefficient between P_1 and the Ailments*

Scales of r	$(P_1, \breve{D}_1)$	$(P_1, \breve{D}_2)$	$(P_1, \breve{D}_3)$	$(P_1, \breve{D}_4)$	$(P_1, \breve{D}_5)$
$r = 1$	0.9869	0.9529	0.9073	0.4660	0.5638
$r = 2$	0.9885	0.9593	0.9330	0.7109	0.7487
$r = 3$	0.9893	0.9779	0.9657	0.8822	0.8850
$r = 4$	0.9925	0.9892	0.9831	0.9474	0.9439
$r = 5$	0.9952	0.9946	0.9914	0.9737	0.9699
$r = 6$	0.9971	0.9972	0.9954	0.9854	0.9825
$r = 7$	0.9982	0.9985	0.9974	0.9913	0.9892
$r = 8$	0.9989	0.9992	0.9985	0.9945	0.9930
$r = 9$	0.9993	0.9995	0.9992	0.9964	0.9953
$r = 10$	0.9996	0.9997	0.9995	0.9975	0.9968

Table 4. *Correlation Coefficient between P_2 and the Ailments*

Scales of $\boldsymbol{r}$	$(\boldsymbol{P_2},\breve{\boldsymbol{D}}_1)$	$(\boldsymbol{P_2},\breve{\boldsymbol{D}}_2)$	$(\boldsymbol{P_2},\breve{\boldsymbol{D}}_3)$	$(\boldsymbol{P_2},\breve{\boldsymbol{D}}_4)$	$(\boldsymbol{P_2},\breve{\boldsymbol{D}}_5)$
$\boldsymbol{r=1}$	0.5512	0.5976	0.7589	0.9697	0.5914
$\boldsymbol{r=2}$	0.7493	0.8134	0.8625	0.9675	0.7518
$\boldsymbol{r=3}$	0.8863	0.9244	0.9330	0.9726	0.8784
$\boldsymbol{r=4}$	0.9450	0.9634	0.9646	0.9785	0.9361
$\boldsymbol{r=5}$	0.9707	0.9797	0.9797	0.9836	0.9632
$\boldsymbol{r=6}$	0.9831	0.9877	0.9875	0.9876	0.9772
$\boldsymbol{r=7}$	0.9895	0.9921	0.9920	0.9906	0.9851
$\boldsymbol{r=8}$	0.9932	0.9947	0.9946	0.9930	0.9898
$\boldsymbol{r=9}$	0.9955	0.9963	0.9963	0.9947	0.9929
$\boldsymbol{r=10}$	0.9969	0.9974	0.9974	0.9960	0.9949

Table 5. *Correlation Coefficient between P_3 and the Ailments*

Scales of $\boldsymbol{r}$	$(\boldsymbol{P_3},\breve{\boldsymbol{D}}_1)$	$(\boldsymbol{P_3},\breve{\boldsymbol{D}}_2)$	$(\boldsymbol{P_3},\breve{\boldsymbol{D}}_3)$	$(\boldsymbol{P_3},\breve{\boldsymbol{D}}_4)$	$(\boldsymbol{P_3},\breve{\boldsymbol{D}}_5)$
$\boldsymbol{r=1}$	0.8397	0.8685	0.8919	0.4539	0.4209
$\boldsymbol{r=2}$	0.8898	0.9107	0.9015	0.7156	0.6762
$\boldsymbol{r=3}$	0.9325	0.9464	0.9318	0.8631	0.8477
$\boldsymbol{r=4}$	0.9595	0.9682	0.9579	0.9283	0.9209
$\boldsymbol{r=5}$	0.9751	0.9805	0.9743	0.9591	0.9546
$\boldsymbol{r=6}$	0.9842	0.9876	0.9840	0.9750	0.9721
$\boldsymbol{r=7}$	0.9897	0.9919	0.9898	0.9839	0.9819
$\boldsymbol{r=8}$	0.9931	0.9945	0.9933	0.9892	0.9878
$\boldsymbol{r=9}$	0.9953	0.9962	0.9955	0.9925	0.9916
$\boldsymbol{r=10}$	0.9968	0.9973	0.9969	0.9947	0.9941

Table 6. *Correlation Coefficient between P_4 and the Ailments*

Scales of $\boldsymbol{r}$	$(\boldsymbol{P_4},\breve{\boldsymbol{D}}_1)$	$(\boldsymbol{P_4},\breve{\boldsymbol{D}}_2)$	$(\boldsymbol{P_4},\breve{\boldsymbol{D}}_3)$	$(\boldsymbol{P_4},\breve{\boldsymbol{D}}_4)$	$(\boldsymbol{P_4},\breve{\boldsymbol{D}}_5)$
$\boldsymbol{r=1}$	0.9131	0.9437	0.8396	0.5871	0.6874
$\boldsymbol{r=2}$	0.9330	0.9727	0.9312	0.7991	0.8276
$\boldsymbol{r=3}$	0.9614	0.9876	0.9722	0.9144	0.9170
$\boldsymbol{r=4}$	0.9809	0.9945	0.9875	0.9579	0.9574
$\boldsymbol{r=5}$	0.9906	0.9975	0.9940	0.9769	0.9764
$\boldsymbol{r=6}$	0.9952	0.9989	0.9970	0.9863	0.9860
$\boldsymbol{r=7}$	0.9975	0.9995	0.9985	0.9914	0.9912
$\boldsymbol{r=8}$	0.9986	0.9998	0.9992	0.9943	0.9943
$\boldsymbol{r=9}$	0.9992	0.9999	0.9996	0.9962	0.9961
$\boldsymbol{r=10}$	0.9996	≈ 1	0.9998	0.9973	0.9973

For $r=10$, we used ≈ 1 because we approximated all the results to four decimal places.

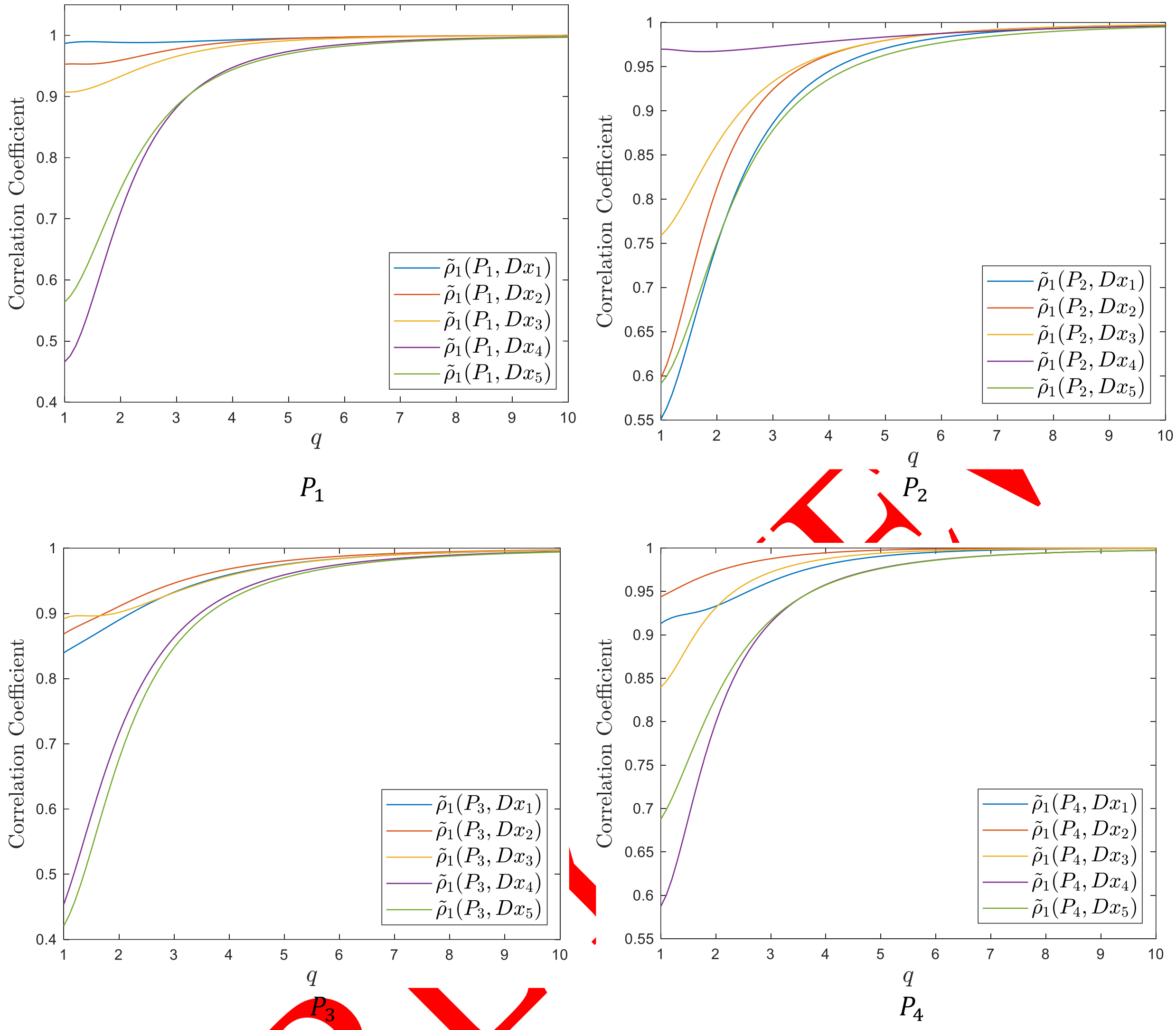

Figure 2. *Patient's Correlation Coefficient with Ailments using our First Technique* $\tilde{\rho}_1$

From the results gotten from (4.1), we deduce the following medical statements using the greatest correlation coefficient between the sick fellows and the ailments, which are:

i. Patient P_1 is sicked of malaria fever for $r = 1,2,\dots,5$, but for $r = 6,7,\dots,10$, we observe that P_1 is suffering from viral fever. However, since a q-ROFS is more equipped to curb uncertainties as r increases, it is most likely that P_1 is suffering from viral fever. To achieve an effective therapy, it will be beneficial to also treat patient P_1 for malaria fever.

ii. Patient P_2 is sicked of stomach problem for $r = 1,2,\dots,5$, but for $r = 6,7,\dots,10$, P_2 is suffering from viral fever. For the same reason in (i), patient P_2 is suffering from viral fever. Notwithstanding, to achieve an effective therapy, it will be beneficial to also treat patient P_2 for stomach problem.

iii. Although for $r = 1$, patient P_3 is suffering from typhoid fever, the patient have to be treated for viral fever because the values of the correlation coefficient between the sick folk and viral fever is the maximum for $r = 2,3,\dots,10$. Besides, q-ROFS is very weak for $r = 1$.

iv. Patient P_4 is suffering from viral fever as the correlation coefficient values between the sick folk and viral fever is the maximum for $r = 1,2,\dots,10$.

Similarly, the correlation coefficient values concerning each sick folks and each illnesses using our second approach i.e., (4.2), are presented in Tables 7-10 and Figure 3.

Table 7. *Correlation Coefficient between P_1 and the Ailments*

Scales of r	$(P_1,\breve{D}_1)$	$(P_1,\breve{D}_2)$	$(P_1,\breve{D}_3)$	$(P_1,\breve{D}_4)$	$(P_1,\breve{D}_5)$
$r=1$	0.9869	0.9529	0.9073	0.4660	0.5638
$r=2$	0.9772	0.9203	0.8705	0.5053	0.5605
$r=3$	0.9684	0.9353	0.9005	0.6865	0.6932
$r=4$	0.9705	0.9576	0.9341	0.8058	0.7938
$r=5$	0.9765	0.9734	0.9575	0.8750	0.8581
$r=6$	0.9825	0.9833	0.9726	0.9157	0.8994
$r=7$	0.9875	0.9894	0.9822	0.9407	0.9267
$r=8$	0.9912	0.9932	0.9884	0.9568	0.9455
$r=9$	0.9939	0.9956	0.9924	0.9676	0.9588
$r=10$	0.9958	0.9972	0.9950	0.9753	0.9686

Table 8. *Correlation Coefficient between P_2 and the Ailments*

Scales of r	$(P_2,\breve{D}_1)$	$(P_2,\breve{D}_2)$	$(P_2,\breve{D}_3)$	$(P_2,\breve{D}_4)$	$(P_2,\breve{D}_5)$
$r=1$	0.5512	0.5976	0.7589	0.9697	0.5914
$r=2$	0.5615	0.6616	0.7438	0.9360	0.5652
$r=3$	0.6962	0.7899	0.8121	0.9201	0.6778
$r=4$	0.7976	0.8616	0.8657	0.9168	0.7680
$r=5$	0.8619	0.9027	0.9024	0.9205	0.8291
$r=6$	0.9025	0.9284	0.9275	0.9277	0.8707
$r=7$	0.9290	0.9459	0.9450	0.9363	0.9000
$r=8$	0.9471	0.9583	0.9577	0.9450	0.9215
$r=9$	0.9599	0.9675	0.9670	0.9532	0.9377
$r=10$	0.9692	0.9744	0.9741	0.9606	0.9502

Table 9. *Correlation Coefficient between P_3 and the Ailments*

Scales of r	$(P_3,\breve{D}_1)$	$(P_3,\breve{D}_2)$	$(P_3,\breve{D}_3)$	$(P_3,\breve{D}_4)$	$(P_3,\breve{D}_5)$
$r=1$	0.8397	0.8685	0.8919	0.4539	0.4209
$r=2$	0.7918	0.8293	0.8127	0.5121	0.4573
$r=3$	0.8108	0.8476	0.8090	0.6430	0.6092
$r=4$	0.8474	0.8787	0.8421	0.7427	0.7192
$r=5$	0.8815	0.9063	0.8780	0.8117	0.7929
$r=6$	0.9090	0.9280	0.9077	0.8591	0.8437
$r=7$	0.9302	0.9445	0.9306	0.8925	0.8800
$r=8$	0.9464	0.9570	0.9476	0.9167	0.9068
$r=9$	0.9587	0.9665	0.9602	0.9346	0.9270
$r=10$	0.9680	0.9737	0.9695	0.9482	0.9424

Table 10. *Correlation Coefficient between P_4 and the Ailments*

Scales of r	$(P_4,\breve{D}_1)$	$(P_4,\breve{D}_2)$	$(P_4,\breve{D}_3)$	$(P_4,\breve{D}_4)$	$(P_4,\breve{D}_5)$
$r=1$	0.9131	0.9437	0.8396	0.5871	0.6874
$r=2$	0.8705	0.9462	0.8672	0.6385	0.6848
$r=3$	0.8886	0.9634	0.9189	0.7645	0.7711
$r=4$	0.9256	0.9780	0.9510	0.8419	0.8402
$r=5$	0.9537	0.9877	0.9706	0.8896	0.8873
$r=6$	0.9715	0.9934	0.9823	0.9204	0.9188
$r=7$	0.9824	0.9966	0.9894	0.9411	0.9402
$r=8$	0.9891	0.9983	0.9935	0.9556	0.9551
$r=9$	0.9931	0.9991	0.9961	0.9659	0.9657
$r=10$	0.9957	0.9996	0.9976	0.9736	0.9734

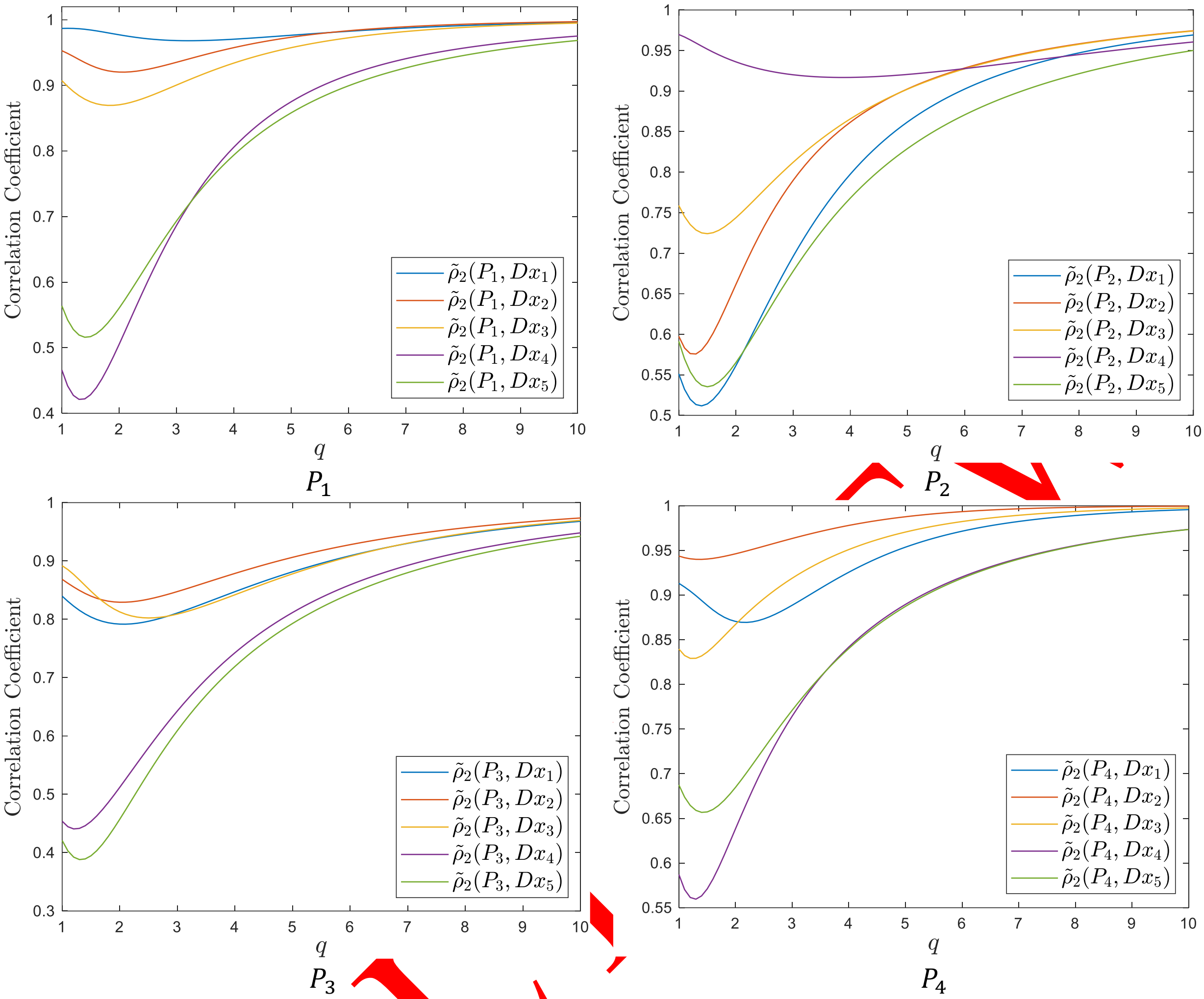

***Figure 3.** Patient's Correlation Coefficient with Ailments using our Second Technique* $\tilde{\rho}_2$

The diagnosis from Tables 7-10 and Figure 3 are:

i. Patient P_1 is sicked of malaria fever for $r = 1,2,\dots,5$. However, for $r = 6,7,\dots,10$, the same is suffering from viral fever. But because a q-ROFS is more fortified to restrain uncertainties as r increases, it is most likely that P_1 should be treated for viral fever. To achieve an effective therapy, it is advisable to likewise treat patient P_1 for malaria fever.

ii. For $r = 1,2,\dots,5$, patient P_2 seems to be suffering from stomach problem, but for $r = 6,7,\dots,10$, P_2 is diagnosed with viral fever. For the same reason in (i), it is most likely patient P_2 is suffering from viral fever. Notwithstanding, to achieve an effective therapy, it will be beneficial to also treat patient P_2 for stomach problem.

iii. Although for $r = 1$, patient P_3 is suffering from typhoid fever, but P_3 has to be treated of viral fever because the values of the correlation coefficient between P_3 and viral fever is the maximum for $r = 2,3,\dots,10$. Besides, q-ROFS is very unreliable for $r = 1$.

iv. For $r = 1,2,\dots,10$, patient P_4 is sicked of viral fever because the correlation coefficient values between the sick folk and viral fever is the maximum.

To foreclose this section, it is needful to state that the diagnoses from both of the novel techniques of CCq-ROFS are equal. However, our first technique produces better correlation coefficient by comparison.

4.2.2. Employment process

In this section, we discuss employment process based on CCq-ROFSs using MCDM approach. Suppose a firm desires to employ a worker, and many qualified applicants applied for the job. The challenge is how to appoint a suitable worker to fill the vacancy where there are more than enough applicants for the positions. The concept of q-ROFSs provides a reliable framework to handle such situation because of its capability in handling hesitations.

Supposing there are some candidates, $C_i(i = 1,2, \ldots n)$ for a job position, and some qualifications $R_j(j = 1,,2, \ldots, n)$ are required for the position. We form the decision matrices $A_k = \{R_j(C_i)\}_{a\times b}$, where $R_j(C_i) = \langle C_{ij}\rangle$ represent q-rung orthopair fuzzy scores for the applicants $C_i(i = 1,2, \ldots n)$ with regards to the required qualifications $R_j(j = 1,,2, \ldots, n)$ for the position.

MCDM Algorithm

Step 1. Devise the q-rung orthopair fuzzy decision matrix (qROFDM) $A_k = \{R_j(C_i)\}_{a\times b}$ given by m-man panel.
Step 2. Compute the mean values of the scores from the m-man panel to obtain a single q-rung fuzzy decision matrix using

$$\bar{A}_k = \frac{\sum_{l=1}^{m} A_k}{m}. \qquad (4.4)$$

Step 3. Obtain the normalized qROFDM, $N = \langle C^*{}_{ij}\rangle_{a\times b}$ for $\bar{A}_k$, where $\langle C^*{}_{ij}\rangle$ are q-rung orthopair fuzzy scores, and N is defined by:

$$\langle C^*{}_{ij}\rangle_{a\times b} = \begin{cases} \langle C_{ij}\rangle \text{ for benefit criterion of } N \\ \langle C_{ij}\rangle \text{ for cost criterion of } N \end{cases} \qquad (4.5)$$

Step 4. Calculate PIS (positive ideal solution), $C^+ = \{C_1^+, C_2^+, \ldots, C_n^+\}$ and NIS (negative ideal solution), $C^- = \{C_1^-, C_2^-, \ldots, C_n^-\}$ by

$$C^+ = \begin{cases} \langle \max\{\xi_{C_i}(R_j)\}, \min\{\eta_{C_i}(R_j)\}\rangle \text{ if } R_j \text{ is the benefit criterion} \\ \langle \min\{\xi_{C_i}(R_j)\}, \max\{\eta_{C_i}(R_j)\}\rangle \text{ if } R_j \text{ is the cost criterion,} \end{cases} \qquad (4.6)$$

and

$$C^- = \begin{cases} \langle \min\{\xi_{C_i}(R_j)\}, \max\{\eta_{C_i}(R_j)\}\rangle \text{ if } R_j \text{ is the benefit criterion} \\ \langle \max\{\xi_{C_i}(R_j)\}, \min\{\eta_{C_i}(R_j)\}\rangle \text{ if } R_j \text{ is the cost criterion.} \end{cases} \qquad (4.7)$$

Step 5. Compute $\rho(C_i,\ C^+)$ and $\rho(C_i,\ C^-)$, respectively using the methods of CCq-ROFSs.

Step 6. Obtain the closeness coefficients for each applicants C_j via:

$$f(C_i) = \frac{\rho(C_i, C^+)}{\rho(C_i, C^+) + \rho(C_i, C^-)}. \qquad (4.8)$$

Step 7. Determine the best ranking of the candidates in declining order of the closeness coefficients.

Step 8. Decide the most suitable candidate for the employment based on the maximum closeness coefficient.

Suppose $\rho(C_i,\ C^+), \rho(C_i,\ C^-) \in [-1,1]$, then we find $\rho_{min}(C_i,\ C^+)$, $\rho_{max}(C_i,\ C^+)$, $\rho_{min}(C_i,\ C^-)$, and $\rho_{max}(C_i,\ C^-)$ to enhance the computations of

$$B^+ = \frac{\rho(C_i, C^+) - \rho_{min}(C_i, C^+)}{\rho_{max}(C_i, C^+) - \rho_{min}(C_i, C^+)}, \tag{4.9}$$

$$B^- = \frac{\rho(C_i, C^-) - \rho_{min}(C_i, C^-)}{\rho_{max}(C_i, C^-) - \rho_{min}(C_i, C^-)} \ . \tag{4.10}$$

before computing the closeness coefficient in this form:

$$f^*(C_j) = \frac{B^+}{B^+ + B^-}. \tag{4.8}$$

Application Example

Assume that four applicants represented by q-ROFSs, $C = \{C_1, C_2, C_3, C_4\}$ are vying for a vacant position in a firm, and the required qualifications is a set $R = \{R_1, R_2, R_3, R_4, R_5\}$, where R_1 is experience, R_2 is team spirit, R_3 is hardworking, R_4 is academic fitness, and R_5 is accountability, respectively. The candidates are interviewed by 3-man panel, and the scores of the interview are presented in Table 11.

Table 11. *Candidates and Qualification Scored by 3-Man Panel*

	C	Experience	Team spirit	Hardworking	Academic fitness	Accountability
1st	C_1	⟨0.2,0.5⟩	⟨0.6,0.1⟩	⟨0.7,0.2⟩	⟨0.8,0.1⟩	⟨0.5,0.2⟩
	C_2	⟨0.3,0.6⟩	⟨0.6,0.2⟩	⟨0.7,0.1⟩	⟨0.6,0.2⟩	⟨0.65,0.3⟩
	C_3	⟨0.3,0.7⟩	⟨0.6,0.1⟩	⟨0.5,0.1⟩	⟨0.5,0.2⟩	⟨0.8,0.1⟩
	C_4	⟨0.4,0.6⟩	⟨0.5,0.3⟩	⟨0.8,0.1⟩	⟨0.7,0.1⟩	⟨0.6,0.3⟩
2nd	C_1	⟨0.4,0.6⟩	⟨0.8,0.1⟩	⟨0.6,0.2⟩	⟨0.8,0.2⟩	⟨0.5,0.2⟩
	C_2	⟨0.3,0.7⟩	⟨0.7,0.3⟩	⟨0.5,0.1⟩	⟨0.7,0.2⟩	⟨0.6,0.3⟩
	C_3	⟨0.2,0.7⟩	⟨0.6,0.1⟩	⟨0.7,0.3⟩	⟨0.6,0.2⟩	⟨0.7,0.2⟩
	C_4	⟨0.3,0.7⟩	⟨0.6,0.3⟩	⟨0.8,0.2⟩	⟨0.7,0.3⟩	⟨0.8,0.1⟩
3rd	C_1	⟨0.3,0.6⟩	⟨0.7,0.2⟩	⟨0.8,0.2⟩	⟨0.7,0.2⟩	⟨0.6,0.2⟩
	C_2	⟨0.4,0.6⟩	⟨0.7,0.1⟩	⟨0.4,0.5⟩	⟨0.6,0.3⟩	⟨0.7,0.3⟩
	C_3	⟨0.2,0.8⟩	⟨0.5,0.4⟩	⟨0.8,0.1⟩	⟨0.7,0.1⟩	⟨0.6,0.1⟩
	C_4	⟨0.1,0.8⟩	⟨0.7,0.1⟩	⟨0.75,0.1⟩	⟨0.6,0.3⟩	⟨0.8,0.1⟩

We apply (4.4) to find the mean of the q-rung orthopair scores by the 3-man panel, and the outcomes are in Table 12.

Table 12. *Candidates and Qualification Scores*

C	Experience	Team spirit	Hardworking	Academic fitness	Accountability
C_1	⟨0.3,0.5667⟩	⟨0.7,0.1333⟩	⟨0.7,0.2⟩	⟨0.7667,0.1667⟩	⟨0.5333,0.2⟩
C_2	⟨0.3333,0.6333⟩	⟨0.6667,0.2⟩	⟨0.5333,0.2333⟩	⟨0.6333,0.2333⟩	⟨0.65,0.3⟩
C_3	⟨0.2333,0.7333⟩	⟨0.5667,0.2⟩	⟨0.6667,0.1667⟩	⟨0.6,0.1667⟩	⟨0.7333,0.1333⟩
C_4	⟨0.2667,0.7⟩	⟨0.6,0.2333⟩	⟨0.7833,0.1333⟩	⟨0.667,0.2333⟩	⟨0.7333,0.1667⟩

Since the cost criterion is Q_1, Table 13 is the normalized qROFDM using (4.5).

Table 13. *Normalized qROFDM*

C	Experience	Team spirit	Hardworking	Academic fitness	Accountability
C_1	⟨0.5667,0.3⟩	⟨0.7,0.1333⟩	⟨0.7,0.2⟩	⟨0.7667,0.1667⟩	⟨0.5333,0.2⟩

C_2	⟨0.6333,0.3333⟩	⟨0.6667,0.2⟩	⟨0.5333,0.2333⟩	⟨0.6333,0.2333⟩	⟨0.65,0.3⟩
C_3	⟨0.7333,0.2333⟩	⟨0.5667,0.2⟩	⟨0.6667,0.1667⟩	⟨0.6,0.1667⟩	⟨0.7333,0.1333⟩
C_4	⟨0.7, 0.2667⟩	⟨0.6,0.2333⟩	⟨0.7833,0.1333⟩	⟨0.667,0.2333⟩	⟨0.7333,0.1667⟩

By (4.6) and (4.7), we get NIS and PIS in Table 14.

Table 14. *NIS and PIS*

NIS/PIS	Experience	Team spirit	Hardworking	Academic fitness	Accountability
C^-	⟨0.7333,0.2333⟩	⟨0.5667,0.2333⟩	⟨0.5333,0.2333⟩	⟨0.6,0.2333⟩	⟨0.5333,0.3⟩
C^+	⟨0.5667,0.3333⟩	⟨0.7,0.1333⟩	⟨0.7833,0.1333⟩	⟨0.7667,0.1667⟩	⟨0.7333,0.1333⟩

Now, we compute the correlation coefficients using (4.1) and (4.2) (for $r = 5$) for each of the applicants with the NIS and PIS, respectively, and the closeness coefficients. The results are contained in Tables 15 and 16.

Table 15. *Correlation Coefficients using (4.1) and Closeness Coefficients*

Candidates	$\tilde{\rho}_1(C_i, C^-)$	$\tilde{\rho}_1(C_i, C^+)$	$f(C_i)$	Ranking
C_1	0.9961	0.9980	0.5005	2nd
C_2	0.9980	0.9956	0.4994	4th
C_3	0.9961	0.9953	0.4998	3rd
C_4	0.9942	0.9980	0.5010	1st

Table 16. *Correlation Coefficients using (4.2) and Closeness Coefficients*

Candidates	$\tilde{\rho}_2(C_i, C^-)$	$\tilde{\rho}_2(C_i, C^+)$	$f(C_i)$	Ranking
C_1	0.9770	0.9884	0.5029	2nd
C_2	0.9897	0.9752	0.4963	4th
C_3	0.9812	0.9717	0.4976	3rd
C_4	0.9696	0.9870	0.5044	1st

From these results, we see that candidate, C_4 is the most qualified for the vacant position. This approach will enhance the employment of the most qualified staff, and by extension boost productivity.

5. COMPARATIVE ANALYSES

The section presents comparative studies for the new CCq-ROFSs methods and the other CCq-ROFSs methods in [54-57] based on recognition principle and MCDM approach to justify the pre-eminence of the new methods.

5.1. Comparison based on Recognition Principle

By applying our methods and the methods in [54-57] with $r = 5$ on the case of medical diagnosis in Subsection 4.1, we get the results in Tables 18-21.

Table 18. *Patient P_1 and Diseases*

Methods	$(P_1, \breve{D}_1)$	$(P_1, \breve{D}_2)$	$(P_1, \breve{D}_3)$	$(P_1, \breve{D}_4)$	$(P_1, \breve{D}_5)$	Diagnosis
ρ_1 [54]	0.9950	0.9912	0.8956	0.3771	0.5081	Malaria fever
ρ_2 [56]	0.5294	0.7637	0.3729	0.2574	−0.0740	Viral fever
ρ_3 [57	0.7946	0.3984	0.4471	0.0560	0.1320	Malaria fever
ρ_4 [57]	0.8934	0.8584	0.6344	0.0733	0.1737	Malaria fever
ρ_5 [55]	0.9702	0.9728	0.9558	0.6527	0.7143	Viral fever
ρ_6 [55]	0.9244	0.9325	0.7647	0.5166	0.6126	Viral fever
$\tilde{\rho}_1$	0.9952	0.9946	0.9914	0.9737	0.9699	Malaria fever

$\tilde{\rho}_2$	0.9765	0.9734	0.9575	0.8750	0.8581	Malaria fever

Table 19. *Patient P_2 and Diseases*

Methods	$(P_2,\breve{D}_1)$	$(P_2,\breve{D}_2)$	$(P_2,\breve{D}_3)$	$(P_2,\breve{D})$	$(P_2,\breve{D}_5)$	Diagnosis
ρ_1 [54]	0.5079	0.5232	0.7457	0.9724	0.5419	Stomach pro.
ρ_2 [56]	-0.0175	0.0954	0.1084	0.7384	0.0521	Stomach pro.
ρ_3 [57	0.2199	0.1101	0.2421	0.7329	0.1991	Stomach pro.
ρ_4 [57]	0.2561	0.3107	0.4500	0.7330	0.2001	Stomach pro.
ρ_5 [55]	0.7778	0.8201	0.8882	0.8809	0.7064	Typhoid fever
ρ_6 [55]	0.5934	0.6218	0.8404	0.8456	0.6256	Stomach pro.
$\tilde{\rho}_1$	0.9707	0.9797	0.9795	0.9836	0.9632	Stomach pro.
$\tilde{\rho}_2$	0.8619	0.9027	0.9024	0.9205	0.8291	Stomach pro.

Table 20. *Patient P_3 and Diseases*

Methods	$(P_3,\breve{D}_1)$	$(P_3,\breve{D}_2)$	$(P_3,\breve{D}_3)$	$(P_3,\breve{D}_4)$	$(P_3,\breve{D}_5)$	Diagnosis
ρ_1 [54]	0.8495	0.9056	0.9420	0.4208	0.3548	Typhoid fever
ρ_2 [56]	0.1248	0.4315	0.2014	0.0328	-0.0304	Viral fever
ρ_3 [57	0.4472	0.1829	0.1801	0.0372	0.0549	Malaria fever
ρ_4 [57]	0.5095	0.5047	0.3273	0.0381	0.0564	Malaria fever
ρ_5 [55]	0.7924	0.8086	0.8349	0.5660	0.5907	Typhoid fever
ρ_6 [55]	0.7262	0.7365	0.7262	0.4666	0.5186	Viral fever
$\tilde{\rho}_1$	0.9751	0.9805	0.9743	0.9591	0.9546	Viral fever
$\tilde{\rho}_2$	0.8815	0.9063	0.8780	0.8117	0.7929	Viral fever

Table 21. *Patient P_4 and Diseases*

Methods	$(P_4,\breve{D}_1)$	$(P_4,\breve{D}_2)$	$(P_4,\breve{D}_3)$	$(P_4,\breve{D}_4)$	$(P_4,\breve{D}_5)$	Diagnosis
ρ_1 [54]	0.9856	0.9644	0.8482	0.5036	0.6819	Malaria fever
ρ_2 [56]	0.7165	0.3388	0.1863	-0.0853	-0.1667	Malaria fever
ρ_3 [57	0.2479	0.3533	0.1396	0.0223	0.0485	Viral fever
ρ_4 [57]	0.7885	0.4638	0.2783	0.0827	0.1806	Malaria fever
ρ_5 [55]	0.9292	0.9387	0.9772	0.7780	0.8202	Typhoid fever
ρ_6 [55]	0.7923	0.8046	0.6678	0.5293	0.6037	Viral fever
$\tilde{\rho}_1$	0.9906	0.9975	0.9940	0.9769	0.9764	Viral fever
$\tilde{\rho}_2$	0.9537	0.9877	0.9706	0.8896	0.8873	Viral fever

The majority of the diagnosis show P_1 has to be treated of malaria fever. However, the diagnosis from the methods in [55, 56] show viral fever, which is again valid because of the relationship between malaria fever and viral fever. For the sake of reliability, the patient should be treated for both diseases with emphasis on malaria fever. P_2 has to be treated for stomach problem and typhoid. The diagnosis shows a serious case of typhoid fever which could leads to abdominal pain due to intestinal problem. P_3 and P_4 are mainly inflicted by viral fever with a number of symptoms of malaria fever and typhoid fever. From Tables 18-21, it can be easily observed that, the new CCq-ROFSs methods give the most precise results with appropriate medical interpretations.

5.2. Comparison Based on MCDM

We apply the new methods and the methods in [54-57] using the MCDM algorithm on the data from Tables 12 and 14, and get the information in Table 22.

Table 22. *Employment Information*

CCq-ROFSs Methods	Order of Employment	Employment Status
ρ_1 [54]	$C_3 \succ C_4 \succ C \succ C_2$	C_3
ρ_2 [56]	$C_4 \succ C_1 \succ C_2 \succ C_3$	C_4

$\boldsymbol{\rho_3}$ [57	$C_4 \succ C_3 \succ C_1 \succ C_2$	C_4
$\boldsymbol{\rho_4}$ [57]	$C_4 \succ C_3 \succ C_1 \succ C_2$	C_4
$\boldsymbol{\rho_5}$ [55]	$C_4 \succ C_1 \succ C_3 \succ C_2$	C_4
$\boldsymbol{\rho_6}$ [55]	$C_4 \succ C_1 \succ C_3 \succ C_2$	C_4
$\tilde{\boldsymbol{\rho}}_1$	$C_4 \succ C_1 \succ C_3 \succ C_2$	C_4
$\tilde{\boldsymbol{\rho}}_2$	$C_4 \succ C_1 \succ C_3 \succ C_2$	C_4

In all the methods except the method in [54], we see that the applicant, C_4 is the most suitable for the employment. By comparing the recognition principle and MCDM approach, it is certain that the MCDM approach is more reliable in the sense that it produces a unified interpretation unlike in the medical diagnosis based on recognition principle.

5.3. Benefits of the New CCq-ROFS Methodologies

It has been demonstrated that the new CCq-ROFSs techniques are more reliable than the current ones in terms of precision. Here are the most notable benefits of the new techniques of CCq-ROFSs:

i. The ability to compute the association concerning any two similar q-ROFSs and the reliability of performance ratings are the two key aspects of the new techniques.
ii. In contrast to the techniques in [54–57], the new techniques perform better as q increases.
iii. Unlike the approaches in [54, 55], the new techniques take into account every distinctive feature of q-ROFSs to prevent error resulting from omission.
iv. Unlike the methods in [54, 55, 57], the new techniques duly satisfy all the conditions of the correlation measure.

6. CONCLUSION

The process of making decisions in uncertain environments requires a thorough understanding of the correlation coefficient concept. q-ROFSs are a generalized class of fuzzy set variants that have the ability to reduce fuzziness and vagueness in judgment calls. In comparison to the current CCq-ROFSs techniques, the two novel CCq-ROFSs techniques introduced in this research are deemed to be more reliable. To validate the value of the new techniques of CCq-ROFSs, we described the new CCq-ROFSs methods to demonstrate how they correspond with the traditional understanding of a correlation coefficient, and provided a relative analysis that demonstrated the superiority of the novel CCq-ROFSs techniques over the existing ones [54–57]. Finally, we used simulated q-ROF information via the recognition principle and the MCDM methodology, respectively, to illustrate the use of the new CCq-ROFSs methods in disease diagnosis and employment procedure. Throughout the study, it is noted that: (i) unlike the CCq-ROFSs techniques in [54–57], the performances of the new CCq-ROFSs techniques increase as q increases; (ii) the new CCq-ROFSs techniques demonstrate reliability in terms of performance rating and ability to estimate correlation concerning comparable q-ROFSs; (iii) the new CCq-ROFSs techniques duly satisfy all the properties of correlation measure; and (iv) the new CCq-ROFSs techniques incorporate all the characteristic features of q-ROFSs to avoid error due to exclusion unlike the CCq-ROFSs techniques in [54, 55, 57]. The new CCq-ROFSs techniques are flexible enough to be used in solving clustering analysis problems. Furthermore, it is possible to alter these novel CCq-ROFSs methods in order to calculate the association concerning certain fuzzy set variations that have additional parameters than qROFSs. The information measures in [58,59] could be extended to the MCDM technique presented in this article for future investigation. Finally, the proposed CCq-ROFSs techniques could be used to discuss earthquake time prediction [60], deep learning models [61], e-commerce [62], and clustering problem [63] in future investigations.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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