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Teacher Practices and Student Understandings in 2D Drawings of 3D Shapes

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Abstract

Geometry holds a pivotal position within the realm of mathematics, yet many students struggle to grasp its concepts, posing challenges for their teachers. In the Republic of Türkiye, the Ministry of National Education has recently recommended the adoption of constructivist approaches to teach geometry. To gauge the efficacy of this instructional approach in actual classrooms, we investigated the practices of Turkish middle school mathematics teachers and their students' experiences in constructing two-dimensional drawings of three-dimensional geometrical shapes during regular lessons. The study involved four teachers from two schools and 199 students aged 12 to 14 years. Multiple data sources, including lesson observations, student worksheets and interviews with teachers and students, were utilized to gain insights into student understandings and whether the pedagogical practices they encountered were associated with this. Based on these findings, we argue that teachers may require additional support for their classrooms to transition into student-centred environments to permit students to actively engage in enriching mathematical exercises, exercise agency, and have opportunities to utilize provided manipulatives and digital tools.

Keywords: Constructivist Approaches, Geometry Education, Middle School Mathematics, Student Experiences, Three-Dimensional Geometrical Shapes, Two-Dimensional Drawings

Öğretmen Uygulamaları ve Öğrenci Anlayışları: 3B Şekillerin 2B Çizimleri Üzerine Bir İnceleme

Öz

Geometri, matematik alanında önemli bir konumu kaplamakla birlikte, birçok öğrenci geometrik kavramları anlamakta zorlanmaktadır, bu da öğretmenler için zorluklar doğurmaktadır. Türkiye Cumhuriyeti Milli Eğitim Bakanlığı, geometriyi öğretmek için yapılandırmacı yaklaşımların benimsenmesini önermiştir. Bu çalışmada, bu öğretim yaklaşımının sınıflardaki etkinliğini değerlendirmek amacıyla, Türk ortaokulu matematik öğretmenlerinin uygulamaları ve düzenli derslerde öğrencilerinin üç boyutlu geometrik şekillerin iki boyutlu çizimlerini oluştururken yaşadıkları deneyimler araştırılmıştır. Çalışmaya iki okuldan dört öğretmen ve 12-14 yaş arasındaki 199 öğrenci katılmıştır. Ders gözlemleri, öğrenci çalışma kağıtları ve öğretmenler ve öğrencilerle yapılan görüşmeler gibi çeşitli veri kaynakları, öğrenci anlayışlarına ve bu anlayışların ilişkilendirildiği pedagojik uygulamalara dair içgörüler elde etmek için kullanılmıştır. Bu bulgulara dayanarak, öğretmenlerin sınıflarında öğrenci odaklı ortamlara geçebilmeleri için ek desteklere ihtiyaç duyabilecekleri savunulmaktadır. Bu, öğrencilere matematiksel alıştırmalara etkin bir şekilde katılmalarına, etki alanı oluşturmalarına ve sağlanan manipülatifleri ve dijital araçları kullanma fırsatlarına sahip olmalarına izin vermek için gereklidir.

Anahtar kelimeler: Yapılandırmacı Yaklaşımlar, Geometri Eğitimi, Ortaokul Matematiği, Öğrenci Deneyimleri, Üç Boyutlu Geometrik Şekiller, İki Boyutlu Çizimler

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INTRODUCTION

Learning about three-dimensional (3D) shapes is widely recognized as a valuable way to enhance students' reasoning and spatial skills, enabling them to compare, manipulate and transform mental images to solve problems (e.g., Battista, 2007). Hence, geometry education programs at the national level, including those in the United States (Common Core State Standards Initiative, 2023), England (Department for Education, 2021), and Türkiye (Ministry of National Education [MoNE], 2024), emphasize the understanding of 3D shapes. However, research suggests that mastering this aspect of geometry poses significant challenges for students and their teachers in middle school (Bakó, 2003; Pittalis & Christou, 2013). This difficulty is reflected in national and international exam results from multiple countries, where students often exhibit suboptimal performance in questions related to the understanding pertaining to characteristics of shapes in both two and three dimensions (Brown et al., 1988; Department for Education, 2019; National Centre for Education Statistics, 2011, 2018; OECD, 2019; Standards and Testing Agency (STA), 2019a, 2019b).

This is not just a recent phenomenon. Over the past three decades, numerous studies have consistently revealed that students typically perform lower than anticipated in geometry, indicating an ongoing challenge (Alghadari et al., 2020; Bhagat & Chang, 2015; Clements, 2003; Clements & Battista, 1992). This situation appears to be stagnant or even deteriorating.

Our crucial aspect of middle school geometry that has been particularly challenging for students is the task of drawing 2D representations of 3D shapes. This difficulty has been observed across multiple countries and persists despite years of research efforts (Cooper & Sweller, 1989; Kali & Orion, 1996; Widder & Gorsky, 2013). Existing research highlights several difficulties students encounter when learning to draw 2D representations of 3D shapes. Studies have consistently shown that students often struggle with mentally visualizing 3D shapes from 2D diagrams, which hinders their ability to draw accurate representations (Pittalis & Christou, 2013). Many students treat 2D representations as if they were themselves 2D, leading to poor performance on coding and decoding tasks (Pittalis & Christou, 2013). Additionally, students may lack the necessary visualization skills to mentally manipulate and perceive 3D shapes based on their 2D representations (Kali & Orion, 1996). These difficulties often result in errors related to improper visualization, incorrect reasoning about cube properties, or both (Fujita et al., 2017).

Students have also been found to face challenges in understanding the conventions and techniques of drawing 2D representations of 3D shapes. They can struggle with using standard tools like isometric drawing paper or fail to grasp the geometrical properties and nets of 3D shapes (Pittalis & Christou, 2013), leading to misconceptions and errors in their drawings.

Recent research has built on this to try and understand the factors that might lead to these difficulties in representing 3D shapes in 2D. Fujita et al.'s (2017) results aligned closely with the four types of student behaviour identified by Pittalis and Christou (2013): attempting to answer questions, responding intuitively or by using the visual data, judging questions as if they were two-dimensional, and being aware of the 3D representation but failing to provide the correct response. These problems can persist in more advanced 3D geometry, as when Fujita et al. (2017) asked students to calculate the diagonal length of a shape drawn inside a cube, common errors included highlighting improper visualization, improper reasoning about cube properties, or a combination of both as the primary causes of the incorrect responses. Overall, the existing research suggests that challenges faced by students when grasping 2D representations of 3D shapes stem from issues with visualization, conceptual understanding, and familiarity with conventions and techniques.

Teachers' beliefs significantly influence the way they conduct their classrooms, and investigating their beliefs regarding teaching about 2D representations of 3D shapes is important to build a full view of geometry teaching and learning (Cross, 2009; Kuzniak & Rauscher, 2011). Existing research suggests that teachers often express a lack of understanding of 3D geometry and perceive it as a challenging topic to teach (see Bakó, 2003; Christou et al., 2006; Parzysz, 1988). Consequently, some teachers avoid teaching it altogether or resort to directed teaching methods rather than student-centered practices (Bakó, 2003). This widespread overall avoidance and instructional approach has been implicated in the difficulties that students face in visualizing 3D shapes from their 2D drawings (Christou et al., 2006; Widder & Gorsky, 2013). When teachers lack a strong understanding of 3D geometry or confidence in teaching it, they may struggle to guide students effectively in developing the necessary visualization skills.

Consequently, our study aims to build upon this prior research that has examined students' understanding and teachers' beliefs related to 2D drawings of 3D shapes in middle school settings to establish possible connections between students' performance and teachers' instructional practices. By relating the two, we hope to gain a deeper understanding of the challenges students face in visualizing 3D shapes from 2D representations and what teachers can do to help them. Thus, the findings from this study will further inform the development of strategies to promote student-centered, conceptually rich learning experiences in geometry classrooms in Türkiye.

Geometry Education in the Researched Context

In Türkiye, the teaching and learning of geometry, particularly 2D and 3D shapes, is a key component of the middle school curriculum. Government policy outlines specific objectives that students are expected to achieve within a limited number (4 to 6) of class hours (Ministry of National Education, 2024), including drawing the orthogonal views (from the top, front, left and right) of 3D shapes constructed from unit cubes, and constructing an isometric drawing corresponding to given orthogonal drawings from the top, front, left, and right (the use of an isometric paper is suggested).

Students' performance is subsequently assessed through a national examination conducted at the conclusion of each academic year.

Both students and teachers have expressed dissatisfaction with the perceived difficulty of the test questions in the national assessment, resulting in poor performance that falls below expectations. Consequently, it appears that the current system is not functioning as intended. Studies have revealed that mathematics teachers in Türkiye tend to adopt a teaching-to-test approach and steer clear of approaches that place students at the forefront (Doruk, 2014; Karaagac & Threlfall, 2004; Saralar-Aras, 2022), which can be attributed to the evaluation system that judges teacher effectiveness based on students' outcomes (Konan & Yılmaz, 2017, 2018). Despite Turkish mathematics teachers expressing belief in the effectiveness of the updated Turkish mathematics curriculum and their reported support for the emphasis on student-centred activities and technology integration, these practices are challenging to observe in natural classroom contexts (Bayrakdar-Çiftçi et al., 2013; Tekalmaz, 2019; Zengin, 2023). Research indicates that Türkiye, like many countries, advocates for a constructivist approach to geometry education. However, the extent to which this approach has been enacted/implemented in Turkish classrooms and its influences/impact on teaching practices and learning outcomes in the teaching of 3D shapes remain unclear.

Constructivism in Türkiye's Mathematics Classrooms

Constructivism has had a significant impact on how mathematics is taught worldwide. Although not without its critics (e.g. Kirschner et al., 2006), many believe that teaching mathematics should not be considered as transfer from teachers to students of mathematical relations, rules, and symbols. Instead, finding a correct answer in mathematics is interpreted as a method emphasising logical thinking, reasoning, and critical problem-solving over rote memorization of algorithms (Simon & Schifter, 1991). Students are seen as learning how to develop a mathematical theory, assess its mathematical correctness, and discover values, processes, and data (Schrifter & Fosnot, 1993). Built upon the notion that students should actively participate in creating mathematics, constructivism tries to help students develop their own ideas through interactions with their classmates, their culture, and tangible objects (Brooks & Brooks, 1999). Constructivism encourages students to apply mathematics to real-world situations because it links mathematics to everyday life (Fosnot & Perry, 1996).

In Türkiye, the new curriculum's emphasis on constructivism, its theoretical significance and purpose are still debated. In some Turkish policy documents, constructivism is often associated with a student-centered approach to education. This means that the focus is placed on addressing the requirements, interests, and capabilities of the students; moreover, instruction is tailored to individual or group needs. In others, constructivism often emphasizes the importance of integrating learning with real-world situations. Learning experiences that are relevant and meaningful to students are believed to enhance understanding and retention of information. However, one undeniable issue with its implementation in the classroom is that it was prescribed independently of other fundamental elements of the educational system. The readiness and suitability of other agents, such as teachers and learning environments for a constructivist mathematics approach, were not considered (Gür & Çelik, 2009). Teachers and schools were insufficiently prepared to incorporate it and achieve the desired outcomes, and numerous studies have found that teachers have difficulties in setting up constructivist learning settings in Turkish classrooms (e.g. Bayraktaroglu, 2011; Cengizhan & Koç, 2017). For example, applying a constructivist approach in the assessment required much more time than teachers had available (Tuncel & Kazu, 2019).

Moreover, the new constructivist curriculum recommends (almost mandates) that technology is used to teach 3D shapes, changing the resources teachers use to teach and their students use to learn in relatively radical ways. Thus, it could be argued that altering the national education curriculum by drawing inspiration from other countries without carrying out in-depth research that explores constructivism in Türkiye is unlikely to be successful. Even more so, if in-service teachers are not given the necessary support to understand and adopt new practices with new resources, they may struggle to effectively incorporate these changes into their teaching.

Consequently, a key focus of our study on the relationship between classroom practices and student understanding of middle school geometry was to explore the extent that recommended constructivist approaches with new technologies were in use in the classrooms.

Originality of the Study

The originality of this study lies in its nuanced exploration of the relationship between teacher practices and student understandings in the context of middle school geometry education in Türkiye. Focusing specifically on the implementation of constructivist approaches recommended by the Ministry of National Education, the study delves into the intricacies of how these pedagogical strategies manifest in actual classrooms. By employing a multi-case study design with a diverse set of teachers and students, the research adds depth to the understanding of the challenges students face in constructing two-dimensional drawings of three-dimensional geometrical shapes. The incorporation of national exam-based questions in the assessment tools enhances the study's relevance and provides insights into the practical implications of classroom practices on students' performance in standardized assessments. Furthermore, the examination of the use of manipulatives and digital technologies, along with the identification of underlying difficulties faced by students, contributes to the broader discourse on effective geometry education. In essence, this study brings a novel perspective to the examination of geometry instruction by intricately examining the interplay between pedagogy, student experiences, and learning outcomes within the unique educational landscape of Türkiye.

The Significance of the Study

The significance of the study lies in its contribution to the understanding of the challenges and potential improvements in middle school geometry education, specifically focusing on the representation of three-dimensional (3D) shapes through two-dimensional (2D) drawings. The following points highlight the significance of the study:

Firstly, the study is conducted in the context of Türkiye, where the Ministry of National Education has recommended the adoption of constructivist approaches in teaching geometry. By investigating the actual practices in middle school mathematics classrooms, the study provides insights into the alignment between recommended approaches and the reality of instruction. The findings can inform educational policies, suggesting potential areas for improvement in implementing constructivist methods. Secondly, understanding the challenges students face in accurately representing 3D shapes in 2D drawings is crucial for enhancing geometry instruction. The study identifies specific errors and underlying difficulties, such as mental visualization and drawing translation. This knowledge can guide curriculum development, teacher training programs, and the creation of resources that address these challenges, ultimately improving students' geometric reasoning and representation skills.

Furthermore, the study explores the extent to which recommended constructivist approaches with new technologies are actually in use in the classrooms. By highlighting any disparities between policy recommendations and actual instructional practices, the study contributes to bridging the gap between educational policies and their implementation. This insight is valuable for education stakeholders, including policymakers, administrators, and curriculum developers. Moreover, the findings of the study indicate that teachers may require additional support to transition their classrooms into student-centered environments that align with constructivist principles. This emphasizes the need for targeted professional development programs for mathematics teachers. Insights into the specific challenges and instructional gaps can guide the design of effective training initiatives, empowering teachers to implement more student-centered and technology-integrated practices.

In addition to these, the study highlights the underutilization of digital technologies in geometry instruction despite their recognized potential. This information can guide efforts to optimize the integration of technology tools, such as 3D geometry software, into the classroom. Recommendations for incorporating dynamic software for constructing and manipulating polycubes can contribute to more effective technology integration strategies. Last but not least, the emphasis on the need for student engagement with manipulatives and the challenges associated with a rule-based, exam-focused pedagogy points towards the importance of creating student-centered learning environments. The study advocates for a shift in instructional practices that allow students to actively engage in enriching mathematical exercises, exercise agency, and utilize provided manipulatives and digital tools.

In conclusion, the significance of this study lies in its potential to inform educational policies, improve instructional practices, and contribute to the professional development of teachers in Türkiye and potentially in other contexts facing similar challenges in middle school geometry education. The findings have practical implications for creating more effective, student-centered, and technology-enhanced learning experiences in the realm of geometry.

METHOD

The study employed a multi-case study approach to investigate the instructional practices of mathematics teachers and the corresponding learning outcomes of their students in relation to representations of three-dimensional (3D) shapes. Two public middle schools and two teachers from each school participated. Geometry education, particularly the representation of three-dimensional shapes through two-dimensional drawings, is known to be a complex area of study. By adopting an explorative approach, the research aims to delve deeply into the multifaceted challenges faced by both teachers and students in this specific aspect of geometry. The explorative nature allows for a nuanced understanding of the intricacies involved.

Research Design

A multi-case study as described by Yin (2009) is well-suited for understanding the pedagogical practices employed by middle school mathematics teachers. This approach enables the researchers to observe, analyze, and compare the practices of different teachers in distinct contexts; it allows for the identification of commonalities and variations in instructional strategies, shedding light on the ways teachers approach the teaching of 3D shape representations (Aberdeen, 2013; Yin, 2009). Moreover, this design enables the collection of rich qualitative data through various sources such as lesson observations, student worksheets, and interviews (Yin et al., 2012). This depth of data allows for a thorough analysis of the complexities inherent in teaching and learning geometry, providing a comprehensive picture of the phenomena under investigation. Thus, this study aimed to address the following research questions to gain insights into the students' performance in representing 3D shapes, the underlying errors, and the pedagogical practices employed in the classroom:

How do students perform when they are tasked with representing 3D shapes, and what specific errors do they make?

What are the underlying misunderstandings or difficulties that contribute to these errors?

What are the pedagogical practices employed in the classroom, and how do they align with the recommended constructivist approach? How might these practices be related to students' performance?

The first question sought to assess the overall performance of students in accurately depicting 3D shapes and identify the specific errors they encountered during this process. By examining the underlying factors that led to the observed errors, the second question intended to identify the specific misunderstandings or challenges students face when working with representations of 3D shapes. Finally, the third question strived to investigate the instructional strategies and approaches teachers use in teaching 3D shape representations, specifically focusing on their alignment with the constructivist approach. Furthermore, it sought to explore the potential relationship between these pedagogical practices and students' performance outcomes. By addressing these research questions, the study aimed to provide valuable insights into the performance, errors, underlying difficulties, and instructional practices related to representing 3D shapes.

Context and Participants

Each school consisted of three to four grade-seventh classes, with an average of 25 to 30 students per classroom. The schools were selected based on their possession of the necessary tools and technology to support the study. All participating students, their parents and teachers provided informed consent before their involvement in the study. The student sample comprised 199 students, including 107 girls and 92 boys, within the age range of 12 to 14 years. Students answered worksheet questions derived from national assessments (section 2.2) and were observed within their respective classrooms. 16 students from this sample (8 girls and 8 boys) took part in an artefact-based interview, enabling an in-depth exploration of their thoughts on the lessons, as well as their thinking processes and solution strategies. They were randomly selected from those who had correctly answered a minimum of two out of ten worksheet questions, ensuring sufficient knowledge to respond to the interview questions, but beyond that, were not stratified by score.

The four teachers involved in the study are referred to pseudonymously as Mr Aras, Ms Ayaz, Ms Onay, and Ms Semin. Their experience of middle school mathematics teaching ranged between 4 to 13 years.

Data Collection

The research employed several methods to generate data, including lesson observations, student worksheets, and artefact-based interviews. The following sections provide an explanation of the methods and the corresponding instruments used.

Lesson Observations

A total of 16 lessons, with four classes each having four lessons, were observed by the first author during regular mathematics classes. The focus of the observations was on the teaching of orthogonal and isometric drawings of cube representations. Before the target lessons, the researcher attended four hours of the teachers' lessons to establish familiarity with both teachers and their students. During the target lessons, the researcher occupied a seat in the rear of the classroom, documenting observations through field notes every 5 minutes. The field notes were structured using an observation protocol with descriptive and reflective observation fields (Creswell, 2007), capturing information about the classroom environment, student and teacher actions. Additionally, the instructional materials employed in the lessons, such as presentations, activity sheets, and pages from books, were gathered and documented.

Students' Works

After the final lesson, students completed a worksheet in two further forty minutes lessons. The worksheet was based on national exam questions and consisted of five orthogonal and five isometric drawings. Orthogonal drawings required students to create four drawings that encompassed four views: top, front, left, and right on a given isometric representation of a cube. In isometric drawing tasks, students were prompted to generate a single isometric representation from given top, front, left, and right views. Sample questions from each type, along with possible correct answers, are provided in Figures 1 and 2. They are the third questions from five and are considered to have medium levels of difficulty.

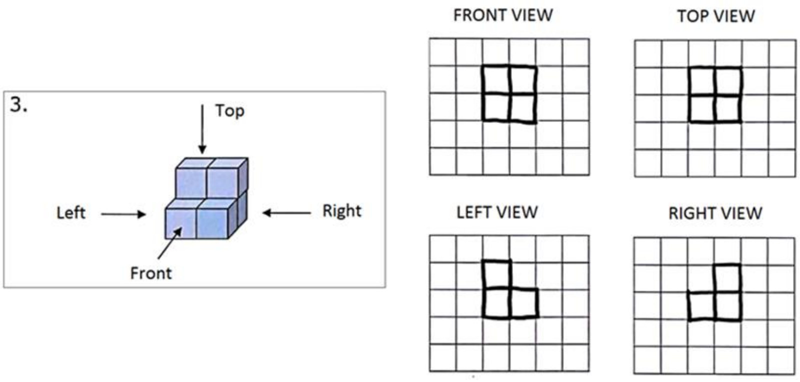


Figure 1. *Q-3 in orthogonal drawings and its possible correct answer*

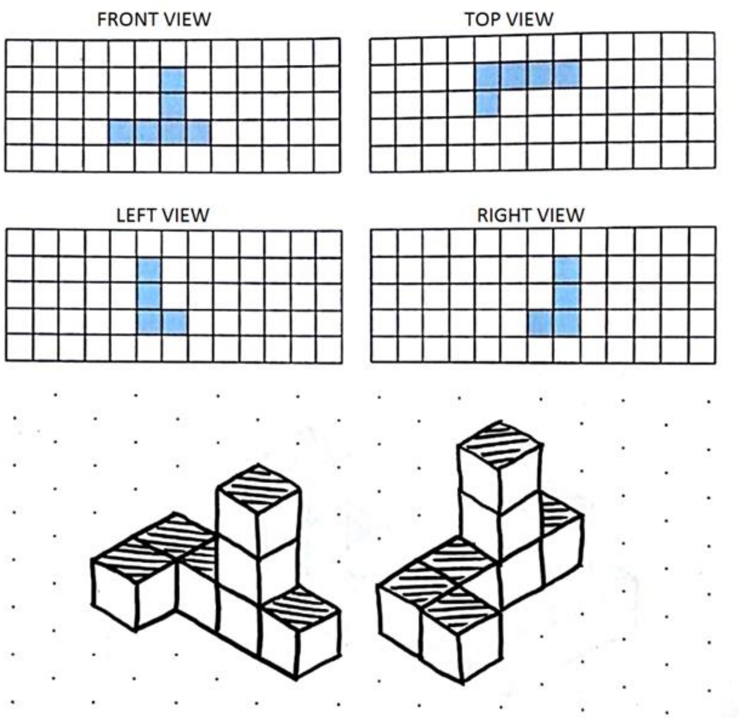


Figure 2. *Q-3 in isometric drawings and its two possible correct answers*

This worksheet had been previously piloted with the students of the same age and necessary modifications had been made to ensure clarity of the questions. The use of national exam questions as the foundation for the worksheet in the study investigating teacher practices and student understandings in 2D drawings of 3D shapes is justified for multiple reasons. Firstly, it ensures alignment with the national curriculum, reflecting the educational objectives set by Türkiye's education authorities. By assessing students' proficiency in representing 3D shapes, the study evaluates core skills outlined in the curriculum. The questions' consistency in difficulty level and format allows for valid comparisons between students, classes, and schools, providing a standardized assessment. Additionally, the policy relevance is heightened as the study explores the effectiveness of current instructional approaches in preparing students for assessments mandated by the education system. The use of national exam questions establishes a link between classroom practices, student understanding, and broader educational standards, providing insights into how well students meet national expectations. While practical and accessible, researchers should remain mindful of potential limitations, such as the influence of exam-oriented content on instructional practices.

Interviews

Individual interviews were conducted to explore students' perspectives on the lessons and their understanding of the subject matter. They took place after the completion of the worksheets and lasted between 15 and 30 minutes. Interviews had two phases. In the first phase, students were asked about their opinions and experiences regarding 2D representations of 3D shapes, including their perceived difficulty and suggested improvements. They were also asked to explain what they had learned about 3D shapes and how they applied the knowledge during the lessons. Questions (translated) asked included:

- "For the last four lessons, you have been working on 2D representations of 3D shapes. Was this topic more difficult compared to other topics in mathematics? Let's assume a scale where one is "too easy", and ten is "too difficult." Where would you place this topic on the scale?
- You mentioned that you find it difficult to learn this topic. Why do you think it is challenging? What changes do you think could make this topic easier?
- If you were to explain what you have learnt about 3D shapes to someone who has not studied this unit yet, what would you say?
- What advice would you give to the next year's class regarding the relationship between the left and right views of the same 3D shape? How did you use this information during the past lessons?"

The second phase aimed to understand students' reasoning behind their incorrect answers on the worksheets. Interview questions were based on the completed worksheets, and students were asked to explain their methods and strategies. In addition to audio recording, notes taken during the interviews included any relevant actions or gestures (e.g., pointing to a specific part of a shape).

Data Analysis

Observations and interviews

A thematic coding strategy was employed to analyze the observations and interviews, aiming to identify patterns in data and report significant findings (Taylor & Ussher, 2001). The analysis process followed the recommended steps by Marshall and Rossman (2011), which involved organizing and coding the data, establishing themes, assessing comprehension and exploring other interpretations and compiling the study findings.

Each interview transcript was carefully reviewed, analyzed and integrated with the data from lesson observations. To ensure a comprehensive understanding, each transcript was read multiple times. Initial coding was performed to capture significant statements and ideas. These codes were then grouped into broader categories, which were refined into themes. During this process, peer debriefing sessions were conducted to review and verify the codes and themes. Member checks were also employed, where participants reviewed the findings to confirm their accuracy and relevance. That is to say, to enhance validity, peer evaluation and member checks were utilized. The researcher was mindful of the potential reactivity, where participants may exhibit different behaviors due to being observed. Table 1 showcases themes, codes and subcodes derived from the study.

Table 1. Themes, Codes and Subcodes of the Study

Themes	Codes	Subcodes
Student difficulties	Visualization	
	Drawing	
Teaching practices	Use of manipulatives	Teacher use
		Student use
	Use of technology	Teacher use
		Student use
	Exam-focused pedagogy	

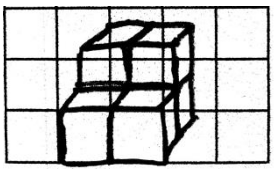
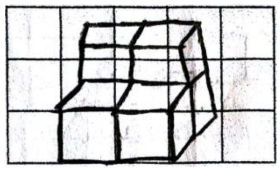
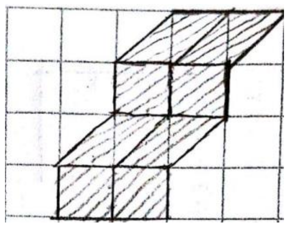
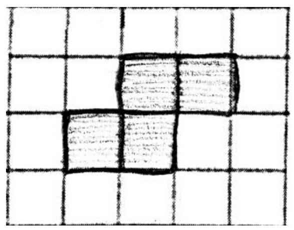
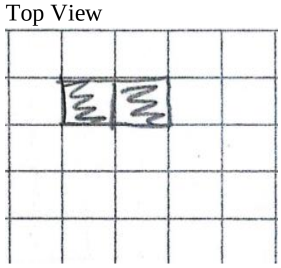
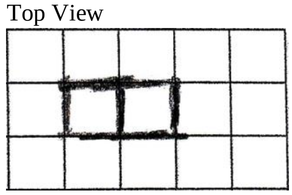
Students' Works

Regarding the students' work, a rubric was developed for each question, outlining all possible correct drawings. The researchers then coded the completed worksheets. No points were assigned for incorrect or missing answers. For each correct aspect of an item (e.g., front, top, left, and right views), one point was given. Therefore, both orthogonal and isometric drawing questions were scored out of 4, resulting in a maximum score of 20. Additionally, the worksheets underwent coding to identify the nature of the errors, which are described and exemplified in Figures 3 and 4. The coding scheme was informed by Fujita et al.'s (2017) and Pittalis and Christou's (2013) codes and modified to suit orthogonal and isometric polycube drawings. The relationship between the codes adopted in this study and these earlier ones is given below, where the codes in brackets indicate the one that would be the one assigned by Pittlis and Christou (2013) first and Fujita et al. (2017) second.

- E1: Redrawing the given shape as the front or a part of it as a side view (judging the 3D shapes as if they were two-dimensional/ two-dimensional)
- E2: Drawing cubes at the back to another column (being aware of the 3D representation but unable to provide the correct answer/implicit conventional)
- E3: Drawing the part only at the very front of the shape (answering based on intuition or using visual data /intuitive)
- E4: Swapping the left and right views (answering based on intuition or using visual data/intuitive)
- E5: Drawing the view upside down (answering based on intuition or using visual data /intuitive)
- E6: Redrawing the provided shapes (judging the 3D shapes as if they were two-dimensional/ two-dimensional)
- E7: Attempting to combine orthogonal views side by side (answering based on intuition or using visual data /intuitive)
- E8: Drawing only one view at a time instead of combining all views (answering based on intuition or using visual data /intuitive)
- E9: Encountering difficulties in establishing links between different components of the shape (being aware of the 3D representation but unable to provide the correct answer /implicit conventional)

An error coding scheme was employed to describe the prevalent errors made by students.

Figure 3 (orthogonal) and Figure 4 (isometric) provide a definition of the error along with two example responses that illustrate it (from Question 3 on both worksheets.).

Description	Example student mistakes	
Redrawing the given shape as the front or as a part of it as a side view (E1)	Front View 	Front View 
Drawing the cubes at the back onto another column (E2)	Front View 	Front View 
Drawing only the part at the very front of the shape (E3)	Top View 	Top View 
Swapping the left and right views (E4)	Left View	Left View

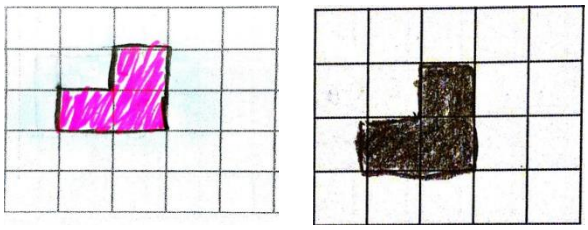
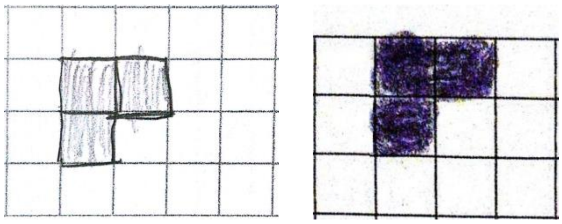
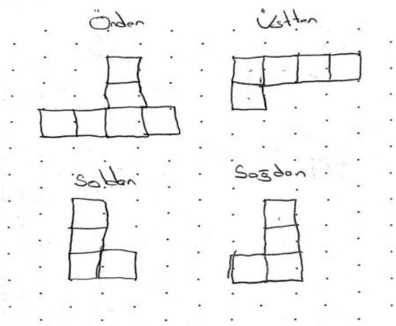
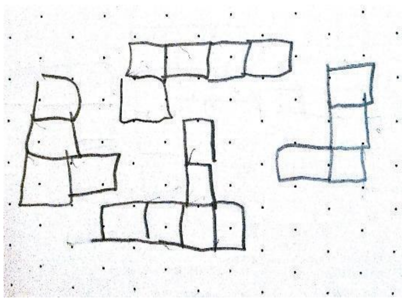
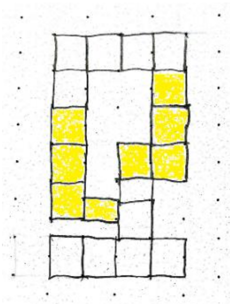
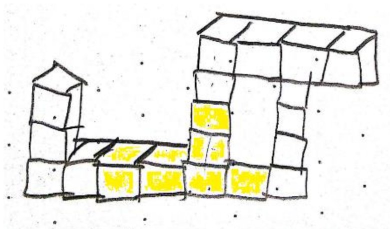
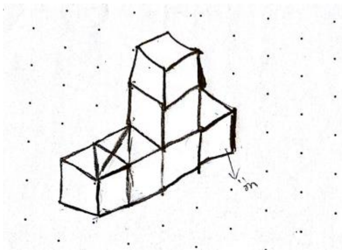
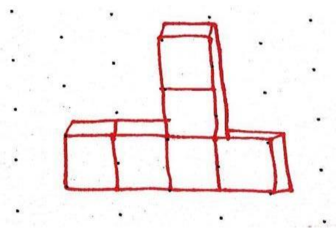
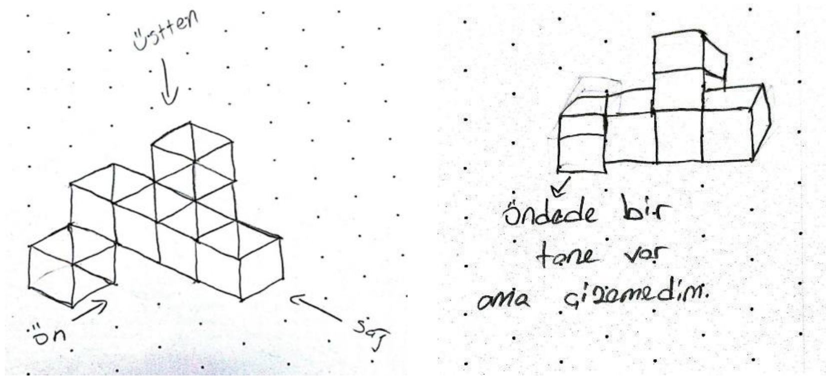
		
	Left View	Left View
Drawing the view upside down (E5)		

Figure 3. *Orthogonal drawing errors*

Description	Example Student Mistakes	
Redrawing the provided shapes (E6)		
Attempting to combine the orthogonal views side by side (E7)		
Drawing only one view at a time instead of combining all views (E8)		

Encountering difficulties in establishing links between different components of the shape (E9)



[a]Önde de bir tane var ama çizemedim (Turkish). = There is one more (cube) in the front but I couldn't draw it (English).

Figure 4. *Isometric drawing errors and linking problems*

A second coder independently coded 20 worksheets selected randomly. The agreement between the coders was assessed, and a very high level of agreement was found (Kappa=0.9, p<.001). Furthermore, an ANOVA was conducted to examine the effect of question type (orthogonal and isometric) on test performance. Detailed results of this analysis are reported in the Results section.

Research Ethics

This study was conducted within the guidelines of the University of Nottingham's Code of Research Conduct and Research Ethics and the School of Education in particular which has adopted Revised Ethical Guidelines for Educational Research (BERA, 2011). Before the submission of the ethics documents, the first researcher needed to obtain a Disclosure and Barring Service (DBS) check to have access to schools and received it with F0104171462 reference number. All information collected was anonymised, confidential and only available to the researcher and her supervisors. Pseudonyms were used throughout the studies to replace teachers' and students' real names.

Particular ethical issues related to the study and ethical permission numbers from the University of Nottingham are noted at the end of the article.

FINDINGS

The findings are organized into three sections. The initial section provides an analysis of students' performance on worksheets, examining their errors when generating orthogonal and isometric drawings of 3D shapes. Subsequently, challenges inherent in the task, potentially contributing to these errors, are outlined. Lastly, an account is provided concerning existing classroom practices that may have influenced the students' misunderstandings and difficulties.

Student Performance and Errors

Question 1: How do students perform when they are tasked with representing 3D shapes, and what specific errors do they make?

An ANOVA was conducted to examine the effect of question type (orthogonal and isometric) on test performance, as shown in Table 2. This analysis revealed a single significant effect of question type: $F(1,198)=265.255$, $p<.001$, $\eta_p^2 = .574$. Specifically, students demonstrated higher performance on orthogonal drawings.

Table 2. Test Scores for Orthogonal and Isometric Drawing

	Orthogonal drawing (/20)		Isometric drawing (/20)	
n	*M*	*SD*	*M*	*SD*
199	11.92	6.31	5.27	5.78

Table 3 and Table 4 provide a detailed breakdown of these errors, including the number and percentages of each error type for orthogonal and isometric drawings. The "students" column denotes the number of students who exhibited each error (e.g., 50 students answered Q1 incorrectly). It should be noted that the sum of the number of errors may not equal the total number of students who made these errors (e.g., 61 ≠ 50 for the Q1), as students could make multiple types of errors per question.

Table 3. Number of Errors for Orthogonal Drawing Questions by Error Type and Question

	Students	E1	E2	E3	E4	E5
Q1	50	29	0	20	N/A	12
Q2	124	26	38	56	N/A	20
Q3	106	22	13	47	25	5
Q4	117	13	37	74	7	6
Q5	151	15	45	29	15	34
Total		105	133	246	47	77

Table 4. Number of Errors for Isometric Drawing Questions by Error Type and Question

	Students	E6	E7	E8	E9
Q1	56	20	0	22	17
Q2	136	31	33	43	34
Q3	131	32	18	55	33
Q4	115	32	24	41	27
Q5	155	29	39	30	39
		144	114	191	150

These tables demonstrate that, as intended, the difficulty level increased with each subsequent question. Generally, Question 1 had the fewest errors, and by Question 5, most of the students could not answer it correctly. Furthermore, no single question was uniquely associated with a specific error type.

For orthogonal drawings, the most common error involved drawing only the front view of the shape, accounting for 40% of all coded errors. On the other hand, drawing an upside-down view constituted only 8% of the errors. In contrast, errors in isometric drawings were more evenly distributed among the four coded error types.

Challenges in Tasks Leading to Errors

This section addresses the challenges inherent in the task that may contribute to the errors discussed in Section 3.1, highlighting their connection to students' difficulties. Generally, it is evident that many students encountered difficulties in recognizing, imagining, and constructing 2D representations of 3D shapes. Over 2/3rds of the interviewed students expressed trouble understanding the topic, rating its difficulty between five and ten on a ten-point scale. For instance, a student stated, "*Well, seven. It is not as difficult as ten, but I have to admit that I still find it pretty challenging."* Moreover, those few students who considered it not difficult demonstrated ambivalence during interviews. One student remarked, "I'm *giving it a two. I am bad at mathematics. I think I couldn't answer most of the worksheet questions correctly. I rarely provide correct answers to my maths teachers, but I believe '3D shapes' is an easy topic for others.*". Consequently, it appears that students generally perceive this topic as demanding.

Analysis of the interviews and students' worksheets revealed two major difficulties underpinning these challenges: 1) mental visualization of cube constructions (experienced by all students) and 2) drawing the visualized shape onto paper (encountered by approximately half of the students).

Mental visualization: All interviewed students reported finding the mental simulation of 3D shapes challenging, struggling to integrate separate views to construct a 3D form in their minds. This challenge was particularly associated with E1, where students redrew the given shape as the front or a part of it as a side view, indicating students were trying to interpret the 3D shape as if it were a two-dimensional representation. Figure 5 illustrates a typical example of a student struggling to mentally combine orthogonal drawings into a single shape resulting in an mistaken drawing which was coded as E8. These sketches demonstrate that their primary struggle lay in mental visualization. Students emphasized the need to visualize the shape before attempting to draw it. Without this mental visualization step, they could not produce the correct response.

"*I just couldn't visualize them [orthogonal drawings] together, you know? Each orthogonal view forms its own shape, but merging them into a single shape is really hard for me.*"

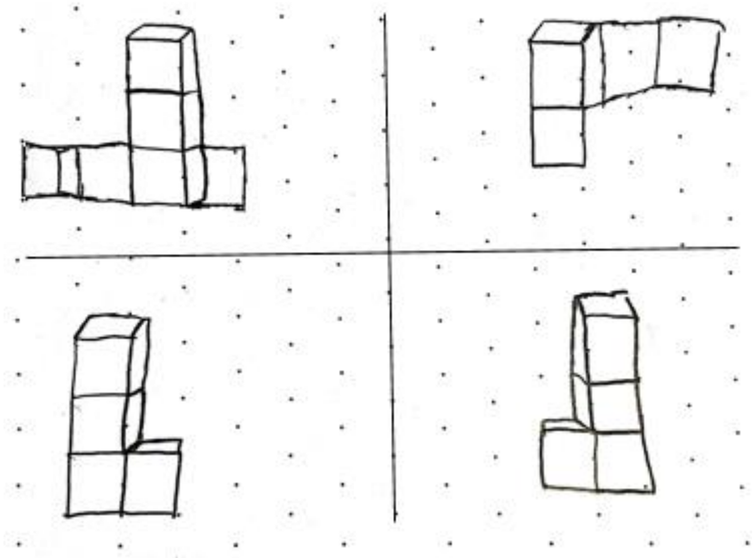


Figure 5. *Sample student drawing for the Q-3 in isometric drawings (E8)*

Drawing the visualized shape on paper: During observed lessons, many students found it challenging to transfer their mental visualizations onto paper. Half of the interviewed students shared similar experiences, noting that while they could visualize the shape in their minds, they encountered difficulties in accurately representing it on paper, and this challenge underpinned a number of errors (E3, E4, E5, E7 and E8).

The following quotation exemplifies an episode from one of the interviews.

"*Researcher: Please talk through how you decided where and how many cubes to draw in this question [Figure 6].*

Student: Hımm… I wanted to draw… Four, five, six… [Counts the cubes previously drawn onto the worksheet.] Seven cubes, but I only drew six cubes.

Researcher: Where should the seventh cube be?

Student: In front of this one [Points to the cube on the far left], but when I draw it, it seems like it is on that cube, not in front of it.

Researcher: I see. Will the shape be complete once you draw it?

Student: Yes, but I don't know how.

Researcher: How many units is the height of this shape?

Student: Three [Points to each cube].

Researcher: I believe what you imagined is perfectly correct, but the drawing is slightly slanted, making it difficult to interpret the shape's intended height. Sometimes, it can be challenging to accurately depict what we visualized. ... Let's start from the beginning and work through it step by step together."

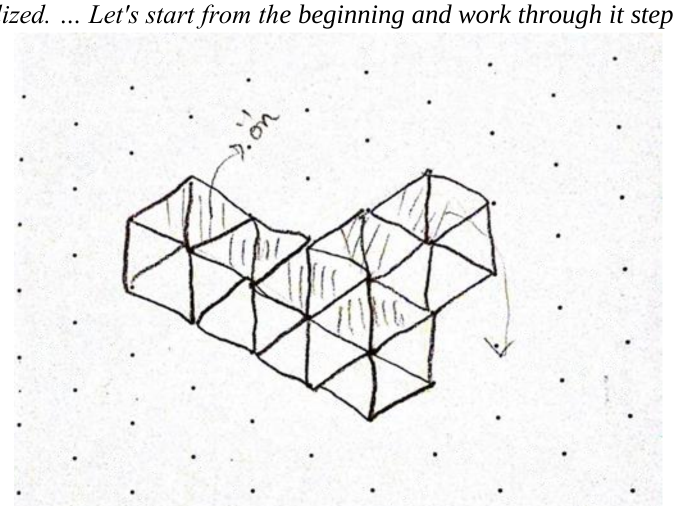


Figure 6. *Sample student drawing for the Q-3 in isometric drawings (E8)*

Figure 6 showcases the quoted student's drawing for Question 3 in isometric drawings, reflecting the student's accurate mental description and shape despite potential evaluation issues raised by mathematics teachers due to the missing cube and incorrect use of isometric paper. This episode is just one example from many, highlighting drawing difficulty as the second primary reason for students' erroneous responses. Taken together Figure 5 and Figure 6 also illustrate how the same error (E8) could challenge in visualization or drawing (or both).

Table 5 illustrates the relationship between students' difficulties and observed errors. Some errors are presented in both sections as they were deemed to relate to difficulties in both drawing and visualization.

Table 5. Errors and Possible Related Difficulties

Visualization	Drawing	Both
Orth – E1 - Redrawing the given shape as the front or as a part of it as a side view	Orth – E4 - Swapping the left and right views	Iso – E7 - Attempting to combine the orthogonal views side by side
Orth – E2 - Drawing the cubes at the back onto another column	Orth – E5 - Drawing the view upside down	Iso – E8 - Drawing only one view at a time instead of combining all views
Orth – E3 - Drawing the part only at the very front of the shape	Iso – E9 - Encountering difficulties in establishing links between different components of the shapes	
Iso – E6 - Redrawing the provided shapes		

In summary, two underpinning challenges encountered by students in the task were mental visualization and translation of that visualization into a drawing. It is important to note that these difficulties are intertwined since students need to visualize and draw simultaneously, and this can be iterative: as students draw, they may change their visualization. However, if a student fails to produce any drawing, it remains unclear whether the student successfully visualized the structure but could not translate it onto paper.

Current Pedagogy and its Relationship to any Difficulties and Misunderstandings

This section begins by briefly describing the main features observed during the lessons. It then examines three key aspects of the pedagogy employed and their potential relationships to the difficulties and misunderstandings experienced by students, as well as the extent to which a constructivist approach was adopted.

All the teachers in this study independently devised their lesson plans, drawing examples from diverse sources, yet adhering to similar lesson structures. Each lesson began by emphasizing previous experiences and competencies. While two teachers (Mr Aras and Ms Onay) summarised previous class content, the others (Ms Ayaz and Ms Semin) checked homework assignments. The teachers then introduced the subject using their chosen examples from external textbooks (Mr Aras and Ms Onay) or the MoNE's suggested workbook questions (Ms Semin and Ms Ayaz).

During the lessons, students were allotted time to tackle particular questions before the teachers demonstrated their own approach by drawing the correct representation on the board. While students were working on their sketches, teachers employed various techniques. For instance, one teacher (Ms Ayaz) used physical manipulatives such as cubes to build 3D shapes on the teacher's desk, while others (Mr Aras and Ms Onay) provided hints and suggestions as they walked around. Afterwards, all teachers invited students to draw on the board, either from volunteers or being selected by the teacher. It was notable that fewer students volunteered to create isometric drawings compared to orthogonal drawings. The teachers outlined a plan for drawing the correct representation and allowed students time to copy it into their notebooks or activity sheets.

In addition to exercises, teachers employed various methods to illustrate essential aspects of the content. Some teachers (Mr Aras and Ms Onay) repeatedly listed steps for making drawings, while others (Ms Semin and Ms Ayaz) advised students to observe their teachers' hand movements during the drawing process.

Lessons concluded with a summary of the covered material using various methods. Mr Aras and Ms Onay used resources provided by the MoNE, such as multiple-choice questions in digital education platforms or overview videos. Ms Semin and Ms Ayaz assigned additional homework from different sources. Particularly, Ms Ayaz consistently displayed a sense of urgency in finishing the planned questions, often assigning the remaining questions as homework. Conversely, Ms Semin frequently managed to cover her intended questions and, as a result, assigned supplementary homework from various external sources after providing a summary of the topics covered in the class. All teachers followed the progression from orthogonal to isometric drawing, aligning with the MoNE's curriculum order; teachers could have chosen to reorder it.

Turning now from this description, three prominent features of the lessons appear to be associated with the subsequent difficulties experienced by students and the extent to which constructivist practices were enacted.

The first feature is the use of manipulatives. Although all teachers acknowledged the value of concrete manipulatives, they predominantly used them themselves, limiting students' opportunities to construct their own models. For instance, Ms Ayaz used cubes measuring 10cm on each edge to construct a three-dimensional shape on her desk. Then, she drew orthogonal views of this cube construction on the classroom board, expecting the students to replicate these drawings. Consequently, the students found themselves reproducing the teacher's renderings rather than transforming the physical cube into a visual representation, arguably missing a vital step that was overlooked in this instructional approach. Only one out of the observed 16 classes involved students creating 3D shapes themselves. This finding from the lesson observations was corroborated by the student interviews, which confirmed they rarely got to use manipulatives themselves in any lesson. This lack of hands-on experience likely contributed to students' difficulties in focusing on the parts of the shapes (e.g., left view and top view) to construct a polycube and resulted in frequent errors, such as swapping left and right views (E 4) or drawing views upside down (E5).

The second feature is a limited and somewhat exam-focused pedagogy, despite teachers' intentions to adopt a constructivist approach. Lessons relied on memorization of rules rather than providing worked examples or encouraging problem-solving by students. Teachers often focussed on drawing only one view at a time to teach the rules, leading to errors such as drawing only one view/drawing views one by one (error 8) or swapping left and right views (error 4), which were so frequent. Students had limited opportunity to observe shapes and develop drawn solutions for themselves. This practice was observed multiple times. During the interviews, students reported that teaching was mainly didactic and that they had a very tight program to cover due to the MoNE's middle school exam. Teachers used the MoNE's past exam questions within their lessons, intending to boost the students' enthusiasm and highlight the topic's relevance for test success. Additionally, teachers (Ms Onay and Mr Aras) often used challenging questions from external sources to motivate students, but these questions proved to be excessively difficult, discouraging student participation. For example, Mr Aras found an external source and circulated copies of two pages from the source to his students. These questions involved orthogonal views of given polycubical shapes, but many had shapes that included more than twice the number of cubes compared to the questions on the MoNE exam. They were frequently so difficult for the students that almost none of them were willing to answer questions when asked.

The last feature pertains to the use of digital technologies. The ministry expects teachers to foster meaningful student learning through constructivist pedagogy and technology integration. Although three teachers (Ms Ayaz, Ms Onay and Mr Aras) acknowledged the potential effectiveness of 3D geometry software packages mentioning in the pre-lesson interviews, they rarely encouraged students to use such technologies. This limited use of technology may have contributed to errors related to redrawing the given shape as the front or as a part of it as a side view (E1), incorrectly positioning parts of shapes (E2) or drawing the part only at the very front of the shape (E3). Dynamic software that allows students to construct and manipulate polycubes could potentially address these visualization difficulties.

Teachers routinely employed video presentations and question sets accessed through the MoNE's EBA Moodle platform and a smartboard to display the solutions to these questions. However, when both teacher and students answered questions on the board, it was treated as a conventional board, and no use was made of its interactive and dynamic features. Considering how beneficial digital technologies can be for geometry learning, it is plausible that more extensive and intensive integration could help ameliorate some of the challenges students encounter in visualizing three-dimensional shapes (section 3.2).

Overall, the observed pedagogical practices, including limited student engagement with manipulatives, a rule-based approach, and underutilization of digital technologies, appear to be associated with the difficulties and misunderstandings experienced by students. These practices deviated from the constructivist approach teachers claimed to embrace.

DISCUSSION AND CONCLUSION

The present study addressed Turkish middle school students' understanding of the drawing of 2D representations of 3D shapes and how existing teaching practices influenced their comprehension. The findings indicated that students' overall performance in drawing 2D representations of 3D shapes was lower than the expectations of both teachers and the National Ministry. Specifically, the average scores on a national assessment-based worksheet were 43%, with many students (n=42) answering all questions incorrectly and only a few (n=8) answering all questions correctly. These results align with previous research conducted in multiple countries over several decades, such as the studies by Christou et al. (2006), Doorman et al. (2020), Parzysz (1988), and Widder and Gorsky (2013), which also reported low achievement levels in this domain.

The study revealed that students faced greater difficulty generating correct isometric drawings (60% correct) than orthogonal drawings (29% correct). This observation aligns with Cooper and Sweller (1989), who identified similar challenges with isometric drawings and attributed them to the difficulty of simultaneously holding multiple components of a polycube in mind to create an isometric drawing. In contrast, when producing orthogonal drawings, students could decompose a polycube and concentrate on one view at a time

An in-depth analysis of the data identified several common errors in both orthogonal and isometric drawings. While coding the worksheets, the study incorporated error types reported in previous studies regarding 2D representations of 3D shapes (Fujita et al., 2017; Pittalis & Christou, 2013). Although the error categories aligned with those in the reviewed studies, the study provided a more precise characterization of error types, for orthogonal and isometric polycube drawings. The detailed categorization of error types offers numerous benefits, including aiding teachers in recognizing these errors and developing effective solutions.

The main difficulties underlying these errors were identified as mental visualizations of 3D shapes and accurately drawing shapes based on these visualizations. "Visualizing a 3D shape from its orthogonal drawings and vice versa" was found to be the primary challenge for students in both orthogonal and isometric drawings of polycubes. This finding is consistent with previous studies that identified similar challenges in various other 2D representations of 3D shapes (Fujita et al., 2017; Parzysz, 1988), thereby indicating that students find isometric drawings more challenging than orthogonal ones. The difficulty lies in the cognitive process of decoding and accurately coding the spatial relationships and dimensions of the shape onto the 2D representation. Isometric drawings require more mental manipulation and visualization to accurately depict the shape's three-dimensional properties in a two-dimensional format. This complexity can result in more errors and inaccuracies compared to orthogonal drawings, where the step-by-step construction of each separate view allows for a more structured approach to represent the shape.

Additionally, many students struggled to accurately draw shapes that they had successfully visualized. Interviews with students confirmed that they frequently visualized shapes correctly but encountered difficulties translating those mental images onto paper. As a result, much erasing and incomplete answers were found, which occurs with findings from other studies (Bishop, 2008). Regarding isometric drawings, while most students could correctly draw the front views of shapes, the responses for other views varied, often resulting in more incorrect answers. The difficulty in isometric drawing may stem from students' lack of familiarity with the oblique convention. This involves depicting the front view of the cube as a square but drawing it with a parallelogram, while the rest of the illustration is shifted from the front.

The third research question examined whether pedagogical practices that students encounter may be related to student performance. It focused on examining whether the teaching practices were, as the Ministry intended, more constructivist in approach and included more technological integration. The findings indicated that three main practices played a role in this regard: teacher-centred pedagogy, exam-focused instruction and limited utilization of classroom technologies. The study uncovered that teachers dominated the use of tools and manipulatives, offering students limited opportunities to interact with concrete materials and develop their understanding autonomously. These findings confirmed the results (e.g. Christou et al., 2006; Widder & Gorsky, 2013), which suggest that visualizing 3D shapes from their 2D images poses obstacles not only for students' learning but also for teachers' instructional approaches.

Furthermore, the findings also shed light on a deficiency in teacher motivation to instruct the topic without an exam-oriented approach. Numerous studies have noted the perceived difficulty of teaching 3D shapes among middle school mathematics teachers, leading them to either neglect this topic or rely on direct instruction rather than student-centred activities (e.g., Bakó, 2003; Kooloos et al., 2022). The participating teachers were aware of the MoNE suggested constructivist approaches; they easily described them and discussed the benefits. However, finding the instances where these approaches were effectively implemented in their classrooms was challenging. The observation revealed that all four teachers incorporated previous national exam questions into their lesson plans. Additionally, the available technology was predominantly used to display videos and assessments provided by the Ministry of National Education at the conclusion of their classes. This practice persisted despite MoNE's recommendation to integrate dynamic geometry software packages into mathematics lessons (MoNE, 2018). No observed teachers allowed students to use dynamic geometry programs on their tablets to imagine and visualize 2D representations of 3D shapes in any of the lessons. The emphasis primarily revolved around memorization and rote application of rules and strategies with the hope of ensuring better and quicker performance of their students in national exams. Consequently, this study confirms that of earlier studies such as Bayrakdar-Çiftçi et al. (2013)

and Tekalmaz (2019), which found that teachers' belief in the efficacy of the revised approach in theory was not reflected in their actual teaching practices.

Implications

The authors are currently developing lessons that offer various depictions of shapes, encompassing prototypes of tangibles such as plastic and moving prisms, as well as constructions on a dynamic geometry tool, to assist students in developing their visualization skills required to visualize these shapes. Considering this, the solution of interacting with real and dynamic 3D models is consistent with best practice advocated elsewhere (Gutierrez, 1996; Hankeln, 2020; Kohen & Orenstein, 2021). Moreover, the most common errors identified in this study will be incorporated into these lessons and introduced to students as “designed mistakes” to be resolved, following the approach employed by Evans and Swan (2014).

The results of this study offer helpful information for teachers, teacher educators, programme developers and policymakers concerning the teaching and learning of cube representations in Türkiye and potentially beyond. Firstly, such research in an authentic educational setting could contribute to enhancing self-awareness among both students and teachers regarding their instructional practices. During the debrief discussions, all participating teachers provided insightful comments about their teaching practices, which can contribute to reflective process and encourage them to challenge their current approaches (Saralar et al., 2018; Balgalmış et al., 2014). Secondly, the findings underscored the importance of incorporating technology courses into teacher education programmes and providing practical activities that enhance teachers' understanding, build their confidence, and offer examples for future lessons (Hegedus & Otálora, 2022; Zengin, 2023). Furthermore, policymakers can be encouraged to ensure that teachers are aware of and benefit from recent study results, such as those provided by the Ministry of National Education. Finally, on a positive note, the circumstances presented by the Techology-supported Learning Scenarios Project to innovate mathematics instruction in Türkiye provides a conducive situation for addressing these challenges (MoNE, 2023).

Limitations and Further Research

The study has some limitations that should be acknowledged. Firstly, its sample size was relatively small, as it was limited to two schools and four teachers. However, despite this limited sample, the schools and teachers involved can be considered typical, and the results are consistent with others that have explored different areas of the mathematics (e.g., Kaya et al., 2016), but the caution should be exercised when applying these findings more broadly.

Additionally, although this study did not provide immediate solutions for the problems reported, future research could benefit from adopting design-based research approaches to develop resources in collaboration with teachers. These approaches could support them in implementing these resources with constructivist approaches during lessons. Techniques such as using theoretical and empirical insights (as seen in Haj-Yahya, 2022) and integrating digital technologies (Huang et al., 2023; Tzafilkou et al., 2023) could enhance the development of sources aimed at helping teachers understand and address students' errors effectively.

Furthermore, the study revealed a lack of meaningful contexts in the tasks designed by teachers, which may have hindered students’ engagement and understanding. However, meaningful contexts could provide students with a purpose for the tasks designed by the teachers and support teachers in building on students' intuitions. To address this, future research could explore constructivist options that leverage students' prior experiences, and their informal reasoning. For example, investigating how students’ understanding of real world contexts about buildings and different perspectives, such as shading in different projections (e.g., shade created by the sun in parallel projection and by lamps in central projection as discussed by Doorman et al., 2020), influences their learning could provide valuable insights, and could be a fruitful direction for further research. Examining the effectiveness of these contextually rich approaches in real classroom settings could offer significant benefits for enhancing mathematics instruction and student engagement.

Statements of Publication Ethics

This study was conducted within the guidelines of the University of Nottingham’s Code of Research Conduct and Research Ethics and the School of Education in particular which has adopted Revised Ethical Guidelines for Educational Research (BERA, 2011). Before the submission of the ethics documents, the first researcher needed to obtain a Disclosure and Barring Service (DBS) check to have access to schools and received it with F0104171462 reference number. Ethics was received on October 19th, Ref: 2017.64.

Researchers' Contribution Rate

Both authors equally contributed to this research by designing, analyzing data, and writing.

Authors	Literature review	Method	Data Collection	Data Analysis	Results	Conclusion
İpek SARALAR-ARAS	☒	☒	☒	☒	☒	☒
Shaaron AINSWORTH	☒	☒	☐	☒	☒	☒

Conflict of Interest

The authors have no conflicts or competing interests to declare.

Availability of Data and Materials

The datasets generated during and/or analyzed during the current study are available from the University of Nottingham's Data Repository, doi: http://doi.org/10.17639/nott.7323.

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