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Research Article

Fractional derivative analysis of Asthma with the effect of environmental factors

Rashid JAN[1], M. S. ZOBAER[2], Şuayip YÜZBAŞI[3], Attaullah[4], Muhammad JAWAD[1], Asif JAN[5]

[1]*Department of Mathematics, University of Swabi, Swabi 23561, Pakistan*
[2]*McGovern Medical School, The University of Texas Health Science Center at Houston,Texas 77030, USA*
[3]*Department of Mathematics, Faculty of Science, Bartin University Tr-74100 Bartın, Türkiye*
[4]*Department of Mathematics, University of Chitral, Chitral, 17200, KP, Pakistan*
[5]*Department of Pathogenic Microbiology & Immunology, School of Basic medical Sciences, Xi'an, 710000, China*

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ABSTRACT

It is observed that the exposure to environmental factors such as indoors and outdoors air pollution, cigarette smoke, and allergens are highly related to asthma attacks. It is also reported that limited exposure to asthma riggers, cure due to medicine, the attacks of asthma can be minimized. In this paper, we formulate the dynamics of asthma with smoking and environmental factors classes in the fractional Caputo-Fabrizio (CF) framework to visualize its dynamical behaviour. We delineate the important properties of the CF derivative for the analysis of our model. The model is then analyzed for the basic properties and the uniqueness and existence of the hypothesized asthma system are investigated via the theory of fixed point. Furthermore, a novel numerical scheme is presented for the solution of our fractional system to illustrate the time series of asthma model. The dynamical behaviour of our asthma model is then highlighted numerically to show the impact of fractional-order ϑ on the system and to visualize the role of input factors on the dynamics of asthma disease.

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INTRODUCTION

It is reported that asthma is a chronic disease that causes the inside walls of the airways of humans and make it hard to breathe. This chronic disease makes it hard to breathe and shorten the breath when the infected individual breathes out. In some cases asthma is a mild annoyance while it may be a big problem for others that disturb their daily life and is life threatening in case of asthma attack. There is no specific treatment for asthma, however, its signs can be monitored. The patients with asthma should consult with the doctors and can modify care according to the condition. The symptoms and signs of asthma are chest tightness or pain, wheezing are coughing attacks, such as flu and cold, infect respiratory system, shortness of breath,

***Corresponding author.**
*E-mail address: suayipyuzbasi@gmail.com

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sleeping problems related to shortness of breath, wheezing or coughing. Furthermore, wheezing in childeren is a common sign of asthma during exhaling, and increasing difficulty breathing. The lack of oxygen in asthma asthma make the patient serious and can feel chest pain or even loss of consciousness. In Figure 1, a systematic flow chart of asthma is given to represent the phenomena of asthma from trigger factors to signs and symptoms of asthma.

It is reported that asthma can be induced not only by air contaminants such as di-isocynates, hexachloroplatinates, etc., but cigarette smoke also triggers asthma [1-3]. Our body can be affected in many ways by smoking which is dangerous to the lungs. This contributes to reduced lung function that makes the lungs vulnerable to asthma triggers. High concentrations of irritants including nitrogen oxides, ammonia, acrolein and formaldehyde are present in cigarette smoke. The expert insisted that lungs damage occurs not only from mainstream smoke but also from side stream smoke which comes to the host through a nearby smoker in the society. Therefore, these unpleasant substances can set off an asthma attack when the smoke of tobacco inhales by a person, whether for personal smoking or passive smoke. More than 15 million children per day are threatened and open to smoke, and more than 1,000,000 children with asthma become more serious after second-hand smoke exposure [4-5]. Also in the cause of asthma, contaminants released in the air such as vanadium and nickel particles from power stations and oil refineries, heavy metal cocktails from SLF burning cement, brick and lime works and dust from open cast coal mining and coal fired cement and other works play important roles [6-8]. Thus, a prone population, due to constant exposure to air contaminants, they become asthmatic. Mathematical models play an important role to explore the dynamics of diseases [9-13, 50-51] and produce significant information about the dynamics of disease for control and prevention [14-17]. Different numerical approaches has been developed by researchers

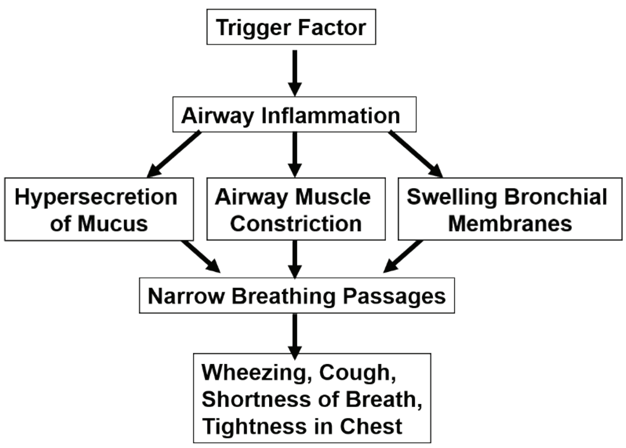


Figure 1. Illustration of systematic flow of asthma disease from trigger factors to signs and symptoms.

to investigate the dynamics of diseases [18-22, 51]. There is limited literature on modeling the dynamics of asthma to visualize, conceptualize and understand the transmission phenomena of asthma [23-27]. The dynamics of asthma has been recently investigated by the researchers in their work from different aspects [28-29]. It is noted that in the development and spread of asthma, continuous exposure to smoking plays a very important role. This very significant factor has been accounted for in our model by considering a different class of smokers.

It is crystal clear that fractional systems give knowledge about the dynamics of various diseases more accurate, broader, precious and precise than integer systems [30,31]. It surpasses integer-order models due to hereditary qualities and memory definition [35-36], moreover, fractional models beterly explore and demonstrate the dynamics between two points. In the literature, it has been demonstrated that fractional models provide the best match to real data [37,38]. These new theories have been efficiently used in physics, economics, biology, engineering and many other fields to model natural processess [39-58]. These operators, on the other hand, have a power law kernel and are limited in their ability to mimic physical situations. Caputo and Fabrizio recently proposed an additional fractional differential operator with an exponential decay kernel to address this challenge. The Caputo-Fabrizio (CF) operator is a revolutionary technique to fractional derivative that has drawn the attention of numerous researchers owing to its non-singular kernel. In addition, the CF operator is ideal for simulating particular types of real-world problems that follow the exponential decay law. Developing a mathematical model employing the CF fractional order derivative has become a fascinating subject of the upcoming research.

We choose to use fractional derivative to investigate the dynamics of asthma with smoking and environmental class, motivated by the accurate results of the fractional derivative. We represent the fundamental results and theory of fractional CF derivative in 1 Section of the article. In the second section, using the CF derivative, a fractional model for asthma infection with smoking and environmental factor compartment is developed. Mathematical skills are then used to investigate the hypothesised asthma model. In Section 3, the uniqueness and existence aof the solution of the formulated FO model of asthma through the fixed-point theorem is presented. The dynamics of proposed asthma model is then investigated with the effect of the different input parameter numerically in Section 4. Finally, in the last section, concluding observations and ideas are offered.

FORMULATION OF THE ASTHMA MODEL

In the evaluation of the dynamics, the total population size $N(t)$ is divided into four subclasses susceptibles class $\mathcal{A}_S(t)$, exposed class $\mathcal{A}_E(t)$, asthma infected class $\mathcal{A}_I(t)$ and smokers class $\mathcal{A}_c(t)$, while the cumulative concentration of pollutants present in the environment is indicated by $\mathcal{A}_p(t)$. The recruitment rate of susceptible and smoker is indicated by Π and Ξ, respectively while the natural death rate is assumed to be μ for all classes of humans population. We supposed that the susceptible population become asthmatic by continuous exposure to air pollutants such as diisocyanates, hexachloroplatinates, etc. The susceptible humans population become exposed at a rate β due to smoking while a fraction γ of susceptible moves to the exposed class due to environmental factors. In the exposed class, two types of transmission has been observed denoted by λ_1 and λ_2. λ_1 is the transmission rate to the infected group from the exposed group due to smokers while λ_2 is the transmission rate from the infected group to the exposed group due to environmental factors. Moreover, we assume η_1 be the rate at which smokers become infected while η_2 is the rate at which smokers become infected through environmental factors. We indicated the asthma infection induced death rate by α in our formulation. We symbolize the emission rate of pollutants by $A(N)$ which depend on population, however, we consider it to be constant. The rate of natural depletion is denoted by ν while smokers emit fumes rate is given by q and the rate at which smoker leave smoking is η. The following system of ODE's describes the dynamics of asthma

$$\begin{cases} \frac{d\mathcal{A}_S}{dt} &= \Pi - \beta\mathcal{A}_S\mathcal{A}_C - \gamma\mathcal{A}_S\mathcal{A}_P - \mu\mathcal{A}_S, \\ \frac{d\mathcal{A}_E}{dt} &= \beta\mathcal{A}_S\mathcal{A}_C + \gamma\mathcal{A}_S\mathcal{A}_P - \lambda_1\mathcal{A}_E\mathcal{A}_P - \lambda_2\mathcal{A}_E\mathcal{A}_C - \mu\mathcal{A}_E, \\ \frac{d\mathcal{A}_I}{dt} &= \lambda_1\mathcal{A}_E\mathcal{A}_P + \lambda_2\mathcal{A}_E\mathcal{A}_C + \eta_1\mathcal{A}_C + \eta_2\mathcal{A}_C\mathcal{A}_P - \alpha\mathcal{A}_I - \mu\mathcal{A}_I, \\ \frac{d\mathcal{A}_C}{dt} &= \Xi - \eta_1\mathcal{A}_C - \eta_2\mathcal{A}_C\mathcal{A}_P - \eta\mathcal{A}_C - \mu\mathcal{A}_C, \\ \frac{d\mathcal{A}_P}{dt} &= A(N) - \nu\mathcal{A}_P + q\mathcal{A}_C, \end{cases} \tag{1}$$

with appropriate initial condition for vector

$$\mathcal{A}_S(0) \geq 0, \mathcal{A}_E(0) \geq 0, \mathcal{A}_I(0) \geq 0, \mathcal{A}_C(0) \geq 0, \mathcal{A}_P(0) \geq 0. \tag{2}$$

It is cristal clear that fractional order systems provide more dependable, deeper, valuable, and accurate data concerning a disease's dynamics. To be more specific, the effect of memory can be captured through fractional operators which is mostly occurs in biological phenomena. Moreover, fractional order parameter make it more flexible for data fitting. Therefore, the above system (1) with (2) of asthma disease is represented in the framework of CF derivative in the following manner

$$\begin{aligned} {}^{CF}_{0}D^{\varsigma}_{t}\mathcal{A}_S &= \Pi - \beta\mathcal{A}_S\mathcal{A}_C - \gamma\mathcal{A}_S\mathcal{A}_P - \mu\mathcal{A}_S; \\ {}^{CF}_{0}D^{\varsigma}_{t}\mathcal{A}_E &= \beta\mathcal{A}_S\mathcal{A}_C + \gamma\mathcal{A}_S\mathcal{A}_P - \lambda_1\mathcal{A}_E\mathcal{A}_P - \lambda_2\mathcal{A}_E\mathcal{A}_C - \mu\mathcal{A}_E; \\ {}^{CF}_{0}D^{\varsigma}_{t}\mathcal{A}_I &= \lambda_1\mathcal{A}_E\mathcal{A}_P + \lambda_2\mathcal{A}_E\mathcal{A}_C + \eta_1\mathcal{A}_C + \eta_2\mathcal{A}_C\mathcal{A}_P - \alpha\mathcal{A}_I - \mu\mathcal{A}_I; \\ {}^{CF}_{0}D^{\varsigma}_{t}\mathcal{A}_C &= \Xi - \eta_1\mathcal{A}_C - \eta_2\mathcal{A}_C\mathcal{A}_P - \eta\mathcal{A}_C - \mu\mathcal{A}_C; \\ {}^{CF}_{0}D^{\varsigma}_{t}\mathcal{A}_P &= A(N) - \nu\mathcal{A}_P + q\mathcal{A}_C; \end{aligned} \tag{3}$$

where ϑ is the order of CF fractional derivative such that $0 < \vartheta \leq 1$. In the upcoming subsection of the article, we represent basic idea and concept of fractional derivative theory that will be utilized in the analysis of our proposed fractional-order model of asthma infection.

Basic Fractional Theory

In this subsection of the article, we will list fundamental results and theory of CF fractional derivative for the analysis of our asthma disease model with environmental factors. We take start from the following theorem:

Definition 1*Assume a function g such that $g \in H^1(a,b)$ with the condition that $a < b$, then the fractional derivative in CF framework [48] is given by*

$$D^{\varsigma}_{t}(g(t)) = \frac{V(\varsigma)}{1-\varsigma}\int_a^t g'(x)\exp[-\varsigma\frac{t-x}{1-\varsigma}]dx, \tag{4}$$

with fractional-order ς with the condition that $\varsigma \in [0,1]$ and $V(\tau)$ indicates normality such that $V(0) = V(1) = 1$ [48]. Here, if $g \notin H^1(a,b)$, then we have the below fractional form

$$D^{\varsigma}_{t}(g(t)) = \frac{\varsigma V(\varsigma)}{1-\varsigma}\int_a^t (g(t) - g(x))\exp[-\varsigma\frac{t-x}{1-\varsigma}]dx. \tag{5}$$

Remark 1*Assume the value $\alpha = \frac{1-\varsigma}{\varsigma} \in [0,\infty)$ and $\varsigma = \frac{1}{1+\alpha} \in [0,1]$, then the above (5) takes the below form*

$$D^{\varsigma}_{t}(g(t)) = \frac{M(\alpha)}{\alpha}\int_a^t g'(x)e^{[-\frac{t-x}{\alpha}]}dx, \tag{6}$$

in which $M(0) = M(\infty) = 1$. Furthermore, we have the below

$$\lim_{\alpha \to 0}\frac{1}{\alpha}\exp[-\frac{t-x}{\alpha}] = \delta(x-t).$$

Here, the definition of fractional integral for CF framework will be represented see in [48].

Definition 2 *Assume a function g, and ς be the fractional-order then the fractional integral is*

$$I^{\varsigma}_{t}(g(t)) = \frac{2(1-\varsigma)}{(2-\varsigma)V(\varsigma)}g(t) + \frac{2\varsigma}{(2-\varsigma)V(\varsigma)}\int_0^t g(u)du, \quad t \geq 0. \tag{7}$$

with the condition $0 < \varsigma < 1$[49].

Remark 2 *Here, we investigate the above mentioned 2 and get the following*

$$\frac{2(1-\varsigma)}{(2-\varsigma)V(\varsigma)} + \frac{2\varsigma}{(2-\varsigma)V(\varsigma)} = 1, \tag{8}$$

which further implies that $V(\varsigma)=\frac{2}{2-\varsigma}$, $0<\varsigma<1$. In the upcoming step, we represent a novel fractional derivative introduced in [49] by using (8) and is as

$$D_t^{\varsigma}(g(t))=\frac{1}{1-\varsigma}\int_0^t g'(x)\exp[\varsigma\frac{t-x}{1-\varsigma}]dx, 0<\varsigma<1.$$

NUMERICAL SCHEME AND ANALYSIS

Here, the solution of the proposed asthma disease model will be investigated for existence through fixed-point theory. We use the concept of CF fractional derivative on the system (3),and get the following

$$\begin{aligned}
&A_S(t)-A_S(0)={}^{CF}_{0}I_t^{\varsigma}[\Pi-\beta\mathcal{A}_S\mathcal{A}_C-\gamma\mathcal{A}_S\mathcal{A}_P-\mu\mathcal{A}_S],\\
&A_E(t)-A_E(0)={}^{CF}_{0}I_t^{\varsigma}[\beta\mathcal{A}_S\mathcal{A}_C+\gamma\mathcal{A}_S\mathcal{A}_P-\lambda_1\mathcal{A}_E\mathcal{A}_P-\lambda_2\mathcal{A}_E\mathcal{A}_C-\mu\mathcal{A}_E],\\
&A_I(t)-A_I(0)={}^{CF}_{0}I_t^{\varsigma}[\lambda_1\mathcal{A}_E\mathcal{A}_P+\lambda_2\mathcal{A}_E\mathcal{A}_C+\eta_1\mathcal{A}_C+\eta_2\mathcal{A}_C\mathcal{A}_P-\alpha\mathcal{A}_I-\mu\mathcal{A}_I],\\
&A_P(t)-A_P(0)={}^{CF}_{0}I_t^{\varsigma}[\Xi-\eta_1\mathcal{A}_C-\eta_2\mathcal{A}_C\mathcal{A}_P-\eta\mathcal{A}_C-\mu\mathcal{A}_C],\\
&A_P(t)-A_P(0)={}^{CF}_{0}I_t^{\varsigma}[A(N)-\nu\mathcal{A}_P+q\mathcal{A}_C].
\end{aligned}\tag{9}$$

Apply the idea presented in [49], the above (9) implies

$$\begin{aligned}
\mathcal{A}_S(t)-\mathcal{A}_S(0)&=\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}[\Pi-\beta\mathcal{A}_S\mathcal{A}_C-\gamma\mathcal{A}_S\mathcal{A}_P-\mu\mathcal{A}_S]\\
&+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t[\Pi-\beta\mathcal{A}_S\mathcal{A}_C-\gamma\mathcal{A}_S\mathcal{A}_P-\mu\mathcal{A}_S]dy,\\
\mathcal{A}_E(t)-\mathcal{A}_E(0)&=\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}[\beta\mathcal{A}_S\mathcal{A}_C+\gamma\mathcal{A}_S\mathcal{A}_P-\lambda_1\mathcal{A}_E\mathcal{A}_P-\lambda_2\mathcal{A}_E\mathcal{A}_C-\mu\mathcal{A}_E]\\
&+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t[\beta\mathcal{A}_S\mathcal{A}_C+\gamma\mathcal{A}_S\mathcal{A}_P-\lambda_1\mathcal{A}_E\mathcal{A}_P-\lambda_2\mathcal{A}_E\mathcal{A}_C-\mu\mathcal{A}_E]dy,\\
\mathcal{A}_I(t)-\mathcal{A}_I(0)&=\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}[\lambda_1\mathcal{A}_E\mathcal{A}_P+\lambda_2\mathcal{A}_E\mathcal{A}_C+\eta_1\mathcal{A}_C+\eta_2\mathcal{A}_C\mathcal{A}_P-\alpha\mathcal{A}_I-\mu\mathcal{A}_I]\\
&+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t[\lambda_1\mathcal{A}_E\mathcal{A}_P+\lambda_2\mathcal{A}_E\mathcal{A}_C+\eta_1\mathcal{A}_C+\eta_2\mathcal{A}_C\mathcal{A}_P-\alpha\mathcal{A}_I-\mu\mathcal{A}_I]dy,\\
\mathcal{A}_C(t)-\mathcal{A}_C(0)&=\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}[\Xi-\eta_1\mathcal{A}_C-\eta_2\mathcal{A}_C\mathcal{A}_P-\eta\mathcal{A}_C-\mu\mathcal{A}_C]\\
&+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t[\Xi-\eta_1\mathcal{A}_C-\eta_2\mathcal{A}_C\mathcal{A}_P-\eta\mathcal{A}_C-\mu\mathcal{A}_C]dy,\\
\mathcal{A}_P(t)-\mathcal{A}_P(0)&=\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}[A(N)-\nu\mathcal{A}_P+q\mathcal{A}_C]+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t[A(N)-\nu\mathcal{A}_P+q\mathcal{A}_C].
\end{aligned}\tag{10}$$

In the next step, we proceed in the following manner

$$\begin{cases}
\mathcal{L}_1(t,\mathcal{A}_S) &= \Pi-\beta\mathcal{A}_S\mathcal{A}_C-\gamma\mathcal{A}_S\mathcal{A}_P-\mu\mathcal{A}_S,\\
\mathcal{L}_2(t,\mathcal{A}_E) &= \beta\mathcal{A}_S\mathcal{A}_C+\gamma\mathcal{A}_S\mathcal{A}_P-\lambda_1\mathcal{A}_E\mathcal{A}_P-\lambda_2\mathcal{A}_E\mathcal{A}_C-\mu\mathcal{A}_E,\\
\mathcal{L}_3(t,\mathcal{A}_I) &= \lambda_1\mathcal{A}_E\mathcal{A}_P+\lambda_2\mathcal{A}_E\mathcal{A}_C+\eta_1\mathcal{A}_C+\eta_2\mathcal{A}_C\mathcal{A}_P-\alpha\mathcal{A}_I-\mu\mathcal{A}_I,\\
\mathcal{L}_4(t,\mathcal{C}_D) &= \Xi-\eta_1\mathcal{A}_C-\eta_2\mathcal{A}_C\mathcal{A}_P-\eta\mathcal{A}_C-\mu\mathcal{A}_C,\\
\mathcal{L}_5(t,\mathcal{C}_E) &= A(N)-\nu\mathcal{A}_P+q\mathcal{A}_C,
\end{cases}$$

Theorem 1 *If the following condition satisfies*

$$0\le(\beta M+\gamma N+\mu)<1,$$

then the kernels $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_4$ and $\mathcal{L}_5$ assures the Lipschitz condition.

Proof. For the proof of Theorem 2, we first take $\mathcal{A}_S$ and $\mathcal{A}_{S1}$, and start from $\mathcal{L}_1$ in the following manner

$$\begin{aligned}\mathcal{L}_1(t,\mathcal{A}_S)-\mathcal{L}_1(t,\mathcal{A}_{S1})&=-\beta\mathcal{A}_S\mathcal{A}_C-\gamma\mathcal{A}_S\mathcal{A}_P-\mu\mathcal{A}_S\\&+\beta\mathcal{A}_{S1}\mathcal{A}_C+\gamma\mathcal{A}_{S1}\mathcal{A}_P+\mu\mathcal{A}_{S1},\end{aligned}$$

this implies that

$$\begin{aligned}\mathcal{L}_1(t,\mathcal{A}_S)-\mathcal{L}_1(t,\mathcal{A}_{S1})&=\beta(\mathcal{A}_{S1}-\mathcal{A}_S)\mathcal{A}_C+\gamma(\mathcal{A}_{S1}-\mathcal{A}_S)\mathcal{A}_P\\&+\mu(\mathcal{A}_{S1}-\mathcal{A}_S)\end{aligned}\tag{11}$$

Here, apply norm on Eq. (11) and simplify, we get the below

$$\begin{aligned}
&\parallel\mathcal{L}_1(t,\mathcal{A}_S)-\mathcal{L}_1(t,\mathcal{A}_{S1})\parallel\le\parallel\beta(\mathcal{A}_{S1}-\mathcal{A}_S)\mathcal{A}_C\parallel+\parallel\gamma(\mathcal{A}_{S1}-\mathcal{A}_S)\mathcal{A}_P\parallel\\
&+\parallel\mu(\mathcal{A}_{S1}-\mathcal{A}_S)\parallel,\le\beta\parallel(\mathcal{A}_{S1}-\mathcal{A}_S)\parallel\\
&\parallel\mathcal{A}_C\parallel+\gamma\parallel(\mathcal{A}_{S1}-\mathcal{A}_S)\parallel\parallel\mathcal{A}_P\parallel+\mu\parallel(\mathcal{A}_{S1}-\mathcal{A}_S)\parallel,\\
&\le\beta M\parallel(\mathcal{A}_{S1}-\mathcal{A}_S)\parallel+\gamma N\\
&\parallel(\mathcal{A}_{S1}-\mathcal{A}_S)\parallel+\mu\parallel(\mathcal{A}_{S1}-\mathcal{A}_S)\parallel,\\
&\le(\beta M+\gamma N+\mu)\parallel\mathcal{A}_{S1}-\mathcal{A}_S\parallel.
\end{aligned}$$

At this stage, we assume $\mu_1=(\beta M+\gamma N+\mu)$, then the following result is obtained

$$||L_1(t,\mathcal{A}_S)-L_1(t,\mathcal{A}_{S1})||\le\mu_1||\mathcal{A}_S(t)-\mathcal{A}_S(t_1)||.$$

Thus the Lipschitz condition is fulfilled for $\mathcal{L}_1$, in addition to this the condition $0\le(\beta M+\gamma N+\mu)<1$ assure the contraction is also satisfied. In the same way, the Lipschitz conditions for the other cases of our system are determined as

$$\begin{aligned}
&\parallel\mathcal{L}_2(t,\mathcal{A}_E)-\mathcal{L}_2(t,\mathcal{A}_{E1})\parallel\le\mu_2||\mathcal{A}_E(t)-\mathcal{A}_E(t_1)||,\\
&\parallel\mathcal{L}_3(t,\mathcal{A}_I)-\mathcal{L}_3(t,\mathcal{A}_{I1})\parallel\le\mu_3||\mathcal{A}_I(t)-\mathcal{A}_I(t_1)||,\\
&\parallel\mathcal{L}_4(t,\mathcal{A}_C)-\mathcal{L}_4(t,\mathcal{A}_{C1})\parallel\le\mu_4||\mathcal{A}_C(t)-\mathcal{A}_C(t_1)||,\\
&\parallel\mathcal{L}_5(t,\mathcal{A}_P)-\mathcal{L}_5(t,\mathcal{A}_{P1})\parallel\le\mu_5||\mathcal{A}_P(t)-\mathcal{A}_P(t_1)||.
\end{aligned}\tag{12}$$

Further simplification of Eq. (10) implies

$$\begin{cases}
\mathcal{A}_S(t) &= \mathcal{A}_S(t)(0)+\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_1(t,\mathcal{A}_S(t))+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_1(y,\mathcal{A}_S(t)))dy,\\
\mathcal{A}_E(t) &= \mathcal{A}_E(t)(0)+\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_2(t,\mathcal{A}_E(t))+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_2(y,\mathcal{A}_E(t)))dy,\\
\mathcal{A}_I(t) &= \mathcal{A}_I(t)(0)+\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_3(t,\mathcal{A}_I(t))+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_3(y,\mathcal{A}_I(t)))dy,\\
\mathcal{A}_C(t) &= \mathcal{A}_C(t)(0)+\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_4(t,\mathcal{A}_C(t))+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_4(y,\mathcal{A}_C(t)))dy,\\
\mathcal{A}_P(t) &= \mathcal{A}_P(t)(0)+\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_5(t,\mathcal{A}_P(t))+\frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_5(y,\mathcal{A}_P(t)))dy,
\end{cases}\tag{13}$$

its recursive form is given by

$$\begin{cases}
\mathcal{A}_{Sn}(t) &= 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_1(t,\mathcal{A}_{S(n-1)})+2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_1(y,\mathcal{A}_{S(n-1)}))dy,\\
\mathcal{A}_{En}(t) &= 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_2(t,\mathcal{A}_{E(n-1)})+2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_2(y,\mathcal{A}_{E(n-1)}))dy,\\
\mathcal{A}_{In}(t) &= 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_3(t,\mathcal{A}_{I(n-1)})+2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_3(y,\mathcal{A}_{I(n-1)}))dy,\\
\mathcal{A}_{Cn}(t) &= 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_4(t,\mathcal{A}_{C(n-1)})+2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_4(y,\mathcal{A}_{C(n-1)}))dy,\\
\mathcal{A}_{Pn}(t) &= 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mathcal{L}_5(t,\mathcal{A}_{P(n-1)})+2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t(\mathcal{L}_5(y,\mathcal{A}_{P(n-1)}))dy,
\end{cases}\tag{14}$$

with proper initial conditions

$$\mathcal{A}_S^0(t)=\mathcal{A}_S(0),\mathcal{A}_E^0(t)=\mathcal{A}_E(0),\mathcal{A}_I^0(t)=\mathcal{A}_I(0),$$
$$\mathcal{A}_C^0(t)=\mathcal{A}_C(0),\mathcal{A}_P^0(t)=\mathcal{A}_P(0).$$

Here, the succeeding difference form is

$$\begin{aligned}
\hbar_{1n}(t) &= \mathcal{A}_{Sn}(t) - \mathcal{A}_{S(n-1)}(t) \\
&= \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\left(\mathcal{L}_1(t,\mathcal{A}_{S(n-1)}) - \mathcal{L}_1(t,\mathcal{A}_{S(n-2)})\right) \\
&\quad + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t \left(\mathcal{L}_1\left(y,\mathcal{A}_{S(n-1)}\right) - \mathcal{L}_1\left(y,\mathcal{A}_{S(n-2)}\right)\right)dy, \\
\hbar_{2n}(t) &= \mathcal{A}_{En}(t) - \mathcal{A}_{E(n-1)}(t) \\
&= \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\left(\mathcal{L}_2(t,\mathcal{A}_{E(n-1)}) - \mathcal{L}_2(t,\mathcal{A}_{E(n-2)})\right) \\
&\quad + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t \left(\mathcal{L}_2\left(y,\mathcal{A}_{E(n-1)}\right) - \mathcal{L}_2\left(y,\mathcal{A}_{E(n-2)}\right)\right)dy, \\
\hbar_{3n}(t) &= \mathcal{A}_{In}(t) - \mathcal{A}_{I(n-1)}(t) \\
&= \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\left(\mathcal{L}_3(t,\mathcal{A}_{I(n-1)}) - \mathcal{L}_3(t,\mathcal{A}_{I(n-2)})\right) \\
&\quad + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t \left(\mathcal{L}_3\left(y,\mathcal{A}_{I(n-1)}\right) - \mathcal{L}_3\left(y,\mathcal{A}_{I(n-2)}\right)\right)dy, \\
\hbar_{4n}(t) &= \mathcal{A}_{Cn}(t) - \mathcal{A}_{C(n-1)}(t) \\
&= \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\left(\mathcal{L}_4(t,\mathcal{A}_{C(n-1)}) - \mathcal{L}_4(t,\mathcal{A}_{C(n-2)})\right) \\
&\quad + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t \left(\mathcal{L}_4\left(y,\mathcal{A}_{C(n-1)}\right) - \mathcal{L}_4\left(y,\mathcal{A}_{C(n-2)}\right)\right)dy, \\
\hbar_{5n}(t) &= \mathcal{A}_{Pn}(t) - \mathcal{A}_{P(n-1)}(t) \\
&= \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}(\mathcal{L}_5(t,\mathcal{A}_{P(n-1)}) - \mathcal{L}_5(t,\mathcal{A}_{P(n-2)})) \\
&\quad + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t (\mathcal{L}_5(y,\mathcal{A}_{P(n-1)}) - \mathcal{L}_5(y,\mathcal{A}_{P(n-2)}))dy.
\end{aligned} \tag{15}$$

Here, we observe the following

$$\begin{cases} \mathcal{A}_{Sn}(t) = \sum_{i=1}^n \hbar_{1i}(t), \mathcal{A}_{En}(t) = \sum_{i=1}^n \hbar_{2i}(t), \mathcal{A}_{In}(t) = \sum_{i=1}^n \hbar_{3i}(t), \\ \mathcal{A}_{Cn}(t) = \sum_{i=1}^n \hbar_{4i}(t), \mathcal{A}_{Pn}(t) = \sum_{i=1}^n \hbar_{5i}(t). \end{cases}$$

Following the same way, we have

$$\begin{aligned}
||\hbar_{1n}(t)|| = \left||\mathcal{A}_{Sn}(t) - \mathcal{A}_{S(n-1)}(t)\right|| = \| 2\tfrac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\big(\mathcal{L}_1(t,\mathcal{A}_{S(n-1)}) \\
- \mathcal{L}_1(t,\mathcal{A}_{S(n-2)})\big) + 2\tfrac{\vartheta}{(2-\vartheta)U(\vartheta)}\int_0^t (\mathcal{L}_1(y,\mathcal{A}_{S(n-1)}) \\
- \mathcal{L}_1(y,\mathcal{A}_{S(n-2)}))dy \|.
\end{aligned} \tag{16}$$

By triangular inequality, Eq. (16) becomes

$$\begin{aligned}
\| \mathcal{A}_{Sn}(t) - \mathcal{A}_{S(n-1)}(t) \| \le 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)} \| \left(\mathcal{L}_1(t,\mathcal{A}_{S(n-1)})\right. \\
\left. - \mathcal{L}_1(t,\mathcal{A}_{S(n-2)})\right) || + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)} \| \int_0^t (\mathcal{L}_1(y,\mathcal{A}_{S(n-1)}) \\
- \mathcal{L}_1(y,\mathcal{A}_{S(n-2)}))dy \|.
\end{aligned}$$

Lipschitz condition leads us to

$$\begin{aligned}
\| \mathcal{A}_{S_n}(t) - \mathcal{A}_{S_{n-1}}(t) \| \le 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_1 \| \mathcal{A}_{S(n-1)} - \mathcal{A}_{S(n-2)} \| \\
+2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_1 \times \int_0^t \| \mathcal{A}_{S(n-1)} - \mathcal{A}_{S(n-2)} \| dy.
\end{aligned}$$

Next, we have

$$\begin{aligned}
\| \hbar_{1n}(t) \| \le 2\tfrac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_1 \| \hbar_{1(n-1)}(t) \| \\
+2\tfrac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_1 \int_0^t \| \hbar_{1(n-1)}(y) \| dy.
\end{aligned} \tag{17}$$

Taking the same steps, we get

$$\begin{aligned}
\| \hbar_{2n}(t) \| &\le 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_2 \| \hbar_{2(n-1)}(t) \| + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_2\int_0^t \| \hbar_{2(n-1)}(y) \| dy, \\
\| \hbar_{3n}(t) \| &\le 2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_3 \| \hbar_{3(n-1)}(t) \| + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_1\int_0^t \| \hbar_{3(n-1)}(y) \| dy, \\
\| \hbar_{4n}(t) \| &\le \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_4 \| \hbar_{4(n-1)}(t) \| + 2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_4\int_0^t \| \hbar_{4(n-1)}(y) \| dy, \\
\| \hbar_{5n}(t) \| &\le 2\tfrac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_5 \| \hbar_{5(n-1)}(t) \| + 2\tfrac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_5\int_0^t \| \hbar_{5(n-1)}(y) \| dy,
\end{aligned} \tag{18}$$

Theorem 2 *Exact coupled-solutions of the fractional-order model (3) of asthma disease exists if the below mentioned condition satisfies. That is one can find t_0 in way that*

$$2\tfrac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_1 + 2\tfrac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_1 t_0 < 1.$$

Proof. As the state variables $\mathcal{A}_S(t)$, $\mathcal{A}_E(t)$, $\mathcal{A}_I(t)$, $\mathcal{A}_C(t)$, and $\mathcal{A}_P(t)$ are bounded. Moreover, we have shown that the Lipschitz condition is fulfilled by the kernels, Eqs. (17) and (18) gives the following by applying the recursive technique

$$\begin{aligned}
\| \hbar_{1n}(t) \| &\le ||\mathcal{A}_{Sn}(0)||[(2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_1) + (2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_1 t)]^n, \\
\| \hbar_{2n}(t) \| &\le ||\mathcal{A}_{En}(0)||[(2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_2) + (2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_2 t)]^n, \\
\| \hbar_{3n}(t) \| &\le ||\mathcal{A}_{In}(0)||[(2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_3) + (2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_3 t)]^n, \\
\| \hbar_{4n}(t) \| &\le ||\mathcal{A}_{Cn}(0)||[(2\frac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_4) + (2\frac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_4 t)]^n, \\
\| \hbar_{5n}(t) \| &\le ||\mathcal{A}_{Pn}(0)||[(2\tfrac{(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_5) + (2\tfrac{\vartheta}{(2-\vartheta)U(\vartheta)}\mu_5 t)]^n.
\end{aligned} \tag{19}$$

As a result, the existence of solutions of the asthma disease model and its continuity are achieved. In addition to this, we will show that that the above is a solution of system (3), and proceed as follows

$$\begin{aligned}
\mathcal{A}_S(t) - \mathcal{A}_S(0) &= \mathcal{A}_{Sn}(t) - \mathcal{M}1_n(t), \\
\mathcal{A}_E(t) - \mathcal{A}_E(0) &= \mathcal{A}_{En}(t) - \mathcal{M}2_n(t), \\
\mathcal{A}_I(t) - \mathcal{A}_I(0) &= \mathcal{A}_{In}(t) - \mathcal{M}3_n(t), \\
\mathcal{A}_C(t) - \mathcal{A}_C(0) &= \mathcal{A}_{Cn}(t) - \mathcal{M}4_n(t), \\
\mathcal{A}_P(t) - \mathcal{A}_P(0) &= \mathcal{A}_{Pn}(t) - \mathcal{M}5_n(t).
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\| H_n(t) \| &= \left\|\frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\left(\mathcal{L}_1(t,\mathcal{A}_{Sn}) - \mathcal{L}_1(t,\mathcal{A}_{S(n-1)})\right)\right. \\
&\quad \left. + \frac{2\vartheta}{(2-\vartheta)U(\vartheta)} \times \int_0^t \left(\mathcal{L}_1(y,\mathcal{A}_{Sn}) - \mathcal{L}_1(y,\mathcal{A}_{S(n-1)})\right)dy\right\|, \\
&\le \frac{2(1-\ell)}{(2-\ell)U(\ell)} \| (L_1(t,\mathcal{A}_{Sn}) - (L_1(t,\mathcal{C}_{S(n-1)})) \\
&\quad \| + \frac{2\vartheta}{(2-\vartheta)U(\vartheta)} \times \int_0^t ||(\mathcal{L}_1(y,\mathcal{A}_S) - \mathcal{L}_1(y,\mathcal{A}_{S(n-1)}))||dy, \\
&\le \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_1 \| \mathcal{A}_{Sn} - \mathcal{A}_{S(n-1)} \| + \frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\mu_1 \| \mathcal{A}_{Sn} - \mathcal{A}_{S(n-1)} \| t.
\end{aligned}$$

Following the technique, we proceed as

$$\| \mathcal{M}1_n(t) \| \leq (\tfrac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)} + \tfrac{2\vartheta}{(2-\vartheta)U(\vartheta)} t)^{n+1} \mu_1^{n+1} a.$$

Then the following is obtain at t_0

$$\| \mathcal{M}1_n(t) \| \leq (\tfrac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)} + \tfrac{2\vartheta}{(2-\vartheta)U(\ell)} t_0)^{n+1} \mu_1^{n+1} a. \quad (20)$$

The above Eq. (20) implies that

$$\| \mathcal{M}1_n(t) \| \longrightarrow 0, \quad n \to \infty.$$

Following the same procedure, we obtain that $\mathcal{M}2_n(t), \mathcal{M}3_n(t), \mathcal{M}4_n(t), \mathcal{M}5_n(t)$ approaches to 0 as n tends to ∞.

In the next step, we focus on the uniqueness of the solution of system (3), on contrast, we assume that $(\mathcal{A}_{S1}(t), \mathcal{A}_{E1}(t), \mathcal{A}_{I1}(t), \mathcal{A}_{C1}(t), \mathcal{A}_{P1}(t))$ is another solution of system (3), then we have

$$\mathcal{A}_S(t) - \mathcal{A}_{S1}(t) = \tfrac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)} (\mathcal{L}_1(t, \mathcal{A}_S) - \mathcal{L}_1(t, \mathcal{A}_{S1})) + \tfrac{2\vartheta}{(2-\vartheta)U(\vartheta)} \times \int_0^t (\mathcal{L}_1(y, \mathcal{A}_S) - \mathcal{L}_1(y, \mathcal{A}_{S1}))\, dy. \quad (21)$$

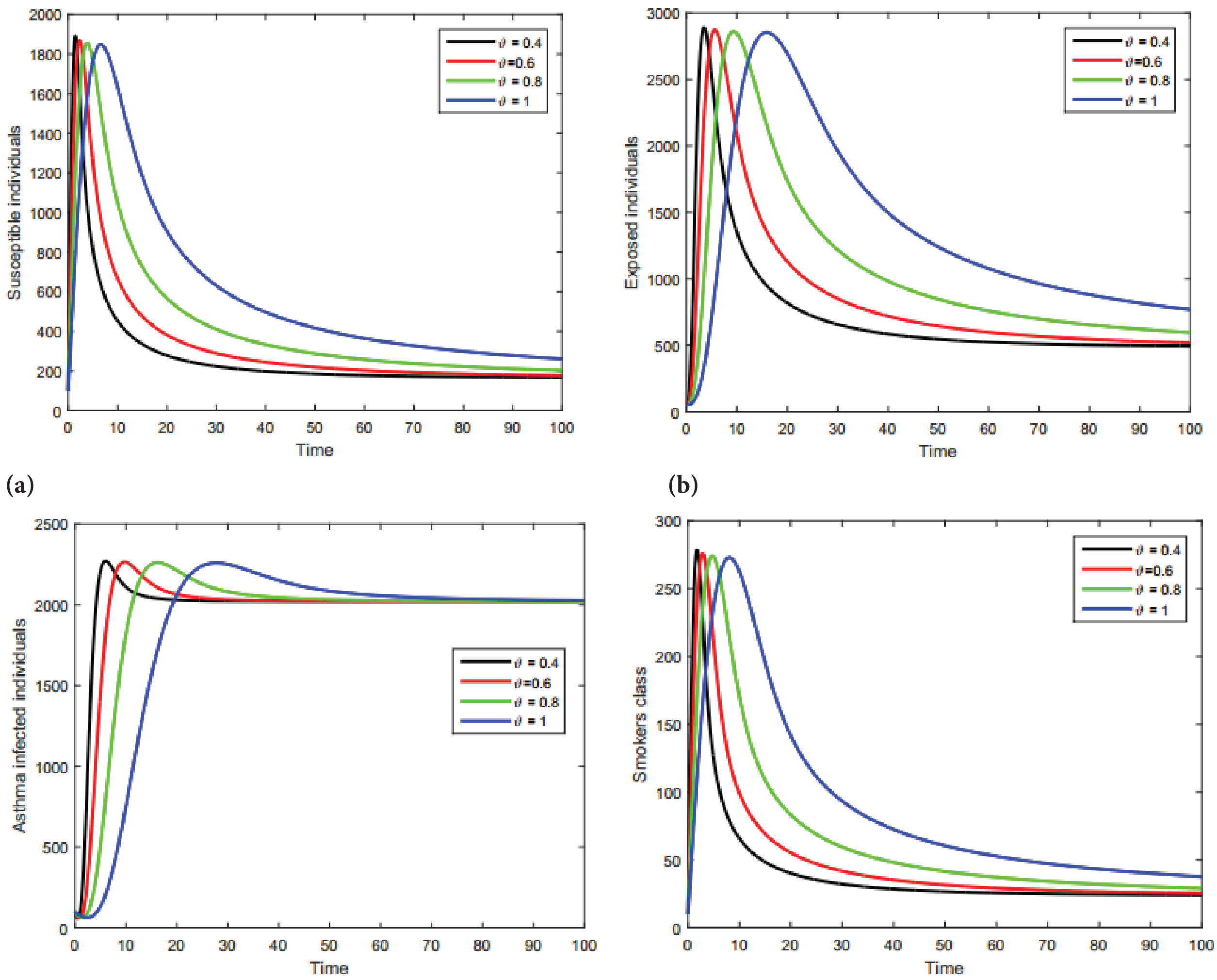
Applying the properties of norm, the above (21) converted into the following

$$\| \mathcal{A}_S(t) - \mathcal{A}_{S1}(t) \| \leq \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)} \| \mathcal{L}_1(t, \mathcal{A}_S) - \mathcal{L}_1(t, \mathcal{A}_{S1}) \| + \frac{2\vartheta}{(2-\vartheta)U(\vartheta)} \times \int_0^t \| \mathcal{L}_1(y, \mathcal{A}_S) - \mathcal{L}_1(y, \mathcal{A}_{S1}) \| \, dy.$$

Here, Lipschitz condition of kernel gives the following

$$\| \mathcal{A}_S(t) - \mathcal{A}_{S1}(t) \| \leq \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)} \mu_1 \| \mathcal{A}_S(t) - \mathcal{A}_{S1}(t) \| + \frac{2\vartheta}{(2-\vartheta)U(\vartheta)} \times \int_0^t \mu_1 t \| \mathcal{A}_S(t) - \mathcal{A}_{S1}(t) \| \, dy,$$

which gives



(a) (b) (c) (d)

Figure 2. Illustration of the time series (solutions pathway) of susceptible, exposed, infected and smokers individuals of the proposed fractional-order model (3) of asthma disease with the variation of fractional-order ϑ.

$$\parallel \mathcal{A}_S(t) - \mathcal{A}_{S1}(t) \parallel (1 - \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_1 - \frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\mu_1 t) \leq 0. \quad (22)$$

Theorem 3 *There exists a unique solution of the fractional-order model (3) of asthma disease if the below condition satisfied*

$$(1 - \frac{2(1-\vartheta)}{(2-\vartheta)U(\vartheta)}\mu_1 - \frac{2\vartheta}{(2-\vartheta)U(\vartheta)}\mu_1 t) > 0. \quad (23)$$

Proof. For the required result, we assume that the above condition (23) holds true then (22) implies that

$$\parallel \mathcal{A}_S(t) - \mathcal{A}_{S1}(t) \parallel = 0,$$

as a result of the uppermentioned, we have

$$\mathcal{A}_S(t) = \mathcal{A}_{S1}(t).$$

Following similar steps, we attain

$$\mathcal{A}_E(t) = \mathcal{A}_{E1}(t), \mathcal{A}_I(t) = \mathcal{A}_{I1}(t),$$

$$\mathcal{A}_C(t) = \mathcal{A}_{C1}(t), \mathcal{A}_P(t) = \mathcal{A}_{P1}(t).$$

RESULTS AND DISCUSSION

We run multiple simulations in this portion of the study to better understand the complex dynamics of the suggested fractional-order asthma model. These simulations are necessary in order to find the most essential input factors that have a substantial impact on asthma prevalence in the general population. The fractional asthma system's dynamical behaviour is quantitatively explored in order to present a more realistic picture of the illness.We assumed the following values of input parameters for simulation purposes: β = 0.0002, γ = 0.00031, μ = 0.014, α = 0.258, λ_1 = 0.0001, λ = 0.00015, η = 0.002, η_1

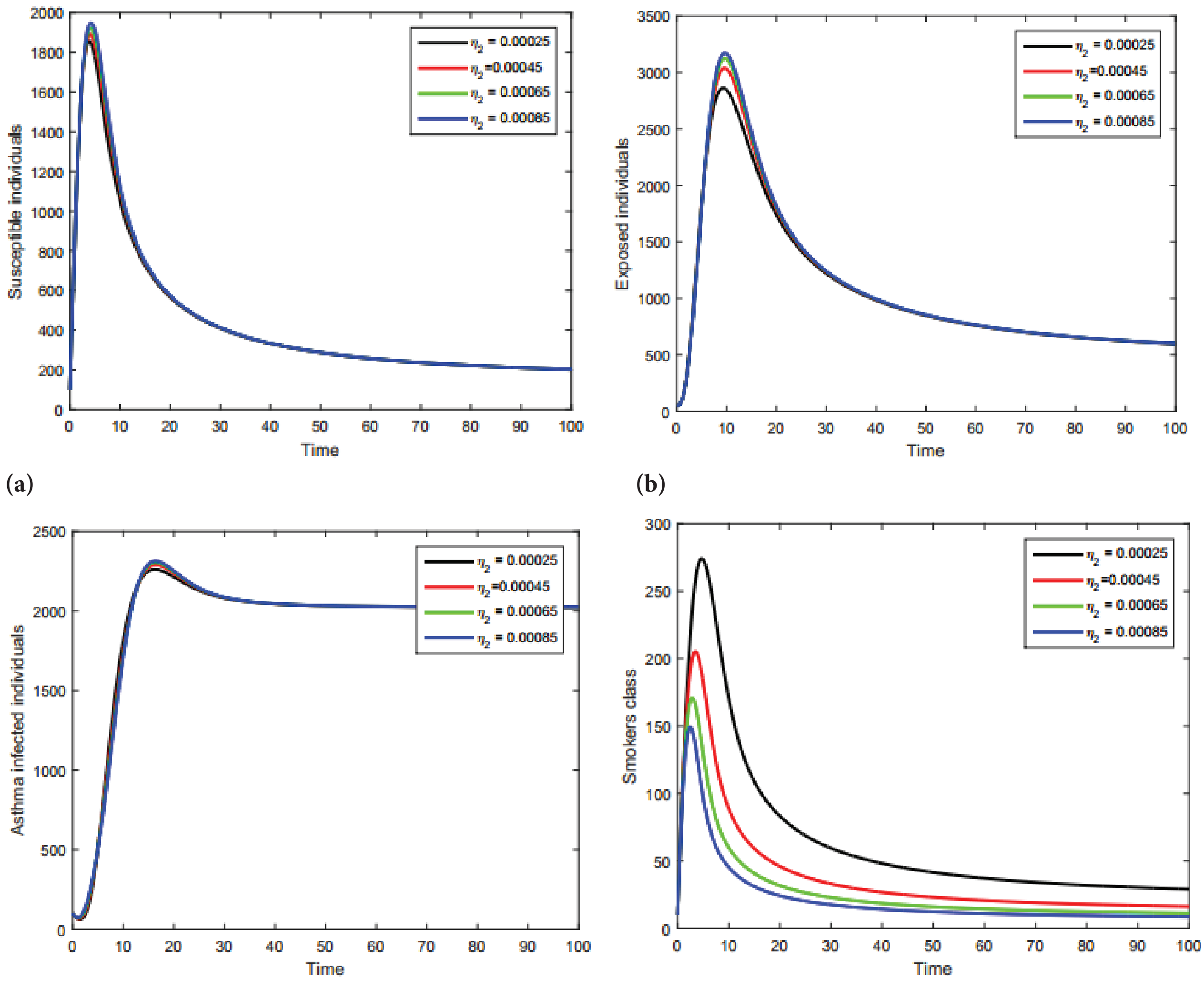


Figure 3. Illustration of dynamical behaviour of our proposed fractional-order model (3) of asthma disease with the variation of input parameter η_2.

= 0.0002, η_2 = 0.00025 and q = 0.0001 where the initial values of state-variables are given by: $\mathcal{A}_S = 100$, $\mathcal{A}_E = 50$, $\mathcal{A}_I = 100$, $\mathcal{A}_C = 10$ and $\mathcal{A}_P = 50$ The time is taken in days in these numerical simulations.

Here, we perform four simulations to conceptualize the solution pathway of the susceptible, exposed, infected and smokers individuals of the asthma system and to show the influence of some input parameters on the dynamical behaviour of the system. In Figure 2, we illustrated the dynamics of asthma with the variation of fractional-order or index of memory ϑ. We observed that the index of memory ϑ has significance influence on solution pathway of asthma model and the control of ϑ can highly control the dynamics of asthma disease. In Figure 3, we have shown the dynamical behaviour of the fractional system of asthma with the variation of the input parameter η_2, i.e., η_2 = 0.00025, 0.00045, 0.00065, 0.00085, which shows that higher the value of η_2 higher will be the level of exposed and infected classes in the community. Thus, we can say that by controlling the smoker class can reduce the infection level of asthma in the community. In third simulation, we demonstrated the solution pathway of the proposed fractional-order model (3) of asthma with the variation of input parameter λ_1, i.e., λ_1 = 0.0001, 0.0003, 0.0005, 0.0007 shown in Figure 4. We noticed in second scenario that the increase of parameter λ_1 highly decreases the level of $\mathcal{A}_E$ while increase the level of asthma infected individuals $\mathcal{A}_I$ in the community. In the last simulation presented in Figure 5, we again illustrated the solution pathway of the fractional asthma system with different values of fractional order, which implies that the endemic level of asthma can be decreased by lowering the fractional order ϑ. Furthermore, we observed that the control of fractional-order can lower the level of infection, therefore, the index of memory ϑ are suggested to the policy makers and medical experts to lower asthma attacks.

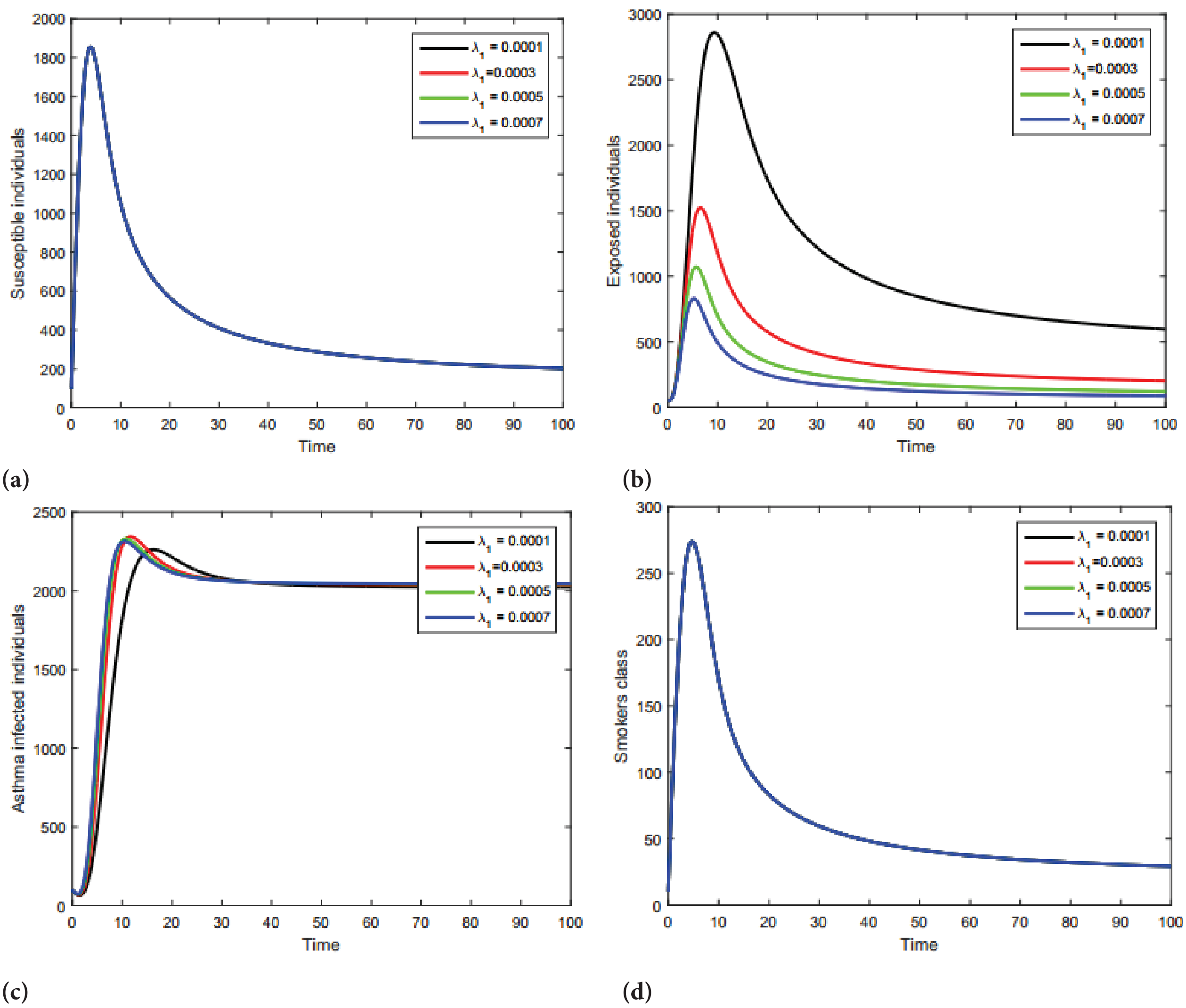


Figure 4. Illustration of the dynamical behaviour of the proposed fractional-order model (3) of asthma disease with the variation of input parameter λ_1.

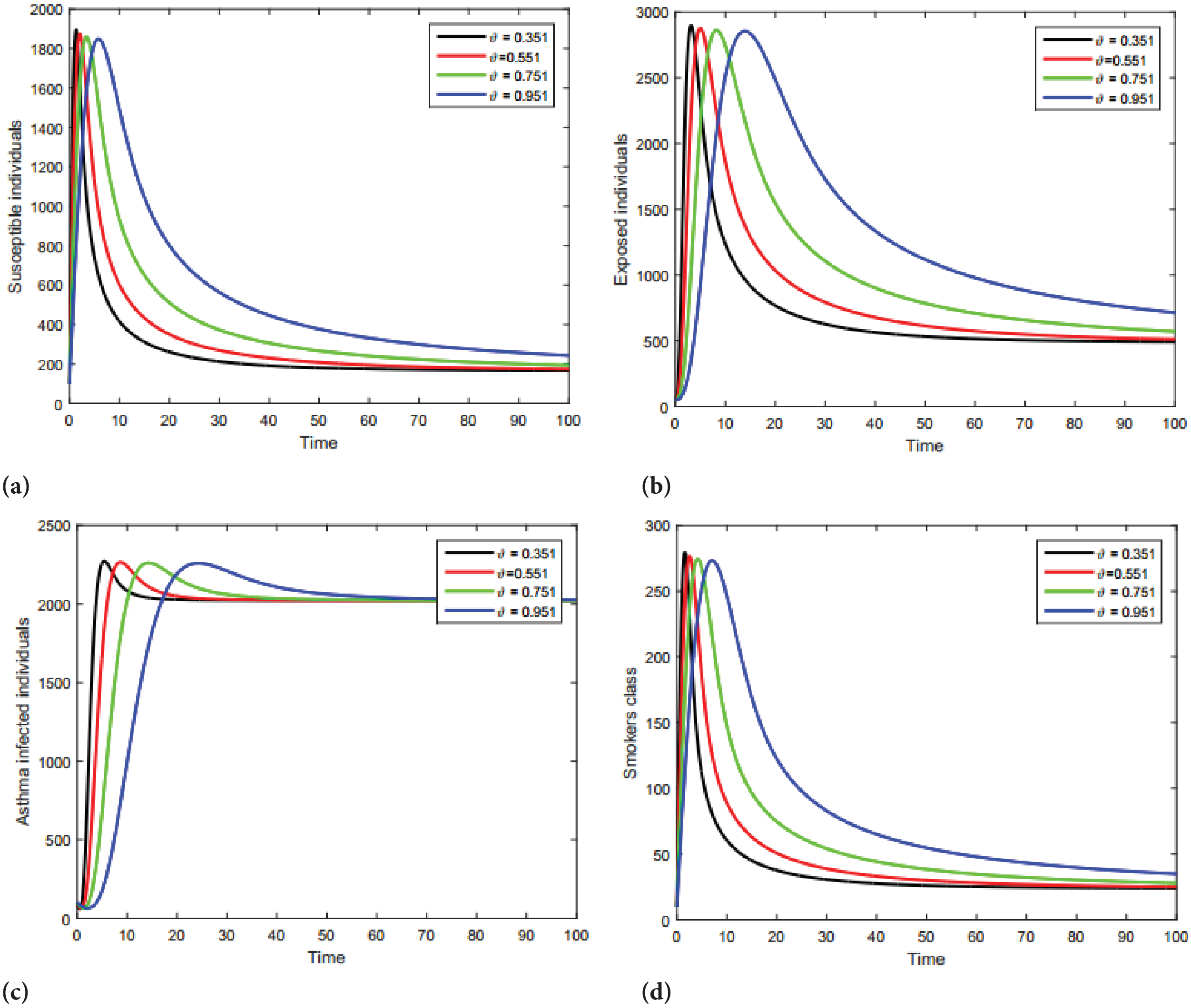


Figure 5. Illustration of the time series (solutions pathway) of susceptible, exposed, infected and smokers individuals of the proposed fractional-order model (3) of asthma disease with the variation of fractional-order ϑ.

CONCLUSION

Asthma is a well-known chronic disease that causes the inside walls of the airways of humans and make it hard to breathe. It is difficult to breath and talk in severe case of asthma which disturbs normal activity of humans. It is reported that smoking and environmental factors are critical and trigger asthma. In this article, we structured a mathematical model for asthma with environmental factors in the fractional derivative framework through CF fractional derivative. The suggested fractional model of asthma disease is then investigated for the basic properties through fractional calculus. The uniqueness and existence of the suggested asthma system are investigated via the theory of fixed point. Furthermore, we represent a novel numerical technique to highlight the time series of fractional-order asthma model. We have shown the dynamical behaviour of different stages of asthma model with variation of fractional-order ϑ. It is predicted that the index of memory ϑ has a positive effect on the system and can be used as a control parameter. Finally, the dynamical behaviour of asthma with input factors are then highlighted numerically to show the influence of several input parameter on the time series of asthma. We suggested to the policy makers that the index of memory or fractional-order can play an important role in the control of asthma. We will further study the epidemiology of asthma to formulate a more comprehensive model for the transmission phenomena of asthma to produce more accurate and precious results with real data in the future work. Moreover, sensitivity test through PRCC method will be performed to determine the most critical input factor for the control and prevention of asthma. We will also introduce some control policies through optimal control theory and impulsive vaccination for the prevention of asthma infection in the society.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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