Sigma J Eng Nat Sci, Vol. 42, No. 1, pp. 121–129, February, 2024

Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2023.00128

Research Article

Finite element contact analysis of the hollow cylindrical roller bearings

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ARTICLE INFO

Article history
Received: 19 October 2023
Revised: 23 November 2023
Accepted: 07 December 2023

Keywords:
Hollow Cylindrical Roller Bearing (HCRB); Penetration; Contact Analysis; Finite Element Method

ABSTRACT

The Hollow Cylindrical Roller Bearings (HCRBs) have some important advantages over solid cylindrical roller bearings such as low weight, good lubrication capability and high bearing stiffness. Additionally, it's also known that maximum contact stress value between the roller and race is less in hollow cylindrical roller bearings compared to solid cylindrical roller bearings. In this paper, the effect of roller's hollow size in Hollow Cylindrical Roller Bearing under a radial load on the maximum contact stress value and contact stress distribution is studied by using finite element method. The maximum tangential tensile stress occurring on the inner surface of the hollow roller is also investigated. Moreover, the effect of the penetration amount, which is a critical parameter for a nonlinear contact problem such as roller-race contact, on the contact stress is taken into consideration, as well. It's seen from the results that tangential tensile stress on the inner surface of the hollow roller has an effect on the value of optimum hollow size. Besides, it is deduced from results that the penetration amount between the contacting surfaces have considerable influence on the contact stresses and it should be taken into consideration when analyzing the roller-race contact problem by using finite element method.

Cite this article as: Bayrak R, Sağırlı A. Finite element contact analysis of the hollow cylindrical roller bearings. Sigma J Eng Nat Sci 2024;42(1):121−129.

INTRODUCTION

The solid cylindrical roller bearings (SCRB) are suitable for high speeds and high radial loads since they have relatively low friction torque and their contact area under the load is wider than that of the ball bearings. The increasing demand for high speeds has led to a reduction in the mass of the rollers in order to reduce the neg-ative impact of the centrifugal force on the contact stresses. The Hollow Cylindrical Roller Bearings (HCRB) are more advantageous in comparison to solid cylindrical roller bearings (SCRB) in terms of having lower contact stresses and more suitable for high speeds. Since the contact stresses are lower under the same radial load, fatigue life of HCRBs is higher than that of SCRBs.

In the literature, there are detailed studies supporting the above-mentioned advantages of HCRBs. Darji and Vakharia [1] investigated experimentally the causes of damage of a hollow cylindrical roller which is under a radial load and is positioned between two flat surfaces. They observed that the cause of damage differed with respect to the hollow size. They stated that the crack starts on the inner surface of the roller having a hollow larger than optimum size, and therefore the damage is caused by the bending of the roller.

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This paper was recommended for publication in revised form by Regional Editor Ahmet Selim Dalkilic

As for a hollow which is optimum size or smaller than optimum size, they emphasized that the damage is caused by the contact stress not by bending effect. Bamberger et al. [2] studied experimentally fatigue damage in hollow cylindrical rollers for two different material (AISI 52100 and AISI M-50). They investigated stresses on the inner surface of the hollow rollers at the time of damage for different hollow sizes and different radial loads. They indicated that the damage in the hollow cylindrical roller (HCR) which is produced from the AISI 52100 bearing steel occurred when the stress on the inner surface of the hole reached 490 MPa. For the AISI M-50 hollow roller, they stated that the damage occurred when the stress on the inner surface of the hole reached 338 MPa. For the hollow cylindrical roller bearing of the cone bit which is generally used in the drilling industry, Han et al. [3] examined the effects of hollow size on von Misses equivalent stresses and contact stresses by using finite element method. They stated that the optimum hollow size is 55% under a certain drilling pressure in terms of contact stresses and von Mises equivalent stresses. Darji and Vakharia [4] calculated the contact stress, von Mises stress, bending stress and deformation using the Finite Element model for different radial loading, different hollow size, and different roller geometry inputs. They also presented a generalized graphical solution that gives the opti-mum hollow size for any material and applied load, regardless of geometry. Abu Jadayil [5] examined the effect of hollow size on von Mises stresses in the contact area in cylindrical hollow rollers under radial and tangential loading using the finite element method and the effect on life using the Ioannides-Harris [6] fatigue life theory.

In addition to the above-mentioned advantages of the HCRBs, there are several studies in the literature in-dicating that the stiffness of HCRBs with preloaded and optimum hollow size is greater than the stiffness of SCRBs with a certain diameter clearance or without preloaded. Darji and Vakharia [7] studied on the optimum hollow size for HCRBs by using finite element method. They emphasized that the reduction in stress concen-trations due to the greater radial displacement is an important advantage of HCRBs. In addition, the authors stated that bearing stiffness increased until a certain hollow size and therefore the load bearing capacity and fatigue life also increased. They emphasized that the optimum value of the hollow size should be 67% by considering the stiffness criterion. Bowen and Bhateja [8] investigated the advantages of the preloaded HCRBs compared to SCRB. They stated that a preload should be applied to the HCRB in order to be more advantageous than SCRB and any of the rollers shouldn't lose its preload under an external load. They stated that the pre-loaded HCRBs had higher rotational accuracy and fatigue life. They also concluded that the optimum hollow ratio should be between 60% and 70%, considering the load carrying capacity and the stresses on the inner surface of the hollow roller. Liu Y et al. [9] investigated the effect of the hollow size of preloaded HCRBs on bearing stiffness by using finite element method. They considered the HCRB as a plane strain problem and investigated the effect of 50% and 80% hollow ratios on bearing stiffness. They stated that hollow size of the roller is critical in terms of bearing stiffness. It is reported by the authors that when the hollow size is too big, the stiffness of the HCRB is too small, which has no advantage over the SCRB. In addition, the authors em-phasized that the stiffness of the HCRB increases with the increase in interference magnitude. In addition to the above studies, various studies were carried out to reduce the bending stress on the inner surface of the HCRBs and to reduce stress concentrations near the roller corners at the contact area. Qishui et al. [10] modeled the inlet and the outlet of the hole conically and called it deep hole hollow roller bearing. Inside of the cavity was filled with polytetrafluoroethylene (PTFE) material, which they called elastic composite hollow roller bearing. They compared bending stresses on the roller for hollow cylindrical roller bearing, deep hole hollow cylindrical roller bearing, and elastic composite roller bearing by using finite element method. They stated that the bending stresses on the roller for elastic cylindrical roller bearing were lower than the bending stresses on rollers for the other two types bearing. For filling materials other than PTFE, they observed that the bending stress values decreased as the modulus of elasticity of the material increased. They also stated that bending stresses increase as the degree of filling increases for elastic composite roller bearing. Qishui et al. [11] improved their previous studies by optimizing the design variable of the deep hole structure in order to reduce stress concentration in contact area. Zhu et al. [12] combined PTFE and hollow cylindrical roller to design a composite cylindrical roller bearing. Contact stress, von Mises and bending stresses were calculated and compared for hollow cy-lindrical roller and composite cylindrical roller under different radial loadings using the finite element method. They examined the effect of the PTFE filling degree of the composite cylindrical roller on contact stress, von Mises and bending stresses, and determined the optimum filling degree for different radial loadings. Solanki and Vakharia [13] investigated the bearing stiffness and bending stresses on the inner surface of the roller ac-cording to hollow size in layered cylindrical hollow roller bearings with finite element method. They stated that the maximum bearing stiffness is obtained when the hollow size is %61 for the layered cylindrical hollow roller bearing which is consist of two embedded hollow cylinders. They emphasized that the bending stress on the inner surface of the layered cylindrical hollow roller were lower than that of the hollow cylindrical roller.

The above-mentioned studies are experimental and numerical. There are very few analytical studies on HCRBs in literature. Harris and Aaronson [14] examined analytically the effect of the hollow ratio in HCRBs on the load distribution and bearing life for different radial loads and different speeds. They stated that as the hollow size increased in HCRBs, the load on the maximum loaded roller decreased and the number of rollers carrying the load increased. They also emphasized that the bearing life increased with the increased number of rollers in the load zone. They stated that HCRBs are lighter than SCRBs and they have better heat removal capability than that SCRBs. They stated that the bending stress and deformation of the roller increased with the increase of the hollow size and therefore these parameters should be taken into consideration when determining the appropriate hollow size. Liu and Shao [15] created an analytical dynamic model of the HCRBs to examine the effect of hollow size, radial load, and spindle shaft speed on bearing vibration. They observed that the vibration amplitude of the roller in the HCRB was lower than the vibration amplitude of the roller in the filled SCRB. They stated that the hollow size, radial load, and shaft speed have influence on the vibration amplitude of the inner ring.

In this study, the effect of the hollow size on the contact stress and edge stress in the Hollow Cylindrical Roller Bearing (HCRB) is investigated by the finite element method considering the bending stress on the roller. In addition to the above studies, it is stated in [16] that penetration is the critical parameter for the results of a contact analysis which is used penalty-based algorithms. By taking the study of [16] into consideration, the effect of penetration amount between the contacting bodies (roller and race) on contact stress and stress con-centration is also investigated in this study, which is not encountered in the literature such a study for a roller bearing contact analysis.

MATERIALS AND METHODS

In this study, the effect of hollow size on bending stress on the roller and on contact stress is investigated by finite element method in static structural module of ANSYS software. Before the analysis of the hollow roller, the analysis of the solid cylindrical roller bearing is performed in order to verify the finite element model. The contact stress from the analysis and analytical contact stress are compared. NU308 type cylindrical roller bearing is preferred for this study. The geometry and the geometric dimensions of the bearing are given in Figure1 and Table 1.

As is known, roller-raceway contact in SCRB does not provide ideal line contact requirements. The length of the contacting bodies must be equal in ideal line contact. However, since the length of the contacting bodies is not equal in roller bearings, there is stress concentration (edge loading) near the roller corners at the contact area. Since this stress concentration (edge loading) is not taken into account in analytical expressions for the ideal line contact assumption, the maximum contact stress obtained from the finite element method is higher than that obtained from the analytical expressions for the ideal line contact.

The contact stress towards the middle of the contact line in finite element analyses and the contact stress values obtained from analytical results are close to each other. Therefore, the verification process is carried out considering this situation.

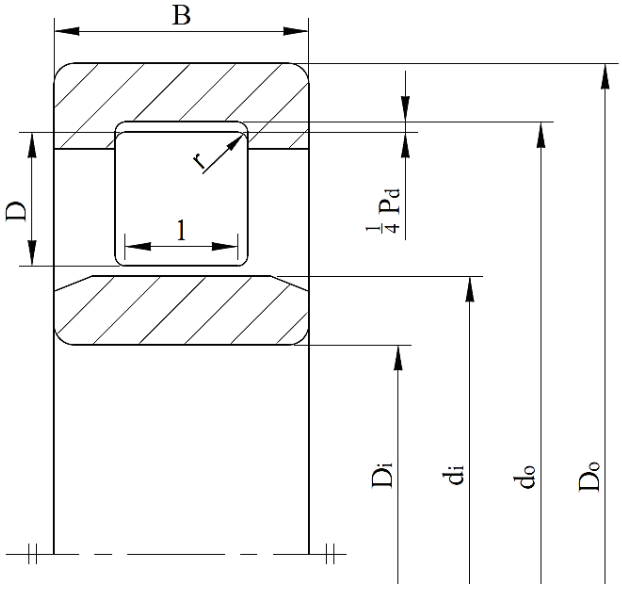


Figure 1. Rolling bearing geometry.

Table 1. The dimensional parameters of NU308 bearing [17,18]

Bore diameter, D_i /mm	40
Inner raceway diameter, d_i /mm	51.9176
Outer raceway diameter, d_o /mm	79.9592
Outer diameter, D_d /mm	90
Bearing width, B/mm	23
Roller effective length l /mm	14.684375
Roller diameter, D /mm	13.9954
Roller corner radius, r /mm	0.5
Number of roller, z	12
Dynamic load rating, C/kN	93

Analytical Expressions

The analytical expressions for contact stress of the ideal line contact condition are as follows [19]:

$$\sigma_{\max} = \frac{2Q}{\pi l b} \tag{1}$$

where σ_{max}, Q, l are maximum contact stress [MPa], radial load applied [N], and effective contact length [mm] (see Figure 1), b=Contact half width [mm]. Contact half width can be obtained as follow [19]:

$$b = \frac{4Q}{\pi l \sum \rho}\left(\frac{\left(1-\varepsilon_1^{\ 2}\right)}{E_1}+\frac{\left(1-\varepsilon_2^{\ 2}\right)}{E_2}\right)^{1/2} \tag{2}$$

where Σρ=Curvature sum, $\varepsilon_{1,2}$= Poisson ratios of contacting bodies, $E_{1,2}$= Young's modulus of contacting bodies. Curvature sum of inner ring and roller can be obtained as follow [19]:

$$\sum \rho_i = \frac{1}{D}\left(\frac{2}{1-\left(D/\left(\left(d_i+d_o\right)/2\right)\right)}\right) \tag{3}$$

where D is the roller diameter and di,o is Inner and outer raceway diameter (see Figure 1).

FE Model of The Solid Cylindrical Roller Bearing

The geometry and mesh model of the NU308 type SCRB are created in the ANSYS based on the dimensional parameters in Table 1. In quasi-static analyses of rolling bearings; when the contact stresses or subsurface shear stresses are examined, the maximum load on a single rolling element is of importance. In the present study, the bearing radial load Fr is assumed to be 7.56 kN (1700 lb) which is the bearing load in helicopter gearbox as stated in Ref. [17]. For radial roller bearings having nominal clearance, the load on the maximum heavily loaded roller Qmax can be calculated with reasonable approximation by Eq. (4) given in Ref. [19]:

$$Q_{\max} = \frac{5F_r}{z\cos\alpha} \tag{4}$$

where F_r is the bearing radial load, z is the number of roller, and α is the contact angle which is zero for cylindrical roller bearing. After substituting the bearing radial load, which is 7.56 kN for the present study, and number of the roller z=12 into the Eq. (4), Q_{max} is equal to 3.15 kN. Therefore the radial load on the most loaded roller is assumed to be about 3 kN in the current study.

One roller-raceway contact (the most loaded roller) is considered. In this direction; after a portion of the inner ring, a portion of the outer ring, and the roller geometry have been created, mesh model of the geometry is generated. (Figure 2). The front and side views of the mesh model of the maximum loaded roller region are shown in Figure 2.

The finite element model is created by defining the symmetry boundary condition in order to save time during solution. In terms of the analysis accuracy and solution time parameters, it has been considered that mesh which is dense in contact area and relatively coarse in away from contact area is the most suitable mesh model. AISI 52100 steel which is a bearing steel is chosen in this study. Material properties of the AISI 52100 are given in Table 2. Linear elastic material behavior is assumed in the current study.

Table 2. Mechanical and physical properties of AISI 52100 [20]

Yield Strength /MPa	1410.17
Young's Modulus / MPa	201.33
Poisson Ratio	0.277
Density/ kg/m^3	7827

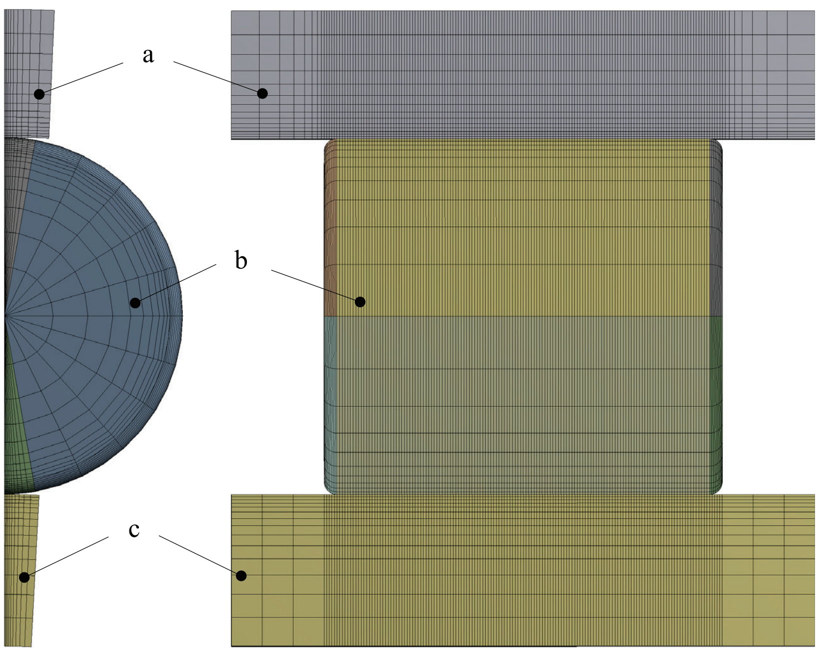


Figure 2. FE model of the roller bearing (a) Outer ring, (b) Roller, (c) Inner ring.

Table 3. Contact settings

Type	Frictional
Friction Coefficient	0.2
Behavior	Asymmetric
Formulation	Augmented Lagrange
Penetration Tolerance	6e-4
Normal Stiffness Factor	9

Frictionless contact type is used between roller and race. The contact settings in the ANSYS for the contact of the roller-inner race or roller-outer race directly affect the contact stress values obtained as a result of the analysis. Some contact settings that are not edited and left as "program controlled" may result in incorrect results. The "penetration" value is of great importance in terms of the accuracy of the results in nonlinear contact problems such a roller bearings. The amount of penetration should ideally be zero, but it is too difficult in a nonlinear contact problem to converge to the solution in such contact settings that will produce zero penetration. On the other hand, especially in finite element contact analyses, unless the amount of penetration is reduced to acceptable levels, the contact stress values obtained may be much lower than the correct values. Considering the penetration parameter, as a result of various trial runs carried out, some contact settings used for this study are given in Table 3.

Table 4. Verification study (Q_{max}=3 kN)

	Contact Stress / MPa	Edge Stress / MPa
FEM	1123.7	2114.7
Hertz (Analytical)	1134.1	-

The amount of penetration as a result of the above settings is in level of 10^{-6} mm. It is observed that the solution could not converge in the current PC conditions for lower penetration level. It is thought that the present penetration level does not affect the accuracy of the results.

Verification Study

As already mentioned; finite element model of the solid cylindrical roller bearing (SCRB) is validated by comparing with the analytical results before the finite element analysis of the HCRB is run. As the ideal line contact assumption is made in the analytical calculations, the peak (edge) stresses due to stress concentration cannot be seen in the results of Hertz (analytical) calculations. For this reason, FEM results taken far from stress concentration region which is near middle of the contact line are compared with the analytical results (Table 4). In the finite element analysis results of this study, the stresses taken from near the middle parts of the contact line are called as "contact stress", and the stresses taken from stress concentration region are called as "edge stress". As seen in Table 4, the results of the finite element analysis performed in ANSYS and the analytical results are near each other.

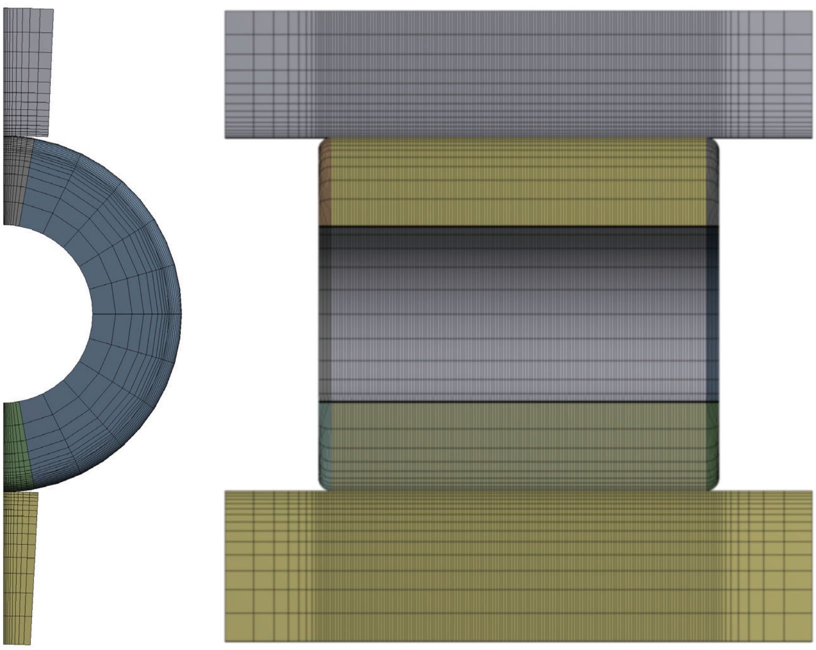


Figure 3. FE model of the hollow cylindrical roller bearing.

Finite Element Model of The Hollow Cylindrical Roller Bearing

For the hollow cylindrical roller bearing (Figure 3), the mesh generation is again carried out after creation of hollows having different sizes on the roller axis. The front and side views of the hollow cylindrical roller bearing (HCRB) whose ratio of the inner diameter of the roller to the outer diameter of the roller is 0.50 can be seen in Figure 3. For all analyses performed for hollow sizes from 0.20 to 0.80, the mesh generation process is repeated for each model and the contact settings are updated so that the penetration amount remains the same or close.

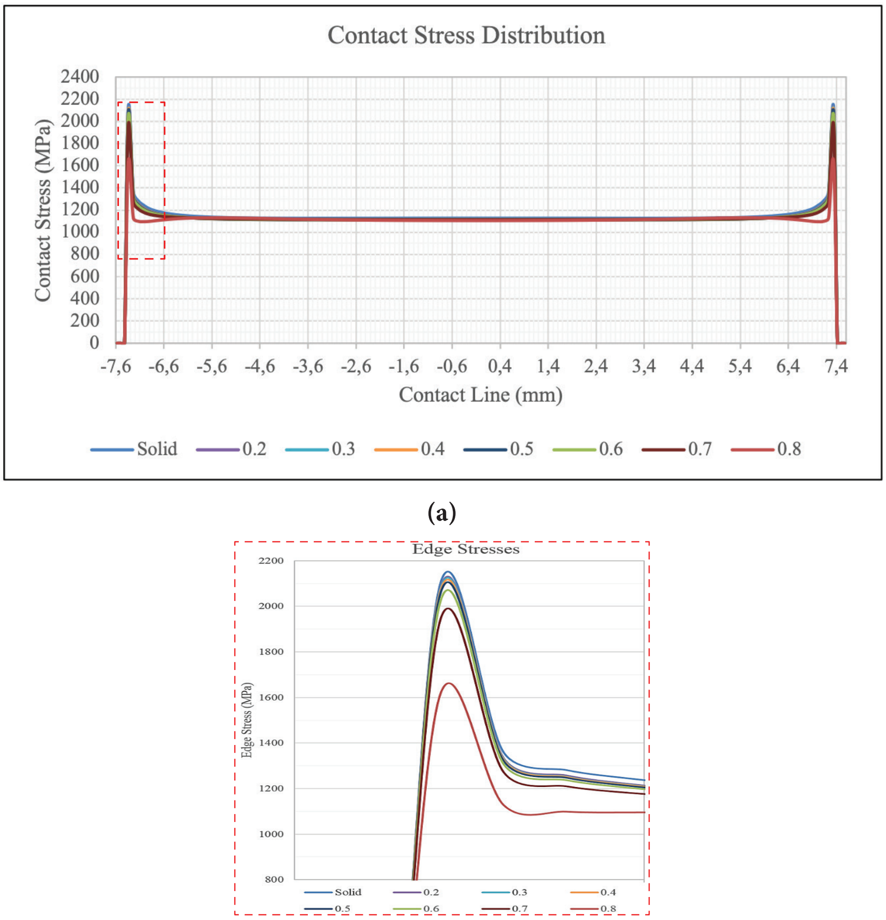
RESULTS AND DISCUSSION

In this study, the contact stresses of HCRBs having different hollow sizes are examined by using FEM in ANSYS (Table 5).

In addition, the maximum tangential tensile stresses on the inner surface of the hollow are investigated to observe any damage caused by bending stresses on the roller in HCRBs (Table 5). It is seen in Table 5 that as the hollow size increases, although there is a slight decrease in the contact stresses, there is a noticeable decrease in the edge stresses.

Table 5. Results of the analysis (Q_{max}=3 kN)

Inner/Outer Diameter Rate	Roller-Inner Raceway Contact Stresses(MPa)		Maximum tangential tensile stresses (MPa)
	$\sigma_{contact}$	σ_{edge}	$\sigma_{tensile}$
0 (solid roller)	1123.4	2114.7	-
0.20	1115.6	2093.9	47.5
0.30	1114.0	2088.0	68.7
0.40	1113.9	2080.2	102.3
0.50	1113.8	2068.9	162.4
0.60	1113.7	2034.6	271.0
0.70	1112.9	1953.7	517.7
0.80	1105.1	1626.9	1244.0



(a)

(b)

Figure 4. Comparison of contact stress distribution for each hollow size including solid roller (a) along the full contact line, (b) focused on the edge region.

Notwithstanding, the maximum tangential stresses on the inner surface of the roller increase as the hollow size increases. It was mentioned in the literature that the maximum tangential tensile stress on the inner surface of the roller must not exceed 490 MPa in terms of the bearing fatigue [2]. Given this situation, it can be em-phasized that the inner/outer diameter ratio for the optimum hollow size should be between 0.60 and 0.70.

Contact stress distribution between the inner race and the most loaded roller having different hollow size can be seen more clearly in Figure 4. It can be seen in Figure 4 that as hollow size increases, stresses in the middle of the contact line don't differ so much for each hollow size. However, there is a visible decrease for the edge stresses when hollow size increases. It is seen that the most decrease is between 0.70 and 0.80 hollow sizes. For 3 kN radial load on the most loaded roller, the hollow size which is 0.70 or 0.80 is optimum for an HCRB. However, bending damage may occur for these hollow sizes as mentioned before. It may be necessary to carry out a FE analysis on HCRB under lighter radial load in future studies in order to investigate whether HCRBs are more useful under light radial loads.

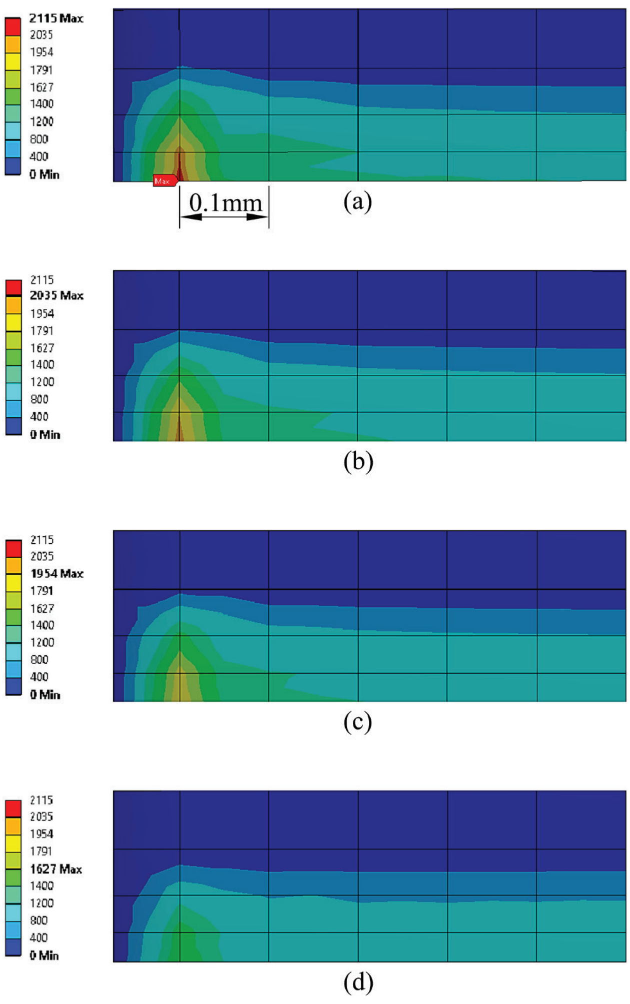


Figure 5. Contact stress distributions in the edge region for: (a)solid, (b)0.60, (c)0.70 and (d)0.80 hollow sizes.

Contact stress distribution of the edge region for the solid roller and for the roller having 0.60, 0.70, 0.80 hollow sizes can be seen in Figure 5. It's seen that stress concentration decreases as the hollow size increases. It's also clear in Figure 5 that the 'dog bone' shape (so named in Ref. [21]) of the contact area changes into rectangular shape as the hollow size increases.

It is mentioned above that some contact settings directly affect the results and if that important settings are left as "program controlled", incorrect results may be obtained. Accordingly, a comparison between the results of the "program controlled" contact settings and the results of the edited contact settings is made. "Penetration Tolerance" and "Normal Stiffness Factor" which are nonlinear contact settings are edited so that an enough penetration level (10^{-6} mm in this study) can be obtained. Subsequently, aforesaid contact settings are left as "program controlled" and analyses are carried out once again. The comparison results for the roller corner area are seen in Figure 6. It is seen clearly in Figure 6 that there is a significant difference between the results from the analysis with program controlled contact settings and from the analysis with edited contact settings. There is lower stress concentration for the analyses with program controlled settings. However, these lower stress concentrations are due to the penetration amount about 10^{-3} mm and 10^{-4} mm levels which are not low enough penetration amounts for a roller bearing contact analysis. When penetration amounts decrease to amounts of 10^{-6} mm by editing the contact settings, contact stresses increase to more correct values. The reason of being almost identical stresses of 0.60 and 0.70 hollow sizes in the analyses with program controlled

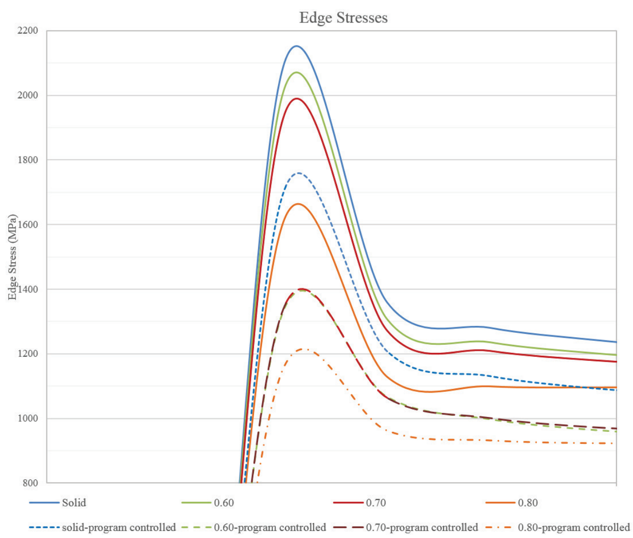


Figure 6. Comparison of "program controlled" contact settings and edited contact settings for the edge region.

contact settings is that ANSYS determined higher penetration value for 0.60 hollow size than for 0.70 hollow size. It means that the comparison for 0.60 and 0.70 hollow sizes may not be correct for the analyses with program controlled contact settings. It's deduced from the results in Figure 6 that it's required to edit the contact settings in this nonlinear contact analysis in order to reduce penetration amount and to obtain more correct contact stresses.

CONCLUSION

In this study, the effect of the roller inner/outer diameter ratio on the contact stresses of the Hollow Cylindrical Roller Bearings (HCRBs) is investigated by using the finite element method. The purpose of the study is to determine the optimum hollow size range which gives the minimum contact stress between inner race and roller. It is also taken into account that as inner/outer diameter ratio of the roller under the same radial load increases, the maximum tangential tensile stresses on the inner surface of the roller increase.

According to the results obtained from the finite element analysis; it has been found that the bending stress on the roller is critical in determining the optimum hollow size due to the fact that increased bending stress values approach the fatigue limit of the bearing material. It has been concluded that the optimum inner/outer diameter should be between 0.60 and 0.70 considering the permissible maximum tangential tensile stress on the inner surface of the rolling element is 490 MPa.

Additionally, in order to emphasize the significance of the penetration amount, the analyses with program controlled contact settings and with edited contact settings are carried out and the results are compared. It's deduced from the results that contact settings in a nonlinear contact analysis are critical in terms of low pene-tration amount and correct contact stress values.

For the further studies; in order to specify a more precise optimum inner/outer diameter ratio, the analyses can be repeated in 0.01 increments between 0.60 and 0.70. In addition, with the purpose of specifying the optimum hollow size for various operating conditions, the analysis can be repeated under different radial loads to determine the optimum hollow size for each external load. In addition, mesh density in the stress concen-tration region may increase to investigate the change in edge stress values.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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