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Search on a Modified Gravity Theory Including Scalar Density Field

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Abstract

In this study, we investigated modified gravity in terms of both scalar and scalar density fields. Subsequently, the results are compared and briefly discussed within the framework of the Friedmann-Robertson-Walker (FRW) metric. We present here focus our attention on investigating a new modified gravitational theory by making use of a weight 2 scalar density field, which may be important to describe a late universe. On this purpose, we derive corresponding equation-of-motion (EoM) for the selected scalar density form in order to reveal cosmological features of our theoretical ground. Consequently we arrived at the new and interesting field equations derived from modified equations of action corresponding to an FRW metric.

Keywords: Modified gravity, scalar density, scalar field.

1. *Introduction*

Many astrophysical and cosmological studies in recent years have revealed that the universe has entered a period of accelerating expansion. The corresponding information and data analyses (Spergel 2003), (Page 2003), (Verde 2003), (Bridel et al. 2003), (Riess et al. 1998), (Perlmutter et al. 1998), (Perlmutter et al.1997), (Perlmutter et al. 1999), (Vishwakarma 2001), (Vishwakarma 2002), (Jain and Taylor 2003), (Dekel et al. 1997), (Viana and Liddle 1999), (Schmidt et al. 1998), (Efstathiou et al. 1999), (Netterfield et al. 2002), (Tonry et al. 2003), (Daly and Djorgovski 2003), (Lahanas et al. 2003), (Seljak et al. 2005), (Riess et al. 2005), (Astier et al. 2006) performed via the WMAP, CMB and supernova datasets indicated some strong evidences for this speedy expansion phenomenon of the cosmos.

Therefore, it is clear that cosmic acceleration has far-reaching implications in contemporary physics. For this reason, scientists have begun to consider the physical process behind this mysterious expansion phase as a fundamental problem of cosmology. However, despite all studies, a proper cause has not yet been fully established. Historically, attempts to modified the General Theory of Relativity (GTR) begin soon after Einstein introduced his theory in 1920. However, especially in the last couple of years, such attempts have increased significantly with the understanding that the universe is entering a period of accelerating expansion. Unfortunately, it is currently not possible to explain this mysterious accelerated expansion in the original form of the GTR. So we need to modified it or come up with new alternative theories. On the other hand, it has been suggested that, if gravity itself, is modified appropriately it could explain this late-time mysterious acceleration (Nojiri and Odintsov 2004), (Capozziello et al. 2003), (Nojiri and Odintsov 2007), (Borowiec 2007), (Brevik and Hurtado 2007), (Sotiriou 2006), (Bertolami et al. 2007), (Li and Barrow 2007). In general, there are two fundamental approaches proposed to explain the reason for this accelerated expansion behaviour (Allemandi et al. 2005): one of them is introducing dark energy with

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negative pressure as the most influential component of the universe and the other one is making use of modified gravity theories. In this context, many studies carried out on the modification of GTR (Boehmer and Tamanini 2013), (Boehmer et al. 2014), (Barrow and Ottewill 1983), (Capozziello and Laurentis 2011), (Dobado and Maroto 1995), (Dvali et al. 2000), (Haghani et al. 2013), (Harko and Lobo 2014), (Harko et al. 2011), (Harko et al. 2011), (Lobo 2008), (Nojiri and Odintsov 2011), (Odintsov and G´omez 2013), (Starobinsky 1980), (Sotiriou and Faraoni 2010). On the other hand, some scientists suggest that dark energy components such as the cosmological constant Λ or quintessence may be causing our universe to accelerate (Padamanabhan 2003), (Parker et al. 2003). it is significant to emphasize here that the idea of using the well-known cosmological constant dark energy model grapples with some significant issues such as the fine-tuning and the cosmic coincidence problems (Weinberg 1972). In addition, there is a fact that the dark energy particle has not been observed yet. As a result, the modifications of the GTR have gained noteworthy momentum in literature (Meng and Wang 2003), (Freese and Lewis 2002), (Salti et al. 2018), (Salti et al. 2016), (Abedi and Salti 2015).

In the GTR, the Einstein-Hilbert action is generally written as given below

$$S_{EH} = \frac{1}{2k^2}\int d^4x\sqrt{-g}\,R. \qquad (1)$$

Here, we have $k^2 = 8\pi G$ and assume natural units $c = \hbar = 1$ for the sake of simplicity. It is well known that the $f(R)$ −theories of gravity extends the Einstein-Hilbert action in the GTR (Sotiriou and Faraoni 2010) to the following one

$$S_{EH} = \frac{1}{2k^2}\int d^4x\sqrt{-g}\,f(R). \qquad (2)$$

Note that, here, (R) is an arbitrary function of the curvature scalar R.

On the other hand, within the framework of the modified GTR, scalar fields provide possible dark energy models, which can describe the late time acceleration. In addition, scalar fields play a considerable role in many fields of physics, such as gravity and cosmology. However, scalar density fields, although very useful in theoretical physics, has not been adequately evaluated. A recent paper shows that using the weight 1 scalar density solution in the FRW metric affects both the Klein-Gordon equation and the Friedmann equation compared to the scalar field (Pirinccioglu and Sert 2012). One can also sees studies containing scalar density fields with different context (Demir and Pak 2009), (Pirinccioglu 2012), (Pirinccioglu 2019).

In this paper, we investigate the recent acceleration phase of the universe using the scalar density field and the scalar field within the framework of modified gravity theory and compare both cases. In addition, we present an original case of a scalar density field to understand the properties of the recent acceleration of the cosmos. This aim may help us to understand the subtleties behind the formation of cosmological structure.

According to cosmological observations, the universe can be described as a homogeneous and isotropic manifold on galactic scales (Friedmann 1922), (Friedmann 1924). In other words, according to the Friedmann approach, which is also known as the FRW metric, the cosmos is isotropic and homogeneous at galactic scales. Thus, the corresponding spacetime fabric has the same behaviour in all directions and everywhere, and the metric is invariant throughout the selected spacetime structure. Therefore in accordance with our purpose throughout this study we will use the Friedmann approach. The four-dimensional FRW spacetime is represented generally by the subsequent line-element

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \qquad (3)$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\varphi^2$. And the parameter k is introduced to explain the spatial curvature of the metric and takes values $k = -1, 0, 1$ in case of the closed, spatially flat, an open universes, respectively. Also, the scale factor is $a(t)$ and r is a radial coordinate on the spatial hypersurfaces.

2. *Equation of Motion in the Scalar Field*

We consider a gravitational action with ϕ scalar field can be given in an action integral as

$$S_{[\phi]} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_{pl}^2 R - f(\phi) + \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}m^2\phi^2\right]. \qquad (4)$$

Here, we have $M_{pl}^2 = \frac{1}{8\pi G}$, where M_{pl} denotes the Planck mass. Taking the variation of action (4) with respect to ϕ, one can get the scalar field equation

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi - m^2\phi + f'(\phi) = 0, \qquad (5)$$

where $f'(\phi)$ is defined as $f'(\phi)=\frac{df}{d\phi}$. In the framework of the FRW metric, considering the homogeneous universe, equation (5) becomes

$$\ddot{\phi}+3H\dot{\phi}-m^2\phi+f'(\phi)=0. \quad (6)$$

Varying equation (4) with respect to metric tensor, $g_{\mu\nu}$, one can get the result

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{1}{2M_{pl}^2}\Big[g_{\mu\nu}f(\phi)-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi+\frac{1}{2}g_{\mu\nu}m^2\phi^2+\nabla_\mu\phi\nabla_\nu\phi\Big]. \quad (7)$$

Here, $G_{\mu\nu}$ is called Einstein Tensor. The right hand side of this equation, which corresponds to the energy-momentum tensor, can be written as

$$T_{\mu\nu}=\frac{1}{2M_{pl}^2}\Big[g_{\mu\nu}f(\phi)-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi+\frac{1}{2}g_{\mu\nu}m^2\phi^2+\nabla_\mu\phi\nabla_\nu\phi\Big], \quad (8)$$

where $T_{\mu\nu}$ is the energy-momentum tensor. If both sides of this equation are contracted by $g^{\mu\nu}$, in terms of curvature scalar the equation (8) becomes

$$R=-\frac{1}{2M_{pl}^2}\left[4f(\phi)-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+2m^2\phi^2\right]. \quad (9)$$

Here, R is the Ricci scalar. In the framework of the FRW metric, assuming a homogeneous and isotropic universe with zero spatial curvature equation (9) becomes

$$\dot{H}+2H^2=\frac{1}{12M_{pl}^2}\left[4f(\phi)-\dot{\phi}^2+2m^2\phi^2\right], \quad (10)$$

where the Hubble expansion rate is defined by $H\equiv\frac{\dot{a}}{a}$ and the dot represents the time derivative.

3. Preliminaries: Scalar Density

In this part, we introduce briefly how we can use a scalar density in an EoM. In the subsequent step, within the framework of modified gravity studies, a new EoM, which include a scalar density field with a weight of 2, is proposed.

It is generally known that a scalar quantity is invariant under all coordinate transformations:

$$\phi'(x')=\phi(x). \quad (11)$$

On the other hand, the four-dimensional volume element generally changes as

$$d^4x=\left|\frac{\partial x^\alpha}{\partial x'^\beta}\right|d^4x' \quad (12)$$

under coordinate transformations. Next, the above relation transforms with the Jacobian, determinant of the transformation coefficients, definition as

$$d^4x=Jd^4x'. \quad (13)$$

For the inverse transformation case, we have

$$d^4x'=J^{-1}d^4x, \quad (14)$$

where d^4x has a scalar density of weight -1. It will be useful remind here that the transformation rule can be applied for the metric tensor in a similar way. In this context, the determinant g of the metric tensor can be presented as

$$g(x)=det g_{\mu\nu}(x). \quad (15)$$

Also, the above equation under the general tensor transformation rules, the metric tensor transforms as

$$g'_{\mu\nu}(x')=\frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}g_{\alpha\beta}(x). \quad (16)$$

Taking the determinant of both sides of equation (16), we reach at the following conclusions

$$det g'_{\mu\nu}(x')=det\left(\frac{\partial x^\alpha}{\partial x'^\mu}\right)det\left(\frac{\partial x^\beta}{\partial x'^\nu}\right)det g_{\alpha\beta}(x) \quad (17)$$

$$g'(x')=J^2g(x). \quad (18)$$

Here, the term J^2 causes $g(x)$ to be a scalar density. Note that, here, the factor $g(x)$ behaves like a scalar density of weight 2. Any tensor density of weight-W can be expressed as the multiplication of an ordinary tensor and the factor $g^{\frac{-W}{2}}$ (Weinberg 1972). In our study, both Ω and g are considered as scalar densities with weight of 2, thence

$$\phi=\frac{\Omega}{g} \quad (19)$$

becomes a normal scalar.

4. Scalar Density Field in the FRW Framework

In this section, we start with the Einstein-Hilbert type action integral. Therefore, we present the EoM that must be solved in order to find the general behaviour of cosmological scalar density fields. An action integral from equations (4) and (19) for scalar density fields can be given as

$$S_{[\frac{\Omega}{g}]} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_{pl}^2R - f\left(\frac{\Omega}{g}\right) + \frac{1}{2}g^{\mu\nu}\nabla_\mu(\frac{\Omega}{g})\nabla_\nu(\frac{\Omega}{g}) - \frac{1}{2}m^2(\frac{\Omega}{g})^2\right]. \quad (20)$$

Here, after varying equation (20) with respect to Ω, one can get the result

$$f'\left(\frac{\Omega}{g}\right) + g^{\mu\nu}\frac{1}{g^2}\nabla_\mu\nabla_\nu\Omega - m^2\Omega\frac{1}{g^2} = 0, \quad (21)$$

where $f'\left(\frac{\Omega}{g}\right) = \frac{df}{d\Omega}$. The above equation can be rewritten in the following form

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\Omega - m^2\Omega + g^2f'\left(\frac{\Omega}{g}\right) = 0. \quad (22)$$

So, making use of this conclusion, it is possible to reach at the Friedmann equation by assuming the homogeneity and isotropy cases. In doing so, note that the general definition for the covariant derivative of a scalar density field of weight- W is written as

$$\nabla_\mu\Omega = \partial_\mu\Omega - W\Gamma^\alpha_{\alpha\mu}\,\Omega. \quad (23)$$

As a result, substituting equation (3) in equation (20) yields the following result

$$\ddot{\tilde{\Omega}} + 6\dot{H}\tilde{\Omega} - 9H\dot{\tilde{\Omega}} + 18H^2\tilde{\Omega} - m^2\tilde{\Omega} + g^2f'\left(\frac{\Omega}{g}\right) = 0. \quad (24)$$

Here, we used the transformation $\Omega = \tilde{\Omega}r^4sin^2\theta$, assumed that $g^2f'\left(\frac{\Omega}{g}\right)$ is a spatial constant and focused on the case $k = 0$ describing flat spacetime metric. Remember that the Hubble parameter $H \equiv \frac{\dot{a}}{a}$ is connected with the expansion rate, where the dot represents the time derivative. Now, we are in position to focus on the principle of least action

$$\delta S_{[\frac{\Omega}{g}]} = 0 \quad (25)$$

for our investigation. Hence, variation of the action (20) with respect to the contravariant metric tensor $g^{\mu\nu}$ gives

$$\int d^4x\delta\left[\sqrt{-g}\left(-\frac{1}{2}M_{pl}^2R - f\left(\frac{\Omega}{g}\right) + \frac{1}{2}g^{\mu\nu}\nabla_\mu(\frac{\Omega}{g})\nabla_\nu(\frac{\Omega}{g}) - \frac{1}{2}m^2(\frac{\Omega}{g})^2\right)\right] = 0 \quad (26)$$

and

$$\int d^4x\delta\sqrt{-g}\,[-\frac{1}{2}M_{pl}^2R - f\left(\frac{\Omega}{g}\right) + \frac{1}{2}g^{\mu\nu}\nabla_\mu(\frac{\Omega}{g})\nabla_\nu(\frac{\Omega}{g}) - \frac{1}{2}m^2\left(\frac{\Omega}{g}\right)^2] + \int d^4x\sqrt{-g}[-\frac{1}{2}M_{pl}^2\delta R - \delta f\left(\frac{\Omega}{g}\right) + \frac{1}{2}\delta g^{\mu\nu}\nabla_\mu(\frac{\Omega}{g})\nabla_\nu(\frac{\Omega}{g}) + \frac{1}{2}\delta(g^{-2})\nabla_\mu\Omega\nabla_\nu\Omega - \frac{1}{2}m^2\delta\left(\frac{\Omega}{g}\right)^2] = 0. \quad (27)$$

Using equation (27), the corresponding energy-momentum tensor can be written as

$$T_{\mu\nu} = \frac{1}{M_{pl}^2}[\frac{3}{2}g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha(\frac{\Omega}{g})\nabla_\beta(\frac{\Omega}{g}) + \nabla_\mu\left(\frac{\Omega}{g}\right)\nabla_\nu(\frac{\Omega}{g}) - \frac{3}{2}g_{\mu\nu}m^2\left(\frac{\Omega}{g}\right)^2 + g_{\mu\nu}f(\frac{\Omega}{g}) - 2g_{\mu\nu}(\frac{\Omega}{g})f'(\frac{\Omega}{g})]. \quad (28)$$

Contracting this equation with $g^{\mu\nu}$, one can reach at the relation

$$R = -\frac{1}{M_{pl}^2}\left[\frac{7}{g^2}g^{\mu\nu}\nabla_\mu\Omega\nabla_\nu\Omega - \frac{6}{g^2}m^2\Omega^2 + 4f\left(\frac{\Omega}{g}\right) - \frac{8\Omega}{g}f'\left(\frac{\Omega}{g}\right)\right], \quad (29)$$

where R is the Ricci scalar and expressed now in terms of scalar density. With this statement, we suppose that the universe is isotropic and homogeneous on large scales to arrive at the modified Friedmann equations. Making use of the definition $\nabla_\mu\Omega = \partial_\mu\Omega - W\Gamma^\alpha_{\alpha\mu}\,\Omega$, the last equation can be rewritten as follows;

$$R = -\frac{1}{M_{pl}^2}\left[\frac{1}{a^{12}}\left(7\dot{\tilde{\Omega}}^2 - 84H\tilde{\Omega}\dot{\tilde{\Omega}} + 252H^2\tilde{\Omega}^2 - 6m^2\tilde{\Omega}^2\right) + 4f\left(\frac{\Omega}{g}\right) + \frac{8\tilde{\Omega}}{a^6}f'\left(\frac{\Omega}{g}\right)\right]. \quad (30)$$

Finally, we can express this equation in terms of the Hubble parameter as

$$\dot{H} + 2H^2 = \frac{1}{6M_{pl}^2}[\frac{1}{a^{12}}(7\dot{\tilde{\Omega}}^2 - 84H\tilde{\Omega}\dot{\tilde{\Omega}} + 252H^2\tilde{\Omega}^2 - 6m^2\tilde{\Omega}^2) + 4f(\frac{\Omega}{g}) + \frac{8\tilde{\Omega}}{a^6}f'(\frac{\Omega}{g})]. \quad (31)$$

This equation shows that both of the scalar density and mass density coefficients are proportional to a^{-12}, which means both of these densities affect the accelerating expansion of the universe in a similar way. In addition, it seems that the derivative of the function has a significant influence here, and this effect decreases with a^{-6}.

5. Discussion

According to the results we obtained in this study, when the Friedmann equations for scalar and scalar density fields are compared, it is seen that the scalar density variables decrease rapidly with time according to the scale factor, $a(t)$. In addition, considering the scalar field equations (6), (10) and the scalar density equations (24), (31), it is understood that the scalar density field equations contain more terms. Moreover, the derivative of the function has a significant effect on equations (6), (24) and (31), but, interestingly it has no any effect on the scalar field equation (10). Also, in scalar density, the mass density is affected by the scale factor. However, it is not affected in the scalar field case.

Astrophysical observations have shown that the accelerating expansion of the universe has recently entered a new exotic phase and unfortunately the GTR cannot fully explain the reason of this behaviour. This situation has led scientists to introduce new approaches such as the dark energy models and modified gravity perspectives. In this study, our main aim was to examine the dynamical behaviour of the expansion phase of our universe from a new and unique perspective by looking at the evolutionary processes of the cosmos. Therefore, the field equations of the GTR have been modified by using a scalar density field with weight of 2. Our results may inspire some phenomenological researches for future studies, such as the problem of the existence of neutron stars, gravitational wave astronomy, gravastars and dark energy stars, which are considered also as an alternative interpretation of black holes. Moreover, it is possible to reach additional original cosmological conclusions by discussion thermodynamics laws in our framework.

Lastly, gravitational wave astronomy, which began in recent years with the famous LIGO (the Laser Interferometer Gravitational-Wave Observatory) detections, may be the basis for testing the extended GTR with the results we have obtained in this paper. A recently published paper claims that advanced projects to detect gravitational waves, if their sensitivity is increased, could allow gravitational wave astronomy to perform a precise test for both the GTR and extended theories of gravity (Corda 2009).

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