

A CONSUMER DEMAND CURVE TAKING THE EXPLOITATION EFFECT INTO ACCOUNT UNDER THE MONOPOLISTIC BEHAVIOUR

Yrd. Doç. Dr. Ahmet ÖZÇAM¹

Abstract

In this paper, the monopolistically competitive producers maximize the prices of their products, rather than their profits, that can be offered to consumers with different income levels given their utility levels. The comparative static analysis of the demand of the consumer, in such a case, is derived analytically under a general utility function and is found to have been decomposed into two parts : the substitution effect and the exploitation effect of the monopolistic seller showing the reaction of the consumer to a change in the price of the monopolistic good. A similarity is found with the standard Marshallian demand curve where the consumer's income effect determines the additional demand in excess of the substitution effect. In our model the same change in the consumer's buying capacity is exploited by the monopolistic producer by raising her product's price, and this is called the " Exploitation Effect". A function which is referred to as the " Income-Price Curve " shows the maximum prices that can be charged to the consumer by the monopolistic seller.

Keywords: Consumer Demand, Monopolistic Competition, Income Effect, Exploitation Effect, Income-Price Curve.

Özet

Bu çalışmada, monopolcü rekabetçi firmalar çeşitli gelir seviyesindeki sabit toplam faydaya sahip tüketicilere sunulan ürünlerden elde edilecek karlarından ziyade, bu ürünlerin fiyatlarını maksimize ederler. Böyle bir durumda, tüketicinin talebinin mukayeseli statik analizi genel bir fayda fonksiyonu için incelenmiş, ve iki bölümden oluştuğu bulunmuştur: ikame etkisi ve tüketicinin monopolcü rekabetçi firmanın ürünüünün fiyatını arttırmasına gösterdiği istismar etkisi. Tüketicinin ikame etkisine ilave gelir etkisinin yarattığı standart Marshall tipi talep eğrisi ile bir benzerlik bulunmuştur. Bizim

¹ Yeditepe University, Department of Economics, aozcam@yeditepe.edu.tr

modelimizde, tüketicinin satın alma gücünde oluşan bu değişiklik ürününün fiyatını yükselten monopolcü rekabetçi firma tarafından istismar edilmektedir, ve “ İstismar Etkisi ” olarak adlandırılmıştır. “ Gelir-Fiyat Eğrisi ” adı verilen bir fonksiyon, tüketicie monopolcü rekabetçi firma tarafından ansütilabilecek en yüksek fiyat seviyelerini göstermektedir.

Anahtar Kelimeler: *Tüketici Talebi, Monopolcü Rekabet, Gelir Etkisi, İstismarEtkisi, Gelir-Fiyat Eğrisi.*

1. INTRODUCTION

The motivation for this paper comes from an observation that similar products can be sold to customers at different prices in practice. This may arise from the fact that a product can be presented to a customer with a higher income level via a higher price list provided that the product is presented in a slightly different quality and/or variety aspect. These differences in turn are evaluated by the preferences of the customers. The disparities in income levels and the preferences of the consumers are important factors in determining how high a price a monopolistic seller can charge for its product.

The investigation about a general theory concerning the analysis of monopolistically competitive industries with product variety has been found difficult to analyse due to the fact that there are many types of such markets. Many factors account for this diversity: the degree of heterogeneity of the product examined vis-a-vis the other products and its complementary assortments, the preference and income differences of the consumers, imperfect information/willingness to search for it on the part of the consumers, the investment/cost situations of the firms, the number of buyers/sellers and the time period under consideration. The complex interactions of these factors are thought to determine the possible equilibria in these types of industries.

Salop and Stiglitz (1977) examined the case of heterogeneity of the consumer rationality in making economic decisions within a model of costly information-gathering process about the price of the product. They assumed that the customers obtaining the price information with higher cost generally remained uninformed and bought randomly at stores while those with the perfect price information shopped at stores with the lowest price. The firms were assumed to know the prices charged by the other firms and the distribution of consumer's search costs, and maximized profits facing no uncertainty of information. Salop and Stiglitz derived various market equilibria based on the zero profit maximization rule working on a single good example. They noted that disparity in incomes and/or preferences of the consumers provided further evidence why some customers performed better than others in bargaining in the marketplace. In their model, the consumers with a lower cost in gathering information benefited from the lowest price. In the current paper, the disparity in income levels of the consumers, rather than their search procedures determine how high a price the producers can charge to these customers by using their monopolistic advantage. It seems obvious, and supported by examining such monopolistically competitive markets in practice that the firms charging lower prices without aiming an appropriate degree of product differentiation designed to disperse income levels of consumers, or ignoring their customers' information search are the ones to exit the market first. However, the important question is that whether such economic rents can be obtained from lower profile price

searching customers or from higher income consumers who are able to consume conspicuously and to generate much higher levels of expenditures?

Salop (1979) examined the monopolistic brands which were equally spaced around the circular product space and introduced a second homogeneous outside commodity explicitly which was sold in a competitive market. Without differentiating between the income levels of the consumers, he assumed that they bought one unit of some monopolistic brand if their surplus of utility depending on the distance from their most preferred brand, minus its price outweighed the surplus from the other homogeneous good. The remaining income was spent on the homogeneous product. He derived three regions for the demand curve facing a representative brand: monopoly, competitive and supercompetitive. These demand curves were derived from comparisons of relative surpluses of the consumers from all these products at various prices. As the representative brand started decreasing its price below the consumers' reservation price, it first captured demand from the homogeneous good, and then from its neighbor when its monopoly market overlapped that of its neighbor, and finally the entire market of its neighbor. In this model, the producers were aware of the number and prices of their rivals and made Nash conjectures by choosing a best price given a perception that all other firms held their prices constant. A general equilibrium is searched within these market configurations. Salop found that the overall equilibrium market price fell as the value of product differentiation preferences of all consumers decreased without treating the income differentials of the consumers explicitly. In the current paper of price maximization of the monopolistic seller vis-a-vis consumers on one-to-one basis it is emphasized that the former's price decision behavior mostly depended on the differences in consumers' income levels implicitly assuming that the right degree of product differentiation was already aimed at higher income consumers. Furthermore, it is believed that there are many practical situations in which there are always some high income and high preferred consumers who will not buy a particular brand at all no matter how low its price is. Hence, it seems plausible to think that the appropriate degree of product differentiation in terms of appearance of a product and its complementary assortments, is purposely created by the monopolistic sellers to attire high income consumers in order to charge a higher price and/or to try to make them become loyal customers.

The well-known theory of the utility maximization of a consumer subject to her budget constraint does not explain to which kind of a market the consumer belongs, and how the producers react if they happen to have any monopolistic benefits. Actually in practice, the consumers with higher income levels might be quite happy to be able to purchase the goods that are more expensive and/or of better quality in the marketplace. They may consume conspicuously. The product variation whether in terms of locational advantages and/or product quality allows the firms to charge some prices that are close to their customers' reservation prices. The firms being much more informed about the market situation compared to their customers are generally able to keep their profit margins large enough with respect to those customers that try to differentiate themselves from an ordinary customer with less of an income. This is the only way for the firms in these kinds of industries to barely cover their overall economic costs, or to make some economic profits if the aggregate demand for such products under the general macroeconomic situation permits them. Moreover, a rule such as "zero profit" heavily depends on the business cycle, and

does not seem to qualify to be an overall rule for a specific monopolistic industry that is treated independently from the rest of the economy.

This paper examines the equilibrium market baskets of the consumers given a change in their income levels or a change in their purchasing ability exploited by a monopolistically competitive seller who raises the price of its good. The assumed change in the income level can be thought as representing customers with dispersed income levels. The maximum prices that can be charged to customers with different income levels are shown. The questions of information, quality differentials or the existence of a general equilibrium for the whole market will not be explored. Instead, the heterogeneous commodity preferences and income levels of the consumers are important aspects of the current model. The results for some general quasi-concave consumer utility function will be derived in addition to those for two specific types of utility function calculated numerically as examples.

In the section 2, the monopolistically competitive producer is assumed to maximize the price of a product that can be offered to consumers with different income levels in order to ultimately maximize her profits without endangering a dissatisfaction in consumers' utilities. The producer/seller is perfectly aware of the purchasing power of the consumer. Therefore, the price of the monopolistic product and the income level of the consumer will be changing simultaneously in this case. Salop and Stiglitz (1977) mentioned that there was a limit on the price increases at one store that the consumers will tolerate without leaving. This tolerance level is calculated given the consumers' preferences and income levels in the present model ignoring any differences in the consumers' willingness to gather the extra information needed to switch stores or brands. In a two good world setting, the second good represents all other goods that the consumer demands.

Similarly, given a decrease in the price of the other good enhancing the purchasing power of the consumer, it is shown that the exploitation of the monopolistic seller via a price increase in its good occurs up to the point where the utility of the consumer is kept constant. The consumer reacts by demanding less of the monopolistic good and more of the other good. The monopolist will be reacting to the change in the purchasing power, rather than the consumer with her well known income effect. The substitution effect of the current model coincides with that of the Marshallian demand of the standard model. This section ends with a comparison of the demand of the consumer in a monopolistic setting with those of the Hicksian and Marshallian types.

In the section 3, a Cobb-Douglas type of utility function is used to show a particular case. A function, which is called the "Income-Price Curve" shows the maximum prices that can be charged to the consumer by the monopolistic seller. This function is found to increase with the income level of the consumer and to decrease with the price of the other good and the level of utility. Of course, higher income consumers pay more per unit of the good. Moreover, if the consumer can accept a lower level of utility, then the monopolistic producer uses her exploitation advantage to increase the price of her product. A lower price for the other good produces an increase in the price of monopolistic good again. These cases are thought to be some situations in which the monopolistic producer uses her discriminating power to take advantage of a change in the purchasing ability or the utility of the consumer.

In the section 4, a numerical analysis is carried out with another special type of utility function to illustrate the theoretical findings of the section 2.

2. THE COMPARATIVE-STATIC ANALYSIS OF THE MODEL

2.1. The comparative-static derivatives

The monopolistically competitive producer is assumed to maximize Px , the price of a product x rather than some profit function (revenues minus costs) with respect to the utility, income level of a consumer and the price of the other good y given by $U(x,y)$, m and P_y respectively. The other good y is supposed to represent all other goods that the consumer demands.

$$\max_{x,y} (m - P_y \times y) / x$$

subject to a fixed level of utility, k :

$$U(x, y) = k$$

(1)

Setting up the Lagrangian of the problem, we have :

$$L = (m - P_y \times y) / x + \lambda \times (U(x, y) - k)$$

(2)

The first order conditions (FOC) can be obtained by taking the first partial derivatives with respect

to λ , x , y and equating them to zero :

$$\partial L / \partial \lambda = U(x,y) - k = 0$$

(3)

$$\partial L / \partial x = (-m + P_y \times y) / x^2 + \lambda \times U_x(x, y) = 0$$

(4)

$$\partial L / \partial y = (-P_y / x) + \lambda \times U_y(x, y) = 0$$

(5)

Solving (4) and (5) for λ , we obtain the FOC subject to (3) :

$$\lambda = (m - P_y \times y) / (x^2 \times U_x(x, y)) = P_y / (x \times U_y(x, y)) \quad (6)$$

The bordered Hessian of the problem gives :

$$|\overline{H}| = \begin{vmatrix} 0 & U_x & U_y \\ U_x & L_{xx} & L_{xy} \\ U_y & L_{xy} & L_{yy} \end{vmatrix} \quad (7)$$

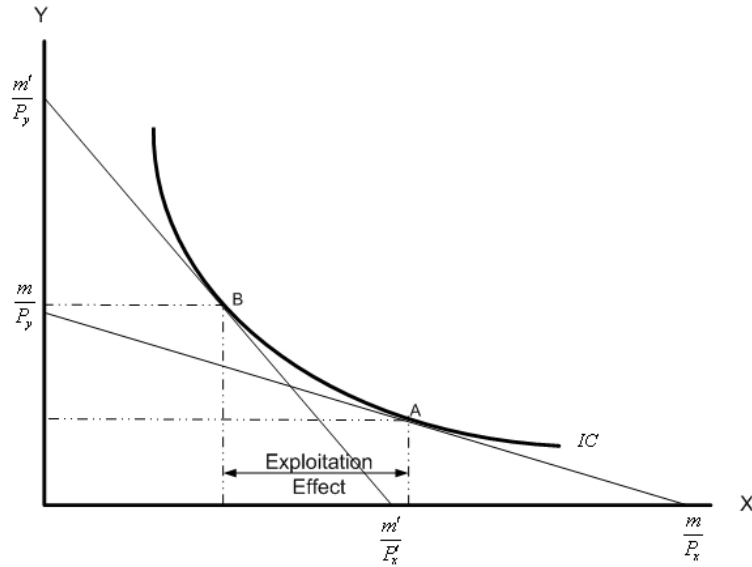
where,

$$\begin{aligned} L_{xx} &= (2(m - P_y \times y) / x^3) + \lambda \times U_{xx}(x, y) \\ L_{xy} &= (P_y / x^2) + \lambda \times U_{xy}(x, y) \\ L_{yy} &= \lambda \times U_{yy}(x, y) \end{aligned} \quad (8)$$

If the bordered Hessian in the equation (7) is positive, then the stationary value of the objective

function (the value of the price of the good x) will assuredly be a maximum and the SOC's for a maximum is fulfilled. The fact that this is true is shown in the Appendix A, making a comparison with the Marshallian demand model in which the consumer maximizes her utility subject to her budget constraint.

Figure 1 : The exploitation effect given a change in consumer's income



Firstly, we would like to investigate the effect of a change in the budget of the consumer on the amount of the first good, x she demands. Differentiating totally the equations (3), (4) and (5) with respect to each variable

(endogenous or exogenous) , setting $dk = dP_y = 0$, and allowing dm to be different from zero :

$$\frac{\partial x}{\partial m} = \frac{\begin{vmatrix} 0 & 0 & U_y \\ U_x & 1/x^2 & L_{xy} \\ U_y & 0 & L_{yy} \end{vmatrix}}{|J|} = -(U_y/x)^2 / |J|$$

$$\equiv T1 < 0$$

(9)

This partial derivative is unambiguously negative. It is the movement from the point A to the point B in the Figure 1. If the producer/seller observes that a particular consumer happens to possess higher income, then she attempts to increase the price of the product x to use her monopolistic advantage without decreasing the consumer's utility level. Our consumer reacts by demanding less of x . We define this as the exploitation effect. Notice that in the standard Marshallian model where the utility is maximized subject to budget constraint and the price of the good x is kept constant, as the consumer's income

level increases she demands more or less of x depending whether the good is normal or inferior while her utility increases. In our model the unambiguous decrease in the demand of x when the consumer's income increases represents another example where perverse effects in the theory turn out not to be unreasonable. Some other known abnormalities which are nicely cited by Varian (1978) are the case of Giffen goods, and the backward-bending supply curve.

Secondly, we examine the effect of a change in the budget of the consumer on the purchase of y :

$$\begin{aligned} \frac{\partial y}{\partial m} &= \frac{\begin{vmatrix} 0 & U_x & 0 \\ U_x & L_{xx} & 1/x^2 \\ U_y & L_{xy} & 0 \end{vmatrix}}{|J|} = (U_x U_y / x^2) / |J| \\ &\equiv T2 > 0 \end{aligned}$$

(10)

This partial derivative is unambiguously positive. The consumer substitutes the good y for the good x, when the producer raises the price of the good x due to an increase in the income of the consumer. It is the reflection of the exploitation effect in (9) on the good y. It is shown as the difference between the points A and B measured on the vertical axis in the Figure 1.

Thirdly, we are interested in investigating how a change in the price P_y of the other good y, affects the demand for the monopolistic good x :

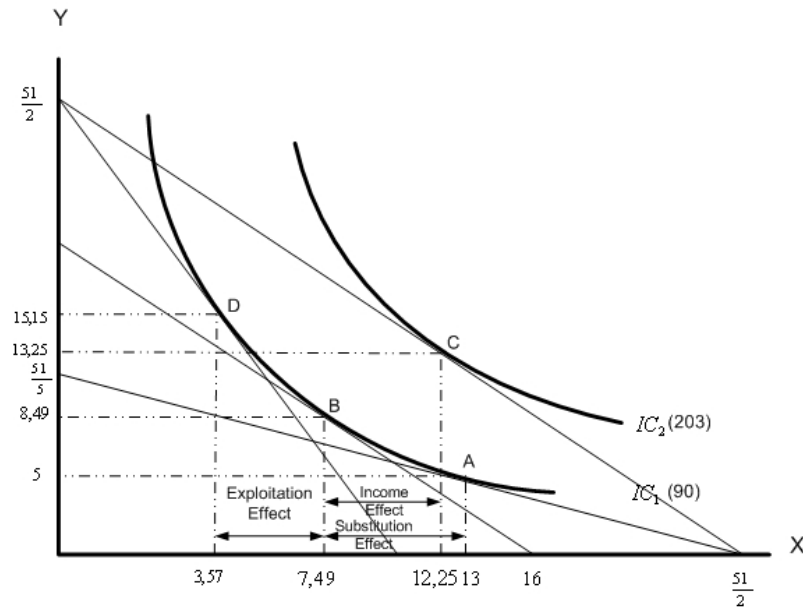
$$\begin{aligned} \frac{\partial x}{\partial P_y} &= \frac{\begin{vmatrix} 0 & 0 & U_y \\ U_x & -y/x^2 & L_{xy} \\ U_y & 1/x & L_{yy} \end{vmatrix}}{|J|} = ((U_x U_y) / x + (y(U_y / x)^2)) / |J| > 0 \\ &= \quad (\partial x / \partial P_y) \quad - \quad y (\partial x / \partial m) \\ &\quad \text{cross substitution effect} \quad \quad \text{exploitation effect} \end{aligned}$$

$$T3 \equiv T4 + T1 > 0$$

(11)

This partial derivative is unambiguously positive. It corresponds to the movement from A to D along IC_1 assuming a decrease in P_y in the Figure 2. It has two components. The first one on the right hand side is the cross substitution effect which is positive, and in the Appendix A it is shown that this is precisely the cross substitution effect of the standard Marshallian model, or the Hicksian demand. This is shown as a movement from A to B along IC_1 in the Figure 2. Therefore it must correspond to an amount of x chosen by the consumer who is assumed to be compensated with some income to make her utility constant at IC_1 after the change in P_y keeping the price of x constant. The second component in (11) is the negative exploitation effect of the equation (9) due to a change in the purchasing power of the consumer resulting from a change in P_y , and is multiplied by $(-y)$ making is also positive. The optimal purchase of x is weighted by the amount of y in the budget of the consumer, since the price of y is changing. This corresponds to a movement from B to D along IC_1 in the Figure 2.

Figure 2 : The consumer's demand given a decrease in the price of the other good



Note that we are working with the cross price effect on the purchase of x, and the exploitation effect works in the same direction as the cross substitution effect in our model. The exploitation effect reinforcing the cross substitution affect differs from the standard model where the sign of the income effect is indeterminate, i.e. depends on the nature of the good.

Therefore, when the price of y decreases, the consumer demands less of x for two reasons :

a) the substitution effect due to the fact that the good y being relatively less expensive keeping the

price of the good x and the utility constant.

b) the exploitation effect where the seller increases the price of the good x, realising that the

purchasing power of the consumer has increased. This effect points out to the monopolistic

behaviour of the seller where she sees the decrease in P_y , as an increase in purchasing

power of the consumer and reacting by raising the price of x without letting the consumer's

utility decrease.

Finally, we examine how the demand for the other good y is affected by a change in its own price :

$$\frac{\partial y}{\partial P_y} = \frac{\begin{vmatrix} 0 & U_x & 0 \\ U_x & L_{xx} & -y/x^2 \\ U_y & L_{xy} & 1/x \end{vmatrix}}{|J|} = ((-U_x^2/x) + (-y \times U_x U_y / x^2)) / |J| < 0$$

$$= \underbrace{(\partial y / \partial P_y)}_{\text{own substitution effect}} - y \underbrace{(\partial y / \partial m)}_{\text{exploitation effect}}$$

$$T5 \equiv T6 + T2 < 0$$

(12)

This partial derivative is unambiguously negative. This corresponds to the change in the amount of y when we trace the movement from A to D through B on the vertical axis in the Figure 2. It has two components. The first one on the right hand side is the own substitution effect which is negative, and in the Appendix A it is shown that it coincides precisely with the own substitution effect of the standard model. The second component in (12) is the positive exploitation effect of the equation (10) due to a change in the purchasing power of the consumer resulting from a change in P_y , and is multiplied by $(-y)$ making it also negative. Again, the optimal purchase of y is weighted by the amount of y in the budget of the consumer, since the price of y is changing.

The point C represents the usual Marshallian demand for the goods x and y when the price of y decreases, i.e. the amounts the consumer would choose if the income is kept constant, and the producer of the good x *does not* change her product's price.

A numerical analysis with a specific utility, income and the prices for both goods is given in section 4.

2.2. A comparison with the Hicksian and the Marshallian demands :

In the standard Marshallian model, the partial differentiation of the Marshallian demand with respect to P_y is :

$$\begin{aligned} (\partial x / \partial P_y)_s &= (\partial x / \partial P_y)_{\text{cross substitution effect}} - y (\partial x / \partial m)_{\text{income effect}} \\ \text{Marshallian demand} & \end{aligned}$$

(13)

where the subscript s refers to the standard model. This is shown as a movement from A to C through B in the Figure 2. In the equation (A4) of the Appendix A, it is shown that the cross substitution term of the standard model, which is the first term on the right hand side of (13) is equal to the cross substitution term, T_4 of our model in (11). Using this equality and plugging the first term on the right hand side of (13) in (11), we have :

$$\begin{aligned} (\partial x / \partial P_y)_s &= (\partial x / \partial P_y)_{\text{our demand}} - (-y (\partial x / \partial m))_{\text{exploitation effect}} \\ \text{cross substitution effect} & \\ &= T_3 - T_1 \end{aligned}$$

(14)

The equation (14) shows that the cross substitution effect of the standard model (which is the Hicksian

demand) corresponds to how much less of good x the consumer would demand when the monopolist raises its price due to a decrease in the price of y, and the exploitation

effect of the consumer's purchasing power is neutralised (subtracted). This is the movement from A to B in the Figure 2. Therefore, this is the consumer's demand for x representing only the substitution effect. Notice that in our model, the monopolist rather than the consumer reacts to the change in the purchasing power.

We can write the Marshallian demand in the equation (13) using (14) as follows:

$$\begin{aligned}
 \frac{(\partial x / \partial P_y)_s}{\text{effect}} &= \frac{(\partial x / \partial P_y)}{\text{Marshallian Demand}} - \frac{(-y (\partial x / \partial m))}{\text{our demand}} - \frac{y (\partial x / \partial m)}{\text{exploitation effect}} - \frac{y (\partial x / \partial m)}{\text{income}} \\
 &= T3 - T1 - y (\partial x / \partial m)
 \end{aligned}
 \tag{15}$$

The Marshallian demand is the amount of x the consumer would like to buy when the monopolist raises the price of x due to a decrease in the price of y, and the exploitation effect is neutralised but the income effect is taken into account. Notice that the income is at the original level at both points C and D in the Figure 2. The movement from D to C involves the annulment of the exploitation effect while the consumer's income effect is kept.

Therefore, we can obtain the Marshallian demand for the good x, if we adjust our demand by two factors:

a) exploitation effect: P_x adjusts to a change in the purchasing power of the consumer keeping

the price of y and the utility constant.

b) income effect: the consumer's income adjusts changing the utility level, keeping both P_x and P_y

constant.

3. A PARTICULAR UTILITY FUNCTION

The monopolistically competitive producer is assumed to maximize the price P_x of a product x, that is offered to a consumer with a Cobb-Douglas utility function, income level, and the price of the other good y given by $U(x, y) = x^a y^{1-a}$, m and P_y respectively.

$$f(k, m, P_y) = \max_{x,y} (m - P_y \times y) / x$$

subject to a fixed level of utility, k :

$$x^a y^{1-a} = k$$

(16)

Setting up the Lagrangian of the problem, we have :

$$L = (m - P_y \times y) / x + \lambda(x^a y^{1-a} - k)$$

(17)

The first order conditions can be obtained by taking the first partial derivatives with respect

to λ , x , y and equating to zero :

$$\partial L / \partial \lambda = x^a y^{1-a} - k = 0$$

(18)

$$\partial L / \partial x = ((-m + P_y \times y) / x^2) + \lambda \times a \times x^{a-1} y^{1-a} = 0$$

(19)

$$\partial L / \partial y = (-P_y / x) + \lambda \times (1 - a) \times x^a y^{-a} = 0$$

(20)

Solving (19) and (20) for y , and substituting into the utility constraint (18), we have the following optimal demand expression for x as a function of the fixed utility level, income, and the price of the

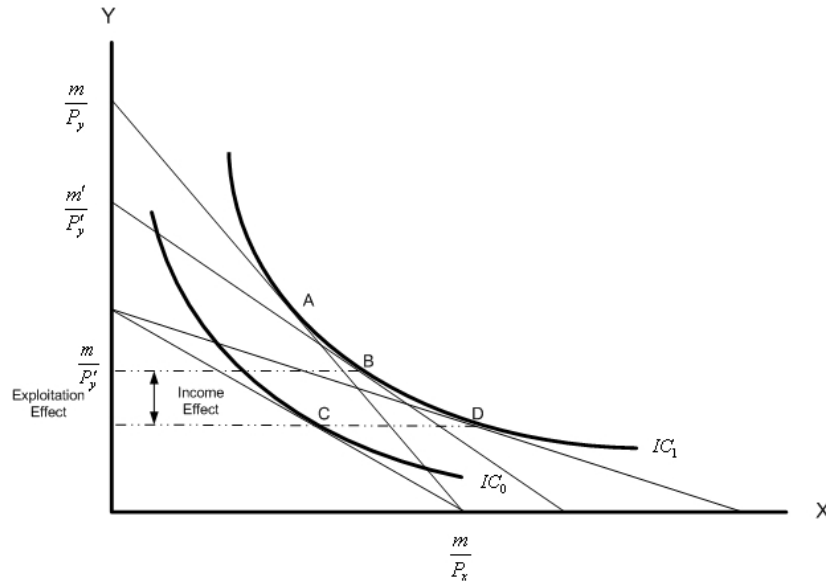
other good :

$$x(k, m, P_y) = k^{1/a} \times (m \times (1 - a) / P_y)^{(a-1)/a}$$

(21)

Our demand for x increases with k , and P_y , and decreases with m . It is free of the price of x , P_x and *differs* from that of a Marshallian type which is a function of the income of the consumer, and the prices of x and y , or that of a Hicksian type which is expressed as a function of a utility level, and the prices of x and y .

Figure 3 : The consumer's demand (under Cobb-Douglas utility) : equality of income and exploitation effects



The partial derivative of the demand function in (21) with respect to the price of y , must be equal to the one that is obtained from (11) of the comparative statics analysis. A calculation using the Cobb-Douglas utility is shown in the Appendix B. This is the movement from the point A to the point D in the case of a price increase in the good y , in the Figure 3. Of course, the optimal values of x calculated directly from (21) and those from (11) will differ from each other as dP_y becomes larger.

The demand for y can be similarly obtained by expressing x as a function of y , and substituting into the utility constraint (18) :

$$y(k, m, P_y) = m \times (1 - a) / P_y \quad (22)$$

Our demand for y corresponds exactly to the Marshallian demand. Hence, with the Cobb-Douglas utility, the income effects and the exploitation effects are exactly equal as shown in the Figure 3. The movement from B to C is the Marshallian income effect on the good y , whereas the movement from B to D is the exploitation effect on the good y . They are equal to each other as measured on the vertical axis.

Finally by substituting the optimal values of x and y which are in the equations (21) and (22) respectively into the objective function in the equation (16), we obtain a function which gives us the maximum prices of the product x that can be charged by the sellers for given values of utility, income and the price of the other product y :

$$f(k, m, P_y) = a \times ((1 - a) / P_y)^{(1-a)/a} \times m^{(1/a)} \times k^{(-1/a)} \quad (23)$$

This is the Income-Price curve. We observe that the maximum price that can be charged to the consumer increases with the income of the consumer and decreases with the price of the other good y and the level of utility. Of course, higher income consumers pay more per unit of the good. Moreover, if the consumer can accept a lower level of utility, then the monopolistic producer uses her exploitation advantage to increase the price of the product x . A lower price for the good y produces an increase in the price of x again.

4. A NUMERICAL ANALYSIS OF THE EXPLOITATION EFFECT

The initial optimum demand of the consumer for the goods x , and y is given by the point A (13,5) in the Figure 2 of the section 2, with the utility function, $U(x,y) = (x+2)(y+1) = 90$, $m = 51$, $P_x = 2$, $P_y = 5$. We have calculated the Marshallian demand to be 12.25 and 13.25 for the two goods respectively which is given by the point C, when P_y decreases down to 2. The utility increases from 90 to 203 when we move from IC_1 to IC_2 . The substitution effect, which is the Hicksian demand (7.49, 8.49) is given by the point B, and the point D (3.57, 15.15) gives the demand of the customer along the IC_1 facing a monopolistic behaviour where the producer maximizes the price of the good x , subject to the consumer's constant level of utility.

5. CONCLUSION

This article inquires into the theory of the utility maximization of a consumer in which one of the two goods is monopolistic. A framework is set up in which the monopolistically competitive producers maximize the prices of their products that can be offered to consumers with different income levels in order to ultimately maximize their profits without affecting the consumers' utilities. The monopolistic powers of the producers are shown as up to

which price level they can increase the prices of their products, given the consumers' incomes, utility levels, and the price of the other good.

In our model, the demand of the consumer is decomposed into two parts. The substitution effect reflects the result of the relative price ratio on the amounts of the two goods demanded, whereas the exploitation effect shows the reaction of the consumer to the change in the price of the monopolistic good. The exploitation effect points out to the monopolistic behaviour of the seller who uses the change in the purchasing ability of the consumer by reacting with a change in the price of her good.

A similarity with the standard model is found. The substitution effect of our model coincides with that of the standard Marshallian demand, or simply the Hicksian demand, given a change in the price of the other good. The difference comes from the use of a

change in the purchasing ability of the consumer. In the standard model, the consumer's income effect determines the additional demand, whereas in our model the same change in the consumer's buying capacity is exploited by the monopolistic producer. Our consumer always reacts by further substituting the other good for the monopolistic one.

The demand of the consumer for the other good does not change at all in our model compared with that of the standard model under a Cobb-Douglas type of utility. In this special case, the usual income effect of the consumer mirrors the exploitation effect of the monopolistic producer. Therefore as far as the other good is concerned, it makes no difference whether the change in the purchasing ability of the consumer is used by herself or the monopolist.

A function called the Income-Price Curve showed the maximum prices that could be charged to the consumer by the monopolistic seller. It was found that this function increased with the income level of the consumer and decreased with the price of the other good and the level of utility. These changes could reflect some situations in which the monopolistic producer exerted her discriminating power to take advantage of a change in the purchasing ability or the utility of the consumer. Higher income consumers facing a fiercer monopolistic power had to pay more per unit of the good due to a possibly higher differentiated product offered to them. Moreover, if the consumer could accept a lower level of utility, then the monopolistic producer used her exploitation advantage to increase the price of her product. Likewise, a lower price for the other good induced the monopolistic producer to increase her price again.

APPENDIX A

In the standard model the consumer maximizes her utility subject to her budget constraint:

$$\max U(x, y)$$

$$x, y$$

subject to a fixed level of income, m :

$$P_x \times x + P_y \times y = m$$

(A1)

The Lagrange multiplier and the bordered Hessian of the standard problem gives:

$$\lambda_s = U_x / P_x = U_y / P_y$$

and,

$$\left| \overline{H}_s \right| = \begin{vmatrix} 0 & P_x & P_y \\ P_x & U_{xx} & U_{xy} \\ P_y & U_{xy} & U_{yy} \end{vmatrix} = 2P_x P_y U_{xy} - P_y^2 U_{xx} - P_x^2 U_{yy} > 0$$

(A2)

where the subscript s in λ_s and $\left| \overline{H}_s \right|$ refers to the standard model in which the Marshallian demand is obtained.

In our model, the Lagrange multiplier and the bordered Hessian were given by (6) and (7). Using (8), in

addition:

$$\begin{aligned} \left| \overline{H} \right| &= -U_x \lambda (U_x U_{yy} - U_y^2 / x - U_y U_{xy}) + U_y \lambda (U_x (U_y / x + U_{xy} - 2U_x / x - U_{xx})) \\ &= \\ &= \lambda (-U_x^2 U_{xx} + 2U_x U_y U_{xy} - U_y^2 U_{xx}) = \lambda_s \left| \overline{H}_s \right| / x > 0 \end{aligned}$$

(A3)

Hence the SOC's are satisfied, as claimed before.

Moreover, the partial differentiation of the optimal amount of x with respect to price of y keeping the utility constant in the standard model, which is the cross substitution term of the Marshallian demand is as follows:

$$\begin{aligned} \frac{(\partial x / \partial P_y)_s}{\text{cross substitution}} &= \frac{\lambda_s P_x P_y}{\left| J_s \right|} = \frac{\lambda_s (U_x / \lambda_s) (U_y / \lambda_s)}{\left(\left| J \right|_x / \lambda_s \right)} \\ &= \frac{\text{cross substitution}}{\text{of the Marshallian model}} \quad \text{of} \\ &= \text{our model} \quad \text{of} \\ &= T4 \end{aligned}$$

(A4)

which is exactly equal to the cross substitution effect of our model, given as the first term, T4 on the right hand side of (11).

It can be shown with similar methods that the own substitution effect of our model given as the first term on the right hand side of (12) is also equal to that of the standard model.

APPENDIX B

The derivative of our demand for the good x , (21) with respect to P_y , in the case of Cobb-Douglas utility of the section 3 is:

$$(\partial x / \partial P_y) = k^{1/a} \times (m(1-a))^{(a-1)/a} \times ((1-a)/a) \times P_y^{((1-2a)/a)} > 0$$

(B1)

Whereas the comparative static derivative from (11) is:

$$(\partial x / \partial P_y)_{CS} = \frac{(U_x U_y / x) + y(U_y / x)^2}{|J|} > 0$$

(B2)

Using $a=0.8$, $P_x=2$, $P_y=0.4$, $m=10$, $x=4$ and $y=5$, our calculation shows that the derivatives both in (B1) and (B2) are exactly equal to 2.5 in the case of Cobb-Douglas utility.

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