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Chaotic and Quasi-periodic Regimes in the Covid-19 Mortality Data

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ABSTRACT It has been reported by World Health Organization (WHO) that the Covid-19 epidemic due to the Sar-Cov-2 virus, which started in China and affected the whole world, caused the death of approximately six million people over three years. Global disasters such as pandemics not only cause deaths but also bring other global catastrophic problems. Therefore, governments need to perform very serious strategic operations to prevent both infection and death. It is accepted that even if there are vaccines developed against the virus, it will never be possible to predict very complex spread dynamics and reach a spread pattern due to new variants and other parameters. In the present study, four countries: Türkiye, Germany, Italy, and the United Kingdom have been selected since they exhibit similar characteristics in terms of the pandemic's onset date, wave patterns, measures taken against the outbreak, and the vaccines used. Additionally, they are all located on the same continent. For these reasons, the three-year Covid-19 data of these countries were analyzed. Detailed chaotic attractors analyses were performed for each country and Lyapunov exponents were obtained. We showed that the three-year times series is chaotic for the chosen countries. In this sense, our results are compatible with the results of the Covid-19 analysis results in the literature. However, unlike previous Covid-19 studies, we also found out that there are chaotic, periodic, or quasi-periodic sub-series within these chaotic time series. The obtained results are of great importance in terms of revealing the details of the dynamics of the pandemic.

KEYWORDS
Chaotic
Quasi-periodic
COVID-19
Largest lyapunov exponent
Time delay
Phase space
Embedding dimension

INTRODUCTION

Humanity has faced the Covid-19 epidemic, which is the biggest global disaster after the Second World War and has surrounded the whole world. The pandemic was declared by the World Health Organization (WHO) on March 11, 2020, due to the coronavirus epidemic that started in China and affected the whole world (World Health Organisation 2020). As of March 26, 2023, 761 million people were infected with coronavirus and 6.8 million people died (World Health Organisation 2023). With the beginning of mass deaths, all governments and WHO are trying to control and prevent the spread of Covid-19. As it is known until the Covid-19 vaccine was found, all countries of the world tried to prevent the spread of this virus with a series of measures such as curfews and travel restrictions. One of the important steps to controlling the spread of Covid-19 was the mathematical modeling of the pandemic and its analysis. With the acquisition of vaccines, efforts were made to prevent the Covid-19 epidemic. As of April 2023, 69.9 percent of the world's population had at least one COVID-19 vaccine. However, despite vaccines, new virus types have emerged and caused new spreading waves. Fortunately, the end of the pandemic process, which lasted approximately three years, was announced by WHO in May 2023.

Modeling a pandemic is important for two reasons. The first of these is to find or understand the mathematical model of the spreading dynamics of the pandemic. The other is to make model-based predictions and develop strategies to take preventive measures against the pandemic. Various models have been supposed to carry out the spread dynamics of infectious diseases. One of the popular methods is the compartment method proposed by

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Kermack and McKendrick (Kermack and McKendrick 1927). In this method, the entire population is divided into different compartments: i) people who are prone to the disease; ii) people who are already infected and can spread the infection; iii) people who have already recovered and have developed the immune system.

This model is called as SIR model in the literature. After the Covid pandemic started, many mathematical and simulation models are proposed for the study of COVID-19 based on SIR model (Schaffer 1985; Olsen *et al.* 1988; Hethcote *et al.* 1989; Earn *et al.* 2000; Kumar *et al.* 2019; Machado *et al.* 2020; Gumel *et al.* 2004; Livadiotis 2020; Youssef *et al.* 2020; Ahmetolan *et al.* 2020). However, it is known that these models are not sufficient to predict the course of the pandemic. The validity of most predictive models relies on numerous parameters, involving biological and social characteristics often unknown or highly uncertain.

To fully understand the dynamics of the spread of such a pandemic, it is necessary to analyze the data set consisting, for example, of the number of people infected or lost their lives. Does the time series correlate? Does the time series consist of unpredictable data? Is there a pattern in the data set? The answers to these questions are important in understanding the dynamics of diffusion. Many studies have been conducted to answer these questions. For example, Mangiarotti et al showed that there are chaotic attractors in the Covid-19 data of China, Japan, South Korea, and Italy (Mangiarotti *et al.* 2020). These findings indicate that the number of people infected and those who lost their lives in the pandemic is unpredictable.

It also points out that it is necessary to include the chaos theory to understand the dynamics of the pandemic. It has been previously reported that the Mexican flu and Ebola and dengue epidemics contained chaotic patterns (Speakman and Sharpley 2012; Mangiarotti *et al.* 2016; Agusto and Khan 2018). Additionally, it is also possible to see new studies in the literature supporting that the Covid-19 pandemic has chaotic spreading dynamics (Jones and Strigul 2021; Borah *et al.* 2022; Abbes *et al.* 2023; Russell *et al.* 2023; Mashuri *et al.* 2023; Wang *et al.* 2023; Debbouche *et al.* 2022; Sapkota *et al.* 2021; Gonçalves 2022).

As it is known, many parameters affect virus spreading. The most important of these are new virus variants arising from the Sars-Cov-2 virus. This causes the data to be superimposed. Therefore, it requires detailed analysis to determine the character of the wave. For example, the data may include quasi-periodic or chaotic signals. Quasi-periodic signals of this type are known as *weak* signals in the literature. These weak signals can be detected with the help of chaotic oscillators (Wang *et al.* 1999; Wang and He 2003; Liu *et al.* 2007; Raj *et al.* 1999; Birx and Pipenberg 1992). However, in this study, we will analyze data as a whole and sub-series to detect quasi-periodic and chaotic regimes.

Since Covid-19 remains a potential, careful analysis of available data remains important. Even if the pandemic were to be officially declared over when we look at the records of the WHO and the Coronavirus Resource Center, it is evident that the COVID-19 outbreak still persists at a low level(World Health Organisation 2023; Coronavirus Resource Center 2024). It should not be forgotten that the world is always under the threat of a pandemic. Understanding the dynamics of the spread is crucial to combating any outbreak. Throughout history, uncontrollable pandemics have inflicted greater damage on nations than wars, and in some cases, entire states have collapsed due to epidemics. The fight against infectious diseases is not merely an epidemic issue but a strategic concern for countries. Therefore, analyzing Covid-19 data is still important to carry out the dynamics of the pandemic. In the present study, we will analyze Covid-19 data of Türkiye, Germany, Italy, and United Kingdom in detail to discuss the spreading dynamic. We will analyze the phase spaces and calculate Lyapunov exponents for these countries' time series and different time intervals. As a main contribution, in the present works, we will show that three years Covid-19 data for the chosen countries are chaotic, and, it is the first time, we will show that the chaotic, periodic or quasi-periodic sub-series embedded as a sub regimes in these chaotic pandemic time series.

The study is organized as follows: In Section II, we briefly introduce the mathematical techniques and algorithms for the analysis of a time series. In Section III, we presented three years Covid-19 mortality data with sub-peak periods and mortality data for Türkiye, Germany, Italy, and the United Kingdom. In Section IV, we give numerical results in detail for four countries. We plotted attractors in the phase spaces and computed Lyapunov exponents of the time series of Covid-19. In this section, we show that Covid-19 data have chaotic attractors and positive Lyapunov exponents in some time intervals while they have quasi-periodic solutions in some time intervals. Finally, in the last chapter, the discussion and conclusion are given.

CHAOTIC TIME SERIES ANALYSIS

Time series

It is known that a time series is a series of data points indexed in time order. Time series can be obtained from data produced by a physical system, but also from discrete or a differential equation. While the discrete systems can be expressed as $\mathbf{x}_{n+1} = \mathbf{f}(\mathbf{x_n})$, the continuous systems can be expressed in the differential form as $\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}(\mathbf{x})$ with three or more degrees of freedom $\mathbf{x}(t) = [x_1(t), x_2(t), ..., x_m(t)]$. The time series we are interested in here is the Covid-19 mortality series of four different countries. This series consists of three years of data. Our main aim is to reveal whether these series are chaotic or not. As we will show below, we will do this both for the entire series and by dividing the series into subdivisions. We will use the same method of analysis for both cases. To perform chaotic analysis, we will need knowledge of the phase space and the Lyapunov exponent. These details will be given briefly below.

According to the classical approach of chaos theory, for a time series to be chaotic, it must be sensitive to the initial condition and be unpredictable. Since the Covid epidemic contains dynamic variables that depend on time, it should also be taken into account that it is sensitive to physical factors that change over time. However, the best way to see chaotic behavior in the data set is to perform phase space analysis and calculate the Lyapunov exponent. We will calculate these quantities using Matlab. However, we would like to briefly present the background of the calculation.

Attractor Reconstruction

Reconstruction of phase space is very important to see the dynamic behavior of the given time series. To figure out the trajectory from a given time series is a big challenge. Fortunately, the delay time-coordinate embedding method laid by Takens (Takens 1981). The delay-coordinate method can be given as follows. From a measured time series $x(k) = x(t_0 + k\Delta t)$ with Δt being the sampling interval, the following vector quantity of m components is constructed:

$$\mathbf{x}(t) = \{x(t), x(t+\tau), ..., x(t+(m-1)\tau)\} \tag{1}$$

where $t = t_0 + k\Delta t$, τ is the delay time which is an integer multiple of Δt and m is the embedding dimension. To plot a phase space of a given time series, it is necessary to determine the delay time τ and embedding dimension m. Once these two parameters are determined, the reconstructed vector $\mathbf{x}(t)$ can accurately represent the trajectory of the unknown attractor. We will not go into calculation details here. It can be seen in the details of computing these quantities in Ref.(Takens 1981).

Lyapunov Exponent

The Lyapunov exponent is the most important quantity used to determine whether chaotic behavior exists in a dynamic system. A positive Lyapunov exponent is the strongest sign that indicates that there is chaos in the system. On the other hand, a negative Lyapunov exponent represents fixed points while a zero Lyapunov exponent denotes a limit cycle or a quasiperiodic orbit.

The Lyapunov exponent of a dynamical system or time series represents the rate of exponential divergence of an orbit from perturbed initial conditions. For example, consider an m-dimensional discrete map $x(j)$ $(j = 1, 2, ..., m)$. Let $x_n(j)$ be its state at time n. By adding $\delta x(j)$ to the $x_n(j)$, we set an new state as $x'_n(j) = x_n(j) + \delta x(j)$. The distance between two states changes exponentially with time

$$\|\delta x_n(j)\| \sim e^{\lambda t} \|x_{n-1}(j)\| \tag{2}$$

Then the maximal Lyapunov exponent λ_{max} can be obtained from Eq.(2) as

$$\lambda_{max} = \lim_{N \to \infty} \frac{1}{N} \sum_{j=0}^{N} \ln \frac{\|\delta x_n(j)\|}{\|\delta x_{n-1}(j)\|} \tag{3}$$

where $\|\delta x_n(j)\| = (\sum_j^m \delta x_n(j)^2)^{1/2}$. By using this approximation can be computed Lyapunov exponent for the dynamical systems. However, it is quite difficult to use this method in a time series analysis. Various methods have been developed to calculate the Lyapunov exponent in time series (Rosenstein *et al.* 1993; Wolf *et al.* 1985) and other methods (Meranza-Castillón *et al.* 2019; Arellano-Delgado *et al.* 2017). In this study, we will calculate Lyapunov exponents using Matlab (Inc. 2023) which based on the algorithm given in Ref.(Rosenstein *et al.* 1993). In this algorithm process, firstly time delay τ and embedding dimension m are computed to construct the phase space for the time series data, and then, the distance between two trajectories starts at different states.

COVID-19 MORTALITY TIME SERIES OF FOUR COUNTRIES

In this study, as we mentioned in the introduction we will analyze the COVID-19 mortality data of Türkiye, Germany, Italy, and the United Kingdom, respectively. The data of the countries between 2020 and 2022 will be used in the analysis. The data were taken from public data of the World Health Organization and Our World in Data sites (Our World in Data Organisation 2023; World Health Organisation 2023).

Three years Covid-19 mortality data for four countries are given in Fig.1. As can be seen from Fig.1 pandemic peaks occur at different time intervals in the time series of four different countries. To conduct a systematic analysis of the data of these four countries, we divided the three-year time series into six sub-divisions for the sake of simplicity. Each peak was represented with a different color, and the start and end dates of the peaks were given in the panels. In Fig.1 the area under the peaks gives the number of people who died during that peak period. These numbers are also given in Table 1. On the other hand, it should be noted that the highest peaks were considered when determining the peak range for each country. For example, if there was no major peak in one country and there was a high peak in another country, it was evaluated as if there was a peak in the same period. We paid attention to this generality when separating these compartments. However, the analysis of peaks is independent of the number of peaks.

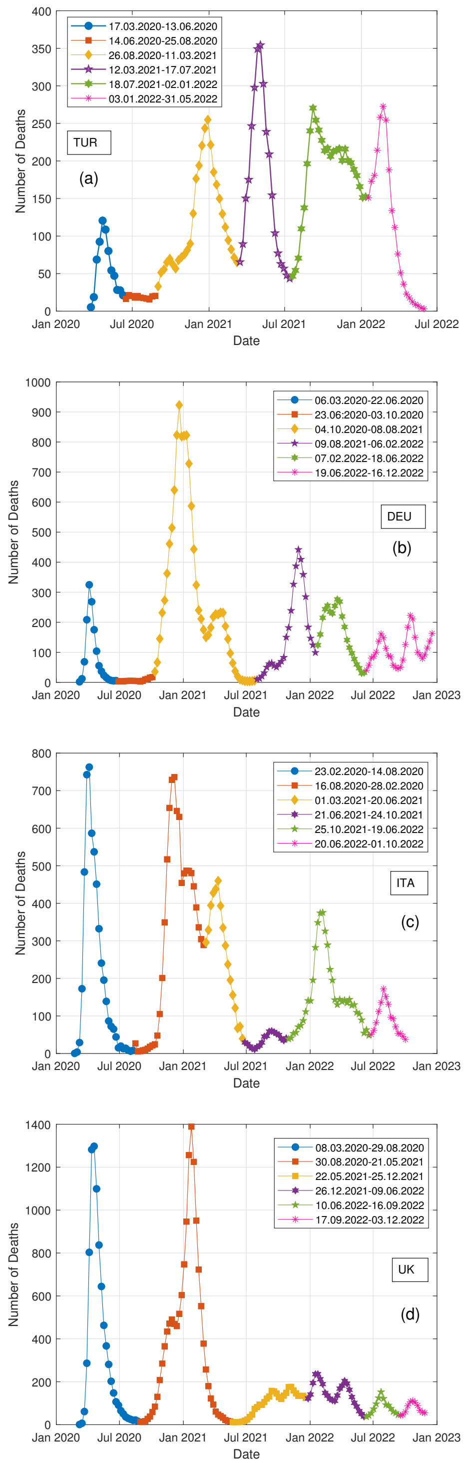

Figure 1 Mortality time series between 17.03.2020 − 31.05.2022 due to Covid-19: In (a) Türkiye, in (b) Germany, in (c) Italy, in (d) United Kingdom.

The number of deaths for each peak period for four countries is given in Table 1. It can be seen that the number of deaths varies

Table 1 COVID-19 waves and mortality number of four countries.

	1st		2nd		3rd		4th		5th		6th	
	Day	Deaths	Day	Deaths	Day	Deaths	Day	Deaths	Day	Deaths	Day	Deaths
Türkiye	89	4792	73	1371	198	23127	128	21198	169	32147	149	16330
Germany	106	8887	103	649	309	82247	182	26935	132	21472	181	19708
Italy	176	35265	197	62307	112	29571	126	4556	238	35895	105	9429
United Kingdom	175	57858	265	96248	218	27398	166	23374	99	7786	78	5596

dramatically within the same peak intervals. The fact that these numbers are very different from each other can be considered to vary depending on many parameters such as the elderly population, isolation strategies, and vaccination. In this study, we would like to analyze the character of the time series, not the numbers in different time intervals. Therefore, firstly, we performed the phase space analyses for three years of data for each country, and the character of the time series was determined by calculating Lyapunov exponents. Subsequently, we separately analyzed all epidemic peaks for each country. Similarly, we discussed the phase space Lyapunov exponents for each pandemic peak. Analysis results are given below.

CHAOS ANALYSIS OF THE COVID-19 MORTALITY DATA

Türkiye

The time series showing the number of deaths due to COVID-19 in Türkiye between 2020 and 2022 is given in Fig 1(a). We obtained the embedding dimensions and delay time for this time series using the method presented in Section Chaotic time series analysis. With the help of this information, we constructed the phase space of the time series in Fig 2 (a). Although not visible in great detail, it can be seen that more than one orbit exists in the phase space. These orbits may indicate the presence of a chaotic attractor. But the attractor is not very clear, as in Lorenz, for example. Orbits may indicate the existence of a periodic or quasi-periodic solution. To see whether the orbit is chaotic or not, we calculated the Lyapunov exponent of the series with the help of MATLAB (Inc. 2023) and gave the result in Fig 2 (b).

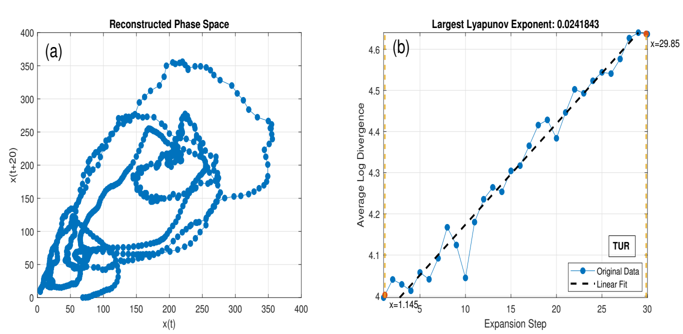

Figure 2 COVID-19 data set of Türkiye's reported deaths time series between 17.03.2020 − 31.05.2022. Embedding dimension $d = 3$ and time delay $\tau = 20$. In (a) phase space representation, in (b) Lyapunov exponent.

As can be seen from Fig 2 (b) the Lyapunov exponent for COVID-19 three-year mortality data is positive which indicates data has chaotic behavior.

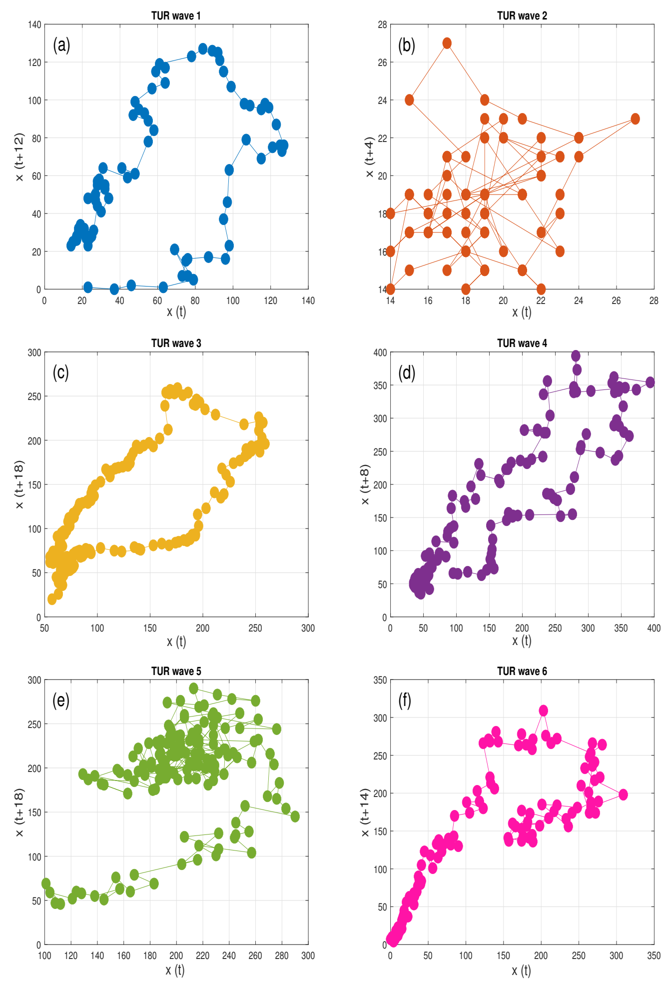

Figure 3 Reconstructed phase space of Türkiye's waves. In (a) 1^{st} wave, in (b) 2^{th} wave, in (c) 3^{th} wave, in (d) 4^{th} wave, in (e) 5^{th} wave, in (f) 6^{th} wave.

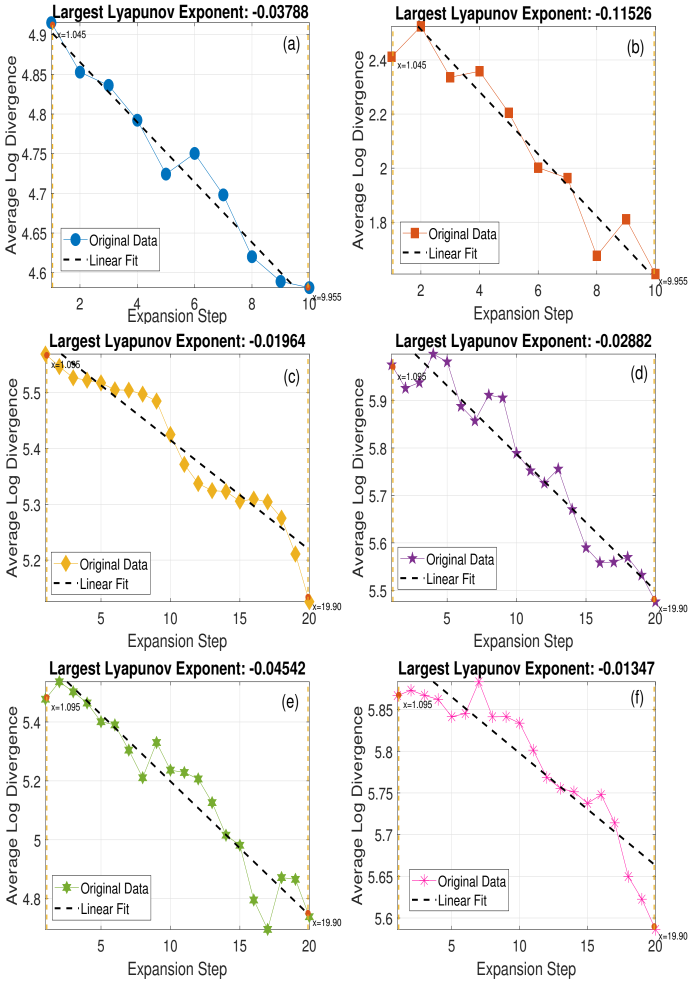

Figure 4 Largest Lyapunov exponent of Türkiye's wawes. In (a) 1^{st} wave, in (b) 2^{th} wave, in (c) 3^{th} wave, in (d) 4^{th} wave, in (e) 5^{th} wave, in (f) 6^{th} wave.

On the other hand, to analyze the local region in the time series, we first computed the embedding dimensions and delay times for the sub-time series corresponding to each peak, and we separately plotted the phase space diagrams for these sub-time series in Fig 3. As can be seen from this figure the single trajectory is seen in all sub-panels in Fig 3. One can see that the presence of these single orbits may indicate aperiodic orbits of the sub-time series. Lyapunov exponents of these sub-time series were calculated and given in Fig 4. As can be seen from Fig 4 all sub-time series of Türkiye have different negative Lyapunov exponents. These interesting results show that while the three-year time series of Covid-19 data is chaotic, the behavior of the sub-time series in the same period is not chaotic for Türkiye. This result is meaningful as it indicates that a time series consisting of quasi-periodic signals sub-sets can produce chaotic dynamics when evaluated as a whole.

Germany

Similarly and using the same systematics, we analyzed the three-year data of the Germany time series shown in Fig 1(b). We determined the delay time for this time series and plotted the phase space as can be seen in Fig 5(a). Contrary to Türkiye's data, we can say that there are more orbits around attractors in Germany's data. We can see from Fig 5(b) that this attractor is chaotic. Indeed, the Lyapunov exponent of this time series is positive. While the Largest Lyapunov Exponent (LLE) value is 0.038 for Germany, this value is around 0.028 for Türkiye. This difference indicates that Germany's Covid-19 time series is more chaotic than Türkiye's time series.

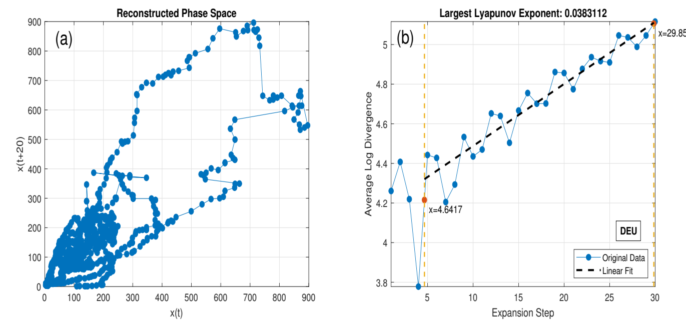

Figure 5 COVID-19 data set of Germany's reported deaths time series between 09.03.2020 − 19.06.2022. Embedding dimension $d = 3$ and time delay $\tau = 20$. In (a) phase space representation, in (b) Lyapunov exponent.

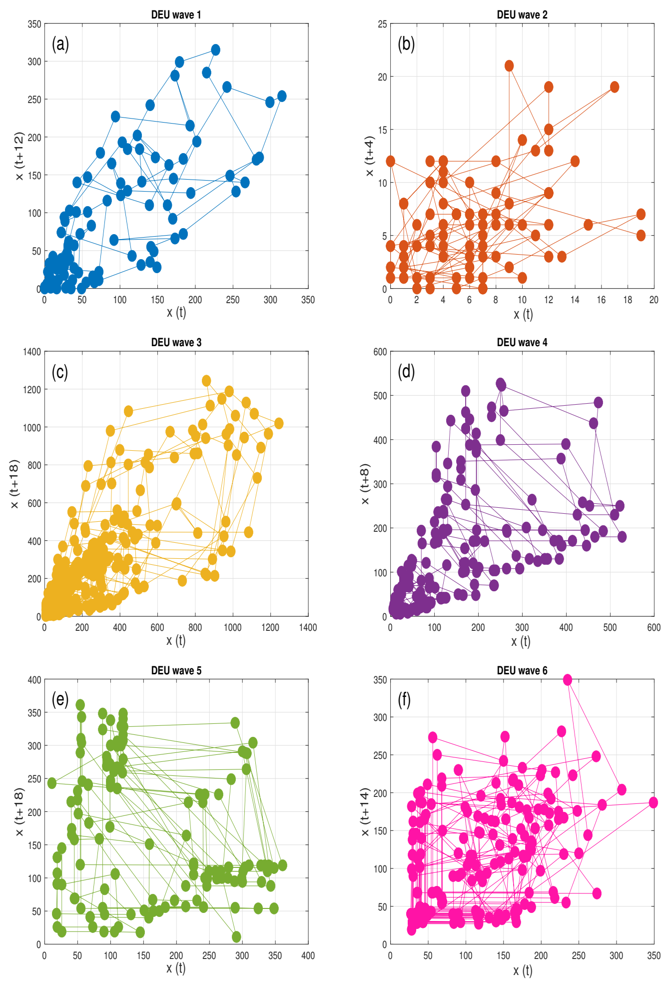

Figure 6 Reconstructed Phase Space of Germany's Waves. In (a) 1^{st} wave, in (b) 2^{th} wave, in (c) 3^{th} wave, in (d) 4^{th} wave, in (e) 5^{th} wave, in (f) 6^{th} wave.

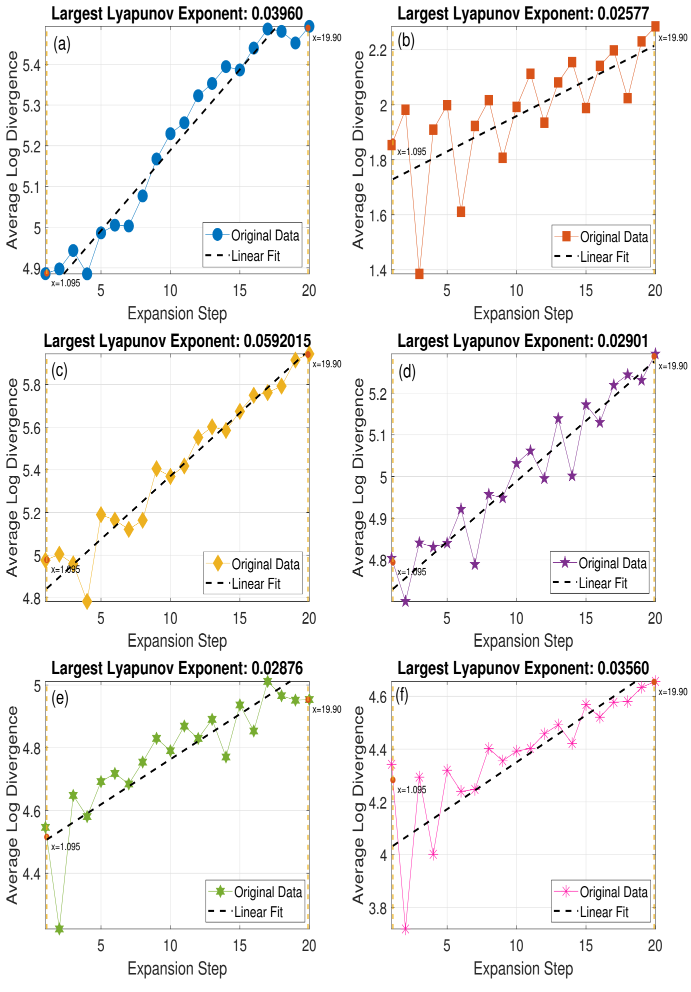

Figure 7 Largest Lyapunov exponent of Germany's waves. In (a) 1^{st} wave, in (b) 2^{th} wave, in (c) 3^{th} wave, in (d) 4^{th} wave, in (e) 5^{th} wave, in (f) 6^{th} wave.

To analyze the time series of each independent peak in Germany's Covid-19 data given in Fig. 3(b), we computed embedding dimensions and delay times for each sub-data. We separately plotted the phase space diagrams for these sub-time series in Fig. 6. As can be seen from this figure more trajectories are seen in all sub-panels in Fig 6. These multi-orbits may indicate chaotic orbits of the sub-time series. Lyapunov exponents of these sub-time series were calculated and given in Fig. 7. As can be seen from Fig. 7 all sub-time series of Germany have different positive Lyapunov exponents. These interesting results show that the three-year time series and all sub-series of Covid-19 data of Germany are chaotic.

Italy

Similarly, we compute the delay time for the three-year data of the Italy time series shown in Fig. 1(c). The chaotic attractor for this data is given in Fig. 8(a). It can be seen that there is more than one trajectory in this phase space. Additionally, we obtained the Lyapunov exponent for this data and plotted it in Fig. 8(a). The value of the Lyapunov exponent for Italy is 0.0037 which is close to the value of Germany.

To see detailed phase space attractors of the sub-series for Italy's Covid-19 data given in Fig 3(c), we computed embedding dimensions and delay times for each sub-data. We separately plotted the

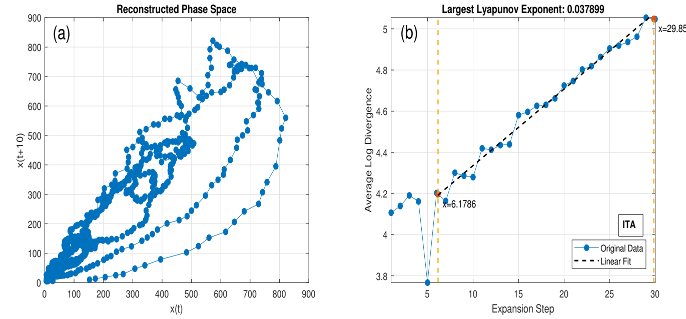

Figure 8 COVID-19 data set of Italy's reported deaths time series between 21.02.2020 − 2.10.2022. Embedding dimension $d = 3$ and time delay $\tau = 10$. In (a) phase space representation, in (b) Lyapunov exponent.

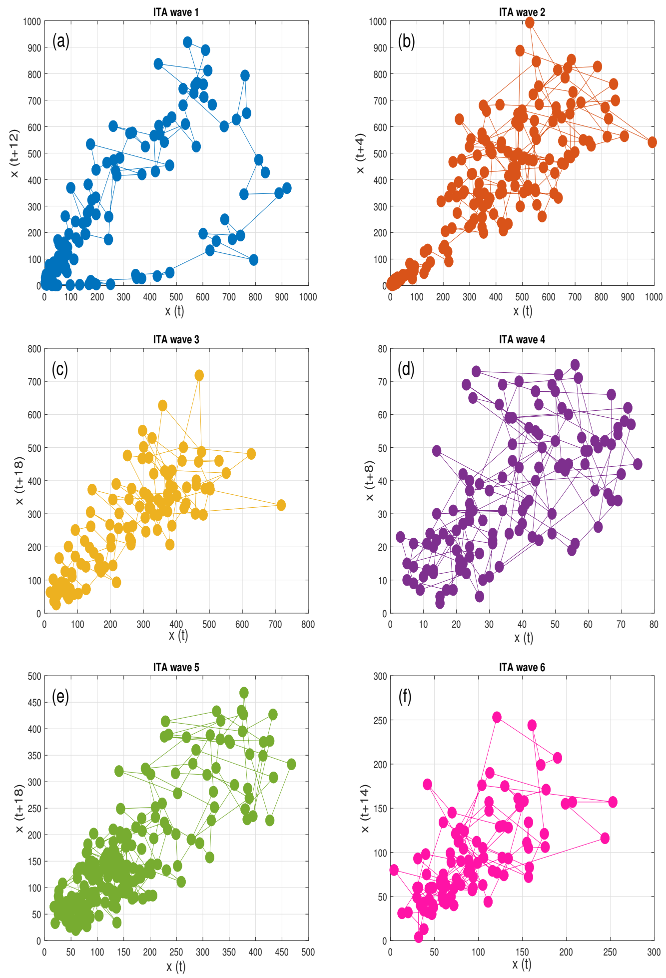

Figure 9 Reconstructed phase space of Italy's waves. In (a) 1^{st} wave, in (b) 2^{th} wave, in (c) 3^{th} wave, in (d) 4^{th} wave, in (e) 5^{th} wave, in (f) 6^{th} wave.

phase space diagrams for these sub-time series in Fig 9. As can be seen from this figure more trajectories are seen in all sub-panels in Fig 9. Although there appear to be attractors in the phase space diagrams, it is difficult to say that the character of the time series can be fully understood from the orbits in the phase space. To see the dynamics of the sub-time series, Lyapunov exponents of the

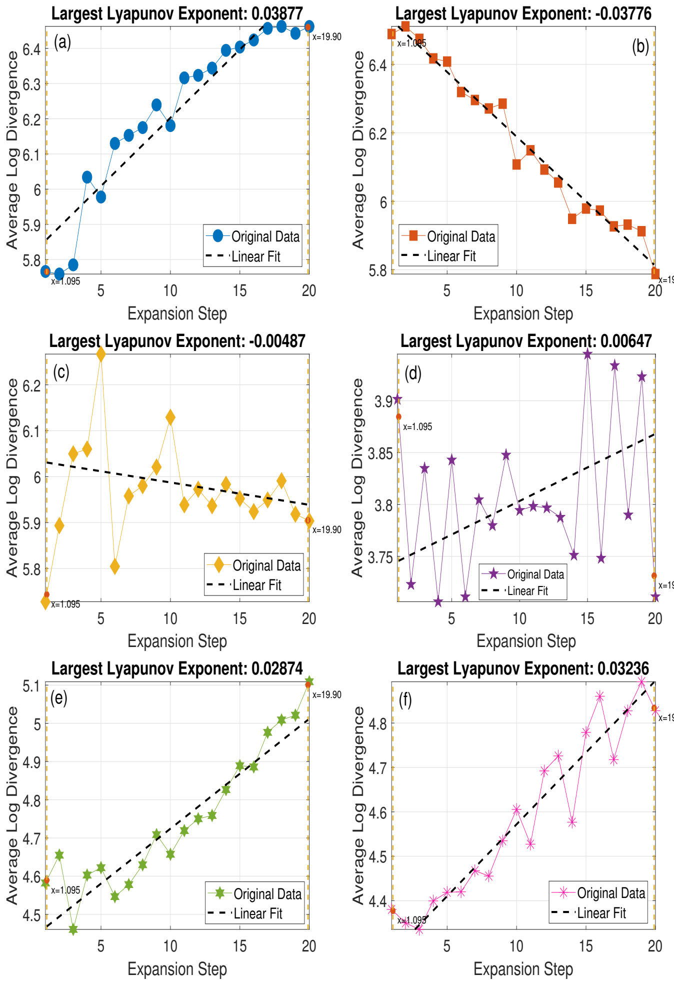

Figure 10 Largest Lyapunov exponent of Italy's waves.In (a) 1^{st} wave, in (b) 2^{th} wave, in (c) 3^{th} wave, in (d) 4^{th} wave, in (e) 5^{th} wave, in (f) 6^{th} wave.

sub-time series were calculated separately and given in Fig 10. Interestingly, the second and third peaks have a negative Lyapunov exponent, while the others have a positive exponent. These results indicate that the three-year chaotic Italy series consists of a combination of chaotic and quasi-periodic sub-series.

United Kingdom

Finally, we compute the delay time for the three-year data of the United Kingdom time series shown in Fig. 1(d). The attractor for this data is given in Fig. 11(a). It can be seen that there is more than one trajectory in this phase space. Additionally, we obtained the Lyapunov exponent for this data and plotted it in Fig. 11(b). The value of the Lyapunov exponent for the United Kingdom is 0.029 which is close to the value of Türkiye.

Obtaining embedding dimensions and delay times for all sub-series for United Kingdon's Covid-19 data given in Fig 1(d). We separately plotted the phase space diagrams for these sub-time series in Fig 12. As can be seen from Fig 12 while the orbits are more distinct in the first two panels, however, the orbits are intertwined in the others. To reveal the dynamics of the sub-time series, Lyapunov exponents were calculated separately and given in Fig 13.

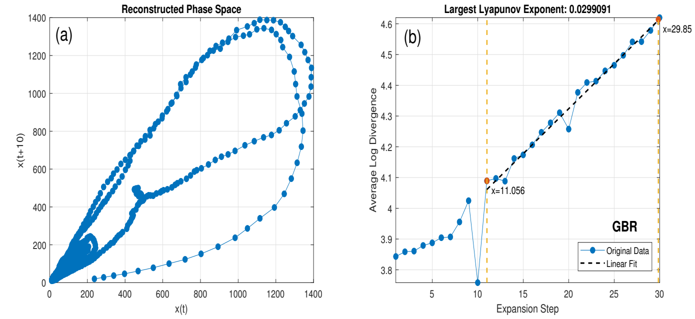

Figure 11 COVID-19 data set of United Kingdom's reported deaths time series between 08.03.2020 − 03.12.2022. Embedding dimension $d = 3$ and time delay $\tau = 10$. In (a) phase space representation, in (b) Lyapunov exponent.

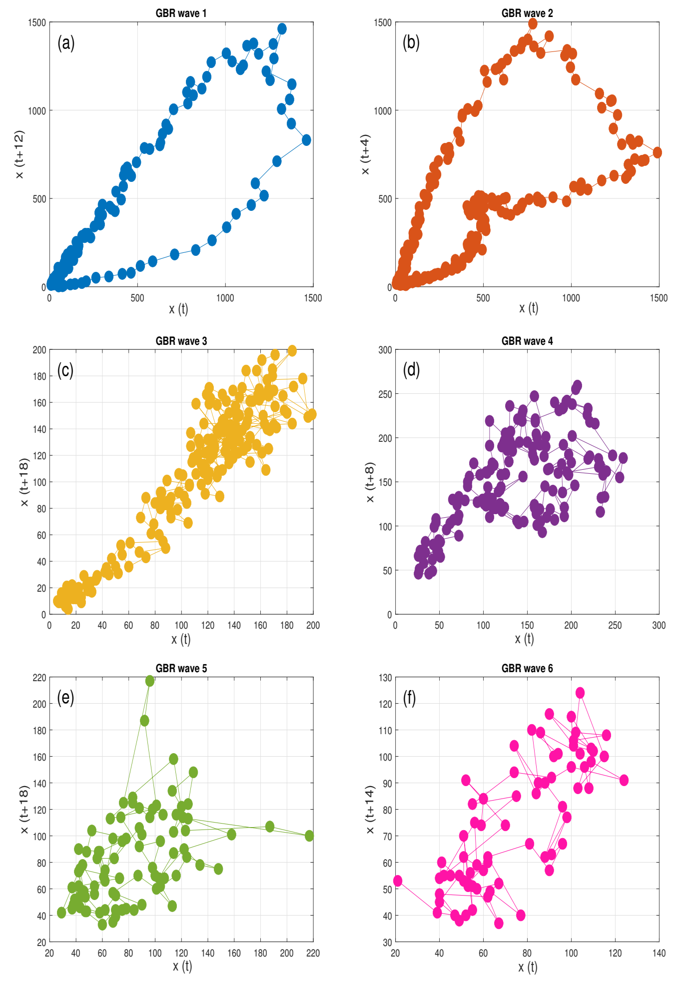

Figure 12 Reconstructed Phase Space of United Kingdom's waves. In (a) 1^{st} wave, in (b) 2^{th} wave, in (c) 3^{th} wave, in (d) 4^{th} wave, in (e) 5^{th} wave, in (f) 6^{th} wave.

Surprisingly, one can see that all sub-time series of the United Kingdom have a negative Lyapunov exponent. While the entire series is chaotic, the subseries behave as quasi-periodic. These results are similar to Türkiye's results.

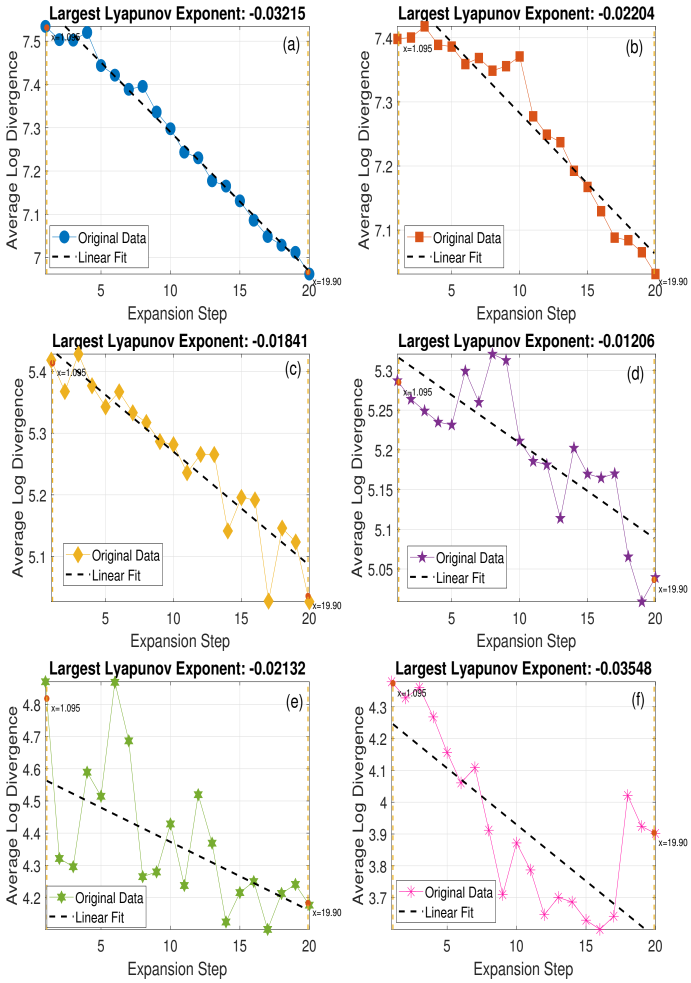

Figure 13 Largest Lyapunov exponent of United Kingdom's waves. In (a) 1^{st} wave, in (b) 2^{th} wave, in (c) 3^{th} wave, in (d) 4^{th} wave, in (e) 5^{th} wave, in (f) 6^{th} wave.

CONCLUSION

As we mentioned in the introduction, it is very difficult to predict and make predictions about the course of the pandemic due to reasons such as its multi-parameter-dependent dynamics, the emergence of new variants, and the impact of vaccine applications. So far, it has been possible to obtain limited information about the course of the pandemic through model-based or statistical analysis-based studies. The most important possible reason for this may be that the pandemic dynamics are chaotic. Therefore, in this study, to see the presence of chaotic patterns in the Covid-19 data, we analyzed the Covid-19 mortality data of Türkiye, Germany, Italy, and the United Kingdom for three years by using the data of the WHO.

We plotted phase space diagrams of three-year mortality data of four countries and obtained Lyapunov exponents. We found positive Lyapunov exponents for all countries, which indicates phase space trajectories of the Covid-19 data are chaotic. These significant numerical results support the studies that suggest that the Covid-19 pandemic has chaotic dynamics. On the other hand, we considered the subset of data corresponding to the spreading peaks of mortality data in the time interval for three years.

Surprisingly, we found that some of the sub-time series of these countries exhibit chaotic or quasi-periodic behavior. This interesting result was reported for the first time in this study. This reveals that there may be quasi-periodic -*weak*- regimes within a chaotic time series. These findings are important for a more detailed understanding of epidemics with chaotic spread dynamics.

If we summarize the results, analysis has revealed that while the Covid-19 epidemic in Türkiye was chaotic over three years, however, no peak that emerged in this period was chaotic. For example, the situation is quite different in Germany. While the three-year data in Germany behaves chaotically, it can be seen from the figure that all independent peaks in this time interval are also chaotic.

The situation in United Kingdom is the same as in Türkiye. As can be seen from the figure, all peaks are chaotic. However, in the Italy, the second and third peaks are periodic or quasi-periodic, while the others are chaotic. As is known, positive Lyapunov exponents indicate that the series behaves chaotic. In the analysis, we saw that time series that behave chaotically take different positive values. These values can be thought to reflect the degree of chaoticness of the system.

As a result, by analyzing Covid-related deaths from four countries, we showed that the series is chaotic as seen as seen Figs.2(b), 5(b), 8(b) and 11(b). In this sense, our results are compatible with the results obtained in the previous studies (Jones and Strigul 2021; Borah *et al.* 2022; Abbes *et al.* 2023; Russell *et al.* 2023; Sapkota *et al.* 2021; Gonçalves 2022). However, unlike previous Covid-19 studies, we also found out that there are chaotic, periodic or quasi-periodic sub-series within these chaotic time series. These new and novel results are reported for the first time in this study. Here we analyzed data from four countries, however, one can estimate that the time series of Covid-19 in the other countries have similar dynamics.

It can be assumed that a pandemic is a catastrophic event that occurs within a complex system (Aydiner 2020). Therefore, by its nature, the pandemic is expected to be chaotic. Indeed, it has been confirmed in the present study and previous studies that the Covid-19 pandemic is chaotic (Jones and Strigul 2021; Borah *et al.* 2022; Abbes *et al.* 2023; Russell *et al.* 2023; Sapkota *et al.* 2021; Gonçalves 2022). However, it is interesting to find periodic or quasi-periodic regimes in chaotic time series. For example; all sub-series of Türkiye and United Kingdom in Figs.4 and 13 are quasi-periodic, not chaotic.

Similarly, two sub-series for Italy in 10(b) and (c) are also quasi-periodic. Quasi-periodic regimes may indicate that the correlations between daily mortality values goes to zero which means daily mortalities are relatively independent each other.

Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical standard

The authors have no relevant financial or non-financial interests to disclose.

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