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QUALIFICATION PROBLEMS WITH AN UPPER BOUND

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ABSTRACT

We study qualification problem introduced by Kasher and Rubinstein (1997) and introduce an upper bound on the number of people who can be qualified. Following Abizada and Tapkı (2015), we analyze *consistency* requirement for this model. We introduce *Priority Based Liberal rule,* which is an extension of the Liberal rule, which has been analyzed widely in the literature. We characterize Priority *Based Liberal Rule* based on *consistency* and *unanimity.*

Keywords: Consistency, liberal rule, group identification, qualification problems, unanimity.

JEL Classification: D70, D71, D72

1. INTRODUCTION

We study the group identification (or qualification) problem introduced by Kasher and Rubinstein (1997). Consider a group of people, who need to make a collective decision on who among them are qualified to be experts in a certain field. Usually there is an upper bound on number of people who can be qualified. Each person has an opinion about his qualification as an expert in this field. Opinions are dichotomous: a person either thinks the other person is qualified or is not. Once everyone shares their opinions, the question is how to aggregate them in order to make a decision on who among these people are qualified.

A rule is a systematic way of aggregating opinions of people into a collective decision. We introduce a generalization of well-known *Liberal Rule, Priority Based Liberal Rule, which* works as follows: a person is qualified if (i) he qualifies himself as an expert and (ii) the upper bound for qualified people is greater than the number of people who have higher priority than this person and consider themselves as qualified.

We analyze *consistency* requirement. To understand the intuition, suppose a rule has been applied and a collective decision has been made. After a while, suppose we need to make the same type of qualification decision. However, suppose some of the people who were present for the initial decision, are not present this time. *Consistency* requires that the decision made about the remaining people should be the same as the initial decision made about them.

We characterize *Priority Based Liberal Rule* using *consistency* together with a *unanimity* requirement which says (a) if everyone unanimously agree on qualification of a person, then that person should be qualified if the upper bound for qualification is sufficient, (b) if there are more such people than the upper bound, then the qualified people should be a subset of such people, (c) if everyone unanimously agree on disqualification of a person, then that person should be disqualified.

The rest of the paper is organized as follows: in Section 2, we mention the related literature. In Section 3, we provide our model, our methodology and our result. In Section 4, we conclude.

2. LITERATURE REVIEW

Qualification problem is introduced by Kasher and Rubinstein (1997). Authors provide axiomatic characterization of liberalism, where qualification of a person depends only on his opinion about himself. Several other papers consider axiomatic analysis of this problem: Sung and Dimitrov (2005) show that five axioms used in Kasher and Rubinstein (1997)'s characterization of the "liberal" aggregator are not independent and prove that only three of their original axioms are necessary and sufficient for the results; Çengelci and Sanver (2010) study the same model and characterize voting rules satisfying *monotonicity, independence, self-duality* and a weaker version of *anonymity*.

Liberal Rule is widely studied in the literature and several characterizations of this rule has been provided. Samet and Schmeidler (2003) characterize a class of voting rules which they call consent rules. This class contains liberalism at one extreme and majoritarianism, where personal opinions about the qualification of an individual are aggregated according to the majority rule, at the other extreme. Ju (2013) characterizes *Liberal rule* using exclusive self-determination and affirmative self-determination together with other plausible requirements. Although, intuitively these two requirements seem similar to our *consistency* requirement, Abizada and Tapkı (2015) show that they are independent.

Consistency idea has been formulated and studied for different models. It has been analyzed extensively in the context of bargaining by Lensberg (1988), single-peaked preferences by Thomson (1994), coalitional form games by Peleg (1986) and Hart and Mas-Colell (1989), taxation by Aumann and Maschler (1985) and Young (1987), cost allocation by Moulin (1985), and matching by Sasaki and Toda (1992).

In Abizada and Tapkı (2015), we analyze qualification problems and introduce two *consistency* requirements for this model. Also, we propose two new characterizations of the *Liberal Rule* based on these *consistency* requirements. Differently from that work, in this paper we introduce an upper bound on the number of qualified people. In real life, there are so many situations requiring a limit on number of qualified people. For example, there cannot be more than certain number of people in a dissertation committee or a project.

3. DATA AND METHODOLOGY

In this paper, we theoretically analyze qualification problems with an upper bound on the number of qualified people and we axiomatically characterize *Priority Based Liberal Rule* using *consistency* and *unanimity* requirements. Now, we will introduce our model and our characterization.

Let $\mathbb{N}$ be the infinite set of "potential" people. Let ***I*** be the class of finite subsets of $\mathbb{N}$ with cardinality of at least two. Each person $i \in \mathbb{N}$ has an opinion about qualifications of all the people, including himself. For each pair $i, j \in \mathbb{N}$, let $\boldsymbol{P_{ij}} \in \{0,1\}$ be the **opinion of person $\boldsymbol{i}$ about person j**, where $\boldsymbol{P_{ij}} = 1$ means that i considers j as qualified, and $\boldsymbol{P_{ij}} = 0$ means that i considers j as disqualified. Given $\mathrm{I} = \{i_1, i_2, \dots, i_{|I|}\} \in I$, and person $j \in I$, let $P_j^I \equiv (P_{i_1 j}, P_{i_2 j}, \dots, P_{i_{|I|} j})$ be the vector of opinions of all people in *I* about person *j*. For each $\mathrm{I} = \{i_1, i_2, \dots, i_{|I|}\} \in I$, let $\boldsymbol{P^I} = \left(P_{i_1}^I, P_{i_2}^I, \dots, P_{i_{|I|}}^I\right)$ be the **opinion profile for *I*** or **opinion matrix for *I***. Let $\mathcal{P}^I$ be the set of all possible opinion matrices for *I*. For each $\mathrm{I} = \{i_1, i_2, \dots, i_{|I|}\} \in I$, let $q^I \in \{1,2, \dots, |I|\}$ be an upper bound on the number of people that can be qualified, that is, at most q^I people can be qualified.

A **qualification problem for *I* with an upper bound** is a pair of an opinion profile for *I*, P^I and an upper bound on the number of people that can be qualified q^I, that is (P^I, q^I). A **(qualification) decision for *I*** is a vector of 0's and 1's, $x \equiv \left(x_{i_1}, \dots, x_{i_{|I|}}\right) \in \{0,1\}^{|I|}$ where 1 in i^{th} component means that the person i is **qualified** and 0 in i^{th} component means that the person i is **disqualified**. For each problem (P^I, q^I), a **rule $\boldsymbol{\varphi}$** makes a decision for I, i.e.

$$\varphi : \bigcup_{\substack{\mathrm{I} \in I \\ q^I \in \{1,2,\dots,|I|\}}} \mathrm{P}^I \times q^I \to \{0,1\}^{|I|}$$

such that $x_{i_1} + \dots + x_{i_{|I|}} \leq q^I$.

For each qualification problem (P^I, q^I) and each rule φ, let $Q(\varphi(P^I, q^I)) \equiv \{i \in I: \varphi_i(P^I, q^I) = 1\}$ be the set of people who are qualified by φ, at (P^I, q^I). Similarly, let $DQ(\varphi(P^I, q^I)) \equiv \{i \in I: \varphi_i(P^I, q^I) = 0\}$ be the set of people who are disqualified by φ, at (P^I, q^I). Let π be a strict priority relation which is a complete ordered list of agents. That is, if for $i, j \in \mathbb{N}$, $\pi(i) < \pi(j)$, then person i has higher priority than person j. For each I∈ I and $i \in I$, let $\mathrm{Pr}^{I,\pi}(i) = \{j \in I: \pi(j) < \pi(i)\}$ be the set of people who have higher priority than i in I.

We define the *Priority Based Liberal rule* which is an extension of standard *Liberal rule* that has been widely studied in the literature. It works as follows: a person i is qualified if he considers himself as qualified and if the upper bound for qualified people is greater than the number of people who have higher priority than i and who consider themselves as qualified. Otherwise, he is disqualified.

Priority Based Liberal Rule, $\boldsymbol{\varphi^{L,\pi}}$: For each (P^I, q^I) and $i \in I$,

$$\varphi_i^{L,\pi}(P^I, q^I) = \begin{cases} 1 & \text{if } P_{ii} = 1 \text{ and } \sum_{j \in \mathrm{Pr}^{I,\pi}(i)} P_{jj} < q^I \\ 0 & \text{otherwise} \end{cases}$$

4. FINDINGS AND DISCUSSIONS

We define two plausible properties of a rule. Let φ be a rule. Before the requirement, for each I∈ I and each $P^I \in \mathcal{P}^I$, let $I^1(P^I) = \{i \in I: P_i^I = (1, \ldots, 1)\}$ be the set of people such that everyone unanimously agree on qualification.

Our requirement states the following: (i) if everyone unanimously agree on qualification of a person, then that person should be qualified if the upper bound for qualification is sufficient, (ii) if there are more such people than the upper bound, then the qualified people should be a subset of such people, (iii) if everyone unanimously agree on disqualification of a person, then that person should be disqualified. Formally,

Unanimity: For each qualification problem (P^I, q^I),

(i) if $|I^1(P^I)| \leq q^I$, then each $i \in I^1(P^I), \varphi_i(P^I, q^I) = 1$,
(ii) if $|I^1(P^I)| > q^I$, then $Q(\varphi(P^I, q^I)) \subseteq I^1(P^I)$ and $|Q(\varphi(P^I, q^I))| = q^I$,
(iii) each $i \in I$ such that $P_i^I = (0, \ldots, 0), \varphi_i(P^I, q^I) = 0$.

Before defining our next requirement, we need to define some notations. For each pair I *and* I' with $I' \subset I$, each $P^I \in \mathcal{P}^I$, let $P^I|_{I'} \in \mathcal{P}^{I'}$ be the P^I-reduced opinion profile for I', which is obtained from P^I by deleting opinions of people in I\I' and opinions of everyone about them. To illustrate this point, let I= {i_1, i_2, i_3} and I'= {i_1, i_2}. Also let be P^I as follows

$$P^I = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Then, we define $P^I|_{I'}$ as follows: $P^I\big|_{I'} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

Next we define our *consistency* (robustness) requirement. Let the rule make a (qualification) decision for a group of people. Let some people (qualified or disqualified) accept the decisions made about them and leave. If the rule is applied to the problem with remaining people where only opinions of the remaining people are considered, then decision about each remaining person should be same as it was before. Before defining it formally, we have to define a reduced problem. If some people accept the decisions made about them and leave and if some of them are qualified, then we need to update the upper bound for the remaining qualified people. For each problem (P^I, q^I), and $I' \subset I$, let $r_{I'}^{\varphi}(P^I, q^I) = (P^I|_{I'}, q^I - |(I/I') \cap Q(\varphi(P^I, q^I))|$ be the reduced problem. Formally,

Consistency: For each pair I and I' with $I' \subset I$, each (P^I, q^I), and each $i \in I'$, we have $\varphi_i(P^I, q^I) = \varphi_i\left(r_{I'}^{\varphi}(P^I, q^I)\right)$.

This is the natural application of *consistency* idea to our model. But as our next result shows, this version of *consistency* turns out to be very strong: we show that the only rule that satisfies *consistency* together with very mild *unanimity* requirement is *Priority Based Liberal rule*.

Theorem. A rule φ is *consistent* and *unanimous* if and only if it is a *Priority Based Liberal rule*, $\varphi = \varphi^{L,\pi}$ for some π.

Proof. Let φ be a rule satisfying *consistency* and *unanimity*. Let (P^I, q^I) be a problem. Let π^{φ} be a strict priority relation for I such that each $k \in Q(\varphi(P^I, q^I))$ and l $\in DQ(\varphi(P^I, q^I))$, $\pi^{\varphi}(k) < \pi^{\varphi}(l)$. Let $i \in I$.

First, suppose $P_{ii}^I = 1$ and $\sum_{k \in Pr^{I,\pi}(i)} P_{kk}^I < q^I$. Suppose for a contradiction, $\varphi_i(P^I, q^I) = 0$. Note that by the choice of π^{φ}, since $\varphi_i(P^I, q^I) = 0$, $\pi^{\varphi}(i) \neq 1$.

Case 1. Suppose there is $j \in I$ such that $P_{ji}^I = 1$. Then, let $I' = \{i, j\}$ and $r_{I'}^{\varphi}(P^I, q^I) = (P^I|_{I'}, q^I - |(I/I') \cap Q(\varphi(P^I, q^I))| \equiv (P^{I\prime}, q^{I\prime})$. Note that, since $\sum_{k \in Pr^{I,\pi}(i)} P_{kk}^I < q^I$, $q^{I'} \geq 1$. By *consistency*, $\varphi_i(P^{I'}, q^{I'}) = 0$.

Case 1.1. Let $\pi^{\varphi}(i) < \pi^{\varphi}(j)$. Since $q^{I\prime} \geq 1$, by *unanimity*, $\varphi_i(P^{I'}, q^{I'})$ =1, which is a contradiction to *consistency*.

Case 1.2. Let $\pi^{\varphi}(j) < \pi^{\varphi}(i)$. Since $\sum_{k \in Pr^{I,\pi}(i)} P_{kk}^I < q^I$, by the choice of π^{φ}, $q^{I'} \geq 2$. Then, by *unanimity*,

$\varphi_i(P^{I'}, q^{I'})$ =1, which is a contradiction to *consistency*.

Case 2. Suppose for each $l \in I$ such that $P_{li}^I = 0$. Let $j \in I$ and $I' = \{i, j\}$. Let $k \in \mathbb{N} / I$ and $I'' = \{i, j, k\}$. Suppose $P_k^{I''} = (0,0,0)$ and $P_{ki}^{I''} = 1$. By *unanimity*, $\varphi_k(P^{I''}, q^{I'}) = 0$.

Case 2.1. Let $\pi^{\varphi}(i) < \pi^{\varphi}(j)$. By Case 1, $\varphi_i(P^{I''}, q^{I'}) = 1$. By *consistency*, $\varphi_i\left(r_{I'}^{\varphi}((P^I, q^I)\right) = \varphi_i(P^{I'}, q^{I'}) = \varphi_i\left(r_{I'}^{\varphi}((P^{I''}, q^{I'})\right) = 1$, a contradiction.

Case 2.2. Let $\pi^{\varphi}(j) < \pi^{\varphi}(i)$. Then, $q^{I''} \geq 2$. Then, by *consistency*, $\varphi_i\left(r_{I'}^{\varphi}((P^I, q^I)\right) = \varphi_i(P^{I'}, q^{I'}) = \varphi_i\left(r_{I'}^{\varphi}((P^{I''}, q^{I'})\right) = 1$, a contradiction.

Second, suppose $P_{ii}^I = 1$ and $\sum_{k \in Pr^{I,\pi}(i)} P_{kk}^I \geq q^I$. If $\varphi_i(P^I, q^I) = 1$, then by the choice of π^{φ}, each $j \in Pr^{\pi^{\varphi},I}(i)$, $j \in Q(\varphi((P^I, q^I))$. But then $|Q(\varphi((P^I, q^I))| > q^I$, a contradiction. Thus, $\varphi_i(P^I, q^I) = 0$.

Lastly, suppose $P_{ii}^I = 0$. Since the proof of this case is very similar to first part, we omit it.

Therefore, $\varphi = \varphi^{L,\pi}$ for $\pi = \pi^{\varphi}$.

5. CONCLUSION

We study group qualification problem. Differently from the earlier literature, we introduce an upper bound on the number of people that can be qualified. We extend the results in Abizada and Tapkı (2015) to our model with proper adjustments in definitions of the requirements and the rules. We propose a new rule, *Priority Based Liberal,* which is an extension of standard *Liberal rule* in the literature and characterize this rule using *consistency* requirement together with *unanimity.*

In this model, we assume that each person either thinks that the other person is qualified as an expert or not. However, he may have neutral opinion about qualification of some person. Extending this model by allowing neutral opinions is an open question.

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