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## MEASURING THE AGENCY COSTS OF DEBT: A SIMPLIFIED APPROACH

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### KEYWORDS

Capital structure, agency costs, investment distortion, trade-off model.

### ABSTRACT

This paper provides a model with which the agency costs of debt can be quantitatively analyzed. The traditional bankruptcy cost model in theories of corporate capital structure cannot explain actual financial leverage. This model extends the bankruptcy cost model by considering the agency costs. Simulating this model reveals several features. One is that it can realize likely optimal capital structure for actual firms. The other is that the agency costs of debt have a strong impact on optimal financial leverage though they are not very large. Furthermore, this paper also attempts tests to investigate whether this model fits behavior of actual firms. For more than 500 firms listed on the Tokyo Stock Exchange, parameters of the model can be appropriately estimated, and our measures of the agency costs of debt are almost consistent with past empirical research concerning agency costs hypotheses..

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## 1. INTRODUCTION

These days the agency costs advocated by Jensen and Meckling (1976) have become a popular concept in investigating corporate capital structure. It is well known what causes agency costs and how they affect financial leverage. However, there are few quantitative studies that focus on agency costs for actual firms: For example, how do we measure their agency costs? How big are their agency costs? How strongly do agency costs influence their capital structure?

This paper constructs a model that enables us to make a quantitative analysis of the agency costs of debt. By fitting this model to data about actual firms, unknown parameters within the model are estimated, and the amounts of the agency costs of debt are computed. We investigate whether or not the calibration given in this paper is appropriate. The purpose of this paper is to test whether a corporate capital structure model grasps actual financial behavior with a simple method.

There are two kinds of agency costs: One is between debtholders and shareholders, the other is between external shareholders and internal managers. The model in this paper considers quantifying the former. Hereafter, we designate this as the agency costs of debt. These are caused by two incentives: Debt overhang and asset substitution. According to Jensen and Meckling (1976), a firm mitigates incentives through monitoring and bonding activities, which give rise to their execution costs. Since such activities cannot perfectly obviate these incentives, firm's earnings decline further owing to the incentives that remain. This decline can be interpreted as another cost, called residual loss. The agency costs of debt are the sum of the

execution costs and the residual loss that occurs when the firm is leveraged. We calculate the agency costs of debt in this paper.<sup>1</sup>

The theory of asset pricing in capital markets is essential to fit a capital structure model to behavior of real firms. These days there are several models which depend on the continuous time risk-neutral method for security valuation. Although a continuous time framework is helpful in modeling agency costs together with security valuation, difficulty remains concerning applicability to actual firms. We employ the single period CAPM for pricing securities. A model in this paper is so simple that we can estimate unknown parameters from actual firm's data straightforwardly.

Simulating this model reveals several features. One is that it can realize likely optimal capital structure for an actual firm. The other is that there is a negative correlation between firm's earnings and its debt ratio. Furthermore, there are two observations concerning the amount of the agency costs. First, the agency costs of debt have a strong impact on optimal financial leverage. Second, the agency costs of debt are not very large, which suggests that they do not seriously damage economic welfare. These characteristics about the agency costs have been already pointed out by Parrino and Weisbach (1999) and Parrino, Poteshman, and Weisbach (2005). This paper confirms them using a more simplified method with valid security valuation.

This paper also attempts two tests to investigate whether this model fits behavior of actual firms. We sampled more than 500 firms listed on the Tokyo Stock Exchange 1st section and which belonged to manufacturing industries. The first test is whether this model follows the debt ratio of actual firms and whether parameters estimated by this model are appropriate. For almost all firms, this model is able to make its optimal debt ratio correspond to the actual one observed from data, and moreover, the estimated values of the model's parameters do fit well with data.

The second test is to ascertain the validity of our quantitative measure for the agency costs of debt calculated using this model. In corporate finance, there are many empirical studies in which firms' debt ratios are cross-sectionally regressed on some explanatory variables. These days, when interpreting these estimation results, some hypotheses based on agency costs have been generally accepted. If these hypotheses are true, then the quantitative measure of this model would need to be consistent with them. Since we have not found any contradiction with these hypotheses, we conclude that the model in this paper is very successful in its application to actual firms.

This paper is summarized as follows. Section 2 digests prior research on a quantitative approach to agency costs. Section 3 formulates the valuation of debt and equity. Section 4 models agency costs and proposes a measure for them. Section 5 simulates this model and demonstrates its features. In Section 6, several regressions are conducted in order to test our model's validity. Section 7 concludes this paper.

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<sup>1</sup> It is easy to extend this model into the generalized one which includes the agency costs between shareholders and managers as well, assuming a utility function on the part of managers. We do not think that such a generalization is useful. As Stulz (1990) and Berkovitch and Israel (1996) pointed out, debt has the effect of mitigating agency costs between shareholders and managers. Strictly speaking, the method provided here is to quantify the mixture of pure agency costs associated with debt and the effect of mitigating them when a firm becomes leveraged. When different materials are confused, measurement of the agency costs becomes obscure. This is why this paper focuses only on the agency costs of debt.

## **2. PRIOR RESEARCH**

This paper draws on the theory of optimal capital structure that disputed the irrelevancy theorem of Modigliani and Miller (1958), and that presumed that a firm decides its capital structure as the result of optimal decision-making. The most representative model in the 1970s was the bankruptcy cost model. This derived optimal capital structure from balancing advantages and disadvantages associated with debt: The trade-off between tax shields and bankruptcy costs came under consideration. The economic implications of the model were clear, and it was possible to undertake a quantitative analysis of the capital structure of actual firms using the CAPM with which securities were priced and in which investors were assumed to be risk averse. The most famous research into the traditional bankruptcy cost model is Kim (1978). Warner (1977) points out a defect in the model.

The agency costs hypothesis is one of the optimal capital structure theories because, according to Jensen and Meckling (1976), a firm or manager makes an optimal decision regarding capital structure. However, the hypothesis depends on an assumption that is quite different from previous model's. The biggest difference concerns the assumption about firm's earnings before interest and taxes (EBIT). The traditional bankruptcy cost models assumed that the distribution of EBIT remained unchanged even if capital structure altered. On the other hand, the agency costs hypothesis presumes that capital structure determines the distribution of EBIT. Hence, the agency costs hypothesis that allows the distribution to change makes it easier to come up with a new way of thinking that is able to undertake an interpretation of behavior of actual firms. There are many studies that take this standpoint: Myers (1977), Long and Malitz (1985), Jensen (1986), Stulz (1990), Berkovitch and Israel (1996), and Lang, Ofek, and Stulz (1996). We can say that these studies are qualitative in that they provide several important implications.

These primary models that initially proposed agency costs often ignored security valuation. They assumed that a discount rate in pricing securities was zero, and that an expected cash flow at the end of a period was equal to a security price. However, it is impossible to study agency costs quantitatively without asset pricing methods to security valuation. Mello and Parsons (1992) and Leland (1998) developed models that enabled quantitative research into agency costs. They depend on risk-neutral security valuation in a continuous time framework. Morellec (2004) and Parrino, Poteshman, and Weisbach (2005) are significant steps toward grasping how to measure agency costs. While it is not regarded as an agency costs model, Goldstein, Ju, and Leland (2001) provides a path-breaking trade-off theory of capital structure in that dynamic debt restructuring is considered. Strebulaev (2007) attempts to investigate whether these continuous time models fit financial behavior for actual firms.

Parrino and Weisbach (1999) employed another approach under which some estimation was possible for actual firms. Their model is similar to that of the current paper in that it considers over-investment and under-investment as incentives for agency costs within a discrete time framework. However, their formulation is quite different from that used in this paper. The difference lies in security valuation. We wonder whether their method of calculating the cost of capital maintains capital market equilibrium. It is necessary to confirm that what Parrino and Weisbach (1999) showed is appropriate in terms of a different method.

This paper constructs a model that enables us to make a quantitative analysis of the agency costs of debt. This model must be so simple that we can fit it to data about actual firms. This is the reason why we depend on the single period CAPM in pricing securities. Once cash flows to equity and debt are formulated, the CAPM derives security values from the cash flows. These days the single period CAPM is not as popular as a continuous time model. We believe that the

single period CAPM is still a useful tool for security valuation in corporate finance if we regard one period as a very long term, such as 10 years.

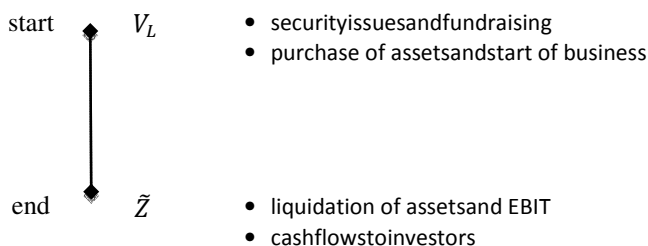
Valuation through an asset pricing theory is concerned with a security value at the beginning of a period. The agency costs are caused by manager's discretionary behavior that will become apparent during the period. An asset pricing theory assumes capital markets to be perfect, which means that, being aware of what the manager will do, investors price securities at the beginning of the period. The manager pursues his or her own objectives, and this gives rise to agency costs. Unable to be verified, the manager's behavior is not enforceable by investors through contracts. All investors can do is to forecast what the manager will do along his or her objectives. On the other hand, capital markets can influence the manager; he or she must accede to security valuation by investors. Under these suppositions, we formulate a security value so as to model the agency costs and provide our measures to quantify them.

### 3. VALUATION OF EQUITY AND DEBT

In order to measure the agency costs of debt, we discuss a one-period model, which is summarized in Figure 1. At the beginning of the period, a firm is founded and issues debt and shares of stock. The firm purchases assets and starts up in business. Investors are debtholders and shareholders. The person who makes decisions for the firm is called a manager, who works on behalf of the shareholders. At the end of the period when the firm is liquidated, EBIT over the period and proceeds from the sale of the assets are distributed among the investors. The values of equity and debt issued at the beginning of the period are denoted as  $S_L$  and  $B$ . The sum of  $S_L$  and  $B$  is a firm value  $V_L$ .

The debt in this model, which is a senior claim, promises a payment  $L$  to debtholders at the end of the period.  $L$  consists of the principal and interest on the debt. The sum of the EBIT and the liquidation value is  $\tilde{Z}$ , which is the cash flow of the firm distributed to debtholders and shareholders at the end of the period.  $\tilde{Z}$  is a random variable that follows a normal distribution  $N(\mu_Z, \sigma_Z^2)$ . If its realized value  $Z$  is greater than  $L$ , the firm pays  $L$  to the debtholders first, then corporate income taxes are paid, and the residual is paid to the shareholders as dividends. However, if  $Z$  is less than  $L$ , the firm is in default and goes bankrupt. Then, bankruptcy costs that amount to  $K$  are incurred. This paper assumes bankruptcy costs to be proportional to the firm value,  $K = kV_L$ .

Figure 1: Time Structure of the Model



This expresses the time structure of this one-period model.  $V_L$  is a firm value at the beginning of a period.  $\tilde{Z}$  is a cash flow distributed among investors at the end.

Suppose that corporate income tax is an asymmetric type of tax loss offset provisions. Asymmetric income tax is such that taxable income is charged at the rate  $\tau$  if and only if it is positive. If taxable income is negative, the tax payment is zero. Taxable income is calculated as  $Z - V_L - (L - B)$ , where  $Z - V_L$  is earnings from business activities and  $L - B$  is an interest expense that is deductible. When the realized value of  $\tilde{Z}$  is greater than  $V_L + L - B$ , the tax payment amounts to  $\tau[\tilde{Z} - V_L - (L - B)]$ . The calculation of taxable income in this model is similar to that in a traditional bankruptcy cost model.

Shareholders' cash flow at the end of the period,  $\tilde{Q}_{LS}$ , is formulated as

$$\tilde{Q}_{LS} = \begin{cases} \tilde{Z} - L - \tau(\tilde{Z} - V_L - [L - B]) & \text{for } Z \geq V_L + L - B, \\ \tilde{Z} - L & \text{for } V_L + L - B > Z \geq L, \\ 0 & \text{for } L > Z. \end{cases} \quad (1)$$

Since shareholders have limited liability, this means  $S_L = V_L - B > 0$ .  $V_L + L - B$  is always greater than  $L$ . There are three equations for  $\tilde{Q}_{LS}$ , depending on whether  $Z$  is greater than  $V_L + L - B$  or  $L$ . The first equation is the case where  $Z \geq V_L + L - B$  and where taxable income is positive. Then the firm pays debtholders  $L$ , pays the income tax, and gives shareholders the remainder as dividends. In the second equation, the taxable income is negative but the firm does not go bankrupt. Hence the firm does not have to pay income tax. The cash flow  $\tilde{Z}$  is divided between debtholders and shareholders. The third equation designates the case of  $Z < L$ , which makes the firm bankrupt. In this case,  $\tilde{Z}$  belongs to the debtholders, and the shareholders get nothing.

Debtholders' cash flows are represented as  $\tilde{Q}_{LB}$ , the formula for which depends on whether the promised payment of debt  $L$  is greater than the bankruptcy costs  $K$ . In the case where  $L > K$ ,  $\tilde{Q}_{LB}$  is

$$\tilde{Q}_{LB}^{(L>K)} = \begin{cases} L & \text{for } Z \geq L, \\ \tilde{Z} - K & \text{for } L > Z \geq K, \\ 0 & \text{for } K > Z. \end{cases} \quad (2)$$

The superscript shows  $L > K$ . When  $Z \geq L$ , debtholders receive the promised payment  $L$ . When  $Z$  is less than  $L$ , the firm goes bankrupt and  $\tilde{Z}$  belongs to the debtholders who have to incur the bankruptcy costs  $K$ . If  $Z$  is less than  $K$ , debtholders' cash flow from the firm becomes zero because of their limited liability.<sup>2</sup>

In the case where  $K \geq L$ , the formula for  $\tilde{Q}_{LB}$  changes into

$$\tilde{Q}_{LB}^{(K \geq L)} = \begin{cases} L & \text{for } Z \geq L, \\ 0 & \text{for } L > Z. \end{cases} \quad (3)$$

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<sup>2</sup> As long as shareholders and debtholders are limited liable, any claims charged on a firm are cancelled unless it has cash to fulfill them.  $\tilde{Z}$  is assumed to be normally distributed and  $Z$  can be negative. What does the negative  $Z$  mean? According to the Absolute Priority Rule, wages paid employees are senior to payments to debtholders, taxation, and shareholders. A negative value of  $Z$  is regarded as the situation where firm's cash flow acquired through its business and liquidation is short of its payroll. Shareholders and debtholders have no obligation to overcome the shortage. Since nobody covers it, the deficit, which is equal to the value of  $Z$ , is written off. When a firm goes bankrupt, the sum of cash flows to shareholders and debtholders is not always equal to the value of  $Z$ .

The first equation is the case where there is no bankruptcy. In the second equation, the bankruptcy occurs and there is no cash flow because of limited liability.<sup>3</sup>

The equity value  $S_L$  and the debt value  $B$  at the beginning of the period are derived from their cash flows at the end of the period. This paper applies the CAPM in pricing securities. The certainty equivalent approach in the CAPM can be applied to their valuation:

$$S_L = \frac{E(\tilde{Q}_{LS}) - \lambda cov(\tilde{R}_M, \tilde{Q}_{LS})}{1 + R_F}, \tag{4}$$

$$B = \begin{cases} \frac{[E(\tilde{Q}_{LB}^{(L>K)}) - \lambda cov(\tilde{R}_M, \tilde{Q}_{LB}^{(L>K)})]}{(1 + R_F)} & \text{for } L > K, \\ \frac{[E(\tilde{Q}_{LB}^{(K \geq L)}) - \lambda cov(\tilde{R}_M, \tilde{Q}_{LB}^{(K \geq L)})]}{(1 + R_F)} & \text{for } K \geq L, \end{cases} \tag{5}$$

where  $R_F$  is a riskless interest rate,  $\tilde{R}_M$  is the rate of return on the market portfolio, and

$$\lambda = \frac{E(\tilde{R}_M) - R_F}{\sigma(\tilde{R}_M)^2}.$$

Means and covariances that appear in Equations (4) and (5) are computed through partial moment formulas:

$$E(\tilde{Q}_{LS}) = \mu_Z[1 - \tau + \tau F(V_L + L - B) - F(L)] + \sigma_Z^2[f(L) - \tau f(V_L + L - B)] - L[1 - F(L)] + \tau(V_L + L - B)[1 - F(V_L + L - B)],$$

$$cov(\tilde{R}_M, \tilde{Q}_{LS}) = cov(\tilde{R}_M, \tilde{Z})[1 - \tau + \tau F(V_L + L - B) - F(L)],$$

$$E(\tilde{Q}_{LB}^{(L>K)}) = L[1 - F(L)] - K[F(L) - F(K)] + \mu_Z[F(L) - F(K)] - \sigma_Z^2[f(L) - f(K)],$$

$$cov(\tilde{R}_M, \tilde{Q}_{LB}^{(L>K)}) = cov(\tilde{R}_M, \tilde{Z})[F(L) - F(K) + Kf(L)],$$

$$E(\tilde{Q}_{LB}^{(K \geq L)}) = L[1 - F(L)],$$

$$cov(\tilde{R}_M, \tilde{Q}_{LB}^{(K \geq L)}) = cov(\tilde{R}_M, \tilde{Z})Lf(L),$$

where  $F(\cdot)$  is the cumulative distribution function of the normal distribution  $N(\mu_Z, \sigma_Z^2)$  and  $f(\cdot)$  is its density function.

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<sup>3</sup> Who is going to pay the bankruptcy costs  $K$  in the case of  $K > L$ ? When a firm goes bankrupt, debtholders obtain  $Z$  and bear  $K$ . Then  $Z - K$  is negative since  $K$  is larger than  $Z$ . If the debtholders are burdened with all of  $K$ , the negative value of  $Z - K$  means that they pay extra money out of their pocket, which violates their limited liability. The limited liability ensures that debtholders are free from any additional outlays except their initial investment  $B$ . A shortfall of  $|Z - K|$  dollars debtholders do not have to pay is not charged on any other investors and, in other words, is written off. This is the reason why a cash flow to debtholders is assumed to be zero when a firm goes bankrupt in the case of  $K \geq L$ .

Although Equations (4) and (5) are formulations of  $S_L$  and  $B$ , they are not solutions of  $S_L$  and  $B$ . The cash flows depend on  $V_L$ , which is the sum of  $S_L$  and  $B$ , and the right-hand sides of Equations (4) and (5) include  $S_L$  and  $B$  through  $V_L$ . Although  $S_L$  and  $B$  cannot be analytically solved from these equations, the values of  $S_L$  and  $B$  that satisfy Equations (4) and (5) can be computed. We focus on these numerical solutions in later sections.

#### 4. MODELING THE AGENCY COSTS OF DEBT

In this section, we consider modeling the agency costs of debt. The valuation of equity and debt in Section 3 is premised on the CAPM, which assumes that capital markets are perfect and that investors have perfect information. It is in the probability distribution parameters,  $\mu_Z$  and  $\sigma_Z$ , that this model reflects managerial discretion that causes agency costs. After issuing securities, the manager runs the firm according to his or her own targets so that  $\mu_Z$  and  $\sigma_Z$  can reach the most preferred values. The manager's objective in this model is to maximize the wealth of the shareholders. On the other hand, anticipating the manager's decisions, the investors correctly forecast the values of  $\mu_Z$  and  $\sigma_Z$  that the manager will select. This is the meaning of "perfect information" in this model.

$\mu_Z$  and  $\sigma_Z$  might be observable but cannot be verified. Unable to be verified, they are not enforceable by investors through contracts.<sup>4</sup> All investors can do is to forecast what the manager will do along his or her objectives. While the manager might promise these values, these promises are not enforceable and not necessarily trusted by the investors. In valuing the securities, they anticipate the values of  $\mu_Z$  and  $\sigma_Z$ , which the manager will decide.

We know from the means and the covariances of Equations (4) and (5) that  $S_L$  and  $B$  are functions of several parameters:  $L$ ,  $\mu_Z$ ,  $\sigma_Z$ ,  $k$ ,  $\tau$ ,  $\lambda$ ,  $R_F$ , and  $cov(\tilde{R}_M, \tilde{Z})$ . What the manager is able to control directly in his or her decision-making is assumed to be  $L$ ,  $\mu_Z$ , and  $\sigma_Z$ . There are other parameters that he or she influences indirectly. For example, the ratio of bankruptcy costs to a firm value,  $k$ , depends on what kinds of assets the firm comprises. The systematic risk in the capital market,  $cov(\tilde{R}_M, \tilde{Z})$ , can be an objective for the manager. We assume that the parameters other than  $L$ ,  $\mu_Z$ , and  $\sigma_Z$  are given and constant. The equity and debt values are denoted as

$$S_L = S_L(L, \mu_Z, \sigma_Z),$$

$$B = B(L, \mu_Z, \sigma_Z).$$

With these functions, the agency costs of debt are formulated as follows. At the beginning of a period, the manager chooses firm's capital structure to maximize the firm value.<sup>5</sup> The capital structure is derived from  $L$ , which is

<sup>4</sup> It is usually assumed in contract theories that  $L$  is verified, but that  $Z$ , a realized value of  $\tilde{Z}$ , is not. Many models use this assumption; for example, see Hart and Moore (1998). In this paper we assume that auditing works well for listed firms and that their  $Z$  is also verifiable. Its verifiability does not necessarily mean that  $\mu_Z$ , the expected value of  $\tilde{Z}$ , is also verifiable.

<sup>5</sup> The reason why a firm must decide its capital structure to maximize a firm value is discussed in (Kane, Marcus, and McDonald, 1984,1985). Fischer, Heinkel, and Zechner (1989) and Goldstein, Ju, and Leland (2001) adopt this discussion to derive optimal capital structure. This paper also follows it. A traditional bankruptcy cost model maintains that maximizing a firm value makes shareholders' wealth maximized. (Kane, Marcus, and McDonald, 1984,1985) advocate the maximization of a firm value because of no arbitrage in equilibrium. The cash flow equations in Section 3 of this paper are similar to those in a traditional bankruptcy cost model. However, one of differences lies in this point.

$$L^* = \arg \max_L \{S_L(L, \mu_Z, \sigma_Z) + B(L, \mu_Z, \sigma_Z)\}. \quad (6)$$

During the period just after the beginning, the manager behaves so as to maximize the equity value. Then, agency costs between shareholders and debtholders arise. One of the reasons behind the agency costs is asset substitution, which enables the equity value to increase at the sacrifice of the debt value, with the firm taking more risk in the management. Through the incentive of asset substitution, the value of  $\sigma_Z$  is chosen by the manager, which leads to the maximization of  $S_L$ .<sup>6</sup>

$$\sigma_Z^* = \arg \max_{\sigma_Z} S_L(L^*, \mu_Z, \sigma_Z) \quad (7)$$

Capital markets being perfect, the manager's incentive in asset substitution during the period is predicted exactly by the investors at the beginning of the period. Thus, they can price securities using  $\sigma_Z^*$ , which is designated in Equation (7). On the other hand, since the manager selects the capital structure according to investors' valuation, Equation (6) must be rewritten as

$$L^* = \arg \max_L \{S_L(L, \mu_Z, \sigma_Z^*) + B(L, \mu_Z, \sigma_Z^*)\}. \quad (8)$$

Mathematically, if the value of  $\mu_Z$  is given,  $L$  and  $\sigma_Z$  are solved from Equations (7) and (8), from which two first-order conditions are derived. These are the functions of  $L$  and  $\sigma_Z$ , the values of which can be solved endogenously with the given value of  $\mu_Z$ .

How is  $\mu_Z$  decided? We assume following constraint about  $\mu_Z$ . Suppose that the expected cash flow of an unleveraged firm is  $\mu_Z^U$ , which for the manager is given.  $\mu_Z$  is regarded as a function of  $\mu_Z^U$  and  $L$ . There are two factors that have opposing effects of  $L$  on  $\mu_Z$ . One is that  $L$  has a positive effect through the tax saving by which an increase in  $L$  raises the firm value. The other is that  $L$  has a negative effect because an increase in  $L$  causes the agency costs to be aggravated.

The incentives that are known as asset substitution and debt overhang give rise to agency costs. Even if asset substitution reduces EBIT, the manager can conduct business that makes the firm sufficiently riskier to increase the equity value. The debt overhang leads the manager to abandon business that improves the EBIT yet might decrease the equity value owing to leakage into debt. If a firm is unleveraged, asset substitution and debt overhang never arise, and all the activities that increase the EBIT are undertaken. As a result, the value of  $\mu_Z^U$  is decided. However, if the firm is leveraged and has to pay  $L$  at the end of the period,  $L$  reduces  $\mu_Z$  to below  $\mu_Z^U$  through these incentives.

In order to formulate  $\mu_Z$  as a function of  $\mu_Z^U$  and  $L$ , the real investment behavior of a firm should be factored into the agency costs model, and this is too complicated to be tractable. Instead of modeling the firm's investment, we assume that  $\mu_Z$  is a linear function of  $L$  as the result of the incentives that cause the agency costs:

$$\mu_Z = \mu_Z^U + aL.$$

If the effect of the tax saving is greater than that of the agency costs,  $a$  is positive. If  $a$  is negative, the effect of the agency costs predominates. The purpose of this model is to account

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This model is by no means a bankruptcy cost model. Full discussion about their differences is available on request to authors.

<sup>6</sup> Note that the optimal value of  $\sigma_Z$  can exist as an interior solution because  $\tilde{Q}_{LS}$  has both convex and concave regions in the function of  $\tilde{Z}$ .



for the effect of agency costs, hence the value of  $\alpha$  is assumed to be negative, and the above linear equation is rewritten as<sup>7</sup>

$$\mu_Z = \mu_Z^U - \alpha L \quad (9)$$

for  $\alpha > 0$ . Equation (9) is correctly recognized by investors at the beginning of the period because they have perfect knowledge of the manager's behavior.

In sum, the equity and debt values are functions of three parameters:  $L$ ,  $\mu_Z$ , and  $\sigma_Z$ . The manager determines their values by carrying out his or her objectives, and investors, having perfect knowledge of these, price the securities. As a result, the parameters are endogenously decided using the three equations; (7), (8), and (9). We denote the solutions as  $L^*$ ,  $\sigma_Z^*$ , and  $\mu_Z^*$ . These can be used to rewrite the simultaneous equations:

$$\frac{\partial}{\partial \sigma_Z} S_L(L^*, \mu_Z^*, \sigma_Z^*) = 0, \quad (7')$$

$$\frac{\partial}{\partial L} V_L(L^*, \mu_Z^*, \sigma_Z^*) = 0, \quad (8')$$

$$\mu_Z^* = \mu_Z^U - \alpha L^*. \quad (9')$$

The purpose of the numerical calculation is to determine the three variables that satisfy the above equations.

In Equation (9), new exogenous parameters,  $\mu_Z^U$  and  $\alpha$ , have arisen. Thus, by formulating agency costs, the equity and debt values become functions of  $\mu_Z^U$  and  $\alpha$ :

$$S_L = S_L(\mu_Z^U, \alpha), \quad (10)$$

$$B = B(\mu_Z^U, \alpha). \quad (11)$$

The purpose of this model is to quantify the agency costs associated with debt. Which parameter of the model is useful in measuring agency costs? It is  $\alpha$ .  $L$  is the burden of debt, and any increases in  $\alpha$  mean that the loss of EBIT per unit of debt becomes greater, which renders the agency costs more serious. So,  $\alpha$  is considered to be the marginal effect of the agency costs of debt.

This model obtains the optimums of  $L^*$  and  $\mu_Z^*$ , with  $\mu_Z^U$  and  $\alpha$  given. Large  $\alpha$  does not always lead to a large loss in EBIT. For example, if a firm faces large  $\alpha$ , small  $L^*$  can be optimal because the firm is willing to decrease debt so as to avoid the loss associated with debt. Then,  $\mu_Z^*$  does not deviate from  $\mu_Z^U$  as much. Hence, another quantitative measure is the extent to which the firm incurs ex post loss in EBIT as the result of optimal behavior:

$$LOSS = \frac{\mu_Z^U - \mu_Z^*}{\mu_Z^U}. \quad (12)$$

This is denoted as the loss rate associated with the agency costs of debt.<sup>8</sup>

<sup>7</sup> In the case where  $\alpha = 0$ , this model cannot fit well with observed capital structure because it resembles a bankruptcy cost model. If  $\alpha$  were positive, it would be more difficult to realize actual firm leverage. As  $\alpha$  increases, optimal leverage in the model encourages greater debt and is more markedly different from the actual situation. However, if  $\alpha$  is negative, the model's optimum more nearly approaches an actual firm. See Appendix A about validity of Equation (9).

<sup>8</sup> As was pointed out in Footnote 1, *LOSS* is not a pure measure for the agency costs of debt. There exists another agency cost that occurs between outside shareholders and inside managers. While discharging a debt brings about the agency costs of debt, it alleviates the one between shareholders and managers (Jensen and Meckling, 1976). See also Stulz (1990) and Berkovitch and Israel (1996). Strictly speaking, *LOSS* quantifies a composite of two kinds of the agency costs.

In Section 6 we calculate parameter values  $\mu_Z^U$ ,  $\mu_Z^*$ ,  $\sigma_Z^*$ ,  $L^*$ ,  $\alpha$ , and  $LOSS$  from data of actual firms. Then, we investigate whether or not these are appropriate.

## 5. SIMULATION

This section presents some simulation results of this model. Since the model does not have analytical solutions, it would be difficult to make its characteristics clear without numerical solution methods. Simulation that depends on these could determine what the model is like.

Some parameter values used in the simulation are as follows: One period in this model is 10 years. The corporate income tax rate  $\tau$  is 0.45. Capital market data are computed from April 1985 to March 1994:  $E(\tilde{R}_M) = 0.07706$ ,  $\sigma(\tilde{R}_M) = 0.17885$ , and  $R_F = 0.054$ . These values are based on one year and those that are converted into 10 years are employed in the model.  $\tilde{R}_M$  is the rate of return on TOPIX, and  $R_F$  is the Nikkei long-term bond index. The correlation between  $\tilde{R}_M$  and  $\tilde{Z}$  is assumed to be 0.4.

In order to investigate the effect of agency costs on capital structure, we begin with the simplest case where  $\alpha$  is zero and where  $\sigma_Z$  is given. This corresponds to the assumption that  $\tilde{Z}$  is distributed over  $N(\mu_Z, \sigma_Z^2)$ , which is exogenously given. Then, the model is similar to the bankruptcy cost model. If all the  $V_L$  in the cash flow equations were replaced with  $V_U$ , the unleveraged firm value, the model would become a traditional bankruptcy cost model. Although the economic meaning is very different between the simplest case and the bankruptcy cost model, these valuations are comparable due to similarity in the cash flow equations.

**Table 1: The Effect of Changes in  $k$  in the Case where  $\mu_Z$  and  $\sigma_Z$  are Given**

$k$	$L^*$	Prob	$S_L$	$B$	$V_L$	$B/V_L$
0.1	54.09	0.667	0.41	26.03	26.44	0.985
0.2	42.04	0.201	2.70	23.03	25.73	0.895
0.3	38.09	0.105	4.07	21.35	25.43	0.840
0.4	35.95	0.069	4.92	20.32	25.24	0.805
0.5	34.51	0.051	5.52	19.59	25.10	0.780
0.6	33.44	0.040	5.97	19.02	25.00	0.761
0.7	32.59	0.033	6.34	18.57	24.91	0.745
0.8	31.89	0.028	6.65	18.19	24.84	0.732
0.9	31.29	0.024	6.91	17.87	24.78	0.721

This table presents simulation results in the case where  $\alpha = 0$ . This case corresponds to the assumption that  $\tilde{Z}$  is distributed over  $N(\mu_Z, \sigma_Z^2)$ , which is exogenously given.  $k$  is a bankruptcy cost parameter. When  $k$  is changed from 0.1 to 0.9, an optimal value  $L^*$  that maximizes  $V_L$  is provided for each  $k$  in the table. The equity value  $S_L$ , the debt value  $B$ , and the firm value  $V_L$  are computed under the optimal  $L^*$ . The probability of default is denoted as Prob and the debt ratio as  $B/V_L$ . Suppose that  $\mu_Z^U = \mu_Z = 50.0$  and  $\sigma_Z = 9.487 = 3 \times \sqrt{10}$  are numbers based on 10 years. Suppose that  $\mu_Z = 50.0$  and  $\sigma_Z = 9.487$  are numbers based on 10 years. The standard deviation is obtained from one-year value 3.0 multiplied by  $\sqrt{10}$ . For each  $k$ , which is a bankruptcy cost parameter, the optimal value that maximizes  $V_L$  with respect to  $L$  is provided in Table 1, where the equity, debt, and firm values are computed under the optimal  $L^*$ . The probability of default,  $Pr\{\tilde{Z} < L\}$ , is denoted as Prob and the debt ratio as  $B/V_L$ .

When  $k$  is 0.4, the debt ratio is greater than 0.8. Even if  $k$  is 0.9, the debt ratio is greater than 0.7. These results are similar to those of the bankruptcy cost model. Since an actual debt ratio is less than 0.5 for most firms, it is true that a bankruptcy cost model is not able to fit this. The simplest case of this model demonstrates the fact. It is obvious that simply replacing  $V_U$  with  $V_L$  is not enough for the model to realize actual capital structure because the two equations remain similar.

A traditional bankruptcy cost model shows that an increase in firm's earnings leads to a higher debt ratio. So does the simplest case in our model. As long as  $\mu_Z$  and  $\sigma_Z$  are exogenously given, earnings have a positive correlation with the debt ratio in this model. On the supposition that  $k = 0.4$  and  $\sigma_Z = 9.487$ , Table 2 calculates an optimal  $L^*$  and its debt ratio for each value of given  $\mu_Z$ . It is confirmed that larger  $\mu_Z$  has a larger debt ratio as well as larger  $L^*$ . However, empirical studies observe that earnings and debt have a strong negative correlation, which contradicts the predictions of the models. As shown later, this model permits them to have a negative correlation.

**Table 2: The Effect of Changes in  $\mu_Z$  in the Case where  $\mu_Z$  and  $\sigma_Z$  are Given**

$\mu_Z^U = \mu_Z$	$L^*$	Prob	$S_L$	$B$	$V_L$	$B/V_L$
35.0	24.29	0.129	3.63	13.04	16.67	0.782
40.0	27.91	0.101	4.15	15.36	19.50	0.787
45.0	31.83	0.082	4.57	17.80	22.36	0.796
50.0	35.95	0.069	4.92	20.32	25.24	0.805
55.0	40.21	0.060	5.22	22.90	28.13	0.814
60.0	44.58	0.052	5.49	25.53	31.02	0.823
65.0	49.03	0.046	5.72	28.20	33.92	0.831
70.0	53.54	0.041	5.93	30.90	36.83	0.839

This table presents simulation results in the case where  $\alpha = 0$ . This case corresponds to the assumption that  $\tilde{Z}$  is distributed over  $N(\mu_Z, \sigma_Z^2)$ , which is exogenously given. When  $\mu_Z$  is changed from 35.0 to 70.0, an optimal value  $L^*$  that maximizes  $V_L$  is provided for each  $\mu_Z$  in the table. The equity value  $S_L$ , the debt value  $B$ , and the firm value  $V_L$  are computed under the optimal  $L^*$ . The probability of default is denoted as Prob and the debt ratio as  $B/V_L$ . Suppose  $\sigma_Z = 9.487$  and  $k = 0.4$ .

The next step is to make  $\sigma_Z$  endogenous in the model, which means that asset substitution is considered as agency costs.  $L^*$  maximizes  $V_L$ , and  $\sigma_Z^*$  maximizes  $S_L$ . Table 3 assumes that  $k = 0.4$  and that each value of  $\mu_Z$  is given. Compared with Table 2,  $S_L$  in Table 3 increases. Then,  $B$  decreases in the cases where  $\mu_Z \geq 40.0$  and where  $\sigma_Z^*$  in Table 3 is greater than  $\sigma_Z (= 9.487)$  in Table 2.

The most interesting result in Table 3 is that the debt ratio becomes constant. While the positive correlation between earnings and debt is observed in the case of exogenous  $\sigma_Z$ , this correlation disappears by making  $\sigma_Z^*$  endogenous. This is because the debt value decreases when endogenous  $\sigma_Z^*$  is greater than fixed  $\sigma_Z$ . As an aside, the effect on the debt ratio is not very great. The debt ratio, which is 0.784, remains high, inconsistent with actual values.

**Table 3: The Effect of Changes in  $\mu_Z$  in the Case where Only  $\sigma_Z$  is Endogenous**

$\mu_Z^U = \mu_Z$	$\sigma_Z^*$	$L^*$	Prob	$S_L$	$B$	$V_L$	$B/V_L$
35.0	8.92	24.31	0.115	3.64	13.21	16.85	0.784
40.0	10.19	27.78	0.115	4.16	15.10	19.26	0.784
45.0	11.47	31.25	0.115	4.68	16.99	21.67	0.784
50.0	12.74	34.72	0.115	5.20	18.88	24.08	0.784
55.0	14.01	38.20	0.115	5.72	20.76	26.48	0.784
60.0	15.29	41.67	0.115	6.24	22.65	28.89	0.784
65.0	16.56	45.14	0.115	6.76	24.54	31.30	0.784
70.0	17.84	48.61	0.115	7.28	26.42	33.71	0.784

This table presents simulation results in the case where only  $\sigma_Z$  is endogenous and  $\alpha$  remains equal to zero. When  $\mu_Z$  is changed from 35.0 to 70.0, an optimal pair,  $L^*$  and  $\sigma_Z^*$ , which maximizes  $V_L$  and  $S_L$ , is provided for each  $\mu_Z$  in the table. The equity value  $S_L$ , the debt value  $B$ , and the firm value  $V_L$  are computed under the optimal  $L^*$  and  $\sigma_Z^*$ . The probability of default is denoted as Prob and the debt ratio as  $B/V_L$ . Suppose  $k = 0.4$ .

The effect of agency costs is not only to make  $\sigma_Z$  endogenous but also to distort firm's business through its under-investment or over-investment incentives. Debt deviates  $\mu_Z$  from the potential that would arise with  $\mu_Z^U$  for the unleveraged firm. This correlation is shown in Equation (9). In the previous simulation,  $\mu_Z$  was given. Now,  $\mu_Z^U$  and  $\alpha$  being given,  $\mu_Z$  becomes endogenous in Equation (9). Mathematically,  $\mu_Z^*$ ,  $\sigma_Z^*$ , and  $L^*$  are solved from the three equations (7), (8), and (9). Table 4 gives calibration results for some changes in  $\alpha$  in the case where  $k = 0.4$  and  $\mu_Z^U = 50.0$ .

It is obvious from Table 4 that  $\alpha$  significantly influences capital structure.  $\alpha = 0.1$  lowers the debt ratio to 0.61 from about 0.8 in the case where  $\alpha = 0$ .  $\alpha = 0.15$  reduces the debt ratio to less than 0.5, and  $\alpha = 0.175$  makes it about 0.3, which is appropriate for actual capital structure. While the bankruptcy cost model was not able to reduce the debt ratio to an actual level even given an unrealistically large value of  $k$ , this model derives any values of the debt ratio as optimal capital structure, depending on the value of  $\alpha$ . Capital structure models are not able to provide actual debt ratios without considering the decline in  $\mu_Z$  associated with the agency costs of debt.

**Table 4: Agency Costs of Debt: The Effect of Changes in  $\alpha$** 

$\alpha$	$\mu_Z^*$	$\sigma_Z^*$	$L^*$	Prob	$S_L$	$B$	$V_L$	$B/V_L$	ROA	LOSS
0.025	49.19	13.97	32.41	0.115	5.71	17.49	23.21	0.754	0.112	0.016
0.050	48.50	15.39	29.93	0.114	6.33	16.02	22.35	0.717	0.117	0.030
0.075	47.96	17.00	27.22	0.111	7.08	14.44	21.52	0.671	0.123	0.041
0.100	47.58	18.73	24.24	0.106	7.98	12.77	20.74	0.615	0.129	0.049
0.125	47.38	20.57	20.95	0.099	9.07	10.98	20.05	0.548	0.136	0.052
0.150	47.40	22.49	17.30	0.090	10.38	9.05	19.43	0.466	0.144	0.052
0.175	47.68	24.51	13.23	0.080	11.96	6.93	18.90	0.367	0.152	0.046
0.200	48.28	26.64	8.62	0.068	13.90	4.56	18.45	0.247	0.162	0.034
0.225	48.57	24.53	6.36	0.043	15.18	3.50	18.68	0.187	0.160	0.029
0.250	48.94	23.87	4.26	0.031	16.33	2.39	18.72	0.128	0.161	0.021
0.275	49.62	23.01	1.37	0.018	18.04	0.79	18.83	0.042	0.164	0.007

This table presents simulation results in the case of considering the agency costs of debt. When  $\alpha$  is changed from 0.025 to 0.275, an optimal triad,  $\mu_Z^*$ ,  $\sigma_Z^*$ , and  $L^*$ , which maximizes  $V_L$  and  $S_L$  and which makes Equation (9) hold, is provided for each  $\alpha$  in the table. The equity value  $S_L$ , the debt value  $B$ , and the firm value  $V_L$  are computed under the optimal triad. The probability of default is denoted as Prob and the debt ratio as  $B/V_L$ . ROA is computed as  $(\mu_Z^* - V_L)/(10 \times V_L)$ .  $LOSS$  is an agency costs measure, that is,  $(\mu_Z^U - \mu_Z^*)/\mu_Z^U$ . Suppose  $k = 0.4$  and  $\mu_Z^U = 50$ .

A more interesting result in Table 4 is that the debt ratio was observed to have a negative correlation with  $\mu_Z^*$ . The debt ratio decreases as  $\alpha$  increases, and  $\mu_Z^*$  increases at the same time for  $\alpha > 0.1$ . We can say that the debt ratio is negatively correlated with earnings although there are a few exceptions. If we define earnings as a ratio such as ROA, ROA in Table 4 confirms the negative correlation with debt.

Another interesting point in Table 4 is  $LOSS$ , which was defined in Equation (12). The difference between  $\mu_Z^U$  and  $\mu_Z^*$  becomes the ex post loss in earnings owing to debt. Table 4 calculates  $\mu_Z^*$  with  $\mu_Z^U = 50.0$  and each value of  $\alpha$  given, and  $LOSS$  is estimated. As  $\alpha$  increases,  $LOSS$  also increases initially and then quickly begins to decrease.  $LOSS$  is at most about 5%, which suggests that agency costs are not very serious in terms of economic welfare.

The next simulation of the agency costs model addresses the effect of  $k$ . Table 5 summarizes the calculations when  $k$  is changed from 0.1 to 0.7 with  $\mu_Z^U = 50.0$  and  $\alpha = 0.175$  given. They are similar to those of the bankruptcy cost model in that the debt ratio declines as  $k$  increases. In the cases where  $k \geq 0.5$ , however, this model becomes irrelevant for  $k$ . When  $k$  is 0.6 or 0.7, the results are almost the same. Compared with  $k = 0.5$ , the difference is negligible. When  $k$  is between 0.1 and 0.5, an increase in  $k$  leads to a decrease in the debt ratio and to an increase in  $\mu_Z^*$  and ROA. Thus, a negative correlation also exists between earnings and debt.

**Table 5: Agency Costs of Debt: The Effect of Changes in  $k$**

$k$	$\mu_Z^*$	$\sigma_Z^*$	$L^*$	Prob	$S_L$	$B$	$V_L$	$B/V_L$	ROA	$LOSS$
0.1	44.74	15.07	30.06	0.165	4.95	16.28	21.22	0.767	0.111	0.105
0.2	46.06	18.83	22.50	0.105	8.06	12.11	20.17	0.601	0.128	0.079
0.3	46.97	21.85	17.31	0.087	10.25	9.18	19.43	0.473	0.142	0.061
0.4	47.68	24.51	13.23	0.080	11.96	6.93	18.90	0.367	0.152	0.046
0.5	48.29	27.08	9.76	0.077	13.41	5.08	18.49	0.275	0.161	0.034
0.6	48.30	25.43	9.69	0.064	13.57	5.15	18.73	0.275	0.158	0.034
0.7	48.30	25.42	9.70	0.064	13.57	5.15	18.73	0.275	0.158	0.034

This table presents simulation results in the case of considering the agency costs of debt. When  $k$  is changed from 0.1 to 0.7, an optimal triad,  $\mu_Z^*$ ,  $\sigma_Z^*$ , and  $L^*$ , which maximizes  $V_L$  and  $S_L$  and which makes Equation (9) hold, is provided for each  $k$  in the table. The equity value  $S_L$ , the debt value  $B$ , and the firm value  $V_L$  are computed under the optimal triad. The probability of default is denoted as Prob and the debt ratio as  $B/V_L$ . ROA is computed as  $(\mu_Z^* - V_L)/(10 \times V_L)$ .  $LOSS$  is an agency costs measure, that is,  $(\mu_Z^U - \mu_Z^*)/\mu_Z^U$ . Suppose  $\alpha = 0.175$  and  $\mu_Z^U = 50$ .

Table 6 simulates the effect of  $\mu_Z^U$ .  $\mu_Z^U$  is changed from 35.0 to 70.0 with  $\alpha = 0.175$  and  $k = 0.4$ . An increase in  $\mu_Z^U$  is associated with increases in  $\mu_Z^*$ ,  $\sigma_Z^*$ , and  $L^*$ . It is easy to confirm that these are homogeneous of degree one with respect to  $\mu_Z^U$ . Then, the debt ratio and ROA remain constant. Under the assumption of Equation (9), the effect of  $\mu_Z^U$  upon them is neutral.<sup>9</sup>

From the above simulation results, we can point out the three most interesting characteristics of the model in this paper. First, the optimal capital structure of this model is appropriate for actual financial leverage when the agency costs are considered as  $\alpha \neq 0$ . This  $\alpha$  has much influence on optimal capital structure. Second, this model shows a negative correlation between earnings and debt. Although there are a few cases where the correlation is obscure, no positive correlation as in the bankruptcy cost model is found. Third, *LOSS*, which quantifies the ex post agency costs, is not as serious as we expected.

**Table 6: Agency Costs of Debt: The Effect of Changes in  $\mu_Z^U$**

$\mu_Z^U$	$\mu_Z^*$	$\sigma_Z^*$	$L^*$	Prob	$S_L$	$B$	$V_L$	$B/V_L$	ROA	<i>LOSS</i>
35.0	33.38	17.16	9.26	0.080	8.38	4.85	13.23	0.367	0.152	0.046
40.0	38.15	19.60	10.59	0.080	9.57	5.55	15.12	0.367	0.152	0.046
45.0	42.92	22.06	11.91	0.080	10.77	6.24	17.01	0.367	0.152	0.046
50.0	47.68	24.51	13.23	0.080	11.96	6.93	18.90	0.367	0.152	0.046
55.0	52.45	26.96	14.55	0.080	13.16	7.63	20.79	0.367	0.152	0.046
60.0	57.22	29.41	15.88	0.080	14.36	8.32	22.68	0.367	0.152	0.046
65.0	61.99	31.86	17.20	0.080	15.55	9.01	24.57	0.367	0.152	0.046
70.0	66.76	34.30	18.52	0.080	16.75	9.71	26.46	0.367	0.152	0.046

This table presents simulation results in the case of considering the agency costs of debt. When  $\mu_Z^U$  is changed from 35.0 to 70.0, an optimal triad,  $\mu_Z^*$ ,  $\sigma_Z^*$ , and  $L^*$ , which maximizes  $V_L$  and  $S_L$  and which makes Equation (9) hold, is provided for each  $\mu_Z^U$  in the table. The equity value  $S_L$ , the debt value  $B$ , and the firm value  $V_L$  are computed under the optimal triad. The probability of default is denoted as Prob and the debt ratio as  $B/V_L$ . ROA is computed as  $(\mu_Z^* - V_L)/(10 \times V_L)$ . *LOSS* is an agency costs measure, that is,  $(\mu_Z^U - \mu_Z^*)/\mu_Z^U$ . Suppose  $\alpha = 0.175$  and  $k = 0.4$ .

## 6. APPLICATION TO ACTUAL FIRMS

### 6.1. Validity of the Calibration

The valuation of equity and debt was derived from the CAPM as a function of three variables:  $L$ ,  $\mu_Z$ , and  $\sigma_Z$ . If we suppose the behavior of investors and of the manager as described in Section 4, then these variables are endogenous, and the equity value  $S_L$  and the debt value  $B$  can be reformulated into the functions of two exogenous variables,  $\mu_Z^U$  and  $\alpha$ . In any case, data for  $S_L$  and  $B$  are available for actual firms.  $S_L$  is obtained by multiplying a share price by the outstanding number of shares, and  $B$  is debt on the balance sheet as a proxy. Here Equations (10) and (11) are rewritten as

$$S_L = S_L(\mu_Z^U, \alpha), \tag{10}$$

$$B = B(\mu_Z^U, \alpha). \tag{11}$$

<sup>9</sup> The homogeneity with respect to  $\mu_Z^U$  depends on Equation (9). If instead we assume another equation,  $\mu_Z = \mu_Z^U - \alpha L^2$ , the homogeneity disappears. This shows that an increase in  $\mu_Z^U$  leads to a decrease in the debt ratio.

When the values of  $S_L$  and  $B$  are given, these equations construct a simultaneous equation system with unknown variables,  $\mu_Z^U$  and  $\alpha$ . The estimation of  $\mu_Z^U$  and  $\alpha$  is to calibrate them as a solution of the system.

If this model is to fit an actual firm, the calibration should be made successfully from data, and the computation results must also be appropriate. We have chosen firms listed on the Tokyo Stock Exchange 1st section, and examine their computed values of  $\mu_Z^U$ ,  $\alpha$ ,  $L^*$ ,  $\mu_Z^*$ , and  $\sigma_Z^*$ . All the firms we have selected belong to manufacturing industries.

In this paper, two periods are used in testing the model. One is 10 years from fiscal year 1974 to 1983, and the other is 10 years from 1984 to 1993. The former is denoted as period[1] and the latter as period[2]. The data for  $S_L$  is a share price multiplied by the number of shares outstanding, and the data for  $B$  is interest-bearing debt. For each firm, an average over the period is used for each item. The debt on the balance sheet is the book value. The market value of debt is not available, and we follow the convention that the book value of debt is used in computing the debt ratio.

It is impossible to estimate an appropriate value of the bankruptcy costs parameter  $k$  for each firm. Here,  $k$  is assumed to be 0.3 for all firms. The corporate income tax rate  $\tau$  is 0.45. We calculate  $\lambda$  from capital market data about  $E(\bar{R}_M)$ ,  $\sigma(\bar{R}_M)$ , and  $R_F$  over periods[1] and [2]. The value of  $cov(\bar{R}_M, \bar{Z})$  is converted from the beta coefficient for each firm over the two periods.

In making the above assumptions, we compute five parameters,  $\mu_Z^U$ ,  $\alpha$ ,  $\mu_Z^*$ ,  $\sigma_Z^*$ , and  $L^*$  through this model. The number of firms we analyze is 515 in period[1] and 592 in period[2]. Among them there are 471 firms in period[1] and 578 firms in period[2] for which the computation is successful. These correspond to 91.5% of the total in period[1] and 97.6% in period [2]. The results prove that this model is able to fit well behavior of actual firms.

**Table 7: Summary of the Computation**

	the number of firms	
	period[1]	period[2]
firms we examined (A)	515	592
success in the computation (B)	471	578
percentage (B)/(A)	91.5%	97.6%

Period[1] is 10 years from 1974 to 1983. Period[2] is 10 years from 1984 to 1993.

Next, we investigate whether the estimates are similar to actual numbers. Panel (A) in Table 8 summarizes cross-section statistics for the estimates of the 471 firms in period[1] and the 578 firms in period[2], which were successful in the computation of this model. Panel (B) tabulates some values from financial reports.

$\mu_Z^* - V_L$  and  $\sigma_Z^*$  in Panel (A) of Table 8 are cross-section averages for a mean and a standard deviation of earnings estimated using the model.  $L^* - B$  is the average of the model's interest payment. This model considers 10 years as one period. For example,  $\mu_Z^* - V_L$ , which was estimated from the model, is a lump sum over the 10 years. To compare this with values from a financial report, we have to allocate the lump sum to every year. The figures in the table are

those that were allocated. In any case,  $\alpha$  and  $LOSS$  are the quantitative measures of the agency costs. Their validity will be investigated in the next subsection.<sup>10</sup>

**Table 8: Cross Section Statistics for the Estimates and Real Values**

	period[1]				period[2]			
	Mean	S.D.	Min.	Max.	Mean	S.D.	Min.	Max.
Panel (A)	Calculated values from the model							
$\mu_Z^* - V_L$	0.2203	0.4602	0.0092	4.9007	0.4232	0.7609	0.0194	8.7035
$\sigma_Z^*$	0.0959	0.1725	0.0056	1.6655	0.1955	0.3532	0.0095	3.6280
$L^* - B$	0.0635	0.1429	0.0004	1.5879	0.0540	0.1129	0.0006	1.2014
$\rho_B B$	0.0572	0.1361	0.0004	1.5401	0.0496	0.1027	0.0005	1.0974
$\alpha$	0.2811	0.0691	0.0997	0.4124	0.2523	0.0299	0.1551	0.3161
$LOSS$	0.0781	0.0292	0.0051	0.1490	0.0363	0.0162	0.0028	0.0689
Panel (B)	Data from financial reports							
$ave(EBITDA)$	0.1704	0.3705	0.0023	4.3286	0.2463	0.5600	0.0011	8.5347
$std(EBITDA)$	0.0525	0.1084	0.0018	0.9317	0.0603	0.1102	0.0023	1.4171
$INTPAY$	0.0562	0.1362	0.0008	1.6046	0.0434	0.0966	0.0006	1.0275
$DBR$	0.4605	0.2106	0.0218	0.8406	0.2490	0.1317	0.0141	0.6385
$RDAD$	0.1129	0.1577	0.0000	1.4723	0.1224	0.1567	0.0000	1.4868
$GROW$	0.0597	0.0402	-0.0633	0.2469	0.0165	0.0356	-0.1134	0.1806
$SIZE$	-0.4711	1.1574	-2.9820	3.5171	0.0467	1.1624	-3.2579	3.8837

Period [1] is 10 years from 1974 to 1983. Period [2] is 10 years from 1984 to 1993. Sample size is 471 firms in period [1] and 578 firms in period [2]. Panel (A) summarizes estimates of the model's parameters.  $\mu_Z^* - V_L$  is an expected value of firm earnings.  $\sigma_Z^*$  is a standard deviation of the earnings.  $L^* - B$  is an interest payment on the model.  $\rho_B B$  is another approximation to model's interest payment. These are converted from a lump sum to a one-year value.  $\alpha$  and  $LOSS$  are the quantitative measures of agency costs. Panel (B) summarizes real data calculated from financial reports.  $ave(EBITDA)$  and  $std(EBITDA)$  are an average and a standard deviation over the period for a sample firm's EBITDA.  $INTPAY$  is an interest payment.  $DBR$  is a debt ratio, which is (interest bearing debt)/(firm value).  $RDAD$  is the ratio of intangible to tangible assets.  $GROW$  is the growth rate of firm's assets.  $SIZE$  is a logarithm value of firm's sales.

<sup>10</sup> We allocate a lump sum to every year using the coefficient  $ADJ$  defined as  $ADJ = \frac{R}{(1+R)^N - 1}$ , where one period is  $N$  years and  $R$  is a one-year discount rate. By multiplying  $(\mu_Z^* - V_L)$  and  $(L^* - B)$  by  $ADJ$ , we can obtain one-year values calculated from the model. The discount rate  $R$  used here is the required rate of return, which is also estimated from the model.



**Table 9: Correlations between Real Values and the Model's Estimates**

Panel (A)			Panel (B)		
	dependent variable: $\mu_z^* - V_L$			dependent variable: $\sigma_z^*$	
	period[1]	period[2]		period[1]	period[2]
const.	0.016 (2.32)	0.103 (3.31)	const.	0.018 (5.09)	0.017 (2.00)
<i>ave(EBITDA)</i>	1.197 (3.10)	1.300 (2.00)	<i>std(EBITDA)</i>	1.475 (5.15)	2.948 (10.29)
$\bar{R}^2$	0.928	0.915	$\bar{R}^2$	0.859	0.845
Panel (C)			Panel (D)		
	dependent variable: $L^* - B$			dependent variable: $\rho_B B$	
	period[1]	period[2]		period[1]	period[2]
const.	0.005 (4.65)	0.004 (6.72)	const.	0.001 (1.75)	0.004 (7.31)
<i>INTPAY</i>	1.041 (1.62)	1.150 (6.38)	<i>INTPAY</i>	0.993 (0.35)	1.047 (2.23)
$\bar{R}^2$	0.984	0.970	$\bar{R}^2$	0.986	0.970

This table presents correlations between real values and the model's estimates. In order to ascertain the extent of any correlation, we attempt to regress the model's estimates on real values.  $\mu_z^* - V_L$  is an expected value of firm earnings.  $\sigma_z^*$  is a standard deviation of the earnings.  $L^* - B$  and  $\rho_B B$  are an interest payment on the model. These are model's estimates. Real values are *ave(EBITDA)*, *std(EBITDA)*, and *INTPAY*. *ave(EBITDA)* and *std(EBITDA)* are an average and a standard deviation over the period for sample firm's EBITDA. *INTPAY* is an interest payment. Parentheses give a t-value for a test that a coefficient is equal to 0 for a constant or to 1 for an independent variable. Period[1] is 10 years from 1974 to 1983. Period[2] is 10 years from 1984 to 1993. There are 471 firms in period[1] and 578 firms in period[2].

On the other hand, the actual values that are to be compared with earnings calculated through the model are statistics concerning the EBITDA in financial reports. The EBITDA in this paper is computed by adding an interest payment, income taxes, and depreciation to after-tax earnings. We compute a mean and a standard deviation of the EBITDA over the period for each firm. Then, *ave(EBITDA)* and *std(EBITDA)* are sample averages of the means and the standard deviations. *INTPAY* is the interest payment that is calculated in the same way. Panel (B) in Table 8 summarizes these cross-section statistics for the sample firms. The other variables, *DBR*, *RDAD*, *GROW*, and *SIZE*, will be addressed in the next subsection. *DBR* is a debt ratio,

*RDAD* is an intangible-tangible asset ratio, *GROW* is the growth rate of firm's assets, and *SIZE* is a logarithm of firm's sales.

Compare Panel (A) with Panel (B) in Table 8. In terms of a cross-section average, the estimates from the model are bigger than those from financial reports for the means and the standard deviations in earnings.  $\mu_Z^* - V_L$  is larger than  $ave(EBITDA)$  by 30% in period[1] and by 70% in period[2].  $\sigma_Z^*$  is 2 times as large as  $std(EBITDA)$  in period[1] and 3.3 times as large in period[2]. For the interest payment, however, the model values are almost the same as the actual ones.  $L^* - B$  is 0.064 and 0.054 over periods[1] and [2], and *INTPAY* is 0.056 and 0.043. If the model's interest payment is considered as  $\rho_B B$ , where  $\rho_B$  is a required rate of return on debt, the estimated values are closer to the actual ones.

The cross-section averages show that the estimated values of earnings appear overestimated. There are several definitions of earnings constructed from a financial report. It is clear that we cannot easily decide which definition is the best. Thus, let us ignore the difference in cross-section averages of earnings. More important is the model's fit. In order to examine this, we look at correlations between actual earnings and the model's estimates for the sample firms. Independent variables for regression equations are the actual values from financial reports, and dependent variables are those computed from the model. If the model's computations are appropriate, the fit of the regression must be good. Table 9 summarizes the regression results.

In Panel (A) of Table 9, a dependent variable is the expected earnings computed from  $\mu_Z^* - V_L$  and its regressor is  $ave(EBITDA)$ . Since  $\bar{R}^2$  is greater than 0.9, the fit of the regression is very good. In Panel (B), a dependent variable is the standard deviation of earnings computed from  $\sigma_Z^*$  and its regressor is  $std(EBITDA)$ .  $\bar{R}^2$  is above 0.8. Panels (C) and (D) are the results of the regressions of the interest payments from the model on *INTPAY*.  $L^* - B$  is used in Panel (C) and  $\rho_B B$  in Panel (D). In these cases  $\bar{R}^2$  is 0.97 and 0.98 respectively, so the fit is very good.

From the regression results we conclude that this model is sufficiently adequate to mimic actual firm's behavior. The model's values for each firm are highly correlated with the actual ones. There remains a problem in that some differences in level exist between the model's estimates and the actual earnings.

## 6.2. Validity of the Measure of the Agency Costs

This model provides us with the quantitative measures of the agency costs,  $\alpha$  and *LOSS*, for each firm. How do we know whether these are appropriate? We compare the model's estimates with past empirical research into capital structure. In corporate finance there are many studies to find out which variables are statistically correlated with a debt ratio. When interpreting regression results, the manner of thinking that depends on agency costs is now becoming conventional wisdom. We test whether the model's estimates are consistent with this.

Table 10 shows regression results. Their dependent variable, *DBR*, is a debt ratio that is computed using interest-bearing debt divided by a market firm value. There are three independent variables: *RDAD*, *GROW*, and *SIZE*. *RDAD* is the ratio of intangible to tangible assets. Intangible assets are the sum of research, development, and advertisement expenditure. Tangible assets are the sum of fixed assets. *GROW* is firm's growth, which is the growth rate of its total assets. *SIZE* is firm's size, which is the logarithm of its sales. The values used are averages over each period ([1] and [2]) for each firm. We attempt cross-section regression for the sample firms. This method of estimation is the most standard in empirical studies of capital structure.

**Table 10: Regression of the Debt Ratio**

dependent variable: <i>DBR</i>					
period	const.	<i>RDAD</i>	<i>GROW</i>	<i>SIZE</i>	$\bar{R}^2$
[1]	0.667 (44.31)	-0.419 (-7.44)	-2.377 (-10.4)	0.036 (5.28)	0.391
[2]	0.293 (41.36)	-0.271 (-8.38)	-0.726 (-4.55)	0.019 (4.15)	0.177

This table shows regression results to see how debt ratios are correlated with some variables. The variables we examine are *RDAD*, *GROW*, and *SIZE*. *DBR* is a debt ratio, which is (interest bearing debt)/(firm value). *RDAD* is the ratio of intangible to tangible assets. *GROW* is the growth rate of firm's assets. *SIZE* is a logarithm value of firm's sales. Parentheses give a t-value. Period[1] is 10 years from 1974 to 1983. Period[2] is 10 years from 1984 to 1993. There are 471 firms in period[1] and 578 firms in period[2].

The results are normal and very typical, compared with past studies. All the explanatory variables are significant. *RDAD* and *GROW* have negative coefficients and *SIZE* has a positive one. The most significant is *GROW* in period[1] and *RDAD* in period[2].  $\bar{R}^2$  in period[2] is half that in period[1].

It has now become a conventional viewpoint that the negative coefficients of *RDAD* and *GROW* are highly significant due to the effect of agency costs. When *RDAD* increases, it is more difficult for investors to monitor firm's behavior because its assets get more intangible. Then, the incentive of asset substitution is stronger, which leads to greater agency costs. In order to avoid this loss, a firm tends to reduce its debt. So firms with high values of *RDAD* decrease their debt ratios. This hypothesis, proposed by Long and Malitz (1985), has been the most popular in empirical studies of capital structure. We designate it as Hypothesis 1.

The next hypothesis is about *GROW*. Since a high growth firm has a lot of investment opportunities, which provide it with high earnings, it is likely that the firm will fall into under-investment. Thus, firms that grow faster have greater agency costs owing to their debt overhang than those that grow slowly. So firms with high growth tend to have less debt, and growth and debt are negatively correlated. Jensen (1986) and Stulz (1990) emphasize this correlation in their models, and Lang, Ofek, and Stulz (1996) is the most famous empirical research in this regard. Here, this is designated as Hypothesis 2.

The coefficient of *SIZE* is positive because a large firm incurs much debt as it can reduce the probability of bankruptcy by diversifying its assets. Among empirical studies, it was Bradley, Jarrell, and Kim (1984) who first supported this. Although their thinking has nothing to do with agency costs, it still remains popular today. By associating it to agency costs, we develop the following hypothesis: The agency costs of debt premise the possibility of firm's bankruptcy. If it is not probable that a firm will go bankrupt, no agency costs of debt will occur for that firm. This means that a larger firm will incur lower agency costs because it can reduce its bankruptcy probability, so it can depend to a significant extent on debt. Here, this constitutes Hypothesis 3.

**Table 11: Correlation between Variables**

		dependent variable	
		$\alpha$	<i>LOSS</i>
Hypothesis 1	<i>RDAD</i>	+	-
Hypothesis 2	<i>GROW</i>	+	-
Hypothesis 3	<i>SIZE</i>	-	+
	<i>DBR</i>	-	+

This table shows correlations between agency costs measures and some variables.

How do these hypotheses relate to  $\alpha$  and *LOSS*? When a firm faces large agency costs of debt, a significant economic loss might come about owing to an increase in debt, which means that the firm experiences a large marginal effect of the agency costs,  $\alpha$ . Since the hypotheses predict that an increase in the agency costs of debt decreases the debt ratio, the firm depends less on debt to avoid the agency costs, which might bring about a decline in *LOSS*. Thus, if an explanatory variable has a positive correlation with the agency costs, it is positively correlated with  $\alpha$  and negatively correlated with *LOSS*. Table 11 summarizes correlations of  $\alpha$  and *LOSS* with the explanatory variable that represents each hypothesis. It is also easy to understand the correlations with the debt ratio; *DBR* has a negative correlation with  $\alpha$  and a positive one with *LOSS*.

If the values of  $\alpha$  and *LOSS* computed from the model are valid, they must be correlated with the variables in the hypotheses the way Table 11 shows. In order to test these correlations, we estimate some regression equations.

$$\alpha g(\text{DBR}, \text{RDAD}, \text{GROW}, \text{SIZE}) + \varepsilon$$

$$\text{LOSS} = g(\text{DBR}, \text{RDAD}, \text{GROW}, \text{SIZE}) + \varepsilon$$

The function  $g(\cdot)$  is a linear regression equation.  $\varepsilon$  is a disturbance. The results of the regression over periods[1] and [2] are summarized in Table 12. Among the explanatory variables, we separate *DBR* and others, and attempt two regression equations.

**Table 12: Regression Results Using the Quantitative Measures of Agency Costs**

	dependent variable: $\alpha$				dependent variable: <i>LOSS</i>			
	period[1]		period[2]		period[1]		period[2]	
const.	0.424	0.219	0.306	0.243	0.025	0.101	0.006	0.042
	(178.4)	(39.75)	(372.0)	(152.9)	(14.68)	(54.46)	(20.13)	(49.57)
<i>DBR</i>	-0.310		-0.215		0.116		0.121	
	(-58.50)		(-72.41)		(31.25)		(87.67)	
<i>RDAD</i>		0.108		0.057		-0.052		-0.034
		(6.75)		(7.84)		(-6.58)		(-8.59)
<i>GROW</i>		0.779		0.153		-0.289		-0.081
		(10.23)		(4.27)		(-9.05)		(-4.12)
<i>SIZE</i>		-0.803		-0.427		0.040		0.217
		(-3.28)		(-3.87)		(0.44)		(3.82)
$\bar{R}^2$	0.893	0.323	0.895	0.151	0.694	0.291	0.974	0.167

This table presents regression results using quantitative measures of agency costs.  $\alpha$  and *LOSS* have correlations that the hypotheses claim to observe.  $\alpha$  and *LOSS* are the quantitative measures of agency costs. *DBR* is a debt ratio, which is (interest bearing debt)/(firm value). *RDAD* is the ratio of intangible to tangible assets. *GROW* is the growth rate of firm's assets. *SIZE* is a logarithm value of firm's sales. Parentheses give a t-value. Period[1] is 10 years from 1974 to 1983. Period[2] is 10 years from 1984 to 1993. There are 471 firms in period[1] and 578 firms in period[2].

Regressing  $\alpha$  and *LOSS* on *DBR* shows similar results over periods[1] and [2]. *DBR* is negatively correlated with  $\alpha$  and positively correlated with *LOSS*. The coefficients are highly significant, and *DBR* explains  $\alpha$  and *LOSS* well. These correlations are stronger in period[2] than in period[1]. We conclude that the values of  $\alpha$  and *LOSS* computed from the model are consistent with the hypotheses under which, while facing large agency costs of debt, a firm intends to depend less on debt to avoid the loss caused by the agency costs.

The next step is to regress  $\alpha$  and *LOSS* on *RDAD*, *GROW*, and *SIZE*. The only insignificant coefficient is that of *SIZE* in period[1] for the *LOSS* equation. Other regressors are significant enough to reject at the 1% level. The signs of the coefficients correspond with Hypotheses 1, 2, and 3. We confirm that *RDAD* and *GROW* have a positive correlation with  $\alpha$  and a negative correlation with *LOSS*, and that *SIZE* has a negative correlation with  $\alpha$  and a positive correlation

with *LOSS*. The regression results show that  $\alpha$  and *LOSS* have the correlations that the hypotheses claim to observe. The only exception is Hypothesis 3 over period[1]. Therefore, the quantitative measures of the agency costs of debt calibrated from the model are almost perfectly consistent with past empirical research into capital structure. We conclude that the model is adequate for actual firm behavior.

## 7. CONCLUSION

Although agency costs are currently well known, there are few studies that have tried to quantify them. How serious is the effect of agency costs on the loss of firm's earnings? How strongly do the agency costs of debt influence firm's capital structure? In order to embody the quantity of agency costs, the model constructed in this paper provides measures for them and investigates their effect on capital structure. This model extends the traditional bankruptcy cost model by considering agency costs, and we conclude that it is very well suited to capture financial behavior of actual firms.

Simulation helps to clarify the features of this model. First, the model realizes optimal capital structure that resembles an actual firm. This model overcomes the difficulty that the bankruptcy cost models did not resemble an actual debt ratio. Second, this model shows a negative correlation between firm's earnings and its debt ratio. Empirical studies have observed that they were negatively correlated. Third, the agency costs of debt strongly influence the optimal debt ratio, while the loss derived from agency costs might not be very serious.

Next, using firm data, we investigate whether the model fits actual behavior. Debt ratios observed from market share prices can be optimized in terms of this model. As for unknown parameters, which are to be estimated from the model, these estimates are highly correlated with data that are observed in financial reports. The quantitative measures of agency costs are fully compatible with past empirical studies of capital structure.

### Appendix A: The Assumption of Equation (9)

We assume Equation (9) in this model. This is based on the premise that more debt leads to a decline in  $\mu_z$  due to over-investment and/or under-investment. It is now well-known why the investment distortion arises through two kinds of incentive; debt overhang and asset substitution. In this appendix, instead of making another model, numerical examples which depend on this model confirm that  $\mu_z$  is a decreasing function of  $L$ .

#### A.1: Debt Overhang

We show the debt overhang, which brings about under-investment; the investment which should be executed cannot be implemented.

Firm's business is considered as many projects which affect its earnings. A more profitable project has higher priority. We take up four marginal projects which are the least profitable. Projects from  $a$  to  $d$  in Table 13 are marginal ones a firm faces. Earnings on the firm is assumed to be 293 if no marginal project is executed. A symbol  $O$  denotes this like  $\mu_z^O$ . Table 13 shows an increase in earnings when a project is carried out. Each project requires an expenditure of 2. For example, Project  $a$  raises firm's earnings by 9 if the firm spends 2 as an initial outlay.

**Table 13: Earnings on a Project: Case 1**

$\mu_Z^0$	Projects				$\mu_Z^U$
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	
293	9	7	6	5	320

Numbers are increases in the expected value of firm's earnings from Projects *a* to *d*.  $\mu_Z^0$  is the expected earnings in the case where the firm does not implement any of these projects.  $\mu_Z^U$  is the one when all of them are executed.

We assume that these marginal projects have a positive NPV, and that all of them are executed when the firm is unleveraged. If only Project *a* is adopted, the value of expected earnings of the firm  $\mu_Z$  is 302, which consists of  $\mu_Z^0$  and Project *a*. In order to exclude the effect of asset substitution, a standard deviation of earnings  $\sigma_Z$  is assumed to be proportional to its expected value. Provided  $\sigma_Z^0$  is set to 28.95 in the case of no marginal project,  $\sigma_Z$  becomes 29.84 when only Project *a* is executed.  $\Delta S_L(L, \mu_Z, \sigma_Z)$ , an increase in an equity value due to a project, is equal to firm value increment for an unleveraged firm. From the column of  $L = 0$  in Table 14, Project *a* increases a firm value by 3.87, and its NPV, which is the difference from investment of 2, proves to be positive.

Next is the case where the firm implements Project *b* in addition to Project *a*.  $\mu_Z$  is 309, and  $\sigma_Z$  is 30.54. Since under no debt ( $L = 0$ ) an equity value rises by 3.01, Project *b* has a positive NPV. When Project *c* is added, an increase in the equity value is 2.58. This is 2.15 for Project *d* besides. They are larger than an expenditure of 2, and Projects *c* and *d* have a positive NPV. The firm does not have any other investment opportunity. As long as all these projects are carried out, the firm expects earnings of 320 and a standard deviation of 31.62. These values are  $\mu_Z^U$  and  $\sigma_Z^U$  for the unleveraged firm. A superscript \* in Table 14 denotes that a project should be executed.

**Table 14: Examples of the Debt Overhang**

Projects	<i>I</i>	$\mu_Z$	$\sigma_Z$	$\Delta S_L(L, \mu_Z, \sigma_Z)$					
				$L = 0$	$L = 265$	$L = 272$	$L = 283$	$L = 287$	$L = 289$
<i>O+a</i>	2	302	29.84	3.87*	3.20*	2.87*	2.26*	2.02*	1.90
<i>O+a+b</i>	2	309	30.54	3.01*	2.70*	2.50*	2.09*	1.92	1.83
<i>O+a+b+c</i>	2	315	31.13	2.58*	2.42*	2.28*	1.99	1.86	1.79
<i>O+a+b+c+d</i>	2	320	31.62	2.15*	2.07*	1.98	1.78	1.68	1.63

The most left column indicates projects executed under which  $\mu_Z$  is an expected value of firm's earnings and  $\sigma_Z$  is a standard deviation.  $\Delta S_L(L, \mu_Z, \sigma_Z)$  is an incremental equity value when Projects *a* to *d* are executed in addition. These are equal to an increase in a firm value only for

an unleveraged firm ( $L = 0$ ), and the NPV is the difference between  $\Delta S_L(0, \mu_Z, \sigma_Z)$  and an investment outlay  $I$ . The expected earnings under no marginal projects is  $\mu_Z^0 = 293$ , and a standard deviation is  $\sigma_Z^0 = 28.95$ .

When the firm is leveraged, an increase in an equity value is smaller than that in a firm value because some of the increase in a firm value leaks to debt. Even if the increase in a firm value is more than an investment outlay, the increase in an equity value is not always more than it. If the incremental equity value is less than the expense for a positive NPV project, carrying it out harms the wealth of shareholders. It is ordinary that the increase in the equity value gets smaller for a more leveraged firm. Table 14 shows that more debt converts the marginal projects into unprofitable for shareholders. The incremental equity values for  $L = 265$  are less than those for  $L = 0$ , but still larger than the outlay of 2. As the result that all the projects are carried out,  $\mu_Z$  is the same as  $\mu_Z^U = 320$ .

On the other hand, when debt grows to  $L = 272$ , the increase in equity for Project  $d$  is less than the outlay of 2. Since the firm executes Projects  $a$  to  $c$ , but not  $d$  then,  $\mu_Z$  decreases from  $\mu_Z^U$  to 315. In the case of  $L = 283$ , Project  $c$  becomes unprofitable, and Projects  $a$  and  $b$  are executed. Then  $\mu_Z$  declines more to 309. Furthermore,  $L = 287$  changes Project  $b$  into unprofitable and unexecutable one, and  $\mu_Z$  is equal to 302. Eventually  $L = 289$  makes the incremental value for Project  $a$  smaller than the outlay, which induces that no marginal project is implemented and  $\mu_Z = 293$ . Because of the increase in  $L$  with more debt, projects which have bad profitability gradually drop out, and the expected value of firm's earnings decreases.

Although  $\mu_Z$  is a decreasing function of  $L$ , it is obvious that its function form depends on investment opportunity a firm faces. The function is assumed to be linear in terms of its approximation so that it is easily applied to any firms. It is true that Equation (9) to determine  $\mu_Z$  is superficial and ad hoc. We have tried another function for several firms. A quadratic relation between  $L$  and  $\mu_Z$  leads to the same arguments as this paper. Further research must bring out an effect of function forms on our estimation.<sup>11</sup>

## A.2: Asset Substitution

This section discusses the over-investment which means that a project which should not be executed is adopted through the asset substitution. Examples for simulation are Projects  $A$  to  $D$  in Table 15. If a firm substitutes its assets into more risky ones, its equity could rise in value without regard to their profitability.

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<sup>11</sup> In order to construct a system for  $\mu_Z$ , we have to model business activities that include investment a firm executes. The investment decision-making depends on its opportunity, which is very different between firms. Equation (9) is, as it were, a reduced form of our model. It is more practical to depend on a reduced form equation than to dwell on a structural form system. More important is that values of  $\alpha$  can be estimated for actual firms. Parameter estimates in this paper adopt complicated procedure of nonlinear simultaneous equations. If the effect of  $L$  on  $\mu_Z$  were irrelevant, it would be impossible to obtain a convergent  $\alpha$ .



**Table 15: Earnings on a Project: Case 2**

$\mu_Z^U$	Projects			
	A	B	C	D
320	-5	-10	-15	-20

Numbers are increases in the expected value of firm's earnings from Projects A to D.  $\mu_Z^U$  is the expected earnings for the unleveraged firm.

We suppose that an unleveraged firm implements all the projects which have a positive NPV, and that its expected value and a standard deviation of earnings are  $\mu_Z^U = 320$  and  $\sigma_Z^U = 31.62$ . The above examples of the asset substitution are the projects which make the firm more risky and which are of negative effect on its earnings. Project A affects  $\mu_Z$  by -5 and its execution has  $\mu_Z = 315$ . Then the risk is assumed to get three times as large as a proportion in scale. The standard deviation in Table 16 is 93.39, which trebles 31.13. When  $\sigma_Z$  is proportional to  $\mu_Z$ , its value is 31.13(= 31.62 × 315/320). The risk for each of Projects B, C, and D is derived from the same way. But damage to earnings gets more and more serious. Project B reduces earnings by 10 into  $\mu_Z = 310$ . From Project C,  $\mu_Z$  is 305 with 15 down, and Project D brings down  $\mu_Z = 300$ .

In the debt overhang examples, we posited that investment was cumulative; Project b was executed in addition to Project a, and Project c was done in addition to Projects a and b. Here we suppose that each of Projects A to D is added to the unleveraged firm  $\mu_Z^U$ . Though it is easy to make another example of the asset substitution the way projects are cumulatively carried out, this is not so meaningful because projects to accumulate are unprofitable.  $U + A$  to  $U + D$  in the most left column of Table 16 represent that each project is executed. These four projects require an initial outlay of 2. The increase in an equity value is calculated as  $S_L(L, \mu_Z, \sigma_Z) - S_L(L, \mu_Z^U, \sigma_Z^U)$ . The NPV of a project is the difference between the incremental value of equity for  $L = 0$  and the investment expense of 2. We can easily confirm that NPVs of these projects are negative.

**Table 16: Examples of the Asset Substitution**

Projects	I	$\mu_Z$	$\sigma_Z$	$S_L(L, \mu_Z, \sigma_Z) - S_L(L, \mu_Z^U, \sigma_Z^U)$				
				L = 268	L = 288	L = 300	L = 305	L = 310
U+A	2	315	93.39	-0.76	2.50*	3.92*	4.34*	4.65*
U+B	2	310	91.90	-2.07	1.38	2.92*	3.39*	3.74*
U+C	2	305	90.42	-3.34	0.31	1.97	2.49*	2.89*
U+D	2	300	88.94	-4.57	-0.71	1.07	1.63	2.09*

The most left column represents a project which is executed.  $\mu_Z$  is an expected value of firm's earnings under the project, and  $\sigma_Z$  is a standard deviation.  $S_L(L, \mu_Z, \sigma_Z) - S_L(L, \mu_Z^U, \sigma_Z^U)$  is an incremental equity value for each of Projects A to D.  $I$  is an investment outlay.  $\mu_Z^U = 320$  and  $\sigma_Z^U = 31.62$  are assumed in the case of the unleveraged firm.

In the case of a leveraged firm, debt of  $L = 268$  which leaves the incremental values of equity negative does not make the firm implement Projects A to D. When debt amounts to  $L = 288$ , however, the incremental value for Projects A to C becomes positive. In particular, the one for Project A is larger than 2. If Project A is carried out, the wealth of shareholders increases. Since  $L = 300$  makes the incremental equity value larger than the outlay for Project B as well as Project A, Projects A and B can be implemented. A bad project gets more feasible with more debt. Project C can be implemented under  $L = 305$ .  $L = 310$  makes all these projects executable. We assume that in the examples each project has its own standard deviation for  $L = 268$  to  $L = 310$ ;  $\sigma_Z$  is 88.94 for Project D without regard to  $L$ . Project D, which should not be implemented, can be of benefit to shareholders with a large amount of debt. As the results  $\mu_Z$  decreases due to more debt.

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