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Iterative Computerized Method for Designing Achromatic Optical Systems with Shelled Ball Lens

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Highlights

- An iterative computerized mathematical model for an achromatic-symmetry optical system was proposed.
- A high Sterhl ratio (over 0.96) was achieved for the optical concentric systems.
- The proposed approach was tested on a concentric optical system without aspherical surfaces.

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Abstract

Many optical studies concentrate on designing achromatic lenses to limit the effects of chromatic and spherical aberration. Most of these studies concentrate on manipulating the profile of surfaces with complex aspheric surfaces. In this study another approach is producing, an optical concentric symmetry (spherical ball – spherical shell) system was optically designed and evaluated. The computational method in MATLAB, an iterative mathematical model, and Fermat's and symmetry principles were used to determine the optical design constructive parameters. The validity of the iterative computerized method to design an achromatic optical system was evaluated for three optical designs. These three designs were tested monochromatically and with Fraunhofer wavelengths for infinite conjugate. In addition, the ZEMAX was used to evaluate the optical designs for aberrations and spherochromatism in the optical systems. The spherochromatism analysis was done for three visible lines (Fraunhofer wavelengths) that fall inside the Airy disk and prove the diffracted limit by spot diagram analysis. This radially un-gradient index of symmetry concentric polymeric (three optical designs) was tested in a balanced focal ratio that appeared Sterhl ratio greater than 0.96.

1. INTRODUCTION

Obtaining achromatic singlet lens [1-3] dispersion should be avoided since it creates chromatic aberration. Optically, natural materials' dispersion is determined by their electronic and molecular energy levels, which have limited ability to control. As a result, dispersion management is complicated and must trade off other aberrations [4]. Many optical treatments are available to reduce the dispersion of an optical system, with one of them based on increasing refractive efficiency, the Abbe value [5-7], or by employing a unique aspheric shape to create a spherochromatic singlet lens that minimizes chromatic focus shift and is constructed from a single substrate [8] on latterly time, implementation of an achromatic singlet made of a single optical material, Zeonex E48R, was done as approaches to chromatic singlet lens [9]. For crossing chromatic singlet lens difficulties, a hybrid approach was developed, with the corrector being a deep meniscus lens with an aspheric curvature on the external surface [10].

The aplanatic of the optical system can be evaluated depending on spherochromatism. The spherochromatism is the shift in spherical aberration caused by changing the wavelength of light, which is represented as a ray intercept curve at various wavelengths [11-13]. Most optical aplanatic system treatments depend on symmetrical and Fermat's of equal optical path principles .In a fully symmetrical optical system, distortion, lateral color, and coma aberrations are eliminated [11, 14]. Geometrically, the symmetry shape of the ball lens is optically described as a double-convex-type thick lens, which is the most natural example of an imperfect lens, caused by caustic effect [15-17].

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The ball lens with spherical surfaces the condition of equalized marginal and paraxial optical paths in case of non-paraxial optically is impossible to achieve, occurring caustic effect. In other words, this symmetry shape, a ball lens, cannot function as an optical element (convergence lens). In this study, the breaking of this impossibility is done by approaching of added optimal shell thickness to keep the equalized condition with a symmetry shape in the non-paraxial optical case Figure 1. The novelty of this study is to develop a mathematical method to convert an aberrated symmetry-shaped lens (ball lens) to an achromatic optical system by adding estimated optimal shell thickness as added optical material.

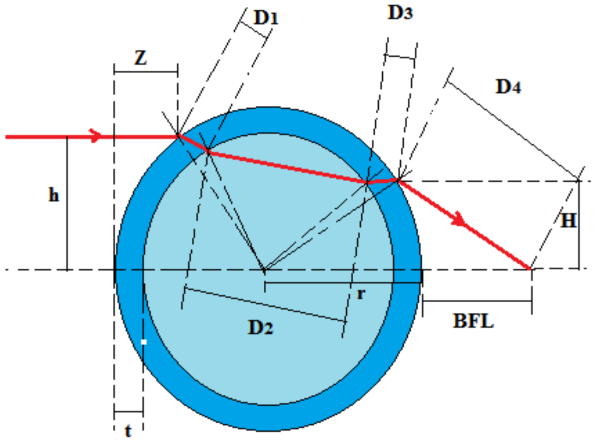

Figure 1. *Optical path through the upper half of one-shell ball lens [18]*

2. OPTIMIZING ADD OPTIMAL SHELL THICKNESS

The computerized steps of designing and evaluating the optical systems are:

- Develop iterative computerized equation by MATLAB program to find optical system constructive parameters (optimal shell thickness for single shell case, and double shells).
- ZEMAX test of optimal constructive parameters of three designs. Achromatic system from single and double shells in range of Abbe wavelengths.
- Effect gradient refractive indices on achromaticity of three optical designs.

In the case of monochromatic and infinite conjugate lenses, Fermat's principle helps in realizing the condition of free spherical aberration. As shown in Figure 2 and illustrated in Equation (1), equivalent paraxial and lateral optical path lengths are necessary for a lens free of spherical aberration [19]

$$Nz + n_1 D_1 + N D_2 = n_1 d_1 + N d_2 \tag{1}$$

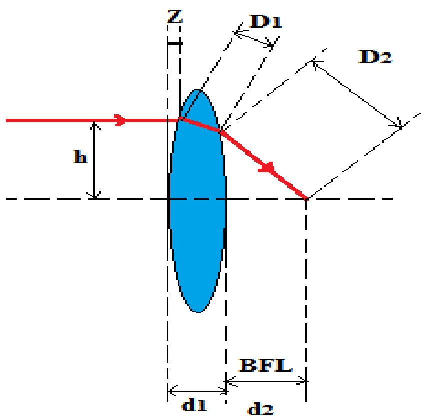

Figure 2. *Optical path parameters through the upper half of a homogeneous convex lens [14]*

The total optical path was determined by the summation of the optical path in every optical material. the total optical path in case of axial of the optical system (POP), Equation (2) [11]. The marginal optical path of the system (MOP) was given by Equation (3) [11]. The absolute value of deference between axial optical path and marginal optical path is optical path difference (OPD), the (OPD) is given by Equation (4) [11, 12]

$$POP = \sum n_i d_i, \tag{2}$$

$$MOP = \sum n_i D_i\ , \tag{3}$$

$$OPD = |POP - MOP|\ \ . \tag{4}$$

where ($n_i\, d_i$) is the axial optical path, ($n_i\, D_i$) is marginal optical path. Optimizing add optimal shell thickness is done by extended Equation (1) of equal optical path principles, Equation (5) [18]

$$t_{optimal} = \frac{z(h) + n_1 D_1(h,t) + n_2 D_2(h,t) + n_1 D_3(h,t) + D_4(h,t) - n_2 r - d_4(t) - OPD}{2(n_1 - n_2)}. \tag{5}$$

In the same way for focused (two-shelled) the paraxial optical path equals the marginal optical path of the lens, Equation (1), can be written in Equation (6). In the case of two- shells with known one shell thickness the added optimal other shell thickness (outer or inner) can evaluated in the same way when OPD minimum summations are reached at marginal ray height (h) [18] Based on that, the Equation (6) [18] was valid to determine this optimal added shell thickness. The optical paths D_2, D_3, D_4, D_5, and D_6 as function of (h, t_1, t_2). In the other optical paths, D_1 is a function (h, t_1), and z (sagittal optical path) is a function only of ray height (h).

$$t_{\ optimal} = \frac{z + n_1 D_1 + n_2 D_2 + n_3 D_3 + n_4 D_4 + n_5 D_5 + D_6 - 2n_2 r - d_6(t_1, t_2) - OPD}{2(n_1 - n_2)}\ \ . \tag{6}$$

2.1. Computerize Added Optimal Shell Thicknesses

The determination of added optimal shell thicknesses of shelled ball lenses requires determining the optical path, the determining optical path that light rays take as they pass through an optical system necessitates a significant numerical computation, which is mostly used in optical system analysis. By using Snell's law at each surface, a ray can be traced. Ray-tracing formulas have been developed in a wide range of ways. The electronic computer is the most commonly utilized one for ray tracing [11]. An iterative mathematical approach for reducing aberration was utilized to obtain the constructive parameters of shelled ball lenses, which can be easily applied to ball lenses [18]. Based on the technique of trigonometric ray tracing [20] to achieve an optical path and Fermat's principle of an equal optical path [21, 22], A symmetry-concentric system's ideal shell thickness was determined using an iterative mathematical process.

The flowchart for determining added optimal shell thickness is illustrated in Figure 3. The computerized process of determining the added optimal shell thicknesses of symmetry shape enables us to determine the other constructive parameters of optical designs. This process was done by MATLAB program to determine three optical design parameters. The constructive parameters of one shell ball lens (design 1) are shown in Table 1 and the other constructive parameters of two shells (designs 2, and 3) were taken from our previous study on testing in the monochrome case [18].

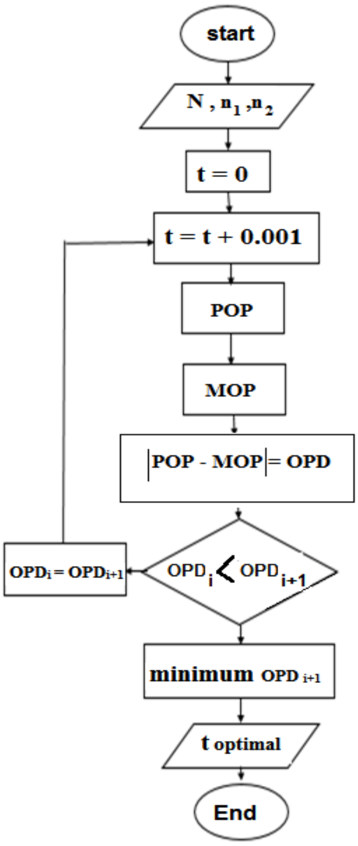

Figure 3. *Flowchart of determining added optimal shell thickness*

Table 1. *The constructive parameters for (one-shell) design 1*

No. surfaces	Curvature radius (mm)	Shell thickness (mm)	Material of shelled lens	Refractive index
1	infinity	-	Air	1
2	25	10.89699	PMMA	1.491756
3	14.10301	28.20602	MEXFLON	1.333044
4	-14.10301	10.89699	PMMA	1.491756
5	-25	-	Air	1

2.2. ZEMAX Simulated Computational Result and Discussion

Significant results were revealed from the three optical designs testing. The results for the three cases were simulated in the ZEMAX (optical design program). Through the lens layout, Figure 4, and longitudinal aberration, Figure 5, this design program displays a high level of acceptability.

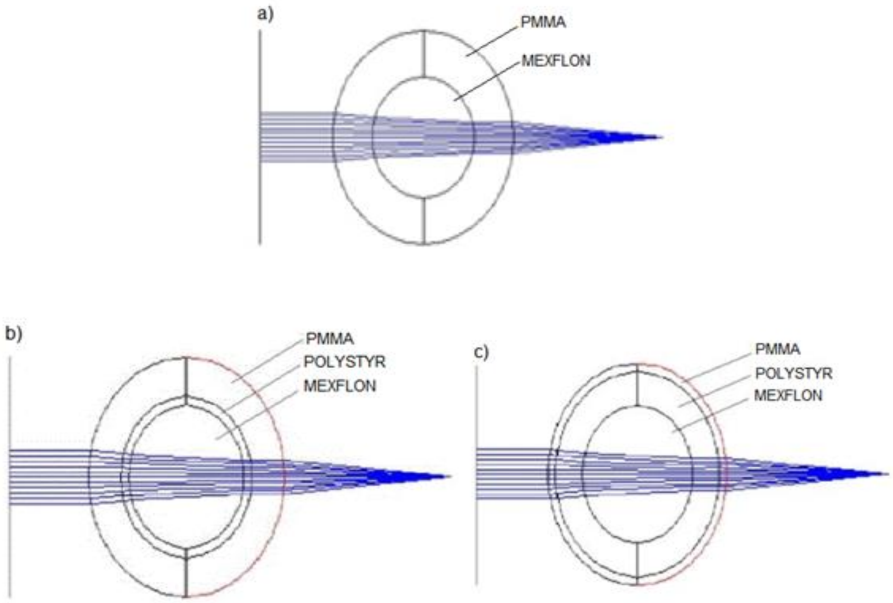

Figure 4. *2-D layout of three optical systems designs: (a) design1, (b) design 2[18], and (c) design 3[18]*

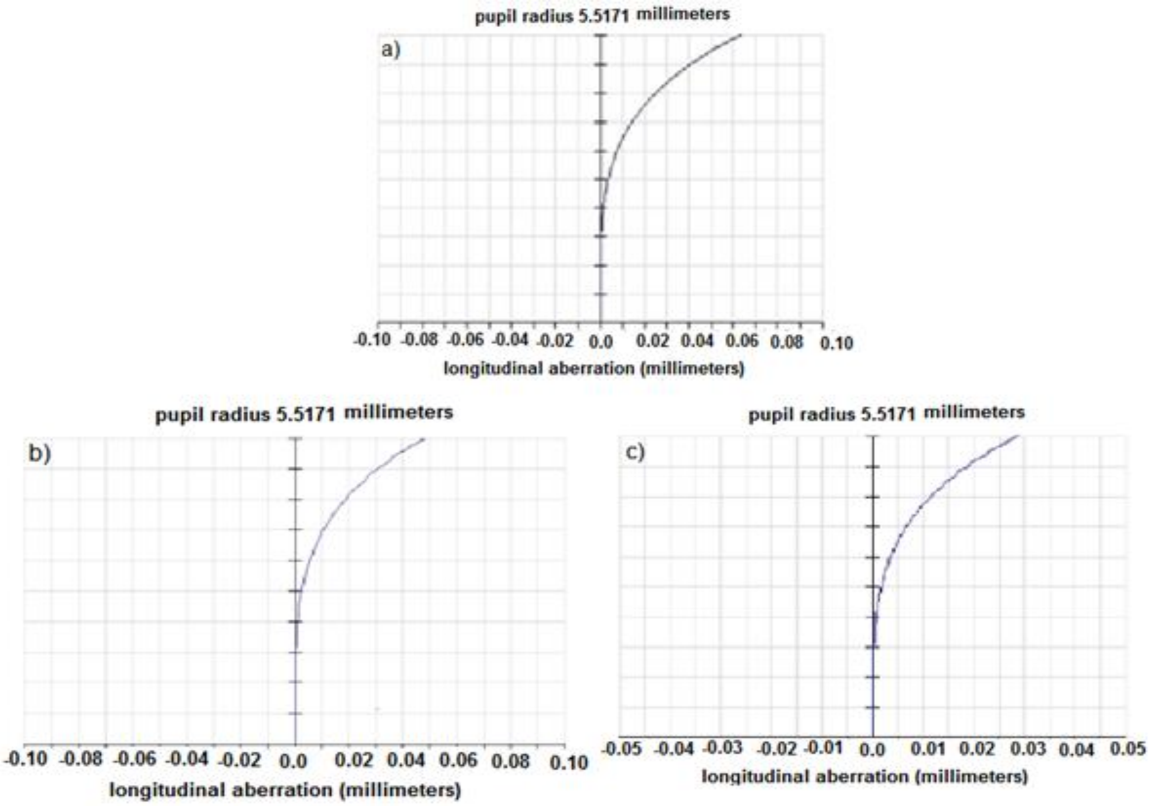

Figure 5. *The longitudinal aberration ZEMAX test of three optical systems designs with constant outer radius: (a) design 1, (b) design 2[18], and (c) design 3[18]*

2.3. Spherochromatic Testing

In optical systems generally, chromatic aberration is eliminated by cemented high and low-dispersion optical materials [11] which is indicated by the Abbe number Equation (7) [8]. In this work, just lower polymer Abbe number materials are used PMMA, POLYSTYR, and MEXFLON to achieve an achromatic optical system with manipulation with a degree of spherochromatic system those testing by Airy disk diffraction. In three design cases, the spherical aberration was monochromatically 0.5876 μm eliminated by determining the added optimal thickness shell while the chromatic aberration was controlled by changing both optical material un-gradient refractive indices, and spherical surface radius curvatures. In other words, the achromatic system was achieved by adjusting hybrid shell thickness /refractive / radius curvatures

$$V_D = \frac{n_D - 1}{n_F - n_C} \tag{7}$$

where n_C, n_D, and n_F are the material's refractive indices for specific lines of the Fraunhofer C, D, and F spectral lines. In the shelled ball lens spherochromatic was achieved for the three cases in these specific lines λ_1 = 0.4861 μm, λ_2 = 0.5876 μm, and λ_3 = 0.6563 μm (Abbe wavelengths) in visible range depending on the Airy radius as spherochromatic indication by spot diagram as shown in Figure 6.

The Total Primary Spherical Aberration Coefficients (T.P.S.A.C.) and Total Transverse Spherical Aberration Coefficients (T.T.S.A.C.) are listed in Table 2 where the other total aberration coefficients are zero This can attributed to satisfying the condition of equalize marginal and paraxial optical paths. This means approximately free spherical aberration. In addition, the shelled ball lens is a symmetrical optical system, so three forms of aberration are eliminated. Lateral color, distortion, and coma are all exactly zero in a fully symmetric optical system [11] to satisfy equalized optical paths and symmetry conditions the zero conics of the shell surfaces prevent any off-axis aberration in case of the infinite conjugate, no specified center curvature surface, this gives potential for using the polymeric shelled lens as aplanatic optical system of balance focal ratio monochromatically.

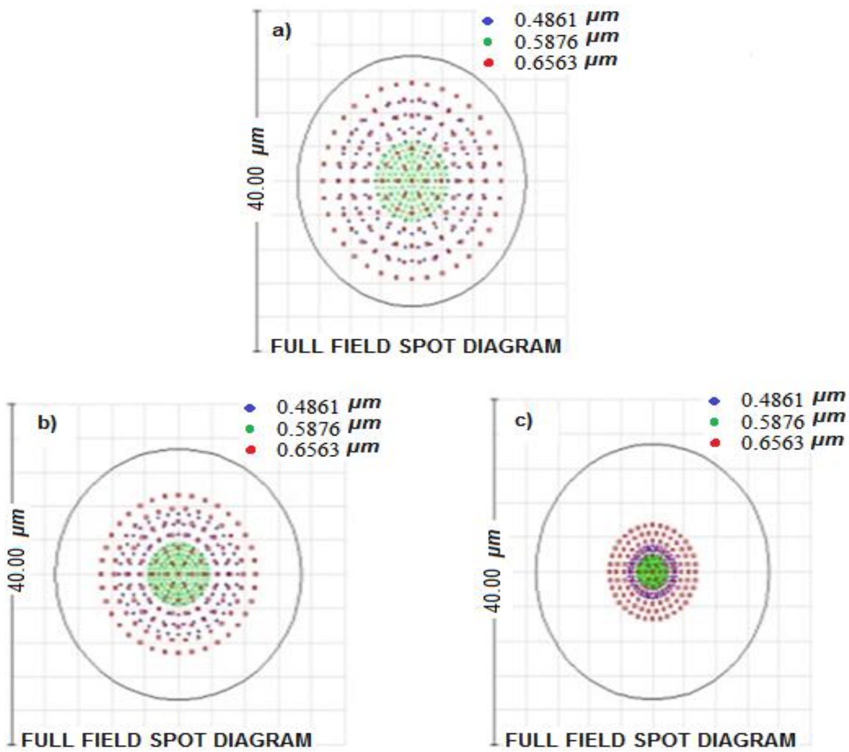

Figure 6. *Spherochromatic ZEMAX testing at a slow focal ratio at three Abbe wavelengths inside Airy disks represent by the black circles*: *(a) design 1, (b) design 2, and (c) design 3*

It is clear from Figure 5, that the effect hybrid of the added shell thickness /refractive indices/radius curvatures caused quite equal the Abbe wavelengths optical paths from paraxial to lateral which was noticed in three cases on residual aberration types (spherical, chromatic). The other aberration types are omitted by the symmetry principle of the shelled ball lens approaching to achromatic system without asphericity surfaces (common sphere shape). The characteristics of achromatic, and symmetry shape give potential to use this optical system in different applications like stereo-scanning [23], wide field of view imaging [24], and antenna [25].

***Table 2**. Summarized result of the three optical designs*

	Size of the image (μm)^2	**The Sterhl ratio**	**The radius of Airy (μm)**	**T.P.S.A.C.**	**T.T.S.A.C.**
design 1	37.37	0.965	4.030	0.000000	0.000002
design 2	37.89	0.971	4.086	0.000001	0.000004
design 3	39.63	0.984	4.273	0.000021	0.000133

3. CONCLUSIONS

This study concluded that the symmetry shape (shelled ball lens) is considered an achromatic optical system when parameters match the three parameters: the added shell thickness, refractive indices, and radius curvatures of spherical surfaces. These three optical parameters essentially affect Total Primary Spherical Aberration Coefficients (T.P.S.A.C.) and Total Transverse Spherical Aberration Coefficients (T.T.S.A.C.) and then the achromaticity of the system. In addition, the shelled ball lens is a symmetrical and concentric optical system. So, other forms of aberration are eliminated. Lateral color, distortion, and coma are all exactly zero.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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In this study, the trial version of the MATLAB program was used.

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