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Some properties of t-intuitionistic fuzzy H_v-rings

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ABSTRACT

This research redefined T-intuitionistic fuzzy H_v-subring of a ring R and obtained some new related properties. Some of their fundamental relation properties were studied. Especially, under idempotent property, it is given that any IFS defined by a subset of H_v is T-IF H_v-subring of a ring if and only if H is a H_v-subring of the ring. Using this property, the main theorem was given as for a T-intuitionistic fuzzy H_v-subring of any ring with continuous t-norm, a factor subring formed using the hyperring is a T-intuitionistic H_v-subring.

Keywords: H_v-rings, fuzzy H_v-group, intuitionistic fuzzy H_v-ideal, t-norm.

T-intuitionistic fuzzy H_v- halkaların bazı özellikleri

OZ

Bu çalışmada, bir R halkası için T-intuitionistic fuzzy H_v-althalka kavramı yeniden tanımlandı ve bazı yeni özellikleri elde edildi. Bu yapıların bazı temel özellikleri çalışıldı. Özellikle, idempotent özelliği altında, $H_v$nin bir alt kümesi ile tanımlı bir intuitionistic fuzzy kümenin T-intuitionistic H_v-althalka olması için gerek yeter koşulun H alt kümesinin bir H_v-althalka olması gerektiği gösterilmiştir. Bu özellik yardımı ile bir halkanın, sürekli t-norm ile tanımlanmış T-intuitionistic fuzzy H_v-alt halkası için, bir hiperhalkanın faktör halkasının yine bir T-intuitionistic fuzzy H_v-halka olduğu çalışmanın ana teoremi olarak verilmiştir.

Anahtar Kelimeler: H_v-halkalar, fuzzy H_v-grup, intuitionistic fuzzy H_v-ideal, t-norm.

1. INTRODUCTION

Zadeh is first researcher who defined the fuzzy set notion of a nonempty set, [10]. After this definition, several author given some generalizations of this structure. Intuitionistic fuzzy sets were defined as two member and nonmember degrees by Atanassov [1]. The hyperstructure theory has been firstly introduced by Marty, [7]. This new field have been worked on modern algebra, also several authors developed it, [9]. Vougiouklis gave the definition of H_v-rings, [9]. H_v-ring is another type algebraic systems which is satisfying the ring structure axioms. So, it satisfied the properties of the concept of ring theory. The special concept of fuzzy subhypergroup especially the fuzzy H_v-group were studied by Davvaz [3]. Davvaz defined the fuzzy H_v-ideal of an H_v-ring. Davvaz, Dudek were firstly defined the intuitionistic fuzzy H_v-ideal of an H_v-ring, [4]. This research redefined T-intuitionistic fuzzy H_v-subring of a ring R using continuous t-norms. After this definition, we obtained more general consequences than the previous studies. We gave a main theorem which is show

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that the property being H_v-subring of a T-intuitionistic fuzzy subring is also moved on the factor rings.

Definition: [10] Let X be a universal set is nonempty then $\mu : X \to [0,1]$ is called a fuzzy set on X. The complement of the fuzzy set μ is the fuzzy set which is given by $1-\mu(x)$ for all $x \in X$, denoted by μ^c.

Definition: [1] X be set. An intuitionistic fuzzy set (IFS) on a set X is an set as follow,

$A = \{<x, \mu_A(x), \nu_A(x)>: x \in X\}$

In here, where $\mu_A(x), (\mu_A : X \to [0,1])$ is the membership degree of x in A, $\nu_A(x), (\nu_A : X \to [0,1])$ is the non-membership degree of x and where μ_A and ν_A satisfy the following condition:

$\mu_A(x) + \nu_A(x) \le 1,$ for all $x \in X$.

We will show an IFS as $A = (\mu_A, \nu_A)$ instead of $A = \{<x, \mu_A(x), \nu_A(x)>: x \in X\}$.

Definition: [2] Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be IFSs in X. Then

1. $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$.
2. $A^c = \{<x, \nu_A(x), \mu_A(x)>: x \in X\}$
3. $A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)>: x \in X\}$
4 $A \cup B = \{<x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)>: x \in X\}$
5. $A = B :\Leftrightarrow A \subseteq B \wedge B \subseteq A$

Definition: [7] Let H be a non-empty set, the H is a hyperstructure with a hyperoperation map $*: H \times H \to P^*(H)$, in here $P^*(H)$ is the set of subsets of H which are non-empty. The $*(x,y)$ is signed by $x*y$. If x element of H and $A, B \subseteq H$, then we define $A*B = \bigcup_{a\in A, b\in B} a*b$, $A*x = A*\{x\}$, $x*B = \{x\}*B$.

Definition: [3] A $(H,*)$ hyperstructure is called a hypergroup if we have the following axioms,

1. $(H,*)$ is a semihypergroup, i.e.
$\forall x,y,z \in H, (x*(y*z)) = ((x*y)*z)$
2. $x*H = H*x = H$ for all x in H

Definition: [8] An H_v-ring is a system if with two hyperoperations on R satisfying the following axioms:

1. $(R,+,\cdot)$ is an H_v -group, for all $a \in R$,
$a + R = R + a = R$
$\forall x,y,z \in H, (x+(y+z)) \cap ((x+(y+z)) \ne \varnothing$
2. $(R,\cdot)$ is an H_v -semigroup, i.e.,
$\forall x,y,z \in R,\ ((x\cdot y)\cdot z) \cap (x\cdot(y\cdot z)) \ne \varnothing$
3. "$\cdot$" is weak distributive to "$+$", i.e., for all $x,y,z \in R$,
$((x+y)\cdot z) \cap (x\cdot z + y\cdot z) \ne \varnothing$
$(x\cdot(y+z)) \cap (x\cdot y + x\cdot z) \ne \varnothing$

Definition: [3] Let H be a set, $(H,\cdot)$ be a hypergroup and let μ be a fuzzy set on H. Then μ called a fuzzy H_v-subgroup of H if the followings are satisfied,

1. $\min\{\mu(x), \mu(y)\} \le \inf_{\alpha \in x\cdot y}\{\mu(\alpha)\}$, for all $x, y \in H$
2. for all elements x,a there exists an element y such that $x \in a\cdot y$ and $\min\{\mu(a), \mu(x)\} \le \mu(y)$

Definition: [3] If $(H,\cdot)$ be an H_v -group and let $\mu \in FS(H)$ then μ is said to be a T-fuzzy H_v -subgroup of H with repect to T-norm T if the followings hold:

1. $T(\mu(x), \mu(y)) \le \inf_{\alpha \in x\cdot y}\{\mu(\alpha)\}$, for all $x, y \in H$
2. for all elements x,a there exists an element y such that $x \in a\cdot y$ and $T(\mu(a), \mu(x)) \le \mu(y)$.

Definition: [4] If μ a fuzzy subset of R and R be an H_v -ring. If the following axioms hold:

1. $\min\{\mu(x), \mu(y)\} \le \inf\{\mu(b) : b \in x+y\}$, for all $x, y \in R$
2. for all elements x,a there exists an element y such that $x \in a+y$ and $\min\{\mu(a), \mu(x)\} \le \mu(y)$
3. for all elements x,a there exists an element b such that $x \in b+a$ and $\min\{\mu(a), \mu(x)\} \le \mu(b)$
4. $\mu(y) \le \inf\{\mu(b) : b \in x\cdot y\}$
($\mu(x) \le \inf\{\mu(b) : b \in x\cdot y\}$) for all $x, y \in R$

then μ is said to be a left (right) fuzzy H_v -ideal of R

Definition: [4] An IFS $A = (\mu_A, \nu_A)$. If we have the following conditions

1. $\min\{\mu_A(x), \mu_A(y)\} \le \inf\{\mu_A(b) : b \in x+y\}$, for all $x, y \in R$
2. for all $x, a \in R$ there exists $y, b \in R$ such that $x \in (a+y) \cap (b+a)$ and
$\min\{\mu_A(a), \mu_A(x)\} \le \min\{\mu_A(y), \mu_A(b)\}$
3. $\mu_A(y) \le \inf\{\mu_A(b) : b \in x\cdot y\}$ (resp., $\mu_A(x) \le \inf\{\mu_A(b) : b \in x\cdot y\}$) for all $x, y \in R$
4. $\sup\{\nu_A(b) : b \in x+y\} \le \max\{\nu_A(x), \nu_A(y)\}$, for all $x, y \in R$
5. for all $x, a \in R$ there exists $y, b \in R$ such that $x \in (a+y) \cap (b+a)$ and
$\max\{\nu_A(y), \nu_A(b)\} \le \max\{\nu_A(a), \nu_A(x)\}$
6. $\sup\{\nu_A(b) : b \in x\cdot y\} \le \nu_A(y)$
(resp., $\sup\{\nu_A(b) : b \in x\cdot y\} \le \nu_A(x)$) for all $x, y \in R$

then A is called a left (resp., right) IF H_v -ideal of R.

Definition: [4] The function $T : [0,1]\times[0,1] \to [0,1]$ if satisfy the followings:

1. $T(x,1) = x$

2. $T(x,y) \le T(x,z)$ if $y \le z$
3. $T(x,y) = T(y,x)$
4. $T(x,T(y,z)) = T(T(x,y),z)$ for all $x,y,z \in [0,1]$

considering a t-norm T on [0,1], set of the elements $\alpha \in [0,1]$ such that $T(\alpha,\alpha) = \alpha$ is denoted by Δ_T. i.e., $\Delta_T := \{\alpha \in [0,1] \ : \ T(\alpha,\alpha) = \alpha\}$

Proposition: [4] Every t-norm T has a property $T(\alpha,\beta) \le \min(\alpha,\beta)$ for all $\alpha,\beta \in [0,1]$

Definition: [4] Let T be a t-norm. if $\mathrm{Im}(\mu) \subseteq \Delta_T$ then it is said that the subset μ of R have idempotent property.

Definition: [6] A t-norm T is continuous if we have
$T\left(\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n\right) = \lim_{n\to\infty} T(x_n,y_n)$
for the $\{x_n\}$, $\{y_n\}$ convergent sequences.

2. ON INTUITIONISTIC FUZZY HYPERSTRUCTURE WITH T-NORM

Definition: Let $(R,+,\cdot)$ be an H_v -ring and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of R. Then $A = (\mu_A, \nu_A)$ is said to be a T-intuitionistic fuzzy H_v -subring of R with respect to t-norm T if the following axioms hold

1. $T(\mu_A(x), \mu_A(y)) \le \inf\{\mu_A(b) \ : \ z \in x+y\}$, for all $x,y \in R$
2. $\sup\{\nu_A(b) \ : \ b \in x+y\} \le 1 - T(1-\nu_A(x), 1-\nu_A(y))$ for all $x,y \in R$
3. for all $x,a \in R$ there exists $y,b \in R$ such that $x \in (a+y) \cap (b+a)$ and $T(\mu_A(a), \mu_A(x)) \le T(\mu_A(y), \mu_A(b))$
4. $T(\mu_A(x), \mu_A(y)) \le \inf\{\mu_A(b) \ : \ b \in x \cdot y\}$, for all $x,y \in R$
5. $\sup\{\nu_A(z) \ : \ z \in x \cdot y\} \le 1 - T(1-\nu_A(x), 1-\nu_A(y))$, for all $x,y \in R$
6. for all $x,a \in R$ there exists $y,b \in R$ such that $x \in (a+y) \cap (b+a)$ and $T(1-\nu_A(a), 1-\nu_A(x)) \le T(1-\nu_A(y), 1-\nu_A(b))$

Proposition: Let T be an t- norm and $A = (\mu_A, \nu_A)$ be an T-intuitionistic fuzzy H_v -subring of R. Let μ_A, $1-\nu_A$ have idempotent property. Then the following sets are H_v -subring of R

$R^w = \{x \in R \ : \ \mu_A(x) \ge \mu_A(w)\}$

$L^w = \{x \in R \ : \ \nu_A(x) \le \nu_A(w)\}$

Proof: Let $x,y \in R^w$. Then $\mu_A(x) \ge \mu_A(w)$ and $\mu_A(y) \ge \mu_A(w)$ Since $A = (\mu_A, \nu_A)$ be an T-intuitionistic fuzzy H_v-subring of R and μ_A have idempotent property, it follows that

$$\inf\{\mu_A(b) \ : \ z \in x+y\} \ge T(\mu_A(x), \mu_A(y)) \ge T(\mu_A(x), \mu_A(w)) \ge T(\mu_A(w), \mu_A(w)) = \mu_A(w)$$

Hence $x+y \subseteq R^w$ implies $x+y \in P^*(R^w)$. Similarly, $x \cdot y \subseteq R^w$ and $x \cdot y \in P^*(R^w)$ exist. Hence $a + R^w \subseteq R^w$ and $R^w + a \subseteq R^w$ for all $a \in R^w$

Now, let $x \in R^w$. Then there exist $y,b \in R$ such that $x \in (a+y) \cap (b+a)$ and

$$T(\mu_A(a), \mu_A(x)) \le T(\mu_A(y), \mu_A(b))$$

Since a , $x \in R^w$, we have

$$\mu_A(w) = T(\mu_A(w), \mu_A(w)) \le T(\mu_A(a), \mu_A(x))$$

and so

$$\mu_A(w) \le T(\mu_A(y), \mu_A(b)) \le \min\{\mu_A(y), \mu_A(b)\}$$

which implies $y \in R^w$ and $b \in R^w$.

This proves that $R^w \subseteq a + R^w$ and $R^w \subseteq R^w + a$. Since $(R,+,\cdot)$ is an H_v -group and $R^w \subseteq R$ then for all $x,y,b \in R^w$,

$$((x+y)+b) \cap (x+(y+b)) \ne \varnothing$$
$$((x+y)\cdot b) \cap (x \cdot b + y \cdot b) \ne \varnothing$$
$$(x \cdot (y+b)) \cap (x \cdot y + x \cdot b) \ne \varnothing$$
$$((x \cdot y)\cdot b) \cap (x \cdot (y \cdot b)) \ne \varnothing$$

Consequently R^w be an H_v -subring of R. If $x,y \in L^w$ afterwards $\nu_A(x) \le \nu_A(w)$ and $\nu_A(y) \le \nu_A(w)$. Since $A = (\mu_A, \nu_A)$ be an T-intuitionistic fuzzy H_v -subring of R and $1-\nu_A$ have idempotent property, it follows that

$$\sup\{\nu_A(b) : b \in x+y\} \le 1 - T(1-\nu_A(x), 1-\nu_A(y)) \le 1 - T(1-\nu_A(w), 1-\nu_A(w)) = \nu_A(w)$$

Hence $x+y \subseteq L^w$.Similarly, we have $x \cdot y \subseteq L^w$. Hence $a + L^w \subseteq L^w$ and $L^w + a \subseteq L^w$ for all $a \in L^w$. Let $x \in L^w$, then there exist $y,b \in R$ such that $x \in (a+y) \cap (b+a)$ and

$$T(1-\nu_A(a), 1-\nu_A(x)) \le T(1-\nu_A(y), 1-\nu_A(b))$$

Since a, $x \in L^w$, we have

$$1-\nu_A(w) = T(1-\nu_A(w), 1-\nu_A(w)) \le T(1-\nu_A(w), 1-\nu_A(x)) \le T(1-\nu_A(a), 1-\nu_A(x))$$

and so

$$1-\nu_A(w) \le T(1-\nu_A(y), 1-\nu_A(b)) \le \min\{1-\nu_A(y), 1-\nu_A(b)\}$$

That signifies $y \in L^w$ and this proves that $L^w \subseteq a + L^w$ and $L^w \subseteq L^w + a$. Since $(R,+,\cdot)$ is an H_v -group and $L^w \subseteq R$ then for all $x, y, b \in L^w$,

$$((x+y)+b) \cap (x+(y+b)) \neq \emptyset$$
$$((x+y)\cdot b) \cap (x\cdot b + y\cdot b) \neq \emptyset$$
$$(x\cdot(y+b)) \cap (x\cdot y + x\cdot b) \neq \emptyset$$
$$((x\cdot y)\cdot b) \cap (x\cdot(y\cdot b)) \neq \emptyset$$

Consequently L^w be an H_v -subring of R.

Proposition: Let H be a nonempty subset of a H_v -ring R and let μ, ν are fuzzy sets in R defined by

$$\mu(x) = \begin{cases} \alpha_0 & , \ x \in H \\ \alpha_1 & , \text{ otherwise} \end{cases}, \quad \nu(x) = \begin{cases} \beta_0 & , \ x \in H \\ \beta_1 & , \text{ otherwise} \end{cases}$$

where $0 \le \alpha_1 < \alpha_0$, $0 \le \beta_0 < \beta_1$ and $\alpha_i + \beta_i \le 1$ for $i = 0,1$. Let μ, $1-\nu$ have idempotent property. Then $A = (\mu, \nu)$ be an T-intuitionistic fuzzy H_v -subring of R $\Leftrightarrow$ H is a H_v -subring of R.

Proof: Suppose that $A = (\mu, \nu)$ be an T-intuitionistic fuzzy H_v -subring of R. Let $x, y \in H$. Then

$$\inf\{\mu(b) : b \in x+y\} \ge T(\mu(x), \mu(y)) = T(\alpha_0, \alpha_0) = \alpha_0,$$

It follows that $x + y \subseteq H$. Similarly, we have $x \cdot y \subseteq H$. Hence $a + H \subseteq H$ and $H + a \subseteq H$, for all $a \in H$. Let $x \in H$ Then there exist $y, b \in R$ such that $x \in (a+y) \cap (b+a)$ and $T(\mu(a), \mu(x)) \le T(\mu(y), \mu(b))$ Since a, $x \in H$, we have

$$a_0 = T(\mu(a), \mu(x)) \le T(\mu(y), \mu(b)) \le \min\{\mu(y), \mu(b)\}$$

which implies $y \in H$ and $b \in H$. This proves $H \subseteq a + H$ and $H \subseteq H + a$. Since $(R,+,\cdot)$ is a H_v-group and $H \subseteq R$ then for all $x, y, b \in H$,

$$((x+y)+b) \cap (x+(y+b)) \neq \emptyset$$
$$((x+y)\cdot b) \cap (x\cdot b + y\cdot b) \neq \emptyset$$
$$(x\cdot(y+b)) \cap (x\cdot y + x\cdot b) \neq \emptyset$$
$$((x\cdot y)\cdot b) \cap (x\cdot(y\cdot b)) \neq \emptyset$$

Therefore H is a H_v -subring of R. Conversely suppose that H is a H_v -subring of R. Let $x, y \in R$. If $x \in R \setminus H$ or $y \in R \setminus H$, then $\mu(x) = \alpha_1$ or $\mu(y) = \alpha_1$ and so

$$\begin{aligned} &\inf\{\mu(b) : b \in x+y\} \\ &\ge \min\{\mu(x), \mu(y)\} = \alpha_1 \\ &\ge T(\mu(x), \mu(y)) \end{aligned}$$

Assume that $x \in H$ and $y \in H$. Then $x + y \subseteq H$ and hence

$$\begin{aligned} &\inf\{\mu(b) : b \in x+y\} \\ &\ge \min\{\mu(x), \mu(y)\} = \alpha_0 \\ &\ge T(\mu(x), \mu(y)) \end{aligned}$$

Let $x, y \in R$. If $x \in R \setminus H$ or $y \in R \setminus H$, then $\nu(x) = \beta_1$ or $\nu(y) = \beta_1$ and so

$$\begin{aligned} &\sup\{\nu(b) : b \in x+y\} \le \beta_1 \\ &= \max\{\nu(x), \nu(y)\} \\ &= 1 - \min\{1-\nu(x), 1-\nu(y)\} \\ &\le 1 - T(1-\nu(x), 1-\nu(y)) \end{aligned}$$

Assume that $x \in H$ and $y \in H$. Then $x + y \subseteq H$ and hence

$$\begin{aligned} &\sup\{\nu(b) : b \in x+y\} \\ &\le \beta_0 = \max\{\nu(x), \nu(y)\} \\ &= 1 - \min\{1-\nu(x), 1-\nu(y)\} \\ &\le 1 - T(1-\nu(x), 1-\nu(y)) \end{aligned}$$

Let $x, y \in R$. If $x \in R \setminus H$ or $y \in R \setminus H$, then $\mu(x) = \alpha_1$ or $\mu(y) = \alpha_1$ and so

$$\begin{aligned} &\inf\{\mu(b) : b \in x\cdot y\} \\ &\ge \min\{\mu(x), \mu(y)\} = \alpha_1 \\ &\ge T(\mu(x), \mu(y)) \end{aligned}$$

Assume that $x \in H$ and $y \in H$. Then $x + y \subseteq H$ and hence

$$\begin{aligned} &\inf\{\mu(b) : b \in x\cdot y\} \\ &\ge \min\{\mu(x), \mu(y)\} = \alpha_0 \\ &\ge T(\mu(x), \mu(y)) \end{aligned}$$

Let $x, y \in R$. If $x \in R \setminus H$ or $y \in R \setminus H$, then $\nu(x) = \beta_1$ or $\nu(y) = \beta_1$ and so

$$\begin{aligned} &\sup\{\nu(b) : b \in x\cdot y\} \\ &\le \beta_1 = \max\{\nu(x), \nu(y)\} \\ &= 1 - \min\{1-\nu(x), 1-\nu(y)\} \\ &\le 1 - T(1-\nu(x), 1-\nu(y)) \end{aligned}$$

Assume that $x \in H$ and $y \in H$. Then $x + y \subseteq H$ and hence

$$\begin{aligned} &\sup\{\nu(b) : b \in x\cdot y\} \\ &\le \beta_0 = \max\{\nu(x), \nu(y)\} \\ &= 1 - \min\{1-\nu(x), 1-\nu(y)\} \\ &\le 1 - T(1-\nu(x), 1-\nu(y)) \end{aligned}$$

Let $x, a \in R$ Since R H_v -ring then there exists $y, b \in R$ such that $x \in (a+y) \cap (b+a)$. If $x \in R \setminus H$ or $a \in R \setminus H$, then $\mu(x) = \alpha_1$ or $\mu(a) = \alpha_1$ and hence $\mu(x) \le \mu(y)$, $\mu(a) \le \mu(b)$. And so

$$T(\mu(a), \mu(x)) \le T(\mu(y), \mu(b))$$

Assume that $x \in H$ and $a \in H$. Since H is a H_v -subring of R, there exists $y, z \in H$, in that $x \in (a+y) \cap (b+a)$. Then $\mu(x) = \mu(y) = \mu(a) = \mu(b) = \alpha_0$ and so

$$T(\mu(a), \mu(x)) \le T(\mu(y), \mu(b))$$

Similarly, we have for all $x, a \in R$ there exists $y, b \in R$ such that $x \in (a+y) \cap (b+a)$ and

$$T(1-\nu(a), 1-\nu(x)) \le T(1-\nu(y), 1-\nu(b))$$

Consequently $A = (\mu, \nu)$ be an T-intuitionistic fuzzy H_v -subring of R.

Definition: [5] Let $(R,+,\cdot)$ be an H_v -ring.The relation γ_R^* is the smallest equivalence relation on R such that the quotient R/γ_R^*, the set of all equivalence classes is a ring. γ_R^* is called the fundamental relation on R and R/γ_R^* is called the fundamental ring.

If Ω denotes the set of all finite polynomials of elements of R, over $\mathbb{N}$ (the set of all natural numbers), then a relation γ_R can be defined on R whose transitive closure is the fundamental relation γ_R^*.

The relation γ_R is as follow; For x,y in R, we write $x\gamma_R y$ if and only if $\{x,y\}\subseteq\Lambda$ for some $\Lambda\in\Omega$.. Suppose $\gamma_R^*(a)$ is the equivalence class containing $a\in R$ .Then both the sum $\oplus$ and the product $\odot$ on R/γ_R^* are defined as follows:

$$\gamma_R^*(a)\oplus\gamma_R^*(b)=\gamma_R^*(c)\ ,\quad \text{for all } c\in\gamma_R^*(a)+\gamma_R^*(b)$$
$$\gamma_R^*(a)\odot\gamma_R^*(b)=\gamma_R^*(d)\ ,\quad \text{for all } d\in\gamma_R^*(a)\cdot\gamma_R^*(b)$$

Here we also denote ω_R the zero element of R/γ_R^*.

Definition: [4] Let $(R,+,\cdot)$ be an H_v-ring and $A=(\mu_A,\nu_A)$ be an left intuitionistic fuzzy H_v-ideal of R. The IFS A/γ_R^* $=\left(\mu_{\gamma_R^*},\nu_{\gamma_R^*}\right)$ is defined as following:

$$\mu_{\gamma_R^*}:R/\gamma_R^*\to[0,1]$$

$$\mu_{\gamma_R^*}\left(\gamma_R^*(x)\right)=\begin{cases}\sup\{\mu_A(a)\ :\ a\in\gamma_R^*(x)\} & ,\gamma_R^*(x)\neq w_R\\ 1 & ,\gamma_R^*(x)=w_R\end{cases}$$

and

$$\nu_{\gamma_R^*}:R/\gamma_R^*\to[0,1]$$

$$\nu_{\gamma_R^*}\left(\gamma_R^*(x)\right)=\begin{cases}\inf\{\nu_A(a)\ :\ a\in\gamma_R^*(x)\} & ,\gamma_R^*(x)\neq w_R\\ 0 & ,\gamma_R^*(x)=w_R\end{cases}$$

Theorem: Let T be a t-norm, continuous and $A=(\mu_A,\nu_A)$ be an T-intuitionistic fuzzy H_v -subring of R. Considering R/γ_R^* as a hyperring, then A/γ_R^* $=\left(\mu_{\gamma_R^*},\nu_{\gamma_R^*}\right)$ is a T-intuitionistic H_v -subring of R/γ_R^*.

Proof: We choose $\gamma_R^*(x),\ \gamma_R^*(y)\in R/\gamma_R^*$. Then we can write:

$$\begin{aligned}
&T\left(\mu_{\gamma_R^*}\left(\gamma_R^*(x)\right),\mu_{\gamma_R^*}\left(\gamma_R^*(y)\right)\right)\\
&=T\left(\sup_{a\in\gamma_R^*(x)}\{\mu_A(a)\}\ ,\ \sup_{b\in\gamma_R^*(y)}\{\mu_A(b)\},\right)\\
&=\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{T\left(\mu_A(a),\mu_A(b)\right)\right\}\\
&\le\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{\inf\{\mu_A(z)\ :\ z\in a+b\}\right\}\\
&\le\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{\sup\{\mu_A(z)\ :\ z\in a+b\}\right\}\\
&\le\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{\sup\{\mu_A(z)\ :\ z\in\gamma_R^*(a+b)\}\right\}\\
&=\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{\mu_{\gamma_R^*}\left(\gamma_R^*(a+b)\right)\right\}\\
&=\mu_{\gamma_R^*}\left(\gamma_R^*(a+b)\right)=\mu_{\gamma_R^*}\left(\gamma_R^*(a)\oplus\gamma_R^*(b)\right)
\end{aligned}$$

Thus the first condition of Definition is provided. If we choose $\gamma_R^*(x),\ \gamma_R^*(y)\in R/\gamma_R^*$. Then we can write:

$$\begin{aligned}
&T\left(1-\nu_{\gamma_R^*}\left(\gamma_R^*(x)\right),1-\nu_{\gamma_R^*}\left(\gamma_R^*(y)\right)\right)\\
&=T\left(1-\inf_{a\in\gamma_R^*(x)}\{\nu_A(a)\},1-\inf_{b\in\gamma_R^*(y)}\{\nu_A(b)\}\right)\\
&=T\left(\sup_{a\in\gamma_R^*(x)}\{1-\nu_A(a)\}\ ,\ \sup_{b\in\gamma_R^*(y)}\{1-\nu_A(b)\},\right)\\
&=\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{T\left(1-\nu_A(a),1-\nu_A(b)\right)\right\}\\
&\le\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{1-\sup\{\nu_A(z)\ :\ z\in a+b\}\right\}\\
&\le\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{1-\inf\{\nu_A(z)\ :\ z\in a+b\}\right\}\\
&\le\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{1-\inf\{\nu_A(z)\ :\ z\in\gamma_R^*(a+b)\}\right\}\\
&=\sup_{b\in\gamma_R^*(y),a\in\gamma_R^*(x)}\left\{1-\nu_{\gamma_R^*}\left(\gamma_R^*(a+b)\right)\right\}\\
&=1-\nu_{\gamma_R^*}\left(\gamma_R^*(a+b)\right)\\
&=1-\nu_{\gamma_R^*}\left(\gamma_R^*(a)\oplus\gamma_R^*(b)\right)
\end{aligned}$$

From above, Definition is verified. Now suppose $\gamma_R^*(x)$ and $\gamma_R^*(a)$ are two arbitrary elements of R/γ_R^*. Since $A=(\mu_A,\nu_A)$ be an T-intuitionistic fuzzy H_v -subring of R: From above, for all $r\in\gamma_R^*(a),\ s\in\gamma_R^*(x)$ there exists $y_{r,s}$, $z_{r,s}\in R$ such that $r\in(s+y_{r,s})\cap(z_{r,s}+s)$ and

$$T\left(\mu_A(r),\mu_A(s)\right)\le T\left(\mu_A(y_{r,s}),\mu_A(z_{r,s})\right)$$

From $r\in(s+y_{r,s})\cap(z_{r,s}+s)$ it follows that

$$\gamma_R^*(s)\oplus\gamma_R^*(y_{r,s})=\gamma_R^*(r)\ ,\ \gamma_R^*(z_{r,s})\oplus\gamma_R^*(s)=\gamma_R^*(r)$$

which implies

$$\gamma_R^*(x)\oplus\gamma_R^*(y_{r,s})=\gamma_R^*(a)\ ,\ \gamma_R^*(z_{r,s})\oplus\gamma_R^*(x)=\gamma_R^*(a)$$

Now if $r_1\in\gamma_R^*(a)$ and $s_1\in\gamma_R^*(x)$, then there exists there exists y_{r_1,s_1}, $z_{r_1,s_1}\in R$ such that

$$\gamma_R^*(s_1)\oplus\gamma_R^*(y_{r_1,s_1})=\gamma_R^*(r_1)$$

and since $\gamma_R^*(r_1)=\gamma_R^*(r)$ we get $\gamma_R^*(s_1)\oplus\gamma_R^*(y_{r_1,s_1})=\gamma_R^*(s)\oplus\gamma_R^*(y_{r,s})$ and therefore $\gamma_R^*(y_{r,s})=\gamma_R^*(y_{r_1,s_1})$. Similarly, we have $\gamma_R^*(z_{r,s})=\gamma_R^*(z_{r_1,s_1})$. So all the $y_{r,s}$, $z_{r,s}$ satisfying $T\left(\mu_A(r),\mu_A(s)\right)\le T\left(\mu_A(y_{r,s}),\mu_A(z_{r,s})\right)$ have the same equivalence class. Now we have:

$$\begin{aligned}
&T\left(\mu_{\gamma_R^*}\left(\gamma_R^*(x)\right),\mu_{\gamma_R^*}\left(\gamma_R^*(a)\right)\right)\\
&=T\left(\sup_{r\in\gamma_R^*(a)}\{\mu_A(r)\}\ ,\ \sup_{s\in\gamma_R^*(x)}\{\mu_A(s)\}\right)\\
&=\sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\left\{T\left(\mu_A(r),\mu_A(s)\right)\right\}\\
&\le\sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\left\{T\left(\mu_A(y_{r,s}),\mu_A(z_{r,s})\right)\right\}\\
&=T\left(\sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{\mu_A(y_{r,s})\}\ ,\ \sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{\mu_A(z_{r,s})\}\right)
\end{aligned}$$

$$\leq T\left(\sup_{y\in\gamma_R^*(y_{r,s})}\{\mu_A(y)\},\ \sup_{z\in\gamma_R^*(z_{r,s})}\{\mu_A(z)\}\right)$$
$$= T\left(\mu_{\gamma_R^*}\left(\gamma_R^*(y_{r,s})\right),\ \mu_{\gamma_R^*}\left(\gamma_R^*(z_{r,s})\right)\right)$$

and Definition is satisfied. Similary, we have

$$T\left(1-\nu_{\gamma_R^*}(\gamma_R^*(a)), 1-\nu_{\gamma_R^*}(\gamma_R^*(x))\right)$$
$$= T\left(1-\inf_{r\in\gamma_R^*(a)}\{\nu_A(r)\}, 1-\inf_{s\in\gamma_R^*(x)}\{\nu_A(s)\}\right)$$
$$= T\left(\sup_{r\in\gamma_R^*(a)}\{1-\nu_A(r)\}, \sup_{s\in\gamma_R^*(x)}\{1-\nu_A(s)\}\right)$$
$$= \sup_{r\in\gamma_R^*(a), s\in\gamma_R^*(x)}\{T(1-\nu_A(r), 1-\nu_A(s))\}$$
$$\leq \sup_{r\in\gamma_R^*(a), s\in\gamma_R^*(x)}\{T(1-\nu_A(y_{r,s}), 1-\nu_A(z_{r,s}))\}$$
$$= T\left(\sup_{r\in\gamma_R^*(a)}\{1-\nu_A(y_{r,s})\}, \sup_{s\in\gamma_R^*(x)}\{1-\nu_A(z_{r,s})\}\right)$$
$$= T\left(1-\inf_{r\in\gamma_R^*(a), s\in\gamma_R^*(x)}\{\nu_A(y_{r,s})\}, 1-\inf_{r\in\gamma_R^*(a), s\in\gamma_R^*(x)}\{\nu_A(z_{r,s})\}\right)$$
$$\leq T\left(1-\inf_{y\in\gamma_R^*(y_{r,s})}\{\nu_A(y_{r,s})\}, 1-\inf_{z\in\gamma_R^*(z_{r,s})}\{\nu_A(z_{r,s})\}\right)$$
$$= T\left(1-\nu_{\gamma_R^*}(\gamma_R^*(y_{r,s})), 1-\nu_{\gamma_R^*}(\gamma_R^*(z_{r,s}))\right)$$

and Definition is satisfied.

If we choose $\gamma_R^*(x)$,

$\gamma_R^*(y)\in R/\gamma_R^*$ then we can write:

$$T\left(\mu_{\gamma_R^*}(\gamma_R^*(x)), \mu_{\gamma_R^*}(\gamma_R^*(y))\right)$$
$$= T\left(\sup_{a\in\gamma_R^*(x)}\{\mu_A(a)\},\ \sup_{b\in\gamma_R^*(y)}\{\mu_A(b)\}\right)$$
$$= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{T(\mu_A(a), \mu_A(b))\}$$
$$\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{\inf\{\mu_A(z) : z\in a\cdot b\}\}$$
$$\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{\sup\{\mu_A(z) : z\in a\cdot b\}\}$$
$$\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{\sup\{\mu_A(z) : z\in \gamma_R^*(a\cdot b)\}\}$$
$$= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{\mu_{\gamma_R^*}(\gamma_R^*(a\cdot b))\}$$
$$= \mu_{\gamma_R^*}(\gamma_R^*(a\cdot b)) = \mu_{\gamma_R^*}(\gamma_R^*(a)\odot\gamma_R^*(b))$$

and Definition is satisfied. Let $\gamma_R^*(x)$,

$\gamma_R^*(y)\in R/\gamma_R^*$. we can write:

$$T\left(1-\nu_{\gamma_R^*}(\gamma_R^*(x)), 1-\nu_{\gamma_R^*}(\gamma_R^*(y))\right)$$
$$= T\left(1-\inf_{a\in\gamma_R^*(x)}\{\nu_A(a)\}, 1-\inf_{b\in\gamma_R^*(y)}\{\nu_A(b)\}\right)$$
$$= T\left(\sup_{a\in\gamma_R^*(x)}\{1-\nu_A(a)\},\ \sup_{b\in\gamma_R^*(y)}\{1-\nu_A(b)\}\right)$$
$$= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{T(1-\nu_A(a), 1-\nu_A(b))\}$$
$$\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{1-\sup\{\nu_A(z) : z\in a\cdot b\}\}$$
$$\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{1-\inf\{\nu_A(z) : z\in a\cdot b\}\}$$
$$\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{1-\inf\{\nu_A(z) : z\in \gamma_R^*(a\cdot b)\}\}$$
$$= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\{1-\nu_{\gamma_R^*}(\gamma_R^*(a\cdot b))\}$$
$$= 1-\nu_{\gamma_R^*}(\gamma_R^*(a\cdot b))$$
$$= 1-\nu_{\gamma_R^*}(\gamma_R^*(a)\odot\gamma_R^*(b))$$

Therefore Definition is satisfied.

3. CONCLUSION

Through the above discussion, we had some properties of T-intuitionistic fuzzy H_v-subring on any ring. The special statement of intuitionistic fuzzy H_v-subrings are intuitionistic fuzzy H_v-ideals. It can be defined T-intuitionistic fuzzy H_v-ideals of a ring and can be studied such type properties.

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