Id: 1428969

Participatory Educational Research (PER)
Vol.12(1), pp. 18-42, January 2025
Available online at http://www.perjournal.com
ISSN: 2148-6123
http://dx.doi.org/10.17275/per.25.2.12.1

Investigation of Eighth Grade Students' Performance on Tasks Involving Statistical Thinking About Measures of Central Tendency

Seyda AYDIN KARACA*
Department of Special Education, Hacettepe University, Ankara, Türkiye
ORCID: 0000-0003-0058-4379

Zeynep Sonay AY
Department of Mathematics and Science Education, Hacettepe University, Ankara, Türkiye
ORCID: 0000-0002-1037-7106

Article history
Received:
31.01.2024

Received in revised form:
01.09.2024

Accepted:
03.12.2024

Key words:
statistical thinking; measures of central tendency; statistics education

Given that students are and will continue to be in constant interaction with statistical data throughout their lives, they must be equipped to read, understand, and interpret such data. As a result, engaging in statistical thinking becomes essential. Statistical thinking can be defined as viewing the statistical process holistically in order to comprehend its underlying purpose. The aim of this study is to investigate eighth-grade students' performance on tasks related to measures of central tendency, and to analyse their statistical thinking processes in detail. This research was designed as a case study, with tasks developed by the researchers focusing on measures of central tendency used during the data collection phase. These tasks were analysed within the framework of the M3ST (Middle School Students' Statistical Thinking), and the students' statistical thinking levels were identified. Concurrently, the solution strategies employed by the students were examined, as well as their performance in solving problems related to measures of central tendency. The analysis revealed that the students' statistical thinking was predominantly at level 3 or level 4 for the components of organizing and reducing data, respectively, while it was at level 2 for the components of representing data and analysing and interpreting data. It was found that the participants possessed procedural knowledge regarding measures of central tendency but struggled to provide explanations or draw inferences about the concepts. Moreover, most of the participants appeared to lack a deep understanding of the fundamental concepts of measures of central tendency. These findings are discussed in the context of the existing literature.

Introduction

Data are used in many areas of daily life. These data may sometimes be collected, analysed, and represented to address a specific problem, or they may present a problem that requires data collection. It is essential to be able to collect, represent, read, analyse, and interpret data accurately in response to any problem encountered. Van de Walle et al. (2013) describe this process as the practice of doing statistics. One of the most critical stages that enables and facilitates interpretation in the statistical process is the data analysis stage. Data

* Correspondency: seydaaydink@gmail.com

analysis can be defined as the process of examining data, collected based on a problem, and represented in an appropriate display format, in order to make an interpretation that leads to a solution. Measures of central tendency are one of the most commonly used tools in the data analysis stage, which is considered the key phase in interpreting data.

Based on the concept of mean, measures of central tendency are as follows: arithmetic mean, median and mode (Bluman, 2012). The concept of mean corresponds to a single number or measure that can represent a large group of numbers and is a tool that provides information about the centre of a data set (Toluk-Ucar and Akdogan, 2009; Van de Walle et al., 2013). Although the concept of the mean is often used interchangeably with arithmetic mean because of the similarity of the words, this usage is not always correct (Watson and Moritz, 2000). Arithmetic mean, which is a measure that gives information about the mean or central tendency of the data group (Randall, 2006), is the summation of the collected data in the data set divided by the number of data values (Bluman, 2012), whereas mean corresponds to the point of balance of the elements in a data set or to the number at which the values in a data set are equalized (Van de Walle et al., 2013). The median, another measure of central tendency, is the value in the middle of the sorted data set (Bluman, 2012). The other measure of central tendency is mode. Mode is the most frequently occurring data value in the dataset (Bluman, 2012).

Measures of central tendency is considered to be one of the most challenging data processing topics addressed in the mathematics curriculum of the Turkish Ministry of Education (Toluk-Ucar and Akdogan, 2009). This is because it requires numerous operations There are learning outcomes addressing the topic of data processing across all grade levels, from first grade to eighth grade (MoNE, 2018) and aims to ensure that students become individuals who can use statistics actively in their daily lives. Individuals need statistical information and statistical thinking because they need to use them in all areas of their daily lives (Bond et al., 2012; Franklin et al., 2005; NCTM, 2000). Many different definitions of statistical thinking have been made by different researchers. Smith (1999) defines statistical thinking as understanding what a statistician does, while according to Chance (2002), it is to see the statistical process as a whole, to ask new questions beyond what has been asked, and to see the big picture.

Conceptual Framework: The identification of individuals’ statistical thinking has been of significant interest in studies in this area. Several models have been created in the related literature with the purpose of determining statistical thinking. The first of these models, developed by Ben-Zvi and Friedlander (1997), consists of four non-hierarchical developmental stages: (a) mode 0- uncritical thinking, (b) mode 1- meaningful use of a representation, (c) mode 2- meaningful handling of multiple representations: developing metacognitive abilities, and (d) mode 3- creative thinking.

Jones et al. (2000) developed a statistical thinking model consisting of data description, data organization, data representation, data analysis and interpretation. Each component includes four levels of thinking. They stated that this model can be used by teachers and program developers to provide instructional information, identify students' statistical thinking, and prepare learning tasks.

The model proposed by Jones et al. (2000) to describe the statistical thinking of sixth, seventh, and eighth-grade middle school students was developed by Money (2002), then it was revised and created the M3ST (Middle School Students' Statistical Thinking) model. The M3ST model consists of four components: describing data, organizing and reducing data, displaying data and analysing and interpreting data. These components were defined as follows (Mooney, 2002, p.28):

Describing Data: It refers to the clear reading of the data presented in tables and graphs. Being able to read the data is the basis for students to make inferences and identify the central tendency.

Organizing and Reducing Data: It involves the organization of the data in a concise manner, categorization and their reduction according to their central tendency and distribution.

Representing Data*:* It includes displaying the data in the form of appropriate graphs.

Analysing and Interpreting Data: It includes determining the central trend of the data, making inferences and predictions from the representations, that is, reading beyond the data.

Each component of the statistical thinking model consists of four hierarchical thinking levels based on Biggs and Collis' (1991) developmental model (Mooney, 2000). The developmental model of Biggs and Collis (1991) consists of five stages: "sensorimotor (from birth), iconic (from around 18 months), concrete symbolic (from around 6 years), formal (from around 14 years) and postformal (from around 20 years)." However, the data in Mooney's (2002) research showed that students only have the first four levels, which Mooney (2002) referred to as idiosyncratic, transitional, quantitative, analytical in statistical thinking. For this reason, Mooney (2002) characterized the statistical thinking framework according to these four levels of thinking, which are described below (Mooney, 2002, p.29).

Level 1 Idiosyncratic: Students are interested in the task but are distracted or mistaken. It is usually limited to the individual's own judgment, which is irrelevant to the data.

Level 2 Transitional: Students use a single partially valid measure to describe the typicality, trend or spread of the data, and the data perspectives and implications are in one direction.

Level 3 Quantitative: Students can generate accurate ideas about the representations, trends, and spreads of data, but may have difficulty integrating data and context.

Level 4 Analytical: Students can use the context to identify the data and the trend of the data, create multiple data representations, use different measures of central tendency, and make inferences appropriate to the context.

Literature Review: Based on models developed for statistical thinking, numerous studies have been conducted to assess students' statistical thinking across various grade levels, ranging from primary school to university (Akkas, 2009; Chick & Watson, 2002; Doluzengin, 2019; Groth, 2003, 2006; Jones et al., 2000; Koparan & Guven, 2013, 2014; Mooney et al., 2001; Mooney, 2002; Ozdemir, 2014). In a study conducted on secondary school students, Akkas (2009) examined students' statistical thinking using the SOLO taxonomy and found that students performed at higher levels when describing data but were mostly at the 2nd and 3rd levels in other areas of statistical thinking. Similarly, Koparan and Guven (2013), in their study with secondary school students, observed that while students tended to focus on the fourth level when describing data, they were mostly at the first level in the other three components of statistical thinking. Altaylar and Kazak (2021) investigated the effectiveness of the Realistic Mathematics Education (RME) approach on sixth-grade students' statistical thinking levels. Their analysis revealed a significant improvement in the statistical thinking processes at Level 4 among students taught using the RME approach, compared to those taught with traditional methods. Yilmaz (2023) conducted a study with fourth-grade students, who were asked to work on tasks based on four different contexts: graphs, averages, distribution, and variance. The study found that students demonstrated the highest proficiency in reading data related to describing data. They also showed competence in evaluating similarities and differences in data and graphs. However, students faced challenges when interpreting concepts related to organizing and reducing data, such as average, distribution, and variation. While these studies have examined various statistical topics appropriate to the students' levels, the current study is particularly important as it offers insights into both specific grade levels and sub-topics of statistics.

The current study was motivated by the need to investigate the statistical knowledge of eighth-grade students, who were approaching the completion of their middle school education. More specifically, the study focuses not only on the measures of central tendency that students will use in their daily, educational, and future lives, but also on how they engage in statistical thinking when applying these measures. The Turkish National Mathematics Curriculum (MoNE, 2018) emphasizes the understanding of mathematical concepts and their practical application in daily life, placing particular importance on higher-order thinking skills such as critical thinking, statistical thinking, metacognition, questioning, making interpretations and estimations, detecting flawed reasoning, problem-solving, and performing mental calculations, rather than merely memorizing isolated facts without an understanding of their real-world application. A review of the relevant literature reveals that there are relatively few studies that explore the measures of central tendency in the context of statistical thinking, and most studies tend to focus primarily on the arithmetic mean. This highlights a gap in the research, suggesting a need for more comprehensive studies that examine students' knowledge and statistical thinking regarding measures of central tendency. Therefore, this study is expected to make a significant contribution to the literature by providing a detailed examination of statistical thinking in relation to measures of central tendency, a key topic in the field of statistics. The aim of this study is to investigate eighth-grade students' performance on tasks related to measures of central tendency and to analyse their statistical thinking processes in depth.

Method

Research Design: The aim of the current study was to examine eighth-grade students' performance on tasks and statistical thinking about measures of central tendency. The case study research design, one of the qualitative research methods, was employed in the present study. Case studies allow in-depth investigation of the target problem (Cresswell, 2013). They are used to gain in-depth insight into how and why a present circumstance is happening, explore complex phenomena (particularly salient in work on socio-environmental systems), and formulate "lessons learned" that may be applicable to future cases (Zainal, 2007).

Study Group: The participants of the current study were thirty eighth-grade students attending a public school in the urban area of Ankara. Twelve of the study group were male and eighteen were female. All participants are from middle socio-economic level. Based on the learning outcomes defined by the national mathematics curriculum of the country where the research was conducted, eighth-grade students need to know how to calculate and interpret measures of central tendency (arithmetic mean, mode, median), and determine the measures of central tendency representing the data set. The participants have been encountering issues related to data handling and data analysis in mathematics lessons since primary school. These topics are also included in mathematics textbooks. The national high school entrance exam also includes questions on these topics. Ethical approval was obtained from Hacettepe University Ethics Committee for this study. Informed consent was obtained from eight-grade students and their parents who would participate in the study. The study group was informed that participation in the study was voluntary and that their identities would be kept confidential. Code names were used instead of students' real names to ensure the confidentiality of their identities such as S1, S2,…, S30. The participants were selected by the course teacher by means of purposive sampling; ten students from among high math achievers, ten from among moderate math achievers and ten from among low math achievements were selected. The students' math course teacher made this selection by examining the students' math course scores. It is accepted that there may be bias in the selection of participants. The math course teacher randomly selected 10 students from students whose math course scores were above 85, 10 students from students whose math course scores were between 55-75, and 10 students from students whose math course scores were below 45. These scores were determined based on the classification in the scoring system. It is accepted that there may be bias in the selection of participants. In order for the findings to be more understandable and to be interpreted in more detail, the participants who were determined to have high mathematics achievement were coded as S1, S2, S3, S4, S5, S6, S7, S8, S9, and S10; the participants who were determined to have moderate mathematics achievement were coded as S11, S12, S13, S14, S15, S16, S17, S18, S19, and S20 and the participants who were determined to have low mathematics achievement were coded as S21, S22, S23, S24, S25, S26, S27, S28, S29, and S30.

Data Collection Tools: The data collection tool was developed by the researcher to reveal levels of statistical thinking in terms of measures of central tendency. The instrument consisted of 16 open-ended questions to elicit data about the following components of statistical thinking proposed by Mooney (2002): describing data, organizing and reducing data, representing data, analysing and interpreting data. Four expert opinions were obtained on the instrument. Two of the experts are researchers who have PhD degrees in mathematics education and are studying on statistics education. The other two are teachers who have master's degrees in mathematics education and have been working as mathematics teachers for more than 5 years. In line with the feedback of the experts, the instrument was revised and finalized. Firstly, a preliminary study was conducted with 5 eighth-grade students. The purpose of the preliminary study was to ensure the reliability and validity of the instrument, and conduct a trial before the main study and to resolve possible problems. The instrument was then administered to thirty eighth grade students in the main study. The instrument was administered to thirty eighth-grade students. The statistical thinking components to which the questions in the instrument are related are presented in Table 1. After students completed the instrument, semi-structured interviews lasting approximately forty minutes were conducted with each student to gain a more detailed understanding of how students understood and thought about measures of central tendency. These interviews provided a deeper understanding and interpretation of the students' solutions to problems related to central tendency measures. These interviews were recorded and later transcribed.

Table 1. The questions addressing the concepts and components of statistical thinking.

Components of Statistical Thinking	Related Concept	Question Number
Organizing and Reducing Data	Arithmetic mean	1, 8, 11
	Median	2, 12
	Mode	3, 10
Representing Data	The Measure of Central Tendency Representing the Data Set	5, 7, 14, 15
Analyzing and Interpreting Data	Arithmetic mean	4, 9, 13
	Median	16
	Mode	6

Data Analysis: The data obtained from the answers given by the participants to the questions in the data collection tool were analysed according to the components and specifications in Mooney's (2002) Middle School Students' Statistical Thinking [M3ST] framework. The data were scored by both researchers. The inter-rater agreement was found to be 95%.

The components and specifications in Mooney's (2002) Middle School Students' Statistical Thinking [M3ST] framework, which is used as the basis for the analysis of the current study, are presented below (Mooney, 2002).

Organizing and Reducing Data

Level 1 – Idiosyncratic:

- Does not make any attempt to organize data (grouping/ordering).
- Cannot define the central tendency of the data.

Level 2 – Transitional:

- Groups or sorts out data so that they are somewhat representative of the data.
- Describes data with developed measures that are somewhat valid.

Level 3 – Quantitative:

- Groups or sorts out data so that they are completely representative of the data, but makes some minor errors.
- Depicts data using a measure of central tendency with an erroneous operation.

Level 4 – Analytical:

- Groups or sorts out data in multiple ways, and data of each method are descriptive.
- Describes data using a valid and accurate measure of central tendency.

Representing Data

Level 1 – Idiosyncratic:

- Cannot construct any representation or constructs an incomplete representation or cannot represent the data.
- Cannot define a partially constructed representation.

Level 2 – Transitional:

- Creates a representation that represents the data but is partially complete or complete but not representative of the data.
- Completes a partially constructed different/unconventional representation.

Level 3 – Quantitative:

- Creates a complete and representative representation of the data. The representation may contain minor errors.

- Accurately completes a partially constructed different/unconventional representation.
- Creates multiple representations based on the same data and context.
- Can switch between representations but representations may contain minor errors.

Level 4 – Analytical:

- Creates a complete representation representative of the data in a manner suitable for the data and context.
- Thoroughly completes a partially constructed different/unconventional representation.
- Seamlessly creates multiple representations based on the same data and context.
- Can switch between representations.

Analysing and Interpreting Data

Level 1 – Idiosyncratic:

- Does not make any comparison between data sets or representations or makes wrong comparisons.
- Makes inferences in contexts that are not based on data or that are completely irrelevant to the data.

Level 2 – Transitional:

- Either makes a single correct comparison between data sets or representations, or makes comparisons that are only somewhat correct.
- Makes inferences based on data. Some inferences may only be somewhat reasonable/appropriate.

Level 3 – Quantitative:

- Makes partial or holistic comparisons between data sets or their representations.
- Makes reasonable/appropriate inferences based on data and context.

Level 4 – Analytical:

- Makes partial or holistic comparisons between data sets or their representations.
- Makes reasonable/appropriate inferences based on data and context from different perspectives.

The following example is provided for the data analysis of the research. The ninth question is on the concept of arithmetic mean and is a question regarding the data analysis and interpretation component. A data group shown with a bar chart is given in the question. It is asked how a data added to this data group will affect the arithmetic mean.
Expected answer for the ninth question: Students are expected to state that the arithmetic mean of the data group will decrease due to the fact that a data smaller than the arithmetic mean of the data group is added to the data group. They are expected to reach this conclusion by including the newly added data in the group and using the arithmetic mean finding procedure or by balancing the columns that correspond to the data in the bar chart.
The framework student answers used to determine and interpret the statistical thinking levels according to the student answers in this question are given below.

It was decided that the statistical thinking of the students who said I don't know and the students who left the question unanswered was at level 1. The students who did not answer were asked basic questions about the measures of central tendency mentioned in the question and the students said that they did not understand, did not know and did not want to answer. Therefore, it could not be observed whether they had knowledge about the arithmetic mean. Based on the data obtained from the teacher about the mathematics course and the general academic performance of these students, it was thought that the students did not have an idea about the measures of central tendency. It was decided that the statistical thinking of the students who said that the arithmetic mean would increase or decrease without doing any calculations or explaining it was at level 2. It was decided that the statistical thinking of the students who said that the arithmetic mean would increase by adding the newly added data to the group's data total and doing the calculation without adding it to the data number was at level 2. It was decided that the statistical thinking of the students who found that the arithmetic mean would decrease by doing calculations or trying it out but did not explain the reason was at level 3. It was decided that the statistical thinking of the students who said that the arithmetic mean would decrease because a smaller data than the arithmetic mean was added to the group was at level 4.

Results

In the current study, eighth-grade students' performance on tasks and statistical thinking was examined in terms of measures of central tendency. The results are categorized and discussed under the relevant statistical thinking component. The participants' answers to the questions in the data collection tool were analysed according to the specifications in Mooney's (2002) statistical thinking framework and both the participants' knowledge of measures of central tendency and their statistical thinking levels were identified. After reporting the findings on the statistical thinking of the eighth-grade students to determine the arithmetic mean, median, mode and the appropriate measure of central tendency representing the data set, general comments on measures of central tendency were discussed. The descriptive findings are presented in Table 2.

Table 2. Descriptive findings.

			Level 1 Idiosyncratic	Level 2 Transitional	Level 3 Quantitative	Level 4 Analytical
Organizing and Reducing Data	Mean	Low	7	2	1	-
		Middle	-	-	3	7
		High	-	-	-	10
	Median	Low	6	1	3	-
		Middle	-	-	2	8
		High	-	-	-	10
	Mode	Low	6	4	-	-
		Middle	-	-	-	10
		High	-	-	-	10
Representing Data	Determining Measures of Central Tendency Representing the Data Set	Low	6	4	-	-
		Middle	-	9	1	-
		High	-	1	7	2
Analyzing and Interpreting Data	Mean	Low	6	4	-	-
		Middle	-	5	4	1
		High	-	-	2	8
	Median	Low	7	3		-

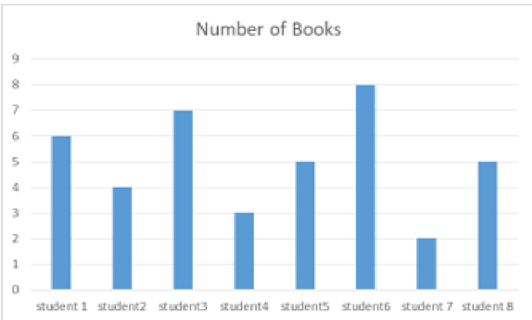
	Middle	4	-	6	-
	High	2	-	8	-
Mode	Low	7	3	-	-
	Middle	-	2	8	-
	High	-	-	5	5

Results and Interpretations of Organizing and Reducing Data: There were seven questions in the data collection tool to elicit information about statistical thinking during the stage of organizing and reducing data. Of these seven questions, three were about the arithmetic mean, two were about the median, and two were about the mode. Most of the participants found the correct answers to the questions on the arithmetic mean, median and mode of the data set. It was observed that the statistical thinking of the participants, who found the correct answer by following the right path, was at level 4, as indicated in the statistical thinking specifications table. Below are the answers and operations of two students to the questions on finding the arithmetic mean of the data set.

The example given in Figure 1 was a question related to the finding of the arithmetic mean and the component of organizing and reducing data. The data set consisted of an even number of data. The student was asked to find the arithmetic mean value of a data set consisting of quantitative data. In Table 3, it provided answers to this question.

8. Librarians are required to conduct research on the students present in the library during the day. They collect the necessary data. The graph given below shows the number of books borrowed by eight students to conduct research from the library. The graph shows the number of the books borrowed by the students to conduct research.

Graph: *Number of Books Students Borrowed from the Library*



The librarians who want to show the data on a bar graph construct the bar graph showing the number of books borrowed by the students as above. The library management wants them to determine the arithmetic mean of the number of books barrowed during that day and then enter it into their daily records. Find the number of books reported to the library management by the librarians via different methods.

Figure 1. Question 8.

The expected answer to the question: The arithmetic mean of the data set is 5. In this question, students can use two methods. First, they can be expected to reach the answer 5 by summing the data in the data set and dividing it by 8, which is the number of data values. Alternatively, they can arrive at the answer 5 by balancing the bars that are suitable for completing each other at 5, using the balance point interpretation of the concept of arithmetic mean.

Table 3. Students' statistical thinking levels for question 8.

	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Student behaviours related to statistical thinking levels in the component of organizing and reducing data	Leaving unanswered S28, S29, S30	Dividing the sum of the data by the median value in the arithmetic mean to find the algorithm S24	Finding the sum of the data wrong by making a calculation error and dividing the number found by the number of data S25	Accurately collecting data and dividing by the number of data S1, S2, S3, S6, S7, S8, S9, S11, S12, S13, S14, S15, S16, S17, S18, S19, S20
	Saying "I do not know" S22, S23, S26, S27	Confusing the arithmetic mean with the median S21		Using the level-equalizing interpretation of the arithmetic mean S4, S10

For these questions, some of the students with low math achievement, S22, S23, S26 and S27, stated that they did not know the answer to the question, while S28, S29 and S30 left the question unanswered. When additional questions such as how do you find the arithmetic mean or what is the arithmetic mean were asked to the students who did not answer, they said that they did not understand and did not want to answer. Therefore, it could not be observed whether they had information about the arithmetic mean. Based on the data obtained from the teacher about the mathematics course and general academic performance of these students, it was thought that the students did not have an idea about measures of central tendency. When the descriptors given in Table 4 for the question in the component of organizing and reducing data were examined, it was concluded that the statistical thinking level of the students who could not give any answers was at level 1. The solution by S24, whose statistical thinking level was observed to be at level 2, is displayed in Figure 2.

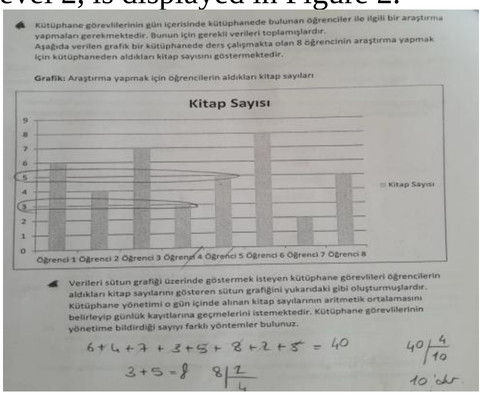


Figure 2. The answer given by S24.

S24 found the arithmetic mean using an algorithm. However, in the algorithm, he/she divided the sum of the data in the data set not by the number of data given, but by the median of the data set. He/she found the median of the data set by taking the arithmetic mean of the numbers in the middle of the group without sorting out the data. The dialogue between the student and the researcher is given below.

Researcher: Can you explain the operations? Why did you do it this way?

S24: Because the arithmetic mean is the division of the sum of the data by the middle number.

It was observed that S24 made the same systematic error in similar questions. The student was found to have a misconception about the concept of the arithmetic mean.

Almost all of the participants used the arithmetic mean algorithm (adding the data in the data set and dividing it by the number of data values in the data set) to find the arithmetic mean while answering the questions. Only two participants (S4 and S10, who were high mathematics achievers) did not use the arithmetic mean algorithm; they reached the answer by balancing the data at one point, based on the conviction that arithmetic mean is the equilibrium point of the data set. The solution of S10 reveals that he/she has balanced the data in the data set at one point and found the arithmetic mean of the data set.

The second question of the data collection tool, which is about finding the median of the data set, is depicted in Figure 3. Additionally, in Table 4, it provided the components for the question.

2. The number of eggs obtained from the chickens in a coop during a week is given in the table below.

Table: *Eggs Obtained from the Chickens in a Coop*

Days of Week	Friday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Number of Eggs	8	12	7	9	6	7	14

- Find the median of the number of eggs obtained from the chickens during a week. (What is median? Can you define it?)

Figure 3. Question 2.

The expected answer to the question: After ordering the data in the data set from the smallest to largest, students are expected to state that the number in the middle of the data set, namely 8, is the median of the data set.

Table 4. Students' statistical thinking levels for question 2.

	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Student behaviours related to levels of statistical thinking in the component	Leaving unanswered S28, S29, S30	Confusing the median with the mode S21	Identifying the data in the middle of the data set without ordering the data in the data set S19, S20, S23,	Ordering the data without errors and determining the median of the data set S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12,

of organizing and reducing data		S24, S25	S13, S14, S15, S16, S17, S18
	Saying “I don’t know”		
	S22, S26, S27		

For this question, some of the students who were low math achievers, namely S22, S26 and S27, stated that they did not know the answer to the question, while S28, S29 and S30 left the question unanswered. When additional questions such as how do you find the median or what is the median were asked to the students who did not answer, they said that they did not understand and did not want to answer. Therefore, it could not be observed whether they had information about the median. Based on the data obtained from the teacher about the mathematics course and general academic performance of these students, it was thought that the students did not have an idea about measures of central tendency.

The solution provided by S21, whose statistical thinking level was observed to be 2 and who confused the arithmetic mean and the median, is given in Figure 4. In Table 5, it provided components and answers for the questions.

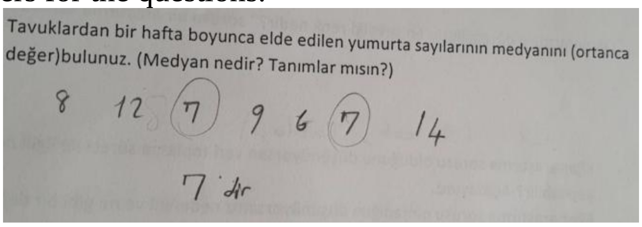


Figure 4. The answer given by S21.

It was observed that this participant (S21) confused the median with the mode and found the mode of the data set. The student used the concepts of mode and median interchangeably. When the answers given by this student to the other questions were assessed, it was seen that the participant systematically confused the mode and median. It was concluded that S21 had a misconception in this specific area. Thus, it was determined that the statistical thinking of the participant was at level 2. On the other hand, the statistical thinking of the participants who left the question unanswered was identified to be at level 1.

The third question of the data collection tool, which was about finding the mode of the data set, is displayed in Figure 5.

3. The number of eggs obtained from the chickens in a coop during a week is given in the table below.

Table: *Eggs Obtained from the Chickens in a Coop*

Days of Week	Friday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Number of Eggs	8	12	7	9	6	7	14

- Find the mode of the number of eggs obtained from the chickens during a week. (What is mode? Can you define it?)

Figure 5. Question 3.

The expected answer to the question: Students were expected to notice that 7 was the most recurring value in the data set and to indicate that 7 was the mode of the data set.

Table 5. Students' statistical thinking levels for question 3.

	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Student behaviours related to the levels of statistical thinking in the component of organizing and reducing data	Leaving unanswered S28, S29, S30	Confusing the median with the mode S21	Correctly giving the mode but making a small mistake in the explanation	Saying that the mode is the most recurring data in the data set S1, S2, S3, S4, S5, S6, S7, S8, S9, Ö10, S11, S12, S13, S14, S15, S16, S17, S18, S19, S20, S25
	Saying "I don't know" S22, S26, S27	Saying that the mode is the largest value in the data set S23, S24		

Most of the participants gave correct answers to the questions requiring the identification of the mode of the data set. It was observed that the participants had good knowledge of how the concept of mode was determined in the data set. It was concluded that the statistical thinking of the students who answered these questions without any problems was at level 4. As mentioned in the previous questions asking for the median, S21 confused the mode and median. On the other hand, S23 and S24 stated that the mode was the data with the largest value in the data set. Thus, it was concluded that the statistical thinking of these participants was at level 2.

Results and Interpretations of Representing Data: There were four questions in the data collection tool to elicit information about how eighth-grade students think statistically about representing data. These questions are also about determining the measures of central tendency representing the data set. Two of these questions have the data set with outliers and the other two are questions where the data in the data set consist of values close to each other. The question given in the example in Figure 6 consists of a data set in which the data are close to each other and the difference between the extreme values is small. The purpose of this question is to examine the reason underlying students' choice of central tendency measure to represent the data set. In Table 6, it provided components and answers for the questions.

15. Mr Mehmet, the owner of a sugar factory, wrote the amount of sugar beet used to produce sugar in his factory for a week in the table given below.

Table: *Amounts of Sugar Beet Used in a Week*

Days of Week	Friday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Quantity (tons)	1500	1000	650	1800	447	1000	2500

Mr Mehmet, at the board of directors meeting, wants to determine the central tendency measure that will best represent the data set for the amount of sugar beet used for one week and to speak on it at the meeting. However, he could not decide which central tendency measure (arithmetic mean, median, mode) would be more appropriate to represent this data set. Which central tendency measure do you think Mr Mehmet should use? ** Explain why.

Figure 6. Question 15.

The expected answer to the question: The students were expected to consider the median because of the difference between the extreme values in the data set.

Table 6. Students' statistical thinking levels question 15.

	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Student behaviours related to the levels of statistical thinking in the components of representing data and analysing and interpreting data	Leaving unanswered S28, S29, S30	Saying that the appropriate measure of central tendency to represent the data set is the arithmetic mean S5, S6, S7, S9, S11, S12, S13, S14, S15, S16, S18, S19, S20, S25		Saying that the appropriate measure of central tendency representing the data set is the median by making the correct explanation S4, S10
	Saying "I don't know" S22, S26, S27	Saying that the appropriate measure of central tendency representing the data set is the mode S21, S23, S24		

	Saying that the appropriate measure of central tendency representing the data set is the median without making any explanation
	S1, S2, S3, S8, S17

The participants who incorrectly determined the measure of central tendency representing the data sets and those who could correctly determine the measure of central tendency representing the data sets but did not explain the reason were placed at level 2. There was no participant at level 3 in these questions. S4 and S10, as in the previous questions, correctly explained both the measure representing the data set and the reason for it. It was determined that the statistical thinking of these two participants was at level 4.

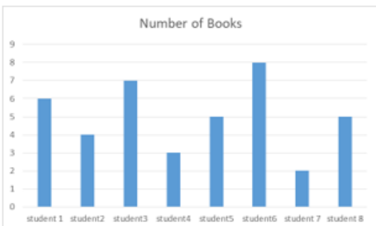
While most of the students were at level 4 of statistical thinking in the questions requiring the determination of the arithmetic mean, median and mode of the data set, the majority of the students were at level 2 in the questions requiring interpretation of the arithmetic mean, median and mode and the questions requiring the completion of the data set. Similarly, the majority of the students were at level 2 of statistical thinking in the questions asking for the identification of the measure of central tendency suitable for the data set.

Results and Interpretations of Analysing and Interpreting Data: There were five questions in the data collection tool to elicit statistical thinking in the area of analysing and interpreting data. Of these five questions, two were about the arithmetic mean, two about the median, and one about the mode. Most of the participants could not provide the correct answer to the questions on analysing and interpreting the arithmetic mean, median and mode of the data set. Based on the statistical thinking specifications table, the statistical thinking of the participants was found to be at either level 2 or 3. Below are the answers and operations of a student to the questions on analysing and interpreting the arithmetic mean of the data set.

The question in Figure 7 is requires interpreting the arithmetic mean belonging to the data set. While there were participants who provided the correct answer in this type of question, there were also participants who could not reach the right answer due to a different and inaccurate way of thinking. In Table 7, it provided components and answers for the questions.

9. Librarians are required to conduct research on the students present in the library during the day. They collect the necessary data. The graph given below shows the number of the books borrowed by eight students to conduct research from the library. The graph shows the number of the books borrowed by the students to conduct research.

Graph: *Number of Books Students Borrowed from the Library*



If one more student comes to the library and barrows one book, how will the number reported to the library management by the librarians change? Explain its reason.

Figure 7. Question 9.

The expected answer to the question: Students were expected to state that the arithmetic mean of the data set would decrease because data smaller than the arithmetic mean of the data set are added to the data set. They were expected to arrive at this result by incorporating the newly added data into the set, using the arithmetic mean finding procedure or balancing the bars that match the data in the bar chart.

Table 7. Students' statistical thinking levels for question 9.

	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Student behaviours related to statistical thinking levels in the component of analysing and interpreting data	Leaving unanswered S28, S29, S30	Saying that the arithmetic mean will increase without performing any operations or giving any reasons S21, S23, S24	Saying that the arithmetic mean will decrease by performing operations but without giving any reasons S7, S9, S11, S12	Saying that the arithmetic mean will decrease as data smaller than the arithmetic mean is added S1, S2, S3, S4, S5, S6, S8, S10, S17
	Saying "I don't know" S22, S26, S27	Saying that the arithmetic mean will increase by adding the newly added data to the sum of the data set but without adding it to the number of data S14, S25	Saying that the arithmetic mean will decrease because a value smaller than the number of data is added S13, S15, S18	
		Saying that the arithmetic mean will decrease without performing any operations and giving any reasons S16, S19, S20		

Participants did not have knowledge of how to think about when some data were added to or removed from a data set. There were many participants who misinterpreted this condition and could not reach the result with the arithmetic mean algorithm. With respect to this type of question, it was observed that the distribution of the various levels of statistical thinking of the participants was close to each other: levels 2, 3 and 4. It was observed that the statistical thinking of the participants who left the question unanswered was at level 1. The participants whose statistical thinking was determined to be at level 2 either gave the wrong answer without providing a reason or reached the wrong conclusion. In Figure 8, the answers and operations provided by a student to the questions requiring interpreting the arithmetic mean of the data set.

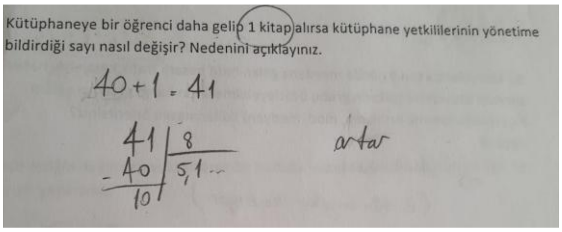


Figure 8. The answer given by S14.

S14, who was a middle mathematics achiever, answered the question of how the arithmetic mean would change if a new data value was added to the data set by making an operation. He/she added the newly added data value to the sum of the data in the data set but applied the arithmetic mean algorithm without adding it to the number of data. For this reason, he/she obtained the wrong answer. The participants, whose statistical thinking was determined to be at level 3, either gave the correct answer without stating a reason or gave the correct answer by stating a wrong reason. The participants whose statistical thinking was revealed to be at level 4 were those who had found the correct answer and provided an accurate explanation of the reason.

One of the questions on analysing and interpreting data was a question related to the median and a slightly higher-order question requiring the completion of the data set. In the question, the median and mode of a data set with missing data were given and the data set was asked to be completed. The question is shown in Figure 9. Additionally, Table 8 provides components and answers to the questions.

16. For a survey conducted in a site where 8 families are known to live, some parts of the paper on which the data of the number of family members were written and erased and could not be read. However, some of the information recorded in the data set prepared on the first day of data collection on how many people these 8 families consisted of is given below.

- Mode = family consisted of 4 people

- Median = family consisted of 4 people

- There are families consisting of at least 2 and at most 8 members in the data set.

Below is given the table containing the data on how many members the families have. Based on the information given above, find the number of members in the family whose number of persons has been deleted in the table below.

Families	Number of People
Family 1	
Family 2	3
Family 3	8
Family 4	
Family 5	4
Family 6	
Family 7	6
Family 8	

Explain how you created the data set.

Figure 9. Question 16.

The expected answer to the question: The students were expected to complete the given data set using the information about the median, mode, and minimum-maximum values provided in the question. Students were expected to complete the data set by noting that the median and mode should be 4, and that the smallest data should be 2 and the largest data should be 8. The correct sample data sets were 2-2-3-4-4-4-6-8, 2-3-4-4-4-5-6-8, and 2-3-4-4-4-4-6-8.

Table 8. Students' statistical thinking levels for question 16.

	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Student behaviours related to the levels of statistical thinking in the component of analysing and interpreting data	Leaving unanswered S28, S29, S30	Confusing the median with the mode and thinking that the mode is the largest value in the set S21	Determining the data in the middle of the data set as the median without ordering the data in the data set, creating a data set suitable for other variables S9, S12, S16, S17, S18	Completing the data set paying attention to all information about the median, mode and minimum and maximum values S1, S2, S4, S8, S10
	Saying "I don't know" S22, S23, S26, S27	Determining the median without ordering the data set and considering the mode as the largest value in the data set S24	Determining the median and mode of the data set without any difficulty but ignoring the information of maximum and minimum values S3, S5, S6, S7, S11, S13, S15	
		Determining the median without ordering the data and ignoring the information about the mode in the question S19, S20, S25		

The participants who left the question unanswered showed a similar attitude to the previous questions, and it was observed that the statistical thinking of these participants was at level 1. S19, S20, S21, S24, and S25 could not find the right answer: S21 confused the median and mode, S24 identified the median without sorting out the data set and thought that the mode was the data with the highest value in the data set, and S19, S20 and S25 disregarded the mode. It was determined that the statistical thinking of these participants was at level 2. It was seen that the majority of the participants had statistical thinking at level 3 because they did not sort out the data in the data set but used the other variables in the question correctly and reached the correct result. It was determined that only five of the thirty participants had statistical thinking at level 4 because they found the correct result by considering all the variables in the question.

In the data set question consisting of qualitative data, which is another question type for analysing and interpreting data, the participants were expected to identify the variable with the highest frequency in the data set, and to realize that this corresponds to the concept of mode. This question is shown in Figure 10. Additionally, in Table 9, it provided components and answers for the questions.

6. For the math project, Bersu prepared and administered a questionnaire to her classmates to determine the type of book they read most. The survey results are shown in the table below.

Table: *Types of books read by the students*

Book Type	Number of Students
Novel	12
Comic	8
Cartoon	5
Biography	2
Poem	3

At the end of her homework, Bersu has to indicate the type of book her classmates read most. Accordingly, with which central tendency measure (arithmetic mean, median, mode) can Bersu determine the type of book that her classmates read most? *** Explain how you think.

Figure 10. Question 6.

The expected answer to the question: The students were expected to state that the genre that was read most was the novel and that the central tendency measure needed to identify this was the mode.

Table 9. Students' statistical thinking levels for question 6.

	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Student behaviours related to the levels of statistical thinking in the component of organizing and reducing data	Leaving unanswered S28, S29, S30	Confusing the mode with the median and giving the value of the median S21, S24	Giving the value of the mode but not explaining its reason S1, S2, S3, S4, S5, S6, S8, S10, S11, S12, S13, S15, S17, S20	Stating that the novel is the most recurring data in the data set and giving the value of the mode
	Saying "I don't know" S7, S9, S14, S16, S18, S19, S25, S22, S26, S27	Giving the value of the arithmetic mean S23		

While the number of participants who left other questions unanswered was the same, the number of participants who left this question unanswered increased. It was concluded that the statistical thinking of the students who left the question unanswered was at level 1. Two of the participants gave the answer “median” because they confused the mode and median and one gave the answer as the arithmetic mean. To answer the question, these three participants identified the genre read the most by using the qualitative data, but they incorrectly identified the measure of central tendency and could not explain the reason underlying this answer. It was revealed that the statistical thinking of these participants was at level 2. Fourteen of the participants stated that the novel was the genre read the most based on the data set and that the measure of central tendency to identify this was the mode. However, they could not explain why they thought so;- they merely said, “the most repetitive data”. For this reason, it was determined that the statistical thinking of these participants was at level 3. There was no participant who was at level 4 in statistical thinking for this question.

Discussion and Conclusion

In the current study, interviews were conducted with a participant group of thirty students by using a data collection tool in order to examine eighth-grade students’ performance on tasks and statistical thinking about measures of central tendency. It was observed that the participants’ statistical thinking levels in the components of representing data and organizing and reducing data, that is, in the operational questions about measures of central tendency, were relatively higher (level 3 and level 4), while their statistical thinking levels were found to be relatively lower (level 2) in the component of analysing and interpreting data, that is, in the questions requiring inferences and interpretations about measures of central tendency and the questions requiring the completion of the data set. Similar to this result, Akkas (2009) concluded that the statistical thinking of secondary school students was at either level 2 or 3. Also, Yilmaz (2023) observed that fourth-grade students demonstrated highest proficiency in reading the data related to the construct of describing data, and they faced challenges when it came to interpreting concepts such as average, distribution, and variation, which are associated with organizing and reducing data.

Statistical thinking processes are evaluated on the basis of statistical information (Toluk-Ucar and Akdogan, 2009). The data collection tool, which was prepared to identify the statistical thinking levels of the participants in terms of measures of central tendency, also measures the statistical knowledge of the participants. It was observed that the statistical thinking of the group of students with low mathematical success was generally at the first level, and only a few of them had second-level statistical thinking in some problems, which can be solved with operational knowledge. It was observed that most of the students in this group could not even complete the operations related to the concepts of measures of central tendency. It was also determined that they did not have sufficient operational knowledge about these concepts. It was observed that the statistical thinking of the group of students with medium mathematical achievement was generally at the third level. It was observed that the statistical thinking of these students was at the fourth level in some problems, which can be solved with operational knowledge. It can be said that most of the students in this group are familiar with the concepts of central tendency measures and have operational knowledge about these concepts. They do not have an idea about the meanings, relationships and purposes of use of these concepts. Conceptual learning enables students to construct knowledge, relate new information to existing knowledge, and thus understand within a broader framework (Bruner, 1960). In this context, it is of great importance to accurately understand the concepts of central tendency measures and to achieve conceptual learning. It has been observed that the statistical thinking of the group of students with high mathematical success is generally at the third level. It has been observed that the statistical thinking of these students is at the fourth level in some problems, which can be solved with operational knowledge. It has been observed that the statistical thinking of S4 and S10 in this group is generally at the fourth level. These students have conceptual knowledge about measures of central tendency and know what these concepts mean.

When the answers given by the participants to the questions about measures of central tendency were evaluated, it was concluded that the conceptual knowledge of the participants regarding measures of central tendency was not sufficient. It was observed that most of the participants had information about how to find measures of central tendency in the data set but they did not have the knowledge about what those concepts mean for the data set and how they should be interpreted. The participants generally reached the result using algorithms without experiencing any problems in the questions that required them to find the arithmetic mean of the data set, but they had great difficulties in answering the questions requiring them to explain how the arithmetic mean changes as a result of the data added to or removed from the data set, and what the arithmetic mean means in the data set. It was revealed that they did not have sufficient conceptual knowledge. It can be said that especially in the component of analysing and interpreting data, students were more successful in reading among data than in reading beyond data (Yilmaz, 2023). Similar to this result obtained in the current study, Konald and Pollatsek (2002) concluded in their study that the students were able to make the necessary calculations for measures of central tendency, but they did not know in which situations they applied them and how they interpreted them. Similarly, Pollatsek et al. (1981) stated that students see dealing with arithmetic mean as a matter of calculation and they have the perception that arithmetic mean is a concept that starts with a simple formula and ends with a value.

Another finding regarding the application of the concept of median was that the participants had identified the data value in the middle of the data set without sorting out the data set. Similar to this finding, Zawojewski and Shaughnessy (2000) highlighted that the results of the study they conducted revealed that only a few of the students were able to determine the median of the given unsorted data set. In the present study, other important findings also emerged in the questions requiring the identification of the measure of central tendency representing the data set, which is one of the objectives of the secondary school mathematics curriculum. The participants are unaware that they need to consider the research question and the distribution of the data when determining the measure of central tendency to represent the data set. They mostly gave random answers to such questions and did not explain the reason for their answers. It is thought that one of the reasons for this situation is that the participants could not evaluate the process of doing statistics as a whole. The participants consider the subjects of statistics as subjects requiring them to perform an operation. This situation is considered as the reason why they could not calculate and interpret measures of central tendency and could not determine the measures of central tendency that would represent the data set. This could be attributed to the fact that students can only recognize and apply the algorithms they know and to the fact that their lessons taken in the former grade levels are not structured in a way to support the statistical process (Ari, 2010). Moreover, they are not aware of the fact that the statistical process starts by formulating the questions and determining the process accordingly (Yilmaz, 2019). Similarly, Groth and Bergner (2006) observed in a study they conducted with the participation of pre-service teachers that they could not choose the appropriate measure of central tendency to represent the data set. Randall (2006) suggested that teachers should not give the impression that the arithmetic mean is always an appropriate

measure to represent the data set when trying to identify of the appropriate measure of central tendency for a data set. To identify the appropriate measure of central tendency to represent the data set, a decision should be made by looking at the research question (Yilmaz, 2019) and the distribution of the data (Toluk-Ucar and Akdogan, 2009). However, the participants tended to give arithmetic mean as their answer not only because they were used to working with arithmetic means from their former grades but also because they had encountered it more often in their daily lives. The reason for this situation may be that the resources used in the courses focus on operational stages. An examination of the sections on central tendency measures in grade 8 textbooks used in schools reveals that definitions provided for these measures describe the operational application of measures of central tendency (Keskin-Ogan and Ozturk, 2019).
The definitions of central tendency measures given below are taken from national textbooks (Altıntas and Keskin, 2019; Erenkus and Eren-Savaskan, 2018; Keskin-Ogan and Ozturk, 2019).

> Arithmetic mean is found by dividing the sum of the data by the number of data.
>
> The most repeated number in a data group is called the mode (peak value). There can be no mode (most repeated) in a data group, as well as more than one mode.
>
> When a data group is sorted from smallest to largest or largest to smallest and an equal number of data is removed from the beginning and the end, the data left in the middle is called the median. If there is not one but two data in the middle, the median value is the arithmetic average of these two data.

Apart from these definitions, there is no conceptual information about measures of central tendency in the course books. Since the definitions in the textbooks are focused on operational stages, students may think that the purpose of measures of central tendency is to find a value belonging to the data set. Thus, correct selection of textbooks and applications in the lessons will not only enable students to learn statistics conceptually, but also help them to develop statistical thinking. Activities can be carried out in classroom environments on the areas of use of central tendency measures, what these measures mean, and which measure is used in which situations. Such applications will ensure that students do not see these concepts as just a process, but learn what the concept means.

In addition, students' dealing with data from an early age (Watson, 2006) and encountering data in their daily lives (Gal, 2000) are important for both conceptual learning and the development of statistical thinking. Since data is everywhere in daily life, students can be provided with data from an early age and their higher-order thinking can be supported on this subject. These practices will be very important both in the development of statistical literacy and in supporting higher-order thinking skills such as critical thinking.
In future studies, middle school students' statistical thinking towards all measures of central tendency can be examined in detail with a case study design. Middle school students' statistical thinking towards graphs, which is a different statistical sub-field, can be examined in detail with a case study design. In the study, only eighth grade students studying at a school in the center of a province were studied. In order to provide diversity in the participant group and to examine and compare the statistical thinking of middle school students studying in different regions (urban, rural etc.), it can be revealed what kind of differences there are in statistics education practices according to the regions. At the same time, experimental studies

can be conducted by making interventions regarding the difficulties and misconceptions that emerged in this study.

Notes

This article is derived from master thesis of Seyda Aydin-Karaca conducted under the supervision of Zeynep Sonay Ay.
It was presented as an oral presentation at "2nd International Science, Education, Art and Technology Symposium (UBEST2021)" held in Izmir between 28-29 May 2021.

References

Akkas, E. N. (2009). *Investigation of the middle school students' statistical thinking* [Master's thesis, Abant Izzet Baysal University]. Council of Higher Education Thesis Center.

Altaylar, B., & Kazak, S. (2021). The effect of Realistic Mathematics Education on sixth grade students' statistical thinking. *Acta Didactica Napocensia,* 14(1), 76-90. https://doi.org/10.24193/adn.14.1.6

Altintas, S., & Keskin, C. (2019). *Mathematics Textbook for Middle School 7th Grade*. Ekoyay Publishing.

Ben-Zvi, D., & Friedlander, A. (1997). Statistical thinking in a technological environment. In J. Garfield and G. Burrill (Eds.), *Research on the role of technology in teaching and learning statistics* (pp. 45-55). International Statistical Institute.

Biggs, J. B. & Collis, K. F. (1982). *Evaluating the quality of learning: The SOLO taxonomy (Structure of the Observed Learning Outcome)*. Academic.

Bluman, A. G. (2012). *Elementary statistics: A step by step approach*, 8th ed. Mc Graw Education.

Bond, M. E., Perkins, S. N., & Ramirez, C. (2012). Students' perceptions of statistics: An exploration of attitudes, conceptualizations, and content knowledge of statistics. *Statistics Education Research Journal, 11*(2), 6-25. https://www.stat.auckland.ac.nz/serj

Bruner, J. S. (1960). *The Process of Education*. Harvard University Press.

Chance, B. L. (2002). Components of statistical thinking and implications for instruction and assessment, *Journal of Statistics Education, 10*(3). https://doi.org/10.1080/10691898.2002.11910677

Chick, H. L., & Watson, J. M. (2002). Collaborative influences on emergent statistical thinking — A case study. *The Journal of Mathematical Behavior, 21*(3), 371-400.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches.* SAGE Publications.

Doluzengin, B. (2019). *The effect of realistic mathematics education on sixth grade students' statistical thinking, achievement motivation and persistence of knowledge* [Master's thesis, Pamukkale University]. Council of Higher Education Thesis Center.

Erenkus, M. A., & Eren-Savaskan, D. (2018). *Mathematics Textbook for Middle School 7th Grade*. Koza Publishing.

Franklin, C., Kader, G., Mewborn, D. S., Moreno, J., Peck, R., Perry, M., and Scheaffer, R. (2007), *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework*, American Statistical Association.

Gal, I. (2000). *Adult numeracy development: Theory, research, practice*. Hampton Press.

Groth, R. E. (2003). *Development of a high school statistical thinking framework.* [Doctoral dissertation, Department of Mathematics Illinois State University]. ProQuest.

Groth, R. E. (2006). An exploration of students' statistical thinking. *Teaching Statistics, 28(*1), 17-21. https://doi.org/10.1111/j.1467-9639.2006.00003.x

Groth, R. E., & Bergner, J. A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning, 8*(1), 37-63. https://doi.org/10.1207/s15327833mtl0801_3

Jones, G. A., Thornton, C. A., Langrall, C. W., Mooney, E. S., Perry, B., & Putt, I. J. (2000). A framework for characterizing children's statistical thinking. *Mathematical Thinking and Learning, 2*(4), 269-307. https://doi.org/10.1207/S15327833MTL0204_3

Keskin-Ogan, A., & Ozturk, S. (2019*). Mathematics Textbook for Middle School 7th Grade*. MoNE Publishing.

Konold, C., & Pollatsek, A. (2002). Data analysis as a research for signals in noisy processes. *Journal for Research in Mathematics Education, 33*, 259-289.

Koparan, T., & Guven, B. (2013). A study on the differentiation levels of middle school students' statistical thinking. *Elementary Education Online, 12*(1), 158-178.

Koparan, T. & Guven, B. (2014). According to the m3st model analyze of the statistical thinking levels of middle school students. *Education and Science*, *39*(171).

Ministry of National Education [MoNE] (2018). *Mathematics Curriculum (Elementary and Middle School 1, 2, 3, 4, 5, 6, 7 and 8th Grades)*.

Mooney, E. S. (2002). A framework for characterizing middle school students' statistical thinking. *Mathematical Thinking and Learning, 4*(1), 23-63, https://doi.org/10.1207/S15327833MTL0401_2

Mooney, E. S., Hofbauer, P. S., Langrall, C. W., Johnson, Y. A. (2001). *Refining a framework on middle school students' statistical thinking.* Proceedings of 141 the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (23rd, Snowbird, Utah, October 18-21, 2001). p437-47.

NCTM. (2000). *Principles and standards for school mathematics*. NCTM.

Ozdemir, S. (2014). *The effect of cooperative learning in statistics course on the academic success and attitude of the students and analyzing their level of statistical thinking* [Master's thesis, Cukurova University]. Council of Higher Education Thesis Center.

Pollatsek, A., Lima, S., & Well, A. D. (1981). Concept or computation: Students' understanding of the mean. *Educational Studies in Mathematics, 12*(2), 191-204.

Randall, G. (2006). An exploration of students' statistical thinking. *Teaching Statistics, 28*(1), 17-21.

Smith, T. M. F. (1999), "Discussion" in response to Wild and Pfannkuch, *International Statistical Review*, 67, 248-250.

Watson J. M. (2006). *Statistical Literacy at School, Growth and Goal.* Lawrence Erlbaum Associates, Publishers.

Watson, J. M., & Moritz, J. B. (2000). The longitudinal development of understanding of average. *Mathematical Thinking and Learning, 2*(1), 11-50, https://doi.org/10.1207/S15327833MTL0202_2

Van de Walle, J. A., Karp, K. S., & Bay-Williams J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th Ed.). Pearson Education, Inc.

Toluk-Ucar, Z. & Akdogan, E. N. (2009). Middle school students' understanding of average. *Elementary Education Online, 8*(2), 391-400.

Yilmaz, N. (2019*). An investigation of pre-service teachers' statistical knowledge for teaching in a practicum-based lesson study* [Doctoral dissertation, Hacettepe University]. Council of Higher Education Thesis Center.

Yilmaz, N. (2023*).* An investigation of 4th grade students' statistical thinking. *Education and Science, 48*(216), 39-66.

Zainal, Z. (2007). Case study as a research method. *Jurnal Kemanusiaan, 9*, 1-6.

Zawojewski, J. S. & Shaughnessy, J. M. (2000). Mean and median: Are they really so easy?. *Mathematics Teaching in the Middle School*, *5*(7), 436-440.