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Algebraic Results on Soft Normed Quasilinear Spaces

Fatma Bulak [1], Hacer Bozkurt[1,*]

[1]*Department of Mathematics, Faculty of Science and Art, Batman University, 72100, Batman, Turkey.*

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Abstract. As novel notions of soft normed quasilinear spaces, we define soft quasilinear dependence, soft quasilinear independence and soft quasi basis. One of the greatest obstacles to the improvement of soft normed quasilinear spaces is the presence of these properties. In this study, we will present the definitions of these significant concepts and give some illustrative examples. Additionally, we demonstrate that the proposed definitions agree with counterparts of similar results in soft linear spaces. Finally, in some soft normed quasilinear spaces, we have studied their singular and regular dimensions just as in quasilinear spaces.

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1. Introduction

The concepts of normed quasilinear spaces, quasilinear operators, and quasilinear spaces were first presented by Aseev in [1]. Next, they obtained a number of findings in normed quasilinear spaces in [6]. Afterwards, they examined quasilinear functions with limited interval values and examined the Hahn-Banach extension theorem for interval valued functions in [10] and [11]. Additionally, it was demonstrated in [19] that a particular class of fuzzy number sequences is a Hilbert quasilinear space. Effortless samples of approximate estimates of deterministic autocorrelation of some semi-non-deterministic signals or signals with imprecise data were provided in [12], and, the authors of [13] describe a mathematical technique for handling non-deterministic signals, called the model interval signal, by utilizing interval-valued functions and they presented the notion of complex interval matrix in [14].

The notion of soft sets was first developed by Molodtsov [16] in 1999. He then demonstrated several uses of this theory in the fields of engineering, economics, and medicine, among others. Subsequently, they introduced multiple operations on soft sets in [15]. The concepts of soft elements and soft real numbers were subsequently presented by Das and Samanta in [7]. Next, they worked in [8] and [9] on soft linear spaces, soft normed linear spaces and some of their properties. Later, they presented soft normed spaces from a different angle in [17] and soft inner product spaces and soft Hilbert spaces in [18].

Drawing from previous research on soft linear spaces and quasilinear spaces, she presented the concepts of soft quasilinear spaces and soft normed quasilinear spaces in [2]. Both soft Hilbert quasilinear spaces and soft inner product quasilinear spaces were later defined in [3]. After that, they focused on a few soft inner product quasilinear

*Corresponding Author
Email addresses: fatmaonatbulak@gmail.com (F. Bulak), hacer.bozkurt@batman.edu.tr (H. Bozkurt)

space features. Soft interval spaces and soft interval sequence spaces are the two types of soft quasilinear spaces that the author described in [5].

As novel notions of soft normed quasilinear spaces, we define soft quasilinear dependence, soft quasilinear independence and soft quasi basis. One of the greatest obstacles to the improvement of soft normed quasilinear spaces is the presence of these properties. In this study, we will present the definitions of these significant concepts and give some illustrative examples. Additionally, we demonstrate that the proposed definitions agree with counterparts of similar results in soft linear spaces. Finally, in some soft normed quasilinear spaces, we have studied their singular and regular dimensions just as in quasilinear spaces.

2. Soft Quasilinear Spaces and Soft Normed Quasilinear Spaces

This part introduces some soft set theory concepts as well as some basic notions such as soft quasilinear spaces and soft normed quasilinear spaces.

Suppose we have a universe Q, a set of parameters P, its power set denoted by $P(Q)$.

Definition 2.1 ([16]). A pair (G, P) is called a soft set over Q, where G is a mapping defined by $G : P \to P(Q)$.

Definition 2.2 ([9]). If $G(\gamma) = Q$ for all $\gamma \in P$, then a soft set (G, P) over Q is considered an absolute soft set symbolized by $\widetilde{Q}$. If $G(\gamma) = \varnothing$ for all $\gamma \in P$, then a soft set (G, P) over Q is considered a null soft set symbolized by Φ.

Definition 2.3 ([7]). Let Q be a non-empty set and P be a nonempty parameter set. After that, the function $q : P \to Q$ is referred to as a soft element of Q. If $q(\gamma) \in G(\gamma)$, $\gamma \in P$, then a soft element q of Q is said to belong to a soft set G of Q, which is indicated by $q \in Q$. Thus, we obtain $G(\gamma) = \{q(\gamma), \gamma \in P\}$ for a soft set G of Q with regard to the index set P. A soft set (G, P) for which $G(\gamma)$ is a singleton set, $\forall\gamma \in P$ can be determined with a soft element by simply establishing the singleton set with the element that it includes $\forall\gamma \in P$.

The set of all soft sets (G, P) over Q will be described by $S\left(\widetilde{Q}\right)$ for which $G(\gamma) \neq \varnothing$, for all $\gamma \in P$ and the collection of all soft elements of (G, P) over Q will be indicated by $SE\left(\widetilde{Q}\right)$.

Now, let us provide a reasonable description of non-soft linear spaces in soft quasilinear spaces.

Definition 2.4 ([2]). Let P be a set of parameters and Q be a quasilinear space. Assume that G is a soft set over (Q, P). If $Q(\gamma)$ is a quasilinear subspace of Q for any $\gamma \in P$, then G is a soft quasilinear space of Q.

With $\widetilde{a}, \widetilde{b}, \widetilde{c}$ and $\widetilde{q}, \widetilde{w}, \widetilde{z}$ soft real numbers and soft quasi vectors of a soft quasilinear space are demonstrated, respectively.

A soft quasi element $\widetilde{q}$ is said to be regular if it has an inverse, i.e., $\widetilde{q} - \widetilde{q} = \widetilde{\theta}$, such that $\widetilde{q}(\gamma) - \widetilde{q}(\gamma) = \widetilde{\theta}(\gamma)$ for any $\gamma \in P$. A soft quasi element $\widetilde{q}$ is referred to as singular if it lacks an inverse.

Let Q and W be two quasilinear spaces over field $\mathbb{R}$, P be a nonempty set of parameters, $\widetilde{Q}$ and $\widetilde{W}$ be the suitable absolute soft quasilinear spaces i.e. $\widetilde{Q}(\lambda) = Q$ and $\widetilde{W}(\lambda) = W$ for every $\lambda \in P$.

Definition 2.5 ([2]). Let $\widetilde{Q}$ be the absolute soft quasilinear space. Then, a mapping $\|.\| : SE\left(\widetilde{Q}\right) \longrightarrow \mathbb{R}(P)$ is said to be soft norm on the soft quasilinear space $\widetilde{Q}$, if $\|.\|$ satisfies the following conditions:

i) $\|\widetilde{q}\| \geq 0$ if $\widetilde{q} \neq \widetilde{\theta}$ for every $\widetilde{q} \in \widetilde{Q}$,

ii) $\|\widetilde{q} + \widetilde{w}\| \leq \|\widetilde{q}\| + \|\widetilde{w}\|$ for every $\widetilde{q}, \widetilde{w} \in \widetilde{Q}$,

iii) $\|\widetilde{\alpha} \cdot \widetilde{q}\| = |\widetilde{\alpha}| \cdot \|\widetilde{q}\|$ for every $\widetilde{q} \in \widetilde{Q}$ and for every soft scalar $\widetilde{\alpha}$,

iv) if $\widetilde{q} \widetilde{\preceq} \widetilde{w}$, then $\|\widetilde{q}\| \leq \|\widetilde{w}\|$ for every $\widetilde{q}, \widetilde{w} \in \widetilde{Q}$,

v) if for any $\varepsilon > 0$ there exists an element $\widetilde{q}_\varepsilon \in \widetilde{Q}$ such that, $\widetilde{q} \widetilde{\preceq} \widetilde{w} + \widetilde{q}_\varepsilon$ and $\|\widetilde{q}_\varepsilon\| \leq \varepsilon$ then $\widetilde{q} \widetilde{\preceq} \widetilde{w}$ for any soft elements $\widetilde{q}, \widetilde{w} \in \widetilde{Q}$.

Soft normed quasilinear space denoted as $\left(\widetilde{Q}, \|.\|\right)$ or $\left(\widetilde{Q}, \|.\|, P\right)$ is a soft quasilinear space $\widetilde{Q}$ with a soft norm$\|.\|$ on $\widetilde{Q}$.

Lemma 2.6 ([4]). *Let $\left(\widetilde{Q}, \|.\|, P\right)$ be a soft normed quasilinear space and a soft quasi norm $\|.\|$ satisfies the condition:*

$$\{\|\widetilde{q}\|(\gamma) : \widetilde{q}(\gamma) = q, \text{ for } q \in Q \text{ and } \gamma \in P\} \text{ is a singleton set.}$$

Then, for every $\gamma \in P$, $\|.\|_\gamma : Q \to \mathbb{R}^+$ defined by $\|q\|_\gamma = \|\widetilde{q}\|(\gamma)$, for every $q \in Q$ and $\widetilde{q}\widetilde{\in}\widetilde{Q}$ such that $\widetilde{q}(\gamma) = q$, is a quasi norm on Q.

Let $\widetilde{Q}$ be a soft normed quasilinear space. Then,

$$h_Q(\widetilde{q},\widetilde{w}) = \inf\left\{\widetilde{r}\widetilde{\geq}0 : \widetilde{q}\widetilde{\leq}\widetilde{w} + \widetilde{q}_1^r, \ \widetilde{w}\widetilde{\leq}\widetilde{q} + \widetilde{q}_2^r, \left\|\widetilde{q}_i^r\right\| \leq \widetilde{r}\right\}.$$

defines soft Hausdorff or soft norm metric on $\widetilde{Q}$.

3. Main Results

This part introduces some algebraic notions on soft quasilinear spaces and provides some examples and theorems.

Definition 3.1. Let $\widetilde{Q}$ be an absolute soft quasilinear space and $\{\widetilde{q}_k\}_{k=1}^n$ are soft quasi vectors of $\widetilde{Q}$. Also, $\{\widetilde{\alpha}_k\}_{k=1}^n$ are soft scalars. The soft quasi vector $\widetilde{q}$ such that $\sum_{k=1}^n \widetilde{\alpha}_k\widetilde{q}_k = \widetilde{q}$ is called the linear combination of $\{\widetilde{q}_k\}_{k=1}^n$. Otherside, the soft quasi vector $\widetilde{q}$ such that $\sum_{k=1}^n \widetilde{\alpha}_k\widetilde{q}_k\widetilde{\leq}\widetilde{q}$ is called the quasilinear combination of $\{\widetilde{q}_k\}_{k=1}^n$.

From the above definition we understand that the set of soft linear combinations corresponding to the scalars $\{\widetilde{\alpha}_k\}_{k=1}^n$ is unique. But, the combination of soft quasilinear corresponding to the same scalars may not be unique. Furthermore, the set of soft quasilinear combinations of a soft vector in a soft quasilinear space may consist of different sets for different parameters. Namely, if γ_1, γ_2 are different parameters of P, then may not always be $\widetilde{q}(\gamma_1) = \widetilde{q}(\gamma_2)$. Again, the images of two different soft quasi vectors of soft quasilinear space under same parameter may be the same.

Definition 3.2. Let $\widetilde{Q}$ be an absolute soft quasilinear space and $\widetilde{M} \subseteq \widetilde{Q}$. Soft quasi span set of $\widetilde{M}$ is

$$QSP\widetilde{M} = \left\{\widetilde{q} \in \widetilde{Q} : \sum_{k=1}^n \widetilde{\alpha}_k\widetilde{m}_k\widetilde{\leq}\widetilde{q}, \ \widetilde{m}_k \in \widetilde{M}, \widetilde{\alpha}_k \text{ are soft scalar every } 1 \leq k \leq n\right\}.$$

If $QSP\widetilde{M} = \widetilde{Q}$, then we say that $\widetilde{M}$ is quasi span**.**

Example 3.3. Let us take soft quasi vector $\widetilde{q} \in \widetilde{\Omega_C(\mathbb{R})}$ such that $\widetilde{q}(\gamma) = [-1,2]$ for a $\gamma \in P$. Assume that $\widetilde{\alpha}(\gamma) = \alpha$ for every soft scalar and for every $\gamma \in P$. From here, soft linear combination set of soft quasi vector $\widetilde{q}$ is $(\widetilde{\alpha q})(\gamma) = \left\{\widetilde{q'}(\gamma) = q \in \Omega_C(\mathbb{R}) : \widetilde{\alpha}(\gamma)\widetilde{q}(\gamma) = \alpha[-1,2] = q, \ \gamma \in P\right\}$ but soft quasilinear combination set of soft quasi vector $\widetilde{q}$ is $(\widetilde{\alpha q})(\gamma) = \left\{\widetilde{q'}(\gamma) = q \in \Omega_C(\mathbb{R}) : \widetilde{\alpha}(\gamma)\widetilde{q}(\gamma) = \alpha[-1,2] \leq q, \ \gamma \in P\right\}$. For example, if we take soft scalar $\widetilde{1}(\gamma) = 1$, then we get that soft quasilinear combination of soft quasi vector $\widetilde{q} \in \widetilde{\Omega_C(\mathbb{R})}$ consists of all soft quasi vector $\widetilde{q'} \in \widetilde{\Omega_C(\mathbb{R})}$ such that $\widetilde{q'}(\gamma) = q \in \Omega_C(\mathbb{R})$ that includes $[-1,2]$ because of $\widetilde{1}(\gamma)\widetilde{q}(\gamma) = [-1,2] \leq q$ for $\gamma \in P$.

Further, if we take $\widetilde{\alpha}(\gamma)\widetilde{q}(\gamma) = \alpha[-1,2] \leq [2,3]$, then we can't find a soft scalar $\widetilde{\alpha}(\gamma) = \alpha$ for $\widetilde{q'}(\gamma) = q = [2,3]$. So, $QSP\{\widetilde{q}\} \neq \widetilde{Q}$.

Example 3.4. Let us take $\widetilde{M} \subset \widetilde{\Omega_C(\mathbb{R})}$ such that $\widetilde{M} = \{\widetilde{m} : \widetilde{m}(\gamma) = \{3\}$ for $\gamma \in P\}$. Again, we take $\widetilde{\alpha}(\gamma) = \alpha$ for every soft scalar and for every $\gamma \in P$ the we find

$$QSP\widetilde{M} = \left\{\widetilde{m'} \in \widetilde{\Omega_C(\mathbb{R})} : (\widetilde{\alpha m})(\gamma) = \alpha\widetilde{m}(\gamma) = \alpha\{3\}\widetilde{\leq}\widetilde{m'}(\gamma), \ \gamma \in P\right\}.$$

Since $\widetilde{\Omega_C(\mathbb{R})}$ is a absolute soft quasilinear space we find $\widetilde{m'} \in \widetilde{\Omega_C(\mathbb{R})}$ such that $\alpha\{3\}\widetilde{\leq}\widetilde{m'}(\gamma)$ for a γ parameter. So,

$$QSP\widetilde{M} = \left\{\widetilde{m'} \in \widetilde{\Omega_C(\mathbb{R})} : (\widetilde{\alpha m})(\gamma) = \alpha\{3\}\widetilde{\leq}\widetilde{m'}(\gamma), \ \gamma \in P\right\} = \widetilde{\Omega_C(\mathbb{R})}.$$

We know from [6] that, the set $[\{a\}]$ quasi span $\Omega_C(\mathbb{R})$. In a soft quasilinear space, if $\widetilde{q}(\gamma) = \{q\} \subset \Omega_C(\mathbb{R})$ for soft scalar $\widetilde{\alpha}(\gamma) = \alpha$ and $q \in \mathbb{R}$, then we say that the soft quasi vector $\widetilde{q} \in \widetilde{\Omega_C(\mathbb{R})}$ will quasi span the soft quasilinear space $\widetilde{\Omega_C(\mathbb{R})}$.

Theorem 3.5. *Let $\widetilde{Q}$ be a soft quasilinear space and $\widetilde{M} = \{\widetilde{q}_1, \widetilde{q}_2, ..., \widetilde{q}_n\} \subset \widetilde{Q}$. Then, $QSP\widetilde{M}$ is a soft quasilinear subspace of $\widetilde{Q}$.*

Proof. We assume that $\widetilde{q}, \widetilde{w} \in QSP\widetilde{M}$ and $\widetilde{\alpha}$ is a soft scalars. From our acceptance there exist soft scalars $\widetilde{\alpha}_k$ and $\widetilde{b}_k$ that we find $\sum_{k=1}^n \widetilde{\alpha}_k\widetilde{q}_k\widetilde{\leq}\widetilde{q}$ and $\sum_{k=1}^n \widetilde{b}_k\widetilde{q}_k\widetilde{\leq}\widetilde{w}$. Since $\widetilde{Q}$ be a soft quasilinear space we obtain

$$\sum_{k=1}^n(\widetilde{\alpha}_k + \widetilde{b}_k)\widetilde{q}_k = \sum_{k=1}^n \widetilde{\alpha}_k\widetilde{q}_k + \sum_{k=1}^n \widetilde{b}_k\widetilde{q}_k\widetilde{\leq}\widetilde{q} + \widetilde{w}$$

and

$$\widetilde{\alpha}(\sum_{k=1}^{n}\widetilde{\alpha_k}\widetilde{q_k}) = \sum_{k=1}^{n}\widetilde{\alpha}\,(\widetilde{\alpha_k}\widetilde{q_k})\,\widetilde{\preceq}\widetilde{\alpha}\widetilde{q}.$$

This gives $\widetilde{q}+\widetilde{w} \in QSP\widetilde{M}$ and $\widetilde{\alpha}\widetilde{q} \in QSP\widetilde{M}$. Thus, we say that $QSP\widetilde{M}$ is a soft quasilinear subspace of $\widetilde{Q}$. □

The result obtained in the above theorem is quite similar to the result in quasilinear spaces.

Definition 3.6. Let $\widetilde{Q}$ be a soft quasilinear space, $\{\widetilde{q_k}\}_{k=1}^{n}$ are soft quasi vectors of $\widetilde{Q}$ and $\{\widetilde{\alpha_k}\}_{k=1}^{n}$ are soft scalars. If

$$\widetilde{\theta}\widetilde{\preceq}\widetilde{\alpha_1}\widetilde{q_1}+\widetilde{\alpha_2}\widetilde{q_2}+...+\widetilde{\alpha_n}\widetilde{q_n}$$

inequality satisfies if and only if $\widetilde{\alpha_1}=\widetilde{\alpha_2}=...=\widetilde{\alpha_n}=\widetilde{0}$, then we say that the soft quasi vectors set $\{\widetilde{q_k}\}_{k=1}^{n}$ is soft quasilinear independent. Otherwise, we say that the soft quasi vectors set $\{\widetilde{q_k}\}_{k=1}^{n}$ soft is quasilinear dependent.

Considering that every soft linear space is a soft quasilinear space with the relation "=", dependency quasilinear independence and dependence concepts in soft linear spaces turn into soft linear independence and dependence concepts.

Theorem 3.7. *In the soft quasilinear space $\widetilde{\Omega_C(\mathbb{R})}$, any soft set with two elements must be quasilinear dependent.*

Proof. Let us take a soft quasi set $\{\widetilde{q},\widetilde{w}\}\subset\widetilde{\Omega_C(\mathbb{R})}$ such that $\widetilde{q}(\gamma)=q\in\Omega_C(\mathbb{R})$ and $\widetilde{w}(\gamma)=w\in\Omega_C(\mathbb{R})$ for every $\gamma\in P$. Here are a few cases:

If $q,w\in(\Omega_C(\mathbb{R}))_r$ then $\widetilde{\theta}(\gamma)\widetilde{\preceq}\widetilde{\alpha_1}(\gamma)\widetilde{q}(\gamma)+\widetilde{\alpha_2}(\gamma)\widetilde{w}(\gamma)=\alpha_1 q+\alpha_2 w$ satisfies for $\widetilde{\alpha_1},\widetilde{\alpha_2}$ other than $\widetilde{0}$ for $\widetilde{\alpha_1}(\gamma)=\alpha_1,\widetilde{\alpha_2}(\gamma)=\alpha_2$.

If $\widetilde{q}\in(\widetilde{\Omega_C(\mathbb{R})})_d$ or $\widetilde{w}\in\widetilde{\Omega_C(\mathbb{R})}_d$ then we get $\widetilde{q}(\gamma)=q$ includes a symmetric interval $[-k,k]$ or $\widetilde{w}(\gamma)=w$ includes a symmetric interval $[-k,k]$. In both cases, the $\widetilde{\theta}(\gamma)\widetilde{\preceq}\widetilde{\alpha_1}(\gamma)\widetilde{q}(\gamma)+\widetilde{\alpha_2}(\gamma)\widetilde{w}(\gamma)=\alpha_1 q+\alpha_2 w$ inequality is not satisfied only $\widetilde{\alpha_1}=\widetilde{\alpha_2}=\widetilde{0}$.

If $\widetilde{q},\widetilde{w}\notin(\widetilde{\Omega_C(\mathbb{R})})_d$ then $\widetilde{\theta}(\gamma)\widetilde{\preceq}\widetilde{\alpha_1}(\gamma)\widetilde{q}(\gamma)+\widetilde{\alpha_2}(\gamma)\widetilde{w}(\gamma)=\alpha_1 q+\alpha_2 w$ inequality is not satisfied only $\widetilde{\alpha_1}=\widetilde{\alpha_2}=\widetilde{0}$. Namely, the $\widetilde{\theta}(\gamma)\widetilde{\preceq}\widetilde{\alpha_1}(\gamma)\widetilde{q}(\gamma)+\widetilde{\alpha_2}(\gamma)\widetilde{w}(\gamma)$ inequality is satisfied within $\widetilde{\alpha_1}$ and $\widetilde{\alpha_2}$ other than $\widetilde{0}$.

Because of these cases in the soft quasilinear space $\widetilde{\Omega_C(\mathbb{R})}$ any soft set with two elements must be quasilinear dependent. This result is quite similar to the result in quasilinear spaces. □

Example 3.8. Let us take a soft quasi vector $\widetilde{q}$ of absolute soft quasilinear space $\widetilde{\Omega_C(\mathbb{R})}$ and $\widetilde{\alpha}(\gamma)=\alpha\in\mathbb{R}$ for every soft scalars $\widetilde{\alpha}$ and $\gamma\in P$. From Definition 3.6, we say that $\widetilde{\theta}\widetilde{\preceq}\widetilde{\alpha}\widetilde{q}$ is soft quasilinear independent in $\widetilde{\Omega_C(\mathbb{R})}$ if and only if $\widetilde{\theta}(\gamma)\,\widetilde{\preceq}\widetilde{\alpha}(\gamma)\widetilde{q}(\gamma)$ is quasilinear independent in $\Omega_C(\mathbb{R})$. Also, if $\widetilde{q}(\gamma)$ includes $\{0\}$ for parameter γ, then $\widetilde{q}$ is soft quasilinear dependent in $\widetilde{\Omega_C(\mathbb{R})}$. Otherwise, we get that $\widetilde{q}$ is soft quasilinear independent in $\widetilde{\Omega_C(\mathbb{R})}$.

Example 3.9. Let us take two soft quasi vectors $\widetilde{q}$ and $\widetilde{w}$ of absolute soft quasilinear space $\widetilde{\Omega_C(\mathbb{R})}$ and $\widetilde{\alpha}(\gamma)=\alpha\in\mathbb{R}$, $\widetilde{\beta}(\gamma)=\beta\in\mathbb{R}$ for every soft scalars $\widetilde{\alpha}$ and $\widetilde{\beta}$ and parameter γ. From Theorem 2.2.4 in [6], we get that $\widetilde{\theta}\widetilde{\preceq}\widetilde{\alpha}\widetilde{q}+\widetilde{\beta}\widetilde{w}$ is soft quasilinear dependent in $\widetilde{\Omega_C(\mathbb{R})}$ since $\widetilde{\theta}(\gamma)\,\widetilde{\preceq}\widetilde{\alpha}(\gamma)\widetilde{q}(\gamma)+\widetilde{\beta}(\gamma)\widetilde{w}(\gamma)$ is quasilinear dependent in $\Omega_C(\mathbb{R})$.

Remark 3.10. In an absolute soft quasilinear space $\widetilde{Q}$, a soft quasi vector $\widetilde{q}$ is soft quasilinear independent if and only if $\widetilde{q}(\gamma)$ is quasilinear independent in Q for $\gamma\in P$.

Remark 3.11. Note that the soft quasilinear combination, soft quasi span, soft quasilinear independency and soft quasilinear dependence of a soft quasi vector is analyzed depending on the parameter $\gamma\in P$. So, when we research at the situation of a soft quasi vector in relation to any of these properties, we can say that it is soft quasilinear dependent or independent under the parameter γ.

In accordance with the above remark, we can give the following theorem.

Theorem 3.12. *Let $\widetilde{Q}$ be a soft quasilinear space and the set $M=\{\widetilde{q_1},\widetilde{q_2},...,\widetilde{q_n}\}$ of soft quasi vectors in $\widetilde{Q}$. Then, M is soft quasilinear independent in $\widetilde{Q}$ if and only if $M(\gamma)=\{\widetilde{q_1}(\gamma),\widetilde{q_2}(\gamma),...,\widetilde{q_n}(\gamma)\}$ is quasilinear independent in Q for every $\gamma\in P$.*

Proof. Assume that $M=\{\widetilde{q_1},\widetilde{q_2},...,\widetilde{q_n}\}$ be a soft quasilinear independent in $\widetilde{Q}$. Then, for every soft scalars $\widetilde{\alpha_1},\widetilde{\alpha_2},...,\widetilde{\alpha_n}$, we obtain

$$\widetilde{\theta}\widetilde{\preceq}\widetilde{\alpha_1}\widetilde{q_1}+\widetilde{\alpha_2}\widetilde{q_2}+...+\widetilde{\alpha_n}\widetilde{q_n}\text{ if and only if }\widetilde{\alpha_1}=\widetilde{\alpha_2}=...=\widetilde{\alpha_n}=\widetilde{0}. \tag{3.1}$$

If we take arbitrary $\gamma_0 \in P$, then we can write $M(\gamma_0) = \{\widetilde{q_1}(\gamma_0), \widetilde{q_2}(\gamma_0), ..., \widetilde{q_n}(\gamma_0)\}$. Moreover, consider that $\widetilde{\alpha_1}(\gamma_0) = \alpha_1, \widetilde{\alpha_2}(\gamma_0) = \alpha_2, ..., \widetilde{\alpha_n}(\gamma_0) = \alpha_n$, then from (3.1), we find $\alpha_1 = \alpha_2 = ... = \alpha_n = 0$. Thus, we find

$$\begin{aligned} \theta &= \widetilde{\theta}(\gamma_0) \widetilde{\leq} \widetilde{\alpha_1}(\gamma_0) \widetilde{q_1}(\gamma_0) + \widetilde{\alpha_2}(\gamma_0) \widetilde{q_2}(\gamma_0) + ... + \widetilde{\alpha_n}(\gamma_0) \widetilde{q_n}(\gamma_0) \\ &= \alpha_1 \widetilde{q_1}(\gamma_0) + \alpha_2 \widetilde{q_2}(\gamma_0) + ... + \alpha_n \widetilde{q_n}(\gamma_0). \end{aligned}$$

From here, we say that $M(\gamma_0) = \{\widetilde{q_1}(\gamma_0), \widetilde{q_2}(\gamma_0), ..., \widetilde{q_n}(\gamma_0)\}$ is quasilinear independent in Q for arbitrary $\gamma_0 \in P$. This gives $M(\gamma) = \{\widetilde{q_1}(\gamma), \widetilde{q_2}(\gamma), ..., \widetilde{q_n}(\gamma)\}$ is quasilinear independent in Q for every $\gamma \in P$.

On the other side, let $M(\gamma) = \{\widetilde{q_1}(\gamma), \widetilde{q_2}(\gamma), ..., \widetilde{q_n}(\gamma)\}$ be quasilinear independent in Q for every $\gamma \in P$. Namely, $\widetilde{\theta}(\gamma) \widetilde{\leq} \alpha_1 \widetilde{q_1}(\gamma_0) + \alpha_2 \widetilde{q_2}(\gamma_0) + ... + \alpha_n \widetilde{q_n}(\gamma_0)$ inequality satisfies if and only if $\alpha_1 = \alpha_2 = ... = \alpha_n = 0$. Let us take $\widetilde{\alpha_1}, \widetilde{\alpha_2}, ..., \widetilde{\alpha_n}$ be any soft scalars such that $\widetilde{\theta \leq} \widetilde{\alpha_1} \widetilde{q_1} + \widetilde{\alpha_2} \widetilde{q_2} + ... + \widetilde{\alpha_n} \widetilde{q_n}$. Then, if we take $\widetilde{\alpha_1}(\gamma) = \alpha_1, \widetilde{\alpha_2}(\gamma) = \alpha_2, ..., \widetilde{\alpha_n}(\gamma) = \alpha_n$ for every $\gamma \in P$, we find

$$\widetilde{\theta}(\gamma) \widetilde{\leq} \widetilde{\alpha_1}(\gamma) \widetilde{q_1}(\gamma) + \widetilde{\alpha_2}(\gamma) \widetilde{q_2}(\gamma) + ... + \widetilde{\alpha_n}(\gamma) \widetilde{q_n}(\gamma).$$

Thus, we get $\widetilde{\alpha_1}(\gamma) = \widetilde{\alpha_2}(\gamma) = ... = \widetilde{\alpha_n}(\gamma) = \widetilde{\theta}(\gamma)$ for every $\gamma \in P$. This gives $\widetilde{\theta \leq} \widetilde{\alpha_1} \widetilde{q_1} + \widetilde{\alpha_2} \widetilde{q_2} + ... + \widetilde{\alpha_n} \widetilde{q_n}$ implies that $\widetilde{\alpha_1} = \widetilde{\alpha_2} = ... = \widetilde{\alpha_n} = \widetilde{0}$. So, $M = \{\widetilde{q_1}, \widetilde{q_2}, ..., \widetilde{q_n}\}$ be a soft quasilinear independent in $\widetilde{Q}$. □

Theorem 3.13. *Let $\widetilde{Q}$ be a soft quasilinear space and the set $M = \{\widetilde{q_1}, \widetilde{q_2}, ..., \widetilde{q_n}\}$ of soft quasi vectors in $\widetilde{Q}$. Then, M is soft quasilinear dependent in $\widetilde{Q}$ if and only if $M(\gamma) = \{\widetilde{q_1}(\gamma), \widetilde{q_2}(\gamma), ..., \widetilde{q_n}(\gamma)\}$ is quasilinear dependent in Q for every $\gamma \in P$.*

Proof. Assume that $M = \{\widetilde{q_1}, \widetilde{q_2}, ..., \widetilde{q_n}\}$ be a soft quasilinear dependent in $\widetilde{Q}$. Then, for every soft scalars $\widetilde{\alpha_1}, \widetilde{\alpha_2}, ..., \widetilde{\alpha_n}$, not equal to $\widetilde{\theta}$, we obtain

$$\widetilde{\theta \leq} \widetilde{\alpha_1} \widetilde{q_1} + \widetilde{\alpha_2} \widetilde{q_2} + ... + \widetilde{\alpha_n} \widetilde{q_n}. \tag{3.2}$$

If we take arbitrary $\gamma_0 \in P$, then we can write $M(\gamma_0) = \{\widetilde{q_1}(\gamma_0), \widetilde{q_2}(\gamma_0), ..., \widetilde{q_n}(\gamma_0)\}$. Moreover, consider that $\widetilde{\alpha_1}(\gamma_0) = \alpha_1, \widetilde{\alpha_2}(\gamma_0) = \alpha_2, ..., \widetilde{\alpha_n}(\gamma_0) = \alpha_n$, are not all $\widetilde{\theta}$ then from (3.2), we find $\alpha_1, \alpha_2, ..., \alpha_n$ are not all 0. Thus, we find

$$\begin{aligned} \theta &= \widetilde{\theta}(\gamma_0) \widetilde{\leq} \widetilde{\alpha_1}(\gamma_0) \widetilde{q_1}(\gamma_0) + \widetilde{\alpha_2}(\gamma_0) \widetilde{q_2}(\gamma_0) + ... + \widetilde{\alpha_n}(\gamma_0) \widetilde{q_n}(\gamma_0) \\ &= \alpha_1 \widetilde{q_1}(\gamma_0) + \alpha_2 \widetilde{q_2}(\gamma_0) + ... + \alpha_n \widetilde{q_n}(\gamma_0). \end{aligned}$$

From here, we say that $M(\gamma_0) = \{\widetilde{q_1}(\gamma_0), \widetilde{q_2}(\gamma_0), ..., \widetilde{q_n}(\gamma_0)\}$ is quasilinear dependent in Q for arbitrary $\gamma_0 \in P$. This gives $M(\gamma) = \{\widetilde{q_1}(\gamma), \widetilde{q_2}(\gamma), ..., \widetilde{q_n}(\gamma)\}$ is quasilinear dependent in Q for every $\gamma \in P$.

On the other side, let $M(\gamma) = \{\widetilde{q_1}(\gamma), \widetilde{q_2}(\gamma), ..., \widetilde{q_n}(\gamma)\}$ be quasilinear dependent in Q for every $\gamma \in P$. Then there is a set of scalars $\alpha_1, \alpha_2, ..., \alpha_n$ not all 0 such that $\widetilde{\theta}(\gamma) \widetilde{\leq} \alpha_1 \widetilde{q_1}(\gamma_0) + \alpha_2 \widetilde{q_2}(\gamma_0) + ... + \alpha_n \widetilde{q_n}(\gamma_0)$. Let us take $\widetilde{\alpha_1}, \widetilde{\alpha_2}, ..., \widetilde{\alpha_n}$ be any soft scalars such that $\widetilde{\alpha_1}(\gamma) = \alpha_1, \widetilde{\alpha_2}(\gamma) = \alpha_2, ..., \widetilde{\alpha_n}(\gamma) = \alpha_n$ for every $\gamma \in P$. So, we say that soft scalars $\widetilde{\alpha_1}, \widetilde{\alpha_2}, ..., \widetilde{\alpha_n}$ not all $\widetilde{\theta}$ for γ parameter. $\widetilde{\theta \leq} \widetilde{\alpha_1} \widetilde{q_1} + \widetilde{\alpha_2} \widetilde{q_2} + ... + \widetilde{\alpha_n} \widetilde{q_n}$. Then, we find

$$\widetilde{\theta}(\gamma) \widetilde{\leq} \widetilde{\alpha_1}(\gamma) \widetilde{q_1}(\gamma) + \widetilde{\alpha_2}(\gamma) \widetilde{q_2}(\gamma) + ... + \widetilde{\alpha_n}(\gamma) \widetilde{q_n}(\gamma).$$

So, $M = \{\widetilde{q_1}, \widetilde{q_2}, ..., \widetilde{q_n}\}$ be a soft quasilinear dependent in $\widetilde{Q}$. □

Definition 3.14. Suppose $\widetilde{Q}$ be a soft quasilinear space and $\widetilde{W} \subseteq \widetilde{Q}$. We say that $\widetilde{W}$ is a soft quasilinear base of $\widetilde{Q}$ if $\widetilde{W}$ is soft quasilinear independent and $QSP\widetilde{W} = \widetilde{Q}$.

Example 3.15. Consider a soft quasi subset $\widetilde{W} = \{\{\widetilde{q}\}\}$ of soft quasilinear space $\widetilde{\Omega_C(\mathbb{R})}$. For $\widetilde{W}$ is a soft quasilinear base of $\widetilde{\Omega_C(\mathbb{R})}$ a necessary and sufficient condition is that $\widetilde{q}(\gamma) \in \widetilde{\Omega_C(\mathbb{R})}_r$ for every $\gamma \in P$.

Corollary 3.16. *For the soft quasilinear space $\widetilde{\Omega_C(\mathbb{R})}$ all soft quasilinear vectors $\{\widetilde{q}\}$ such that $\widetilde{q}(\gamma) \neq 0$ and $\widetilde{q}(\gamma) \in \widetilde{\Omega_C(\mathbb{R})}_r$ are bases.*

Example 3.17. Consider a subset of the singular subspace of $\widetilde{\Omega_C(\mathbb{R})}$ such that

$$F = \{\widetilde{q} : \widetilde{q}(\gamma) = [a, b] \ , a < b, \ a, b \in \mathbb{R}^+ \text{ or } a, b \in \mathbb{R}^-\} \, U \left\{\left\{\widetilde{0}\right\}\right\}.$$

F is soft quasilinear independent set. But, F has not soft quasilinear base because of $QSPF \neq \widetilde{Q}$.

Now, some information and examples will be given about the dimension of soft quasilinear spaces, which are a generalization of quasilinear spaces. We will see here again when the dimension of a soft quasilinear space is defined as regular and singular dimensions, just like in quasilinear spaces.

Definition 3.18. The maximum number of linearly independent elements that a regular subspace of a soft quasilinear space $\widetilde{Q}$ can contain is called the regular dimension of $\widetilde{Q}$. Similarly, the maximum number of quasilinear independent elements that the singular subspace of $\widetilde{Q}$ can contain is called the singular dimension of $\widetilde{Q}$. The regular dimension of the soft quasilinear space $\widetilde{Q}$ is denoted by $r-\dim\widetilde{Q}$ and singular dimension of the soft quasilinear space $\widetilde{Q}$ is denoted by $s-\dim\widetilde{Q}$. If in a soft quasilinear space $r-\dim\widetilde{Q} = s-\dim\widetilde{Q}$ then we will denote the dimension of soft quasilinear space $\widetilde{Q}$ with $\dim\widetilde{Q}$.

If $\widetilde{Q}$ is a soft linear space, it's singular dimension is 0. Therefore, the dimension of $\widetilde{Q}$ is only the regular dimension. Therefore, only the concept of dimension is used instead of regular dimension in soft linear spaces. Similar to quasilinear spaces, If $s-\dim\widetilde{Q} > 0$, $\widetilde{Q}$ is a soft quasilinear space.

Remark 3.19. If $s-\dim\widetilde{Q} = 0$ in a space, this space does not have to be a soft linear space. Let's now illustrate this using an example.

Example 3.20. Let $\widetilde{\Omega_C(\mathbb{R})}_d$ denote the set of all symmetric vectors of the $\widetilde{\Omega_C(\mathbb{R})}$. From definition of $\widetilde{\Omega_C(\mathbb{R})}_d$ we know that $\widetilde{\Omega_C(\mathbb{R})}_d = \{\widetilde{q} : \widetilde{q}(\gamma) = [-a, a],\ a \in \mathbb{R}\}$. If we take $\widetilde{q}_1, \widetilde{q}_2, ..., \widetilde{q}_n \in \widetilde{\Omega_C(\mathbb{R})}_d$, $\widetilde{0 \leq} \widetilde{\alpha}_1\widetilde{q}_1 + \widetilde{\alpha}_2\widetilde{q}_2 + ... + \widetilde{\alpha}_n\widetilde{q}_n$ inequality is not satisfy only for $\widetilde{\alpha}_1 = \widetilde{\alpha}_2 = ...\widetilde{\alpha}_n = \widetilde{0}$. As a result, we cannot find the soft quasilinear independent vectors of $\widetilde{\Omega_C(\mathbb{R})}_d$. Hence, $s-\dim\widetilde{\Omega_C(\mathbb{R})}_d = 0$. This gives $\dim\widetilde{\Omega_C(\mathbb{R})}_d = 0$.

Definition 3.21. It is said that a soft quasilinear space $\widetilde{Q}$ is of finite dimension, if there is a finite set of quasilinear independent soft quasi vectors in $\widetilde{Q}$ that are also associated with $\widetilde{Q}$.

In a soft normed quasilinear space $\tilde{Q}$, if $\widetilde{q}_1, \widetilde{q}_2, ..., \widetilde{q}_n$ are soft quasilinear independent vectors in $\tilde{Q}$, then $\widetilde{q}_1(\gamma), \widetilde{q}_2(\gamma), ..., \widetilde{q}_n(\gamma)$ are quasilinear independent vectors in Q for every $\gamma \in P$. Namely, if $\widetilde{0 \leq} \widetilde{\alpha}_1\widetilde{q}_1 + \widetilde{\alpha}_2\widetilde{q}_2 + ... + \widetilde{\alpha}_n\widetilde{q}_n \Longleftrightarrow \widetilde{\alpha}_1 = \widetilde{\alpha}_2 = ...\widetilde{\alpha}_n = \widetilde{0}$ then $\widetilde{0}_1(\gamma) \leq \widetilde{\alpha}_1(\gamma)\widetilde{q}_1(\gamma) + \widetilde{\alpha}_2(\gamma)\widetilde{q}_2(\gamma) + ... + \widetilde{\alpha}_n(\gamma)\widetilde{q}_n(\gamma) \Longleftrightarrow \widetilde{\alpha}_1(\gamma) = \widetilde{\alpha}_2(\gamma) + ... + \widetilde{\alpha}_n(\gamma) = 0$.

Example 3.22. The soft quasilinear spaces $\widetilde{\mathbb{R}^n}$, $\widetilde{\Omega_C(\mathbb{R}^n)}$, $\widetilde{\Omega_C(\mathbb{R}^n)}_s \cup \{\widetilde{0}\}$ and $\widetilde{\Omega_C(\mathbb{R}^n)}_r$ have the following dimensions:

$$r-\dim\widetilde{\mathbb{R}^n} = n,\ s-\dim\widetilde{\mathbb{R}^n} = 0$$
$$r-\dim\widetilde{\Omega_C(\mathbb{R}^n)} = n,\ s-\dim\widetilde{\Omega_C(\mathbb{R}^n)} = n$$
$$r-\dim\left(\widetilde{\Omega_C(\mathbb{R}^n)}_s \cup \{\widetilde{0}\}\right) = 0,\ s-\dim\left(\widetilde{\Omega_C(\mathbb{R}^n)}_s \cup \{\widetilde{0}\}\right) = n$$
$$r-\dim\widetilde{\Omega_C(\mathbb{R}^n)}_r = n,\ s-\dim\widetilde{\Omega_C(\mathbb{R}^n)}_r = 0.$$

Example 3.23. The dimensions of the $\widetilde{\Omega_C(c_0)}$, $\widetilde{\Omega_C(l_2)}$ and $\widetilde{\Omega_C(l_\infty)}$ soft quasilinear spaces are as follows.

$$r-\dim\widetilde{\Omega_C(c_0)} = \infty \qquad s-\dim\widetilde{\Omega_C(c_0)} = \infty$$
$$r-\dim\widetilde{\Omega_C(l_2)} = \infty \qquad s-\dim\widetilde{\Omega_C(l_2)} = \infty$$
$$r-\dim\widetilde{\Omega_C(l_\infty)} = \infty \qquad s-\dim\widetilde{\Omega_C(l_\infty)} = \infty.$$

Example 3.24. For the soft quasilinear space

$$\widetilde{Q} = \left(\widetilde{\Omega_C(c_0)}\right)_s \cup \{\{0, 0, ..., 0, k, 0, 0, ..., ...\}\, k \in \mathbb{R}\}$$

we find

$$r-\dim\widetilde{Q} = 1 \text{ and } s-\dim\widetilde{Q} = \infty.$$

Example 3.25. Consider the soft quasilinear subspace $M = \left(\widetilde{I\mathbb{R}^2}\right)_s \cup \left\{\widetilde{t} : \widetilde{t}(\gamma) = (t, 0) : t \in \mathbb{R},\ \gamma \in P\right\}$ of soft quasilinear space $\widetilde{I\mathbb{R}^2}$. Let we take $\widetilde{q_1}, \widetilde{q_2} \in M$ such that $\widetilde{q_1}(\gamma) = ((0), [1, 2])$ and $\widetilde{q_2}(\gamma) = ([1, 2], \{0\})$ for a $\gamma \in P$. Then, the inequality

$$\begin{aligned}(0, 0) = \widetilde{0}(\gamma) &\widetilde{\leq} \widetilde{\alpha}_1(\gamma)\widetilde{q_1}(\gamma) + \widetilde{\alpha}_2(\gamma)\widetilde{q_2}(\gamma) \\ &= \alpha_1 \cdot ((0), [1, 2]) + \alpha_2 \cdot ([1, 2], \{0\})\end{aligned}$$

is satisfied only when $\widetilde{\alpha}_1 = \widetilde{\alpha}_2 = \widetilde{0}$. So, the set $\{\widetilde{q_1}, \widetilde{q_2}\}$ is quasilinear independent set in $\widetilde{I\mathbb{R}^2}$. Also, $s-\dim M = 2$ since M is a soft quasilinear subspace of $\widetilde{I\mathbb{R}^2}$.

Furthermore, if we consider the $M_r = \left\{\widetilde{t} : \widetilde{t}(\gamma) = (t,0) : t \in \mathbb{R},\ \gamma \in P\right\}$, then the inequality

$$\begin{aligned}(0,0) = \widetilde{0}(\gamma) &\widetilde{\leq} \widetilde{\alpha}(\gamma)\widetilde{q}(\gamma)\\ &= \alpha \cdot (t,0)\end{aligned}$$

for $\widetilde{q} \in M_r$ is satisfied only when $\widetilde{\alpha} = \widetilde{0}$. Thus, we have $r - \dim M = 1$.

Example 3.26. Let us take a soft quasilinear space $M = \left(\widetilde{Ic_0}\right)_s \cup \{\varnothing\}$. Let us take arbitrary finite singular subset $\{\widetilde{q_1}, \widetilde{q_2}, ..., , \widetilde{q_n}\}$ of M such that $\widetilde{q_1}(\gamma) = \{([1,2], 0, 0, ...)\}$, $\widetilde{q_2}(\gamma) = \{(0, [1,2], 0, ...)\}, ..., \widetilde{q_n}(\gamma) = \{(0, 0, ... [1,2], 0, ...)\}$ for a $\gamma \in P$. Then,

$$(0,0,...,0,...) = \widetilde{0}(\gamma) \quad \widetilde{\leq} \quad \widetilde{\alpha}_1(\gamma)\widetilde{q}_1(\gamma) + \widetilde{\alpha}_2(\gamma)\widetilde{q}_2(\gamma) + ... + \widetilde{\alpha}_n(\gamma)\widetilde{q}_n(\gamma)$$

is satisfied only when $\widetilde{\alpha}_1(\gamma) = \widetilde{\alpha}_2(\gamma) = ... = \widetilde{\alpha}_n(\gamma) = 0$. So, the set $\{\widetilde{q_1}, \widetilde{q_2}, ..., \widetilde{q_n}\}$ is quasilinear independent set in M. Also, $s - \dim M = \infty$. Moreover, $r - \dim M = 0$ because of $M_r = \{\varnothing\}$.

Conflicts of Interest

Author Fatma Bulak declares that she has no conflict of interest. Author Hacer Bozkurt declares that he has no conflict of interest.

Authors Contribution Statement

Both authors have created and written the article.

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