CURRENT TRENDS IN COMPUTING
VOL.1 NO.1 PP. 1-9 (2023)
ISSN: 2980-3152
**Received:** 10. 01. 2023
**Accepted:** 17. 02. 2023

# ONE TO ALL BROADCASTING ALGORITHM FOR HIERARCHICAL HONEYCOMB MESHES

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**Abstract**. In the nature, a honeycomb structure is found frequently and man-made honeycomb structures are used in many different areas due to its features. The use of honeycomb meshes provides advantages when building hierarchical structures. In this paper, one to all broadcasting algorithm are studied for hierarchical honeycomb meshes (HHM) using a new strategy.

## 1. INTRODUCTION

A network consists of nodes and links to connect these nodes together. A hierarchical network design involves dividing the network into clusters, discrete group of nodes. In clusters, nodes are connected to other nodes with cluster links and each cluster has one or more hubs to connect other hubs with backbone links. There are different types of topology to construct hierarchical network clusters and structure of backbone. Like star topology and tree topology, a cluster networks can be sparse meanwhile backbone networks can be denser similar with ring, mesh or fully interconnected in [1].

Liu et al. studied on connectivity and diagnosability of hierarchical star network applied on forbidden faulty sets in [2]. Kwak et al. worked on torus ring on multiprocessor systems, altered hierarchical ring and in this way performance increased in [3]. Kim et al. used hierarchical spanning tree to design network backbone topology with Nash genetic algorithm to improve performance throughout the routing [4]. Rabarijaona et al. used hierarchical mesh tree as a protocol for multi-hop data collection efficiently and ad hoc networks, wireless sensor networks, intelligent transportation system networks, etc. achieved great results in efficient routing at MAC layer to collect multi-hop data in [5]. Moreover, mesh tree is used to design hierarchical wireless network on chip for multi-core systems and the performance

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*Key words and phrases.* Hierarchical honeycomb meshes, Interconnection network, Broadcasting.

obtained as a result of the measurements has quite good performance by Dehghani and Rahimi Zadeh [6]. Another method used for network design is hierarchical cubic mesh. An interconnection network is designed with hierarchical cubic network by Zhao et. al [7], on the other hand Yun and Park [8] presented hierarchical hypercube network is more appropriate than hierarchical cubic network to design interconnection network with massively parallel computers. In addition, in hierarchical network studies honeycomb is used as the structure in ( [9–12].

Honeycomb structure is a pattern inspired from nature so it can be found in organic materials and biological systems. It greatly contributes to the survival and functionality of what it contains in nature. The structural properties of honeycomb such as strong, rigid, lightweight are important features for material design and also its topological properties are used when it comes to network design. Honeycomb structure is also used in satellite component research [13], multiprocessor interconnection networks design [14,15], chemical engineering [16], computer graphics [17], station positioning for cellular phones [18] and in various fields of engineering. Properties of honeycomb mesh are also studied like fractal properties of honeycomb mesh [19,20], honeycomb network in higher dimension [21], topological properties of honeycomb network and communication algorithm [22]. Information about hierarchical structures constructed with different strategies [23,24].

In [9], first, a single unit cell is extracted from the honeycomb structure and then the edges of the hexagons were replaced with smaller hexagons to create a hierarchical structure. In [12], sub-hexagons are iteratively inserted at the corners of the main hexagon and connect the vertexes of two adjacent sub-hexagons using an internal rib. By repeating this operation iteratively, the first and the second order hierarchical structures are constructed (see Fig.1.).

In this study, HHM [23] is reconsidered in section 2. Then, one to all broadcasting algorithm is implemented on HHM by a new strategy different from one to all broadcasting algorithm on hypercube is shown in section 3.

## 2. Hierarchical Honeycomb Meshes (HHM)

$HHM(n) = (V(n), E(n))$ can be constructed hierarchically using six $HHM(n-1)$ graphs where $n = 2, 3, \ldots$. In this paper, to concatenate two strings, the symbol "$\|$" is used. In the Hamming distance definition, summation is a bitwise-XOR operation. Hierarchical fractal structure can be constructed using Honeycomb Networks. In this section, Hierarchical Honeycomb Meshes (HHM) is reintroduced. A hyper hexa-cell (HHC) was defined by Mahafzah et.al in [24] as in the Fig.2.a. In order to reduce the cost in HHC, HHM is defined using fewer edges in Fig.2.b.

**Definition 2.1.** ($HHM(n)$). The description of Hierarchical Honeycomb Mesh is as follows. $HHM(n) = (V(n), E(n))$ for $n > 0$ can be designed as

$$HHM(n) = HHM(n-1) \,\|\, H(1)$$

where

$$\begin{aligned} V(n) &= V(n-1) \,\|\, V(1), \\ V(1) &= \{000, 010, 011, 001, 101, 100\} \end{aligned}$$

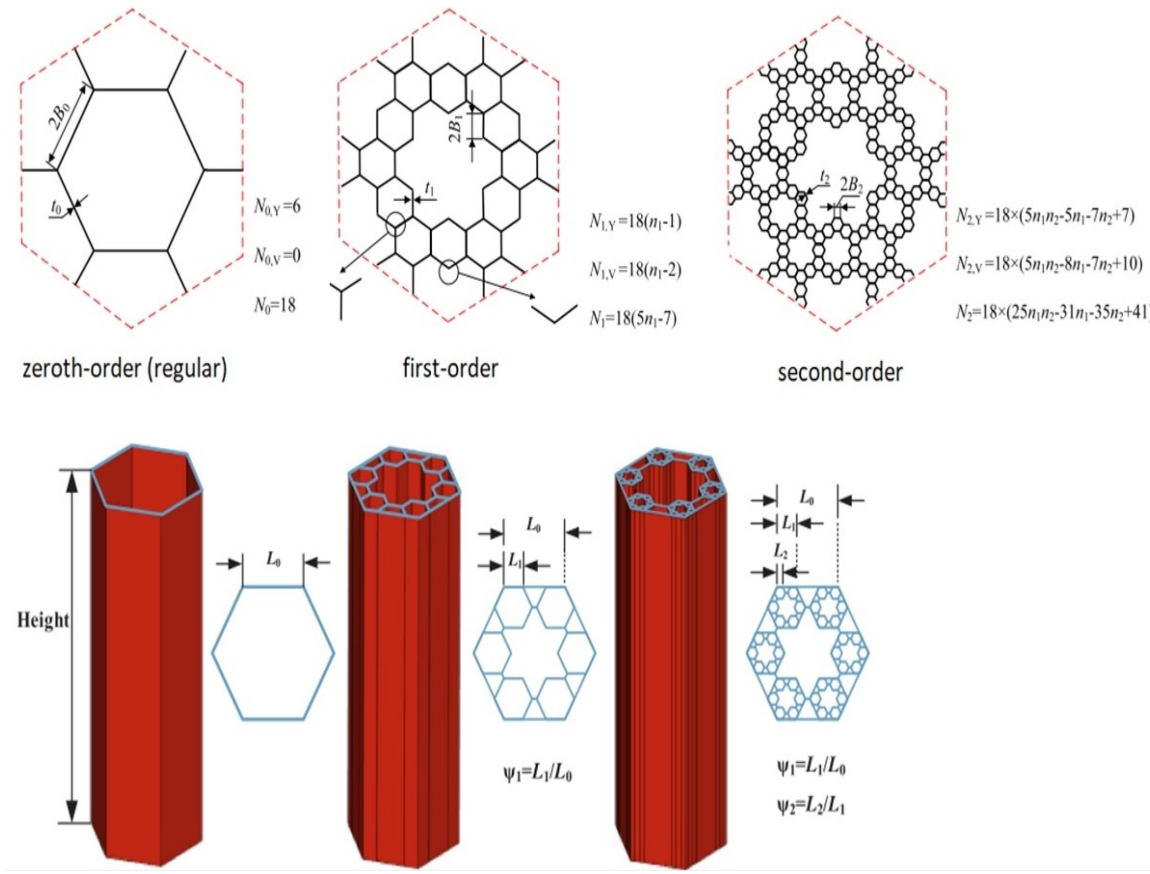


FIGURE 1. Different HHN structures in [9, 12], respectively.

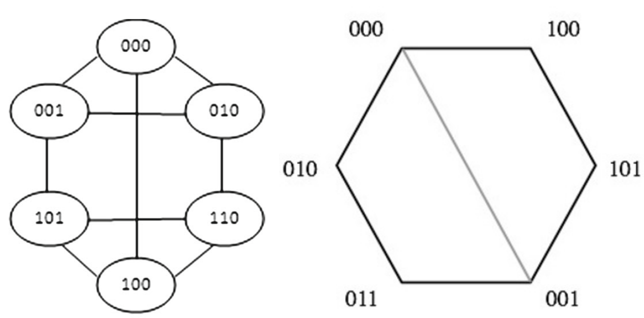


FIGURE 2. a. Labeling of HHC(1) b. Labeling of HHM(1), respectively

and

$$E(n) = \begin{array}{c} 000 \parallel E(n-1) \cup 010 \parallel E(n-1) \cup 011 \parallel E(n-1) \cup \\ 001 \parallel E(n-1) \cup 101 \parallel E(n-1) \cup 100 \parallel E(n-1) \cup E' \end{array}$$

where $E' = \{(e_i, e_j) \mid e_i \oplus e_j = 1\}$, $e_i$ and $e_j$ have same labels except first 3 bits $abc$ in $e_i$ and $dfg$ in $e_j$ while number of bits in label of vertex $m = 0, 3, 6, \ldots$. Assume $v_1$ and $v_2$ are two nodes whose with labels $a_{3k+2}a_{3k+1}a_{3k}\ldots a_1a_0$ and $b_{3k+2}b_{3k+1}b_{3k}\ldots b_1b_0$, respectively. We can get the $i$th order external edges by using one of the following conditions:

$$(a_{3i+2} \oplus b_{3i+2}) \oplus (a_{3i+1} \oplus b_{3i+1}) \oplus (a_{3i} \oplus b_{3i}) = 1$$

where $1 \leq i \leq k$.

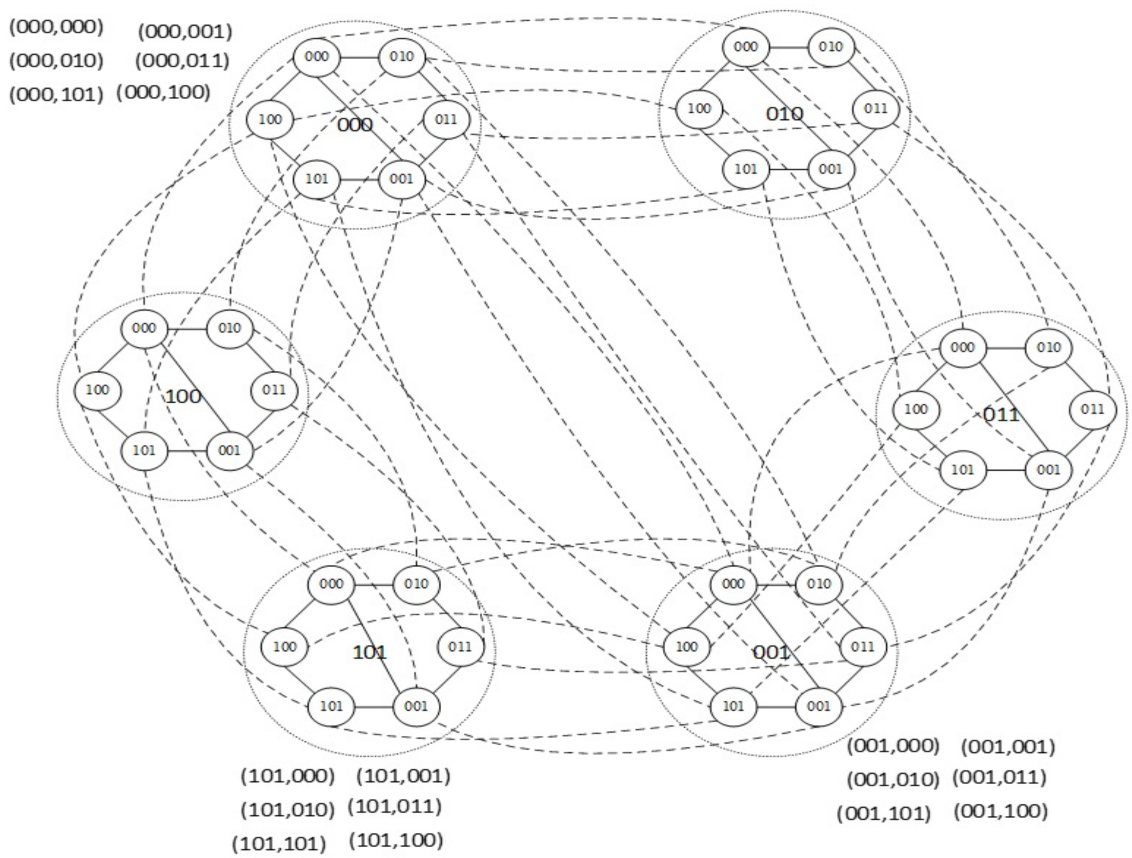


FIGURE 3. Labelling of HHM(2)

In Fig 3 example of $HHM(2)$ is shown. For hexagons in $HHM(2)$, the node's label format is $(L_1, L_2)$; where $L_1$ is the common label for the nodes, while $L_2$ is different for each. $L_1$ in the upper left corner becomes 000 and $L_2$ becomes $000, 001, 010, 011, 101, 100$, respectively in Fig. 3. Labels for other hexagons are written in the same style. The dashed edges are called external edges and the other edges are called internal edges. The figure above has two levels. When it has three levels, the label is in the form of $(L_1, L_2, L_3)$. As the dimension is increased, the level of label is increased with the same logic and for each dimension 3 bits are added to the beginning of the label.

## 3. One to all Broadcasting Algorithms for HHM

One to all broadcasting algorithm of HHM can be easily obtained with the help of hypercube one to all broadcasting algorithm. In this section, a different strategy will be applied. The strategy differ according to the degrees of honeycomb nodes. The degrees of HHM(1) are as follows;

$$degree(A_1) = degree(A_4) = 3, degree(A_2) = degree(A_3) = degree(A_5) = degree(A_6) = 2.$$

where the labels of $A_1, A_2, A_3, A_4, A_5$ and $A_6$ are $000, 010, 011, 001, 101$ and 100, respectively.

There are two one-to-all broadcasting strategies for HHM(1) without confusion. These are;

**Strategy_1:** If the degree of source node is 3;

- move to other node of degree 3,
- two nodes of degree 3 move to four nodes of degree 2.

**Strategy_2:** If the degree of source node is 2;

- move to neighbor node of degree 2,
- first two nodes move to nodes of degree 3,
- two nodes of degree 3 move to other nodes of degree 2.

**Example 3.1.** (a) Let the source node is 000 and use strategy 1 (Fig.4.a);

- 000 $\rightarrow$ 001,
- 000 $\rightarrow$ 010 and 000 $\rightarrow$ 100,
  001 $\rightarrow$ 011 and 001 $\rightarrow$ 101,

(b) Let the source node is 010 and use strategy 2 (Fig.4.b);

- 010 $\rightarrow$ 011,
- 010 $\rightarrow$ 000 and 011 $\rightarrow$ 001,
- 000 $\rightarrow$ 100 and 001 $\rightarrow$ 101.

TABLE 1. Cases to determine the path in the HHM(2)

| Source node | Strategies |
|---|---|
| deg_3, deg_3 | Strategy_1 $\rightarrow$ Strategy_1 |
| deg_3, deg_2 | Strategy_1 $\rightarrow$ Strategy_2 |
| deg_2, deg_3 | Strategy_2 $\rightarrow$ Strategy_1 |
| deg_2, deg_2 | Strategy_2 $\rightarrow$ Strategy_2 |

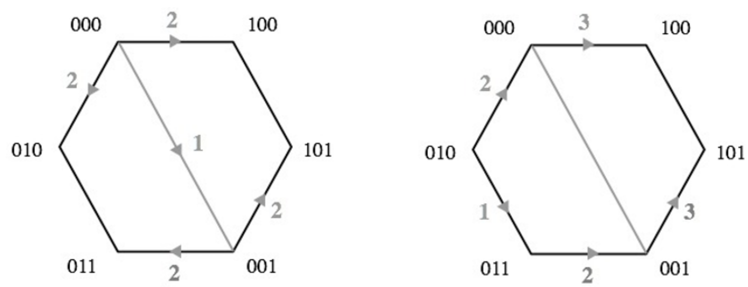


FIGURE 4. a. Strategy_1, b. Strategy_2, respectively

**Example 3.2.** According to the selection of source node in HHM(2), alternative strategies of one to all broadcasting are stated in Table 1. Assume source node in HHM(2) is 000010. The source node consists of two parts as $(000, 100)$ in Fig 3. With the help of Table 1, Strategy_1 and then Strategy_2 is used where $\text{degree}(000) = 3$ and $\text{degree}(010) = 2$, respectively.

**First Step:** The outermost honeycomb is visited.

- 000010 $\rightarrow$ 001010,
- 000010 $\rightarrow$ 010010 and 000010 $\rightarrow$ 100010,
  001010 $\rightarrow$ 011010 and 001010 $\rightarrow$ 101010,

**Second Step:** Each honeycomb inside is visited.

- 000010 $\rightarrow$ 000011,
- 000010 $\rightarrow$ 000000 and 000011 $\rightarrow$ 000001,
- 000000 $\rightarrow$ 000100 and 000001 $\rightarrow$ 000101.
- 001010 $\rightarrow$ 001011,
- 001010 $\rightarrow$ 001000 and 001011 $\rightarrow$ 001001,
- 001000 $\rightarrow$ 001100 and 001001 $\rightarrow$ 001101.
- 010010 $\rightarrow$ 010011,
- 010010 $\rightarrow$ 010000 and 010011 $\rightarrow$ 010001,
- 010000 $\rightarrow$ 010100 and 010001 $\rightarrow$ 010101.
- 011010 $\rightarrow$ 011011,
- 011010 $\rightarrow$ 011000 and 011011 $\rightarrow$ 011001,
- 011000 $\rightarrow$ 011100 and 011001 $\rightarrow$ 011101.
- 100010 $\rightarrow$ 100011,
- 100010 $\rightarrow$ 100000 and 100011 $\rightarrow$ 100001,
- 100000 $\rightarrow$ 100100 and 100001 $\rightarrow$ 100101.
- 101010 $\rightarrow$ 101011,
- 101010 $\rightarrow$ 101000 and 101011 $\rightarrow$ 101001,
- 101000 $\rightarrow$ 101100 and 101001 $\rightarrow$ 101101.

**Algorithm 1:** This algorithm calculates one to all broadcasting for HHM($n$) using new strategies

**Data:** Source: initial node, $n$: hierarchical rank

**Result:** MSG message is transmitted from one node to all nodes

```
Function One_to_All(Source, MSG, n):
    if n == 0 then
        break
    end
    else
        current_node = [ ]
        current_node(0) = Source[1...3]
        if n > 1 then
            remain source = Source[4...3n]
        end
        else
            remain source = Null
        end
        current_node(1) = current_node(0) ⊕2^0
        send MSG to [current_node(1) remain source]
        if current_node(0) = 000 or current_node(0) = 001 then
            // strategy 1: degree of current_node(0) equal to 3
            for i = 2 : 1 : 3 do
                current_node(i) = current_node(0) ⊕2^(i−1)
                send MSG to [current_node(i) remain source]
                current_node(i+2) = current_node(0) ⊕2^(i−1)
                send MSG to [current_node(i+2) remain source]
            end
        end
        else
            // strategy 2: degree of current_node(0) equal to 2
            for i = 2 : 2 : 4 do
                current_node(i) = current_node(i-2) ⊕2^(i/2)
                send MSG to [current_node(i) remain source]
                current_node(i+1) = current_node(i-1) ⊕2^(i/2)
                send MSG to [current_node(i+1) remain source]
            end
        end
    end
    return current_node || One_to_All(remainsource,MSG,n-1)
    End Function
```

**Remark 3.3.** The number of nodes of HHM($n$) is $6^n$. Algorithm 1 provides one to all broadcasting procedures for HHM using strategy_1 and strategy_2 described in example 1 and example 2. In this algorithm; sub-problem size is $n-1$, number of sub-problem is 1, and since the $n-1$ size HHM has $6^{n-1}$ nodes, the cost of extra operations for this algorithm is $6^{n-1}$ for the "||" operator in step 35, which includes a hidden for loop. So, the recurrence relation for algorithm 1 is obtained as $T(n) = T(n-1) + \Theta(6^{n-1})$. By solving this with the substitution method, the time complexity of algorithm 1 is $T(n) = \Theta(6^n)$. As expected, algorithm 1 runs as many times as the number of nodes.

## 4. Conclusion

The use of hypercube one to all broadcasting algorithm in honeycomb meshes, which is the traditional way of obtaining routing algorithms, caused deadlocks. For this reason, the focus of this study is to get one to all broadcasting algorithm of hierarchical honeycomb meshes through new strategy. With the proposed new strategy, the one to all broadcasting algorithm of HHM has been obtained effectively. As future works, a new approach will be tried to be put forward in the all to all broadcasting algorithm for HHM problem.

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