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Research Paper

Numerical Modelling of the Flow Passing through a Rectangular Linear Weir with Flat Crest Shape

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Abstract: The most common hydraulic structures used for discharge measurements in open channels are linear weirs due to their accuracy, simplicity, design, and ease of construction. Linear weirs are also used to increase flow depth, control floods, and regulate flow. Before the actual on-site manufacturing of weirs, their hydraulic properties must be determined with experimental and numerical studies. In this study, experiments were carried out on a rectangular linear weir with dimensions 60 cm width, 30 cm height, 1 cm thickness and a flat crest shape. 2-Dimensional numerical models of the flow passing over the linear weir were created by ANSYS-Fluent. The data obtained from the experimental and numerical model were evaluated in terms of, total head (H_T) and discharge coefficient (Cd) and water surface profile. According to the results, although there are differences between the models in the discharge (Q) – total head (H_T) comparison due to the working principle of the 2-dimensional numerical model, the models gave 92.5 % consistent results in the discharge coefficient (C_d) and dimensionless total head (H_T/P) comparison. In addition, the experimental and numerical models were compared visually, and it was seen that the numerical models of the experiments were created with a high degree of accuracy.

Keywords: *Experimental Modelling, Linear Weirs, Numerical Modelling, Sharp-Crested Weirs*

Introduction

Dams and hydraulic structures are used to supply and control water, which has great importance on human life. Dams are large-sized barrier structures made of different materials and types that block the flow of water and create reservoirs and lakes. Since water is an important resource for human life, studies on the use of water began with the existence of humans. Dams built for many purposes basically consist of structures such as body, reservoir, spillway, bottom outlets and water intake structure. Additionally, depending on the purpose of construction, there may be additional structures such as sedimentation pools, penstocks pipes, energy dissipation pools and energy production facilities.

Weirs are widely used in open channels for flow measurement, flow direction and control. Their main functions include controlling water levels on the upstream and downstream sides as well as maintaining channel stabilization (Kumar *et al*., 2011). They also have different engineering applications, such as the controlling excessive discharge coming from rivers and dams during flood times. Weirs are mainly divided into two types, broad-crested and sharp-crested, according to their geometry and design. The crest thickness (t) of broad-crested weirs increases along the channel direction, and in some cases, the crest thickness may even be greater than the net crest length (L_{net}) of the weir. In sharp-crested weirs, the crest thickness (t) is very small compared to the net crest length (L_{net}). In broad-crested weirs, the critical flow depth occurs on the top of weir, while in sharp-crested weirs, the critical flow depths occurs as it spills over the weir to downstream. The name sharp-crested weir comes from the crest shape of the weir. These types of weirs can also be called linear or rectangular weirs. Rectangular sharp-crested weirs are classified as fully narrowed, partially narrowed and full-width weirs according to the weir opening (Bos, 1978). In a full-width weir, since the channel width (W) and net crest length (L_{net}) are the same, there is no narrowing from the sides, and this can be called a linear weir. Linear weirs are sharp- crested weirs formed from a vertical plate. The thickness (t) of a linear weir placed in a rectangular channel and perpendicular to the flow is equal to the crest thickness along its height. Thanks to these features, there are no lateral contractions in the flow and the flow is two-dimensional. Linear sharp-crested weir in a rectangular open channel is used for flow measurement and controlling discharge in open channels.

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Weirs are useful and widely used devices in flow measurements. The most basic and important parameter in weirs is the discharge coefficient (C_d). Additionally, there are weirs where the flow over the weir is not continuous along the z axis, such as labyrinth weirs, which do not continue in the same plane throughout the channel width. If there is an irregularity in the weir along the channel width, the flow becomes 3-dimensional (Henderson, 1966) and labyrinth weirs are an example of this. Weirs can have rectangular, triangular, trapezoidal, and circular plain view or shapes can be changed for special applications.

The first studies on linear weirs were conducted by Boileau (1854), Horton (1907), Escande and Sabathé (1937), Istomina (1937). Rouse and Reid (1935) conducted mathematical and experimental studies on the design of spillway crest shapes based on the examination of the flow characteristics of a linear sharp-crested weir. Rouse and Kandaswamy (1957) experimentally investigated the weir discharge coefficient (C_d) as a function of H_T/P; where H_T is the total head and P is the weir height. Kindsvater and Carter 1957) presented a comprehensive solution for the flow-discharge capacity of the weir based on experimental results and dimensional analysis. Rajaratnam and Muralidhar (1971) experimentally examined in detail the velocity and pressure distributions in the weir crest region. Han and Chow (1981) developed a model using ideal flow theory to obtain the properties of the flow. Based on experimental results and simplified theoretical evaluations, a general relationship between the discharge coefficient (C_d) and the H_T/P parameter was determined by others. Khan and Steffler (1996) predicted water surface profiles for sharp- crested weirs with inclined upstream faces using a two-dimensional finite element model incorporating continuity and momentum equations. Aydin et al. (2011) conducted experiments on rectangular weirs with a height (P) of 10 cm and 16 different net crest length (L_{net}). Bagheri and Heidarpour (2010) conducted experiments on linear weirs at 3 different weir heights (P) and 6 different net crest length (L_{net}). With the data they obtained, they produced equations showing the profiles of the upper and lower nappe flows from 3 and 4 streams. Arvanaghi and Oskuei (2013) conducted experiments at 3 different weir heights (P) and created numerical models of these experiments. According to the data obtained, despite the increasing discharge (Q), the discharge coefficient (C_d) of sharp- crested weirs remained constant at C_d = 0.7. Qu et al. (2009), used Reynolds Averaged Navier Stokes (RANS) equations to solve flow over a sharp-crested weir numerically. Numerical and experimental results are compared in terms of pressure, velocity and water surface profile. Mahtabi and Arvanaghi (2018), conducted experiments on 3 different weir height (P=10, 15 and 20 cm) with a L_{net}=25 cm net crest length. Experiments show that after HT/P exceed the 0.6, discharge coefficient (C_d) reach maximum value of 0.7. Also, they found consistent results between experiments and numerical model calculated in Fluent.

Experiments conducted to determine the performance of hydraulic structures and water surface profiles can be laborious and time-consuming. In addition, experimental research of hydraulic variables, field tests and finding the necessary equipment also increase the cost. Therefore, numerical simulation of hydraulic problems such as flow over a linear weir using computational fluid dynamics (CFD) can be less costly and practical than laboratory experiments. It can simulate turbulent flow using advanced numerical methods and is used to determine the velocity distribution, water surface profile, flow velocity and some other coefficients. However, to ensure that the numerical modeling results are accurate enough, the numerical solution needs to be validated with experimental data.

In this study, the discharge (Q), and total head (H_T) values are obtained from the flow passing over a weir which has P=30 cm height and have flat crest shape placed in a channel rectangular open channel. Additionally, numerical models of these measurements with the same size as the experiments were created and the results were compared.

Material and Method

Sharp-Crested Weirs

Linear weirs are important hydraulic structures for controlling water in stream regulation. These structures are used to raise water levels and control flow from canals, rivers and other water sources. During heavy rainfall, the upstream water level can be adjusted depending on the weir height (P) of the sharp-crested weir, providing protection against floods (Bos, 1978). As the flow passes over the weir, the flow characteristics change from the subcritical to the supercritical (Henderson, 1966). The water surface profile formed by the flow passing freely over a linear weir is called nappe flow (Figure 1). In the nappe flow, after the water passes over the weir, there is air between the wall and the water. Linear weirs are a type of overflow weirs (Escande & Sabathé, 1937).

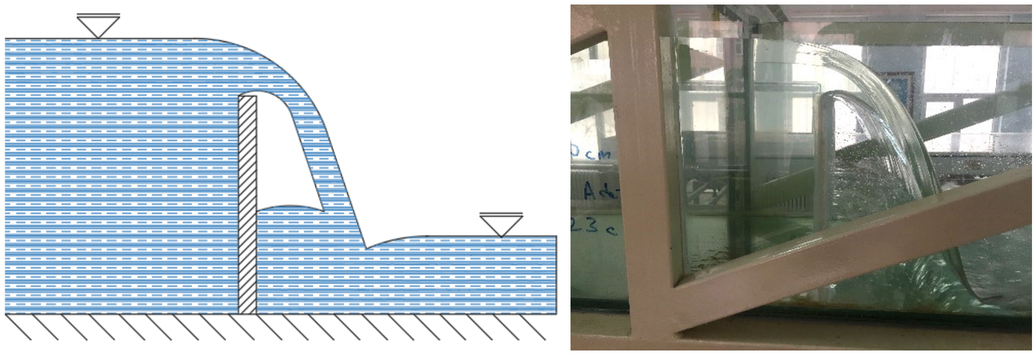


Figure 1. Nappe flow passing over sharp-crested weir.

Aeration of the flow passing over a linear weir can occur in four different ways (Falvey, 2002). These are unvented (viscid) flow (a), partially aerated flow (b), self-aerated or nappe flow (c) and submerged (choked) flow (d), as shown in Figure 2. The flow condition has a significant effect on the discharge coefficient (C_d).

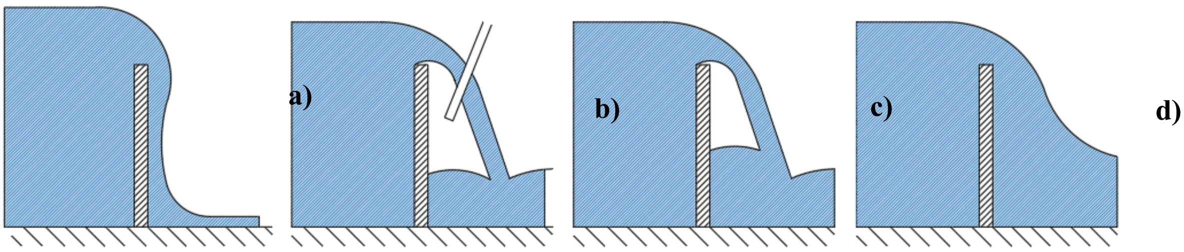


Figure 2. (a) Unvented (viscid) flow, (b) Partially aerated flow, (c) Self-aerated or nappe flow, (d) Submerged (choked) flow

The main parameters to be considered in the design of linear weirs are spillway (channel) width (W), weir height (P), approach velocity (V), total head (H_T) and crest thickness (t) are shown in Figure 3.

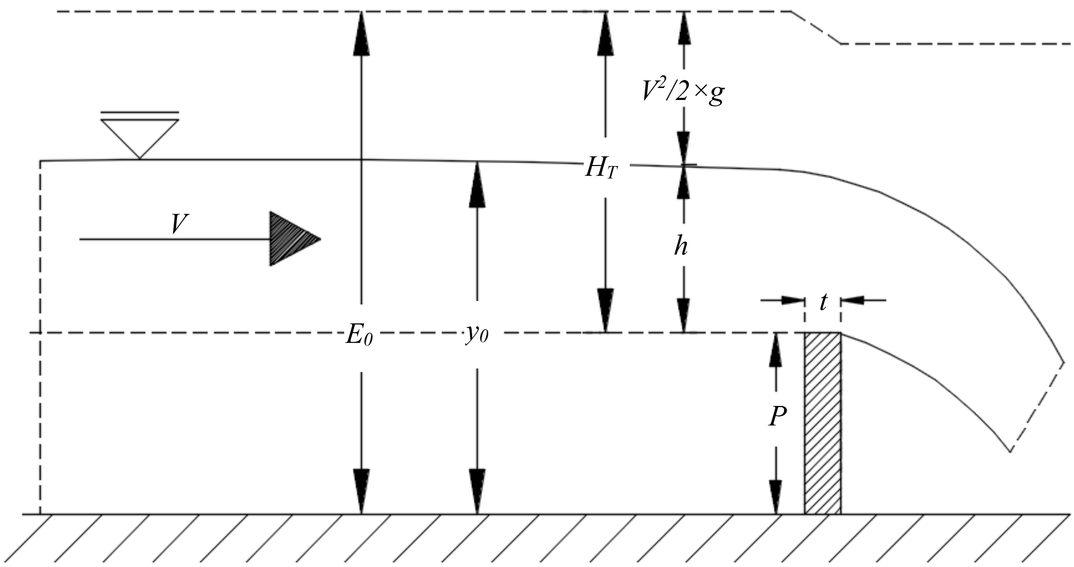


Figure 3. Parameters affecting the flow over a linear (sharp-crested) weir.

Discharge (Q) passing over a sharp- crested weir is calculated with Equation (1). In this equation, it is assumed that H_T is the total head and $H_T = h + V^2/2 \times g$ (Tullis et al 1995).

$$Q = \frac{2}{3} * C_d * L_{net} * \sqrt{2 * g} * H_T^{1,5} \tag{1}$$

where;
Q : Discharge
C_d : Discharge Coefficient
L_{net} : Net Crest Length
g : Gravity
H_T : Total head over the weir.

Crest shape has a significant impact on the flow and discharge performance of linear weirs. Different types of crest shapes that can be used in linear weirs are shown in Figure 4. below. Circular crest shape has better hydraulic performance than sharp- crested crest shape. The profile of the flow as it passes over different crest shapes is different.

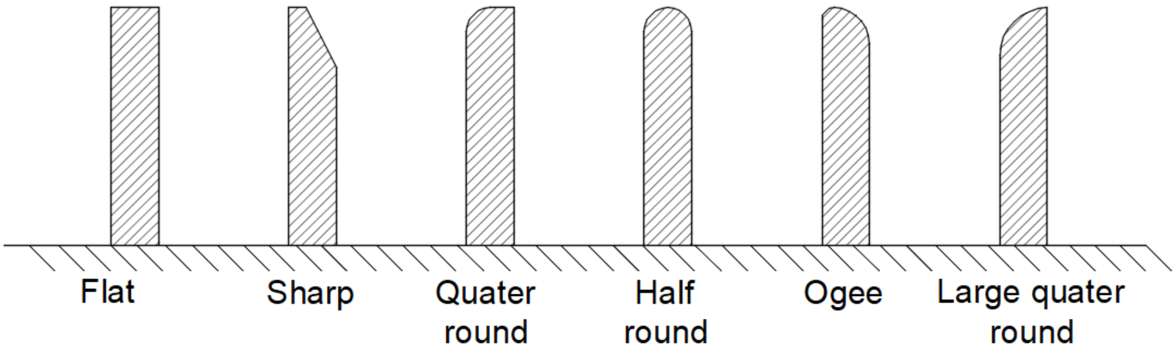


Figure 4. Different weir top crest shapes

Experimental Setup

The open channel system used in linear weir experiments is shown in Figure 5. The open channel system used in the experiments is 6.50 m long, 0.60 m wide and 0.50 m high. Linear weirs were placed 3 m away from the beginning of the channel. The purpose of choosing this location is to minimize the fluctuations in the water coming from the reservoir and prevent the fluctuations from affecting the water surface profile.

Figure 5. Open channel system

In the open channel system, the flow is provided by circulating water between two reservoirs. In the open channel system, discharge (Q) can be adjusted between 1 lt/s and 45 lt/s can be obtained with the help of the frequency converter connected to the pumps. The discharge (Q) circulated in the system is measured with an ultrasonic flowmeter with a accuracy of 0.01 lt/s placed between the pipes after the pumps (Figure 5). Total heads (H_T) of the flow over the weirs placed in the channel were measured with a limnimeter. The upper part of the linear weirs used in the experiments has a flat crest shape. Since there is no rounding at the top of the weirs, the thickness of the crest is equal to the wall thickness (t) of the weir 1 cm. The linear weir used in the experiments was made of plexiglass (acrylic) sheets. The net crest length and height of the weir used in experiments is L_{net} = 60 cm and P = 30 cm respectively (Figure 6).

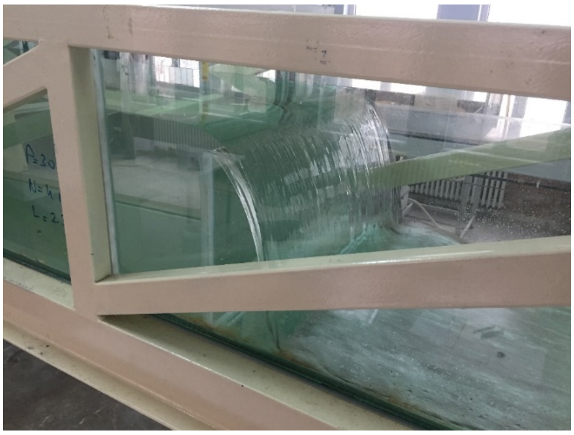


Figure 6. Sharp-Crested weir used in experiments.

10 mm thick plexiglass was used in the manufacture of sharp-crested weir. In the experiments carried out on linear weir, attention was paid to the formation of nappe flow and all experimental data were obtained in the case of nappe flow. Data were not obtained in case of the nappe flow condition did not occur, unventilated (viscid) flow or submerged (choked) flow condition. When the images obtained from the experiments are examined, it is seen that self-ventilated nappe flow occurs in all measurements.

Numerical Modelling

ANSYS-Fluent is a Computational Fluid Dynamics (CFD)-based program used to analyze and optimize the motion and interaction of fluids in hydraulic applications. CFD is a branch of fluid mechanics that uses mathematical and numerical methods to model, numerically analyze and simulate the behavior of fluids (liquids, gases or solid-liquid mixtures). Volume of Fluid (VOF) is used to model a free-surface flow in the numerical solution of fluids. In computational fluid dynamics, mathematical modeling of fluid motion is based on the Navier-Stokes equations (Fluent, 2023). These equations describe the momentum, mass, and energy of the fluid. The motion of the fluid is calculated by the numerical solution of these equations. A grid system is created to calculate the continuous fluid field. This grid is divided into sub-cells (meshes) to describe the movement of the fluid. Momentum and energy calculations of the fluid are made in these cells (mesh) and updated over time. Equations used in solving sets of differential equations obtained from the laws of conservation of momentum, energy and mass.
The conservation of mass equation or continuity equation can be written as Formula 2:

$$\frac{\partial \rho}{\partial t} + \nabla * (\rho \vec{v}) = S_m \tag{2}$$

The continuity equation for 2D axisymmetric geometries is given by Formula 3.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial r}(\rho v_r) + \frac{\rho v_r}{r} = S_m \tag{3}$$

Conservation of momentum in an inertial (non-accelerating) reference frame is explained by Formulas 4.

$$\frac{\partial}{\partial t}(\rho\, \vec{v}) + \nabla * (\rho \vec{v}\, \vec{v}) = -\nabla\, P + \nabla * (\vec{\tau}) + \rho\, \vec{g} + \vec{F} \tag{4}$$

where x axial coordinate, r radial coordinate, v_x axial velocity, v_r radial velocity, ρ density of fluid, P static pressure, g gravity, $\vec{F}$ force and $\vec{\tau}$ shear tensor.

A validated numerical model allows obtaining the hydraulic parameters of a sharp- crested weir without resorting to expensive and time-consuming experimental methods. Additionally, on the same numerical model, geometric parameters, inlet and outlet conditions can be changed and their effects on the flow over the weir can be examined in same numerical model. In order to obtain consistent results from numerical model with the experiments, boundary conditions of the physical model should be entered the program in a way that best suits the environmental conditions. Although the linear weir used in the physical model 3-dimensional, there is no irregularity along the channel width. For this reason, numerical models were created in 2 dimensions.

Geometry-Mesh

In the Fluent, program used in the analysis of the numerical model, the flow area is the total area of containing only water and air. All hydraulic events that occur must remain within this flow area. The 2D flow area used in the numerical models of linear weirs were created in AutoCAD and transferred to Fluent with sat (Standard ACIS Text) format. The dimensions of the flow area used in the numerical model are shown in Figure 7.

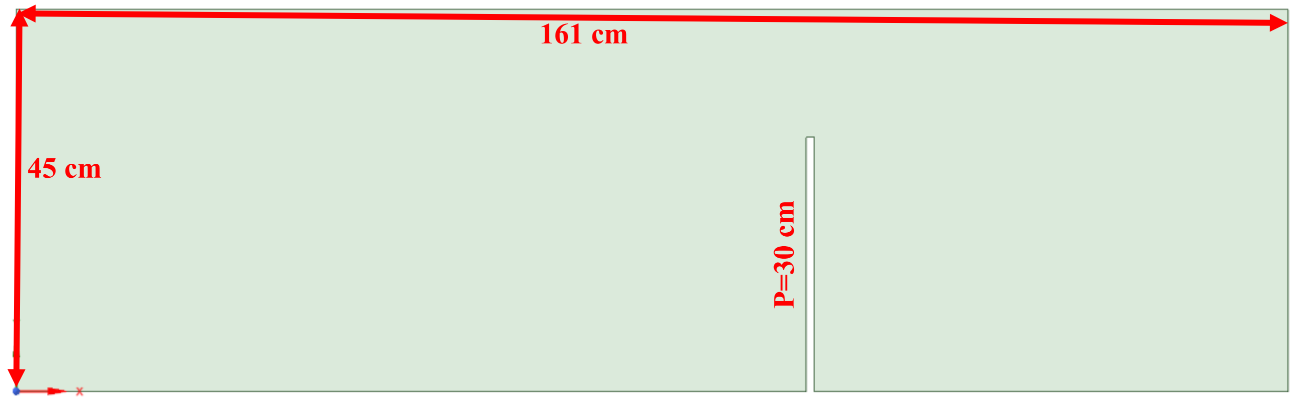


Figure 7. Geometry and dimensions of the flow area used in numerical modeling of linear weirs.

Fluent program divides the flow area into small sub-cells to analyze the movement of the fluid and performs calculations with the finite area method in these cells. Each mesh (cell) is modeled by solving the continuity and Navier-Stokes equations. Increasing the number of meshes in the flow area increases the accuracy of the solution. However, the most important factors that limit the mesh size and the total number of cells that will emerge due to this increase in accuracy are the technical specifications of the computer used and the time it takes to complete the analyses.

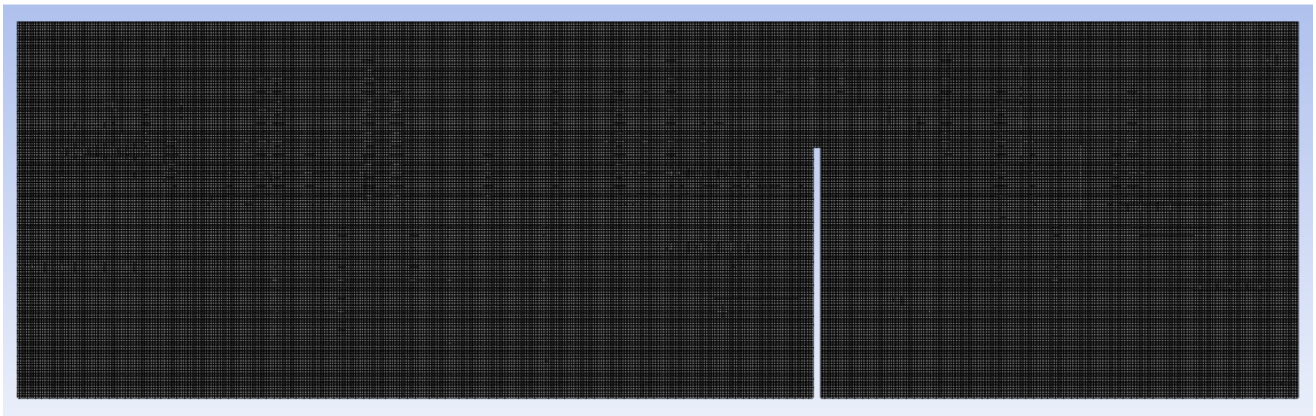


Figure 8. Separation of geometry into mesh (cells)

When dividing the flow area into cells, square-shaped meshes were preferred. The edge size of the square meshes used is 0.0025 m = 2.5 mm (Figure 8). In this study, before determining the optimum mesh size, mesh independence study was carried out using the Grid Resolution method. Before creating numerical models for all discharge (Q) values, 4 different mesh sizes were used for modeling og 5 and 10 lt/sec and the average errors were calculated by comparing the results with the experiments. Average errors were calculated as 12% for 10 mm mesh size, 7% for 5 mm mesh size, 2% for 2.5 mm mesh size and 1% for 1.25 mm mesh size (Figure 9). What the percentage error between the numerical model and the experimental results should be is entirely related to the physics and importance of the hydraulic problem. In this study, it will be sufficient for the numerical model results of sharp-edged weirs to be 5% consistent with the experimental models. Because the sensitivity of the limnimeter used to measure the total weir load on sharp-edged weirs is 1 mm and this value goes up to 10% of cm. There is a possibility of a 10% margin of error during measurements made with a limnimeter. For a effective numerical model to be effective, its results must be below the desired error rates, be consistent with experimental data, and analysis times must not be too long depending on the severity of the problem. While the analysis with a 2.5 mm mesh size took 3 hours to finalize, the model with a 1.25 mm mesh size took 9 hours to finalize. Therefore, it is appropriate to use 2.5 mm mesh size.

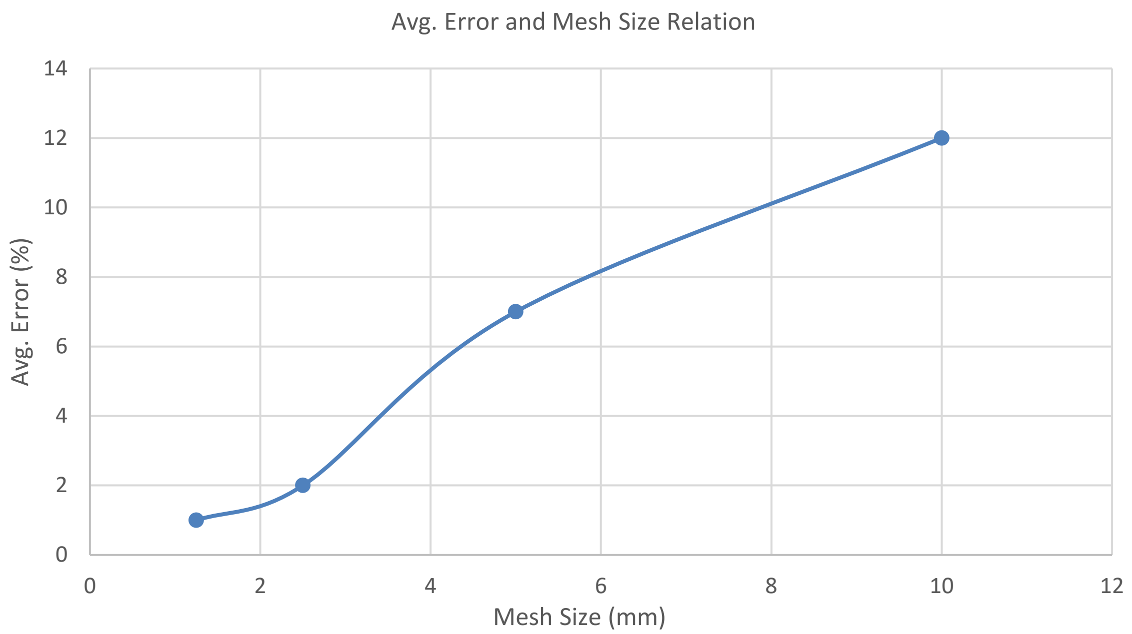
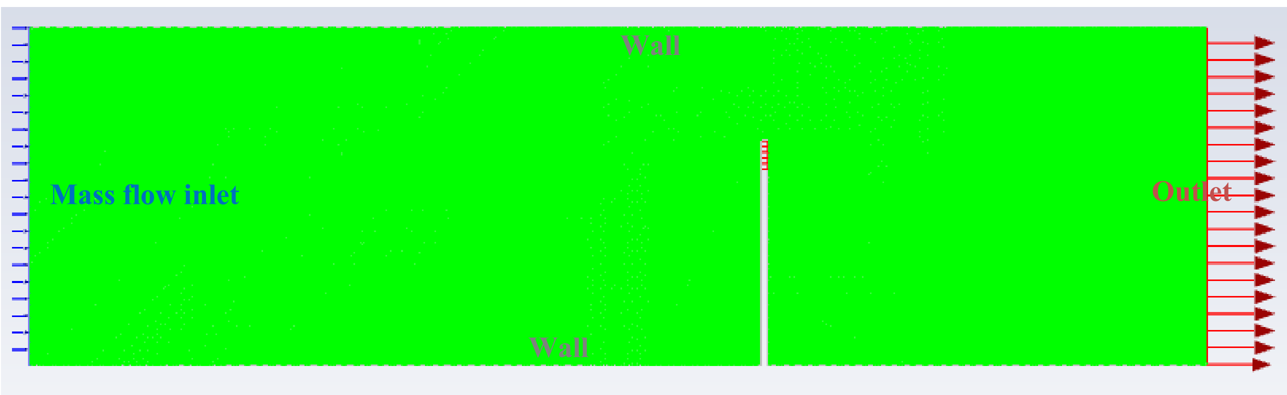


Figure 9. Average error calculation based on mesh size.

Analyzes were carried out with a total of 115440 meshes. While solving a 2D numerical model in the Fluent program, the width along the z-axis in 3D is accepted as 1 m by the program. The discharge (Q) values entered the program will be valid for 1 m width. The "inlet" part, which is the surface where water enters the flow volume, is defined as "mass flow inlet". Flow input is determined in kg/s, that is, lt/s. The "outlet" surface of the flow volume is defined as "pressure outlet". The remaining parts are defined as “wall” (Figure 10).



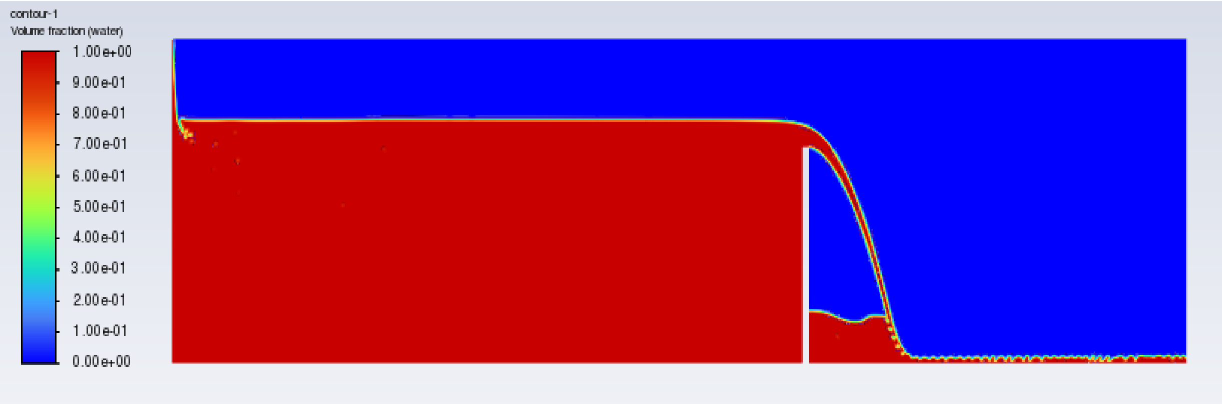


Figure 10. Boundary layer conditions created on the geometry and the image of the model as a result of the analysis.

Setup
Experiments in the laboratory, the free surface flow and the atmosphere above flow represent therefore in the numerical analysis a two-phase free surface flow is aimed. Therefore, by choosing the "Multiphase" model as the VOF (Volume of Fluid) method, it is aimed to consider the presence of both air and water in the flow area. At the same time, the "explicit" solution was preferred over the traditional "implicit" solution to increase the detail of the analysis. The effects of air on water, especially atmospheric pressure, are taken into account with this method. "Time step size" varies between 0.001 and 0.005 seconds depending on the total head (H_T), discharge (Q) and mesh size of the weirs. In the numerical model, the analyzes were continued until difference between the inlet and outlet discharge values become zero, and the total water volume in the flow area was expected to remain constant.

Results and Discussions
Numerical models of the tested linear (sharp-crested) weir were created by using the Fluent module of the ANSYS program. In the experiments, the discharge (Q) was gradually increased from 1 lt/s to 40 lt/s, depending on the Total head (H_T) capacity of the linear weir. Linear weir experiments were repeated for approximately 32 different discharge (Q) values. Since it would not be practical to use all discharge values (Q) in the numerical model, the analyzes were limited to certain discharge (Q) ranges. In this context, analyzes were carried out by selecting 7 8 different discharge (Q) values, Q=5-10-15-20-25-30-35-40 lt/sec, the total head (H_T) values corresponding to these discharge (Q) values were determined in linear weir numerical models. The results obtained from the numerical models were compared with the experimental data in two main aspects. In visual comparison, 2D images obtained from the numerical model were compared with laboratory experiments. Water surface profiles, the way the water spills depending on the features of the weir, and the turbulence that occurs in the downstream part of the weirs after it is spilled, were taken into consideration. In numerical comparison, the results obtained from numerical models and experiments were examined from two different perspectives: total head (H_T) - discharge (Q) and discharge coefficient (C_d) – dimensionless total head (H_T/P). The linear weir for which numerical models were created can be seen in Figure 10. At each discharge (Q) entered in the numerical model, the analyzes took between 3-4 hours, depending on the value of the discharge (Q). In linear weirs, discharge (Q) and total head (H_T) measurements were collected in the presence of a self-aerated nappe flow. No experimental data were collected in airless and adherent flow situations.
Graphical data showing the experimental and numerical model results of linear weirs are shown in Figure 11. The main goal of the graphical comparison is to visually highlight the differences between the discharge (Q) values determined in the numerical model and the differences that occur when the same discharge (Q) values are not found in the experiments. When the results obtained from models were examined, it was seen that the numerical models gave different results than the experiments. The main reason for this is that in the 2-dimensional numerical model, the model width in the 3rd dimension in z-axis is accepted as 1 m by the program default. While the net crest length (L_{net}) of linear weirs is equal to the channel width, L_{net} = 60 cm, the net crest length of the model used in the numerical model is L_{net} = 100 cm.

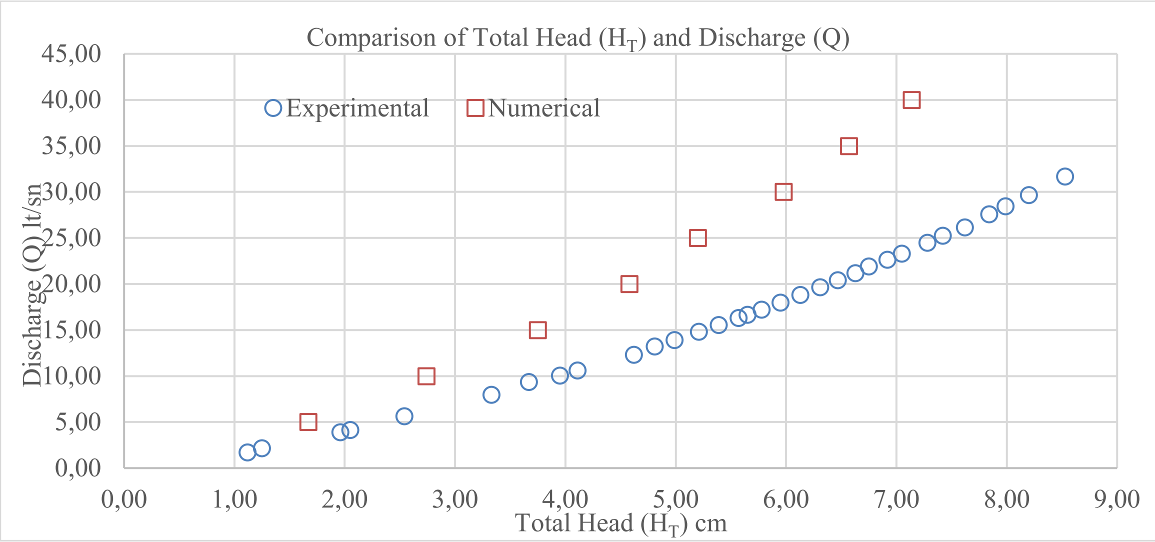


Figure 11. Comparison of experimental and numerical results in terms of total head (H_T) and discharge (Q)

The most important parameter of the discharge equation of linear weirs is the discharge coefficient (C_d). The effect of net crest length (L_{net}) disappears when compared with discharge coefficients (C_d). If a comparison is to be made to determine the hydraulic performance of weirs, the discharge coefficients (C_d) must be compared, and since the results obtained will be independent of the weir height (P), they can be used in comparisons with all weirs. Equation (5), obtained by rearranging Equation (1), was used to compare the discharge coefficients (C_d). Comparison of discharge coefficient (C_d) – dimensionless total (H_T/P) for linear weirs is given in Figure 12.

$$C_d = \frac{Q}{\frac{2}{3} * L_{net} * \sqrt{2 * g} * H_T^{1.5}} \quad (5)$$

Unlike the comparison of total head (H_T) - discharge (Q), numerical and experimental models of linear weir gave close results in the comparison of discharge coefficient (C_d) - dimensionless total head (H_T/P). Since the linear weirs in the numerical models have a longer net crest length (L_{net}) than the weirs in the experiments, they passed more flow at the same total head (H_T). However, this does not show that the weir in the numerical model is more hydraulically efficient. To compare weirs in terms of hydraulic efficiency, discharge coefficients (C_d) must be examined. The net crest length (L_{net}) of the weir is not important in the discharge coefficients (C_d).

ANSYS-Fluent program provides not only numerical results of the analyzes but also visual two and three-dimensional results. These visual results can be used to check whether the numerical model produces results consistent with physical experiments. By focusing on factors such as the path of the flow, the profile of the water as it spills over the weirs, and the conditions of the nappe flows, the performance of the numerical model can be visually examined thanks to the images obtained.

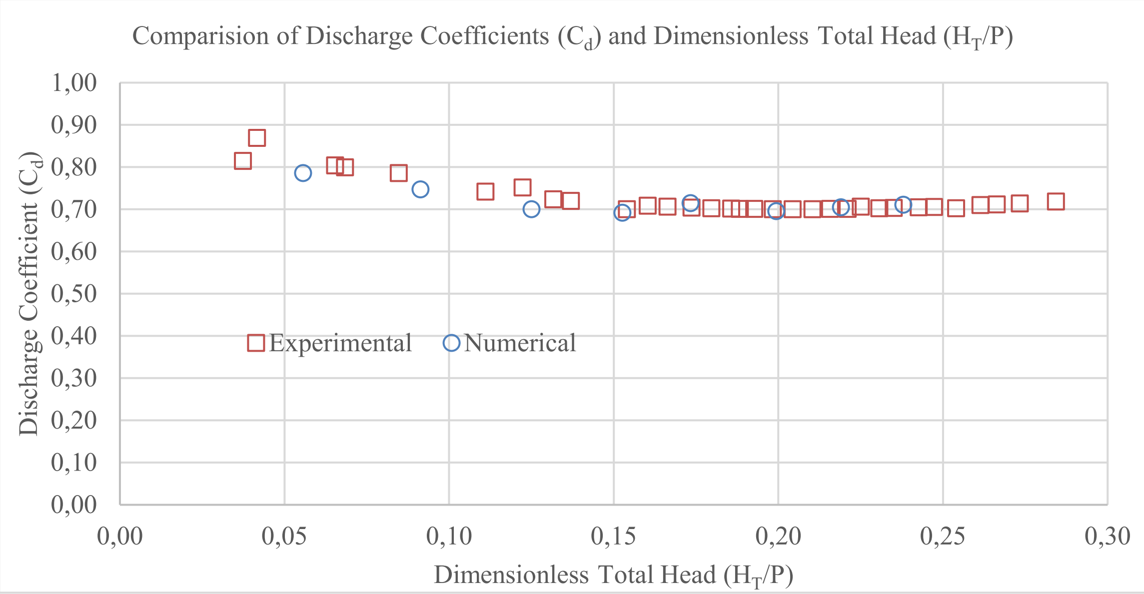


Figure 12. Comparison of experimental and numerical results in terms of discharge coefficients (Cd) and dimensionless total head (HT/P)

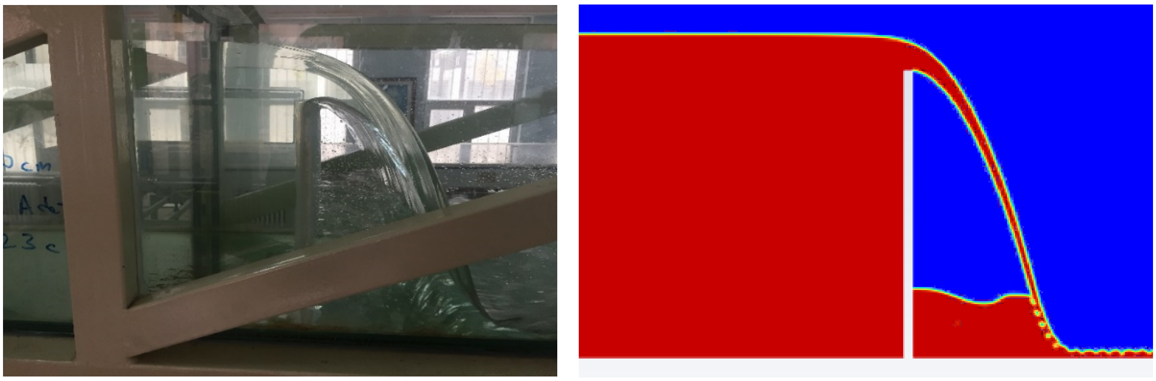


Figure 13. Visual comparison of Experimental and Numerical model

The 2-Dimensional images obtained from the numerical model show great similarities in terms of water surface profiles and flow properties when compared with the experimental results. In both models, water behaves as if it flows in a reservoir behind the weirs and exhibits a similar flow profile over the weirs. No fluctuation or turbulence is observed in the chamber, which indicates that the "boundary condition" has been successfully applied. While the water flows freely over the weirs, the flow is in a semi-aerated state on the downstream side and a distinct nappe flow is observed (Figure 13). The comparison between the 2-dimensional images obtained from the numerical model and the experimental images is presented below.

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