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Nash Equilibrium in Control-Unregistered Employment Decisions Under Incentive and Punishment Strategies

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ABSTRACT

Unregistered employment, which gained attention on the global economic agenda following the ILO's Kenya Report in the 1970s, has become a significant issue on par with unemployment in many countries. Policymakers and social planners have been working to mitigate this problem by intervening in the wage bargaining process between workers and employers through control mechanisms. They implement policies aimed at reducing unregistered employment, including routine audits of companies and the application of penalties and incentives to promote registered employment. These measures not only aim to increase tax and social security revenues for the state but also seek to boost worker productivity by motivating them to secure registered employment. However, a conflict arises between the control objectives of social planners and the profit-maximizing, cost-reducing tendencies of employers. This article explores this conflict by analyzing the employer's decision to opt for registered or unregistered employment against the backdrop of the social controller's punitive and incentivizing actions using a mixed-strategy 2x2 game model. Through this game model, the "best response functions" of both parties are derived, and the Nash Equilibrium is identified to determine the optimal response probabilities of the social planner and employer. The analysis examines the effectiveness of penalties and incentives in reducing unregistered employment based on the strategic interactions of both parties..

Keywords: Informal Employment, Informality, Social Planner, Social Control, Game Theory, Nash Solution.

JEL Classification Codes: C70, J20, J21, H55, H83

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INTRODUCTION

As countries around the world grappled with the aftermath of the oil crisis, two significant developments in the 1980s had a profound impact on the concept of employment. The first was a decline in the registered workforce, leading to a rise in unregistered employment. The second was the weakening of the welfare state model (Agarwala, 2006). These simultaneous trends resulted in workers being unable to access secure wages and adequate social rights, while the state faced reduced tax revenues and other financial inflows, thereby increasing public deficits and creating shortfalls in the social security system (Fidan and Genç, 2013). The phenomenon of unregistered employment, also referred to as the informal sector or informality, first gained attention among economists and policymakers through Hart's influential work in 1973 (Maloney, 1999; Jütting et al., 2008). The extensive literature that followed generally defines informal employment based on its deficiencies compared to formal employment (Williams and Lansky, 2013). In the current era, marked by the effects of the second major global economic crisis, the dualistic structure of labor markets (Esteban-Pretel and Kitao, 2021) has become a critical issue, necessitating targeted solutions and reforms (Leyva and Urriata, 2020; Eichhorst and Marx, 2021).

In both developed and developing countries, registered workers are those with an employment contract officially recorded in the Labor and Social Security registry, which documents their work history. Similarly, legitimate businesses possess a tax identification number, incur registration costs (in terms of time and money), pay payroll taxes, and face termination costs when dismissing employees. Conversely, all employees working for an unregistered company are automatically classified as unregistered. However, even formal companies can engage in informal employment by hiring workers without official contracts. The Ministry of Labor is responsible for conducting inspections to ensure that companies comply with labor laws (Brotherhood et al., 2023). These inspections can be conducted unannounced across all companies. This research article begins by examining this critical area of labor regulation.

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Unregistered employment is widely recognized as a serious economic issue (Elgin et al., 2021; Kanbur, 2017; Heintz, 2021), with its nature, measurement, macroeconomic and microeconomic effects (Ulyssea, 2020; Dellas et al., 2024), and associated productivity challenges (Fredström, 2021; Haanwinckel and Soares, 2021) being subjects of extensive research and analysis. Economists have made numerous contributions to the literature on unregistered employment, exploring it from economic, statistical, sociological, psychological, and various other social science perspectives. This body of research continues to expand. Additionally, several studies have examined the decisions of workers and employers—the two key actors in unregistered employment—focusing on the factors influencing their choices to engage in registered or unregistered work. Game theory and Nash Equilibrium are often employed as analytical tools to assess these decisions, evaluating the costs and benefits for both parties (Nohoua, 2023).

Studies on game theory in the literature predominantly focus on the Nash bargaining equilibrium (Carneiro and Henley, 1998; Bouev, 2005; Araujo and de Souza, 2010; Ciccarone et al., 2016; Sun, 2022). These studies typically examine game models that analyze wage-based strategies between workers and employers, exploring the economic impacts of their strategic interactions. They often aim to explain the macro-level effects of micro-level decisions or how macro-level employment dynamics influence decision-making at the micro-level between employees and employers. In conflict game analyses involving unregistered employment or in wage bargaining scenarios like those described by insider-outsider theory (Çolak and Koç, 2017), the dominant strategies of both parties tend to be well-defined.

In line with these dominant strategies, the worker, who seeks to maximize their benefits, and the employer, who aims to minimize costs, often align on a common approach toward informal employment. The employer prefers to hire informally to lower costs and increase profits, while the worker may accept precarious, unregistered work to meet basic livelihood needs. In this scenario, informal employment emerges as a pure strategy for both parties, resulting in a stable equilibrium. Consequently, such static and non-cooperative games have not typically been modeled in the existing literature. This study distinguishes itself by addressing this gap. It acknowledges the dominant strategies of both the employer and the worker, models the employer's behavior in response to the social planner, and analyzes the optimal strategy using the Nash Equilibrium solution, thereby contributing a novel perspective to the literature.

This study does not focus on the mutual strategies and reactions of employees and employers. Instead, it incorporates the role of a social planner (Parker, 2020), tasked with combating and preventing informal employment, into the game model. The primary objective of this article is to mathematically model the conflict of interest between the state, acting as the social planner, and the employer, specifically examining their motivations and strategies regarding informal employment. By directly involving the state in the game model, the study evaluates the responses of both parties to their chosen strategies using the Nash equilibrium framework. Game theory is particularly effective for mathematically explaining conflicts of interest, the strategies each party may use to maximize their benefits, and their potential decision-making in response to opposing strategies. Thus, the study's contribution lies in proposing a model to re-analyze the labor market dynamics using a Nash equilibrium based on the mixed strategies of the social planner and the employer, focusing on their behavioral strategies.

CONCEPTUAL FRAMEWORK and THEORETICAL BACKROUND

This study is fundamentally centered around two key concepts: unregistered employment and game theory. In this chapter, we will first provide a brief introduction to the concept of unregistered employment, followed by an explanation of the conceptual framework of game theory and its relevance to this study. Given the focus on the intersection of unregistered employment and game theory, aspects such as the measurement of unregistered employment or its broader economic implications will not be explored in detail.

Unregistered Employment

The OECD defined unregistered employment in 1987 as "activities that are legal in nature but not reported to one or more institutions" (Mateman and Renooy, 2001). The European Commission describes it as any paid activity that is legal in itself but not declared to public authorities to evade taxes and social security contributions (European Commission, 2007). Similarly, the ILO broadly defines unregistered employment as encompassing all individuals, whether in primary or secondary jobs, who were employed in unregistered firms or at least one informal sector enterprise during the reference period, regardless of their employment status (Hussmanns, 2004).

More broadly, unregistered employment encompasses a range of work patterns in both the formal and informal sectors, involving economic activities that are not fully captured in statistical records, whether performed by self-employed individuals or wage workers. These activities remain outside public records and official statistics (Uçarı and Koç, 2017; Moghaddasi Kelishomi and Nisticò, 2023). Informal employment can occur in firms of all sizes, not just small enterprises. For instance, in small firms (fewer than 10 employees), up to 40% of workers may be unregistered during audits, while even in large firms (500 or more employees), nearly 10% of the workforce can be unregistered. Interestingly, registered and unregistered workers within the same firm often share similar characteristics in terms of age, education level, and gender distribution. However, unregistered workers tend to earn slightly less. This suggests that the issue is not about fundamentally different types of workers but rather similar individuals working under different employment statuses (Brotherhood et al., 2023).

Unregistered employment has both positive and negative effects (Moghaddasi Kelishomi and Nisticò, 2023). Its consequences extend beyond a specific segment of society, impacting workers, employers, and the state, all of whom are part of the formal economy. On the positive side, unregistered employment can prevent individuals who would otherwise be excluded from the formal job market from remaining unemployed and without income. It can also enhance employer competitiveness by allowing them to hire workers at lower costs. However, the negative effects are significant, including workers' lack of access to secure wages and social rights, and the state's inability to generate sufficient revenue through taxes and other sources. While some positive outcomes may be observed, the negative effects tend to outweigh the positive ones in the short term and can be particularly detrimental in the long term (Yanici Erdal, 2019).

One of the policy tools used to combat informality is the regulation of penalties that compel lawbreakers to change their behavior. Unfortunately, in Turkey, there are no criminal sanctions for employing unregistered workers within the social security system. Instead, the penalties are administrative, related to the failure to issue employment and bonus documents within the legal timeframe (Işıklı, 2015). In contrast, many European countries impose severe sanctions for unregistered employment. For example, Kızılot (2015) highlights that in France, those who employ unregistered workers face fines of 45,000 Euros and up to five years of imprisonment; in the Netherlands, the fine is also 45,000 Euros with a one-year prison sentence; in Finland, the penalty is one year of imprisonment; and in Germany, it can reach up to 500,000 Euros in administrative fines and up to five years in prison. In Poland, businesses that employ unregistered workers lose access to various government incentives, and previously granted aid must be repaid (Kaleli and Karaca, 2019). Beyond fines, the imposition of severe penalties, such as imprisonment, serves as an effective deterrent, prompting employers to change behaviors that disrupt public order (Karaaslan, 2010).

Another tool used in the fight against informality is for countries to reduce financial obligations periodically or continuously, or to provide premium support to encourage the registration of employment (Kanbur, 2017; Elgin et al., 2021; Heintz, 2021). Offering such incentives, along with reducing tax and premium rates on employment—applied unconditionally to all employers—can significantly contribute to increasing registered employment (Kaleli and Karaca, 2019). When countries implement regular and frequent inspection mechanisms, informal sector firms and their employees face a higher risk of detection. The potential for fines upon detection may deter workers and firms motivated by tax evasion from participating in the informal sector. As a result, variations in the size of the informal sector between countries can be partly attributed to differences in tax policies and punishment systems (Kolm and Larsen, 2002; Bíró et al., 2022).

The drivers of informal employment vary from country to country, but it is widely recognized that factors such as high tax burdens, high labor costs, bureaucratic red tape, insufficient labor market inspections, and a lack of trust in government contribute to the growth of this phenomenon. While labor inspectorates play a crucial role in addressing the issue of informal employment, they often lack sufficient resources, tools, procedures, and coordination with other relevant authorities (Görmüş, 2017).

Game Theory and Nash Equlibrium

Game theory was first methodically introduced by Hungarian-American John von Neumann in 1928 in his article *On the Theory of Strategic Games*, where he demonstrated that multiple strategies can be determined for each player in two-person zero-sum games. About 15 years later, John von Neumann and economist Oscar Morgenstern (1944) from the same university published *Economic Behavior and the Theory of Games*, which marked the first time game theory was applied to

economics. However, the most significant contribution to the field came in 1950 from John F. Nash, whose work helped bring game theory to its present prominence. Between 1950 and 1953, Nash published four pivotal papers on game theory. Two of these, based on Cournot's (1838) work on oligopolistic markets, laid the foundation for the Nash equilibrium, named after him (Korolev and Ougolnitsky, 2023; Sarafopoulos and Papadopoulos, 2023). These four papers had a profound impact on the development of game theory. Nash's work expanded game theory beyond zero-sum games to include non-zero-sum games. In his papers *Equilibrium Points in N-Person Games* (1950) and *Non-Cooperative Games* (1951), Nash demonstrated the existence of strategies that ensure Nash equilibrium in non-cooperative games. By reducing these equilibrium strategies to non-contracted games, he elucidated the principles of consensual games (Eichherger, 1997). The mathematical solution methods introduced by game theory offer an alternative to the "ceteris paribus" assumption commonly used by economists, meaning "all other conditions being constant" (Savaş, 2000). Unlike Walras' concept of homo economicus, game theory does not rely on the assumptions of the invisible hand theory (Holler and Klose-Ullmann, 2020). In game theory, "rational behavior" is no longer a constraint on the preferences players can have. Players are free to have a wide range of preference relations, allowing game theory to internalize all kinds of behavior (Koray, 2012; Çolak and Koç, 2023).

In game theory, there are three key concepts that form the foundation of the theory: player, strategy, and payoff, along with the element of information. These concepts are often referred to as the "rules of the game." A player, seeking to maximize their earnings, employs different strategies based on the information available to them (Rasmusen, 2007). A strategy refers to a set of decisions that defines a player's preferences for every possible situation that could arise throughout the game, from start to finish. In other words, a strategy involves setting goals and objectives and determining the steps a person or organization should take to achieve them (Çolak and Koç, 2017). In game theory, strategies are typically categorized into two types: pure strategies and mixed strategies. Pure strategies are those played with certainty, while mixed strategies involve making decisions based on a certain probability (Jordan, 1993).

Conflict situations in real life are influenced by a wide range of complex, interrelated factors, making their analysis challenging and intricate. To address this, new models are developed by simplifying these situations, excluding less significant factors, to make mathematical analysis feasible. These models are referred to as games (Fudenberg and Tirole, 1991). In this context, game theory is defined as the systematic study of strategic interactions, aimed at understanding how economic agents behave in situations where their actions are interdependent and identifying how they can act more effectively (Çolak and Koç, 2023).

The concept of Nash equilibrium, introduced by John Forbes Nash (1950, 1951), is a fundamental solution method that helps players identify the most appropriate strategies (Myerson, 1991). It is defined as a situation in which each player's strategy is the optimal response to the strategies chosen by the other players (Fudenberg and Tirole, 1991). In a Nash equilibrium, no player has an incentive to deviate from their current strategy unilaterally (Carmichael, 2005). Nash equilibrium has two key characteristics: First, each player selects their best strategy based on their expectations of the strategies chosen by others. Second, since switching to an alternative strategy would decrease the player's payoff, no player has the desire to change their strategy in equilibrium (Dutta, 1999; Aydın and Karabacak, 2023).

LITERATURE

In the literature, analyses of Nash Equilibrium have primarily focused on wage markets. However, there is a lack of studies that model the interaction between the social planner or social auditor and the employer. For instance, Carneiro and Henley (1998), in their study "Wage Determination in Brazil: Union Bargaining Power and the Growth of Informal Employment," specifically highlight the role of auditing in combating informality.

In the model developed by Ciccarone, Giuli, and Marchetti (2016) based on the American economy, the unregistered segments, as defined by the Nash market model, are examined in the context of their dynamics within a detailed business model that incorporates search and exchange capacities. The study highlights the differences in wages and hours worked between regular and undeclared employment, showing that a higher degree of informal work correlates with lower average employment, greater employment variability, and lower regular wage variability. According to the findings, in the Nash process, the steady-state ratio is influenced by the minimum wage a worker can accept and the maximum wage a firm can offer. The larger the gap between these values, the greater the employment volatility and the lower the wage volatility (Ciccarone et al., 2016).

In another adaptation of the Nash model, integrated with the DSGE framework and applied to analyze the dynamics of the informal employment surge in China, Sun (2022) confirms the findings regarding the limitations of informal employment, despite its substantial costs. The study reveals a negative correlation between interest rates and the rate of unregistered employment in China. Sun suggests that the limited increase in rates, particularly during economic shocks, is due to the rise in informal employment. In economies affected by shocks, the Nash wage bargaining model indicates that household decisions lean towards informal employment, while firms also opt for unregistered workers (Sun, 2022).

In a study analyzing the emergence of the informal economy within non-competitive labor markets characterized by wage bargaining, Bouev (2005) extends the standard search model of Pissarides (2000) by incorporating both formal and informal sectors. The model demonstrates how government control over informal firms and the corrupt influence of bureaucracy on the formal sector can enable companies to maintain stable coexistence. Bouev (2005) explores market failures and matching frictions in both formal and informal job search processes, highlighting the effects of audits, taxes, bribery in bureaucratic processes, and productivity on workers, employers, and firms in both sectors. The study reveals that these factors lead to a shared interest between the two sectors. These findings are framed within the context of the Nash Bargaining Solution. The model suggests that higher unemployment benefits could contribute to the expansion of the informal sector.

Araujo and de Souza (2010) examined the key factors driving workers and companies toward informality in developing countries, focusing on the dual structure of the labor market. Using an evolutionary game theory approach, they identified excessive regulatory systems as one factor that makes the formal economy unattractive. Their study analyzed the dynamics of entry and exit for workers and firms in both the formal and informal economies, aiming to assess the impact of taxes by examining how economic actors choose between these markets based on expected returns. The study also evaluated the optimal balance between government regulation and enforcement actions (Araujo and de Souza, 2010).

In a similar vein, Çolak and Koç (2017) developed a model to analyze the strategies and reactions between the social planner and the employer. The study focused on punishment and control mechanisms, establishing the Nash equilibrium of the game in the context of punishments and inspection possibilities. The best response values for the parties' potential behaviors, based on opposing strategies, were identified. However, they noted that their model does not encompass all possible scenarios or response strategies (Çolak and Koç, 2017). The primary objective of this study is to construct and analyze the best response functions of the parties within the context of punishment, incentives, efficiency, and the tax spiral.

This study examines the control mechanism that, while not directly influencing the employee's choice between registered or unregistered work in wage and labor bargaining, has a significant impact on the employer's decisions. In other words, it evaluates the mutual strategies and reactions of the employer within the context of the social planner, who acts as a sanctioning authority in the power balance within the game theory framework between the employee and the employer.

METHODOLOGY

Model

As outlined in the introduction, the wage and employment bargaining between the employer and the employee can be modeled as a 2x2 zero-sum game. In terms of the game matrix, the key strategies for both parties are registered work and unregistered work. If the employer offers unregistered employment during the job interview, the likelihood of the employee rejecting the offer is very low, depending on the employee's level of need for work. Conversely, if the employee proposes unregistered employment to the employer, the employer is highly likely to accept the offer. In this scenario, when the game model is established between the employee and the employer, the only dominant strategy is unregistered employment for both parties. The dominant strategy for the employee who has secured a job is to accept the unregistered employment offer, which incurs lower costs for the employer. In this case, when both the employee and the employer agree on unregistered employment, they each maximize their own interests.

In his article "Gift Exchange and Efficiency-Wage Theory: Four Views," Akerlof argued that workers might accept an employer's offer despite low wages and the loss of certain rights if they are facing unemployment, but would be more inclined to accept an offer with higher wages and full employment rights, as noted in Burawoy's (1979) work. He suggested that workers who have social rights would experience increased productivity (Akerlof, 1984). Research in the literature also supports the idea

that registered workers tend to be more productive than unregistered workers (Loayza, 2018; Aberra et al., 2022). Informality is particularly prevalent among low-skilled workers and can negatively affect their motivation (Milica and Milica, 2019; Haanwinckell and Soares, 2021). Moreover, it can harm not only individual productivity but also firm productivity and growth (Endale, 2022; Kelishimo and Nistico, 2023). Additionally, missed tax and premium payments contribute to public sector losses (Mariola, 2022; Haanwinckell and Soares, 2021; Ulyssea, 2023). The main objective of this study is to analyze employer behavior within an employer-social controller 2x2 mixed strategy game, drawing on these insights.

A similar study was previously conducted by Yalama and Çelikkaya (2014) in their work titled "Determination of Optimum Tax Rate, Tax Punishment, and Audit Relationship for Turkey with Game Theory Model," which focuses on tax collection and the determination of the optimal tax rate. By analyzing the Nash Equilibrium and strategies within the model, they calculated the appropriate audit levels, punishment rates, and optimum tax rates for the period. Their findings indicate that as the audit rate increases and rational punishment rates are applied, employers' motivation to evade taxes decreases, as they perceive these measures as threats to their profits (Yalama and Çelikkaya, 2014).

Establishing the Model and Determining Strategies

In the model developed in this study, a mixed strategy game model will be applied to 2x2 static games, and Nash Equilibrium will be determined. This game theory model is also non-cooperative and non-zero-sum. When multiple strategy pairs influence the outcome of the game, a mixed strategy is used (Nohoua, 2023). In mixed strategy games, strategy pairs are represented by probability values, and the sum of these probability values equals 1. If it is not possible to find the equilibrium value using the peak approach within the full strategy method, the mixed strategy method can be employed. In such games, players can ensure a gain that does not fall below a certain level and a loss that does not exceed a certain threshold by selecting and playing their strategies according to a probability distribution. The value achieved through the mixed strategy method is known as the expected value (Straffin, 1996).

The model examines the social planner's response to the company's decisions regarding informal employment and the company's reactions to the social planner's audit decisions. The social planner expects employment to be registered and productivity to be high. This is because the social planner understands that when employment is registered, tax revenues are fully collected, social security premiums are paid in full, strengthening the system. Additionally, this ensures fairness by preventing tax evasion and unfair competition. Registered employment, along with higher wages and the security of future benefits, increases worker productivity, which, in turn, contributes to economic growth and higher production (Carneiro and Henley, 1998; Çolak and Koç, 2017; Parker, 2020).

The employer, aiming to maximize profits, will seek high levels of production from the worker while minimizing labor costs. One way to reduce labor costs is through informal employment, where taxes and social security premiums are avoided. Thus, the employer is inclined to choose unregistered employment. However, this decision comes with trade-offs, as the employer will not be able to extract full performance from the workforce. The worker, knowing they are in a temporary position or in an unstable work environment where they can leave at any time, will struggle to internalize the corporate culture and fully dedicate themselves to the job, leading to a decrease in productivity.

Table 1. Social Planner – Employer 2x2 Game Matrix

		Firm (Employer)	
		Formal	Informal
Social Planner	Audit	a-h-b , v+a+b-w-t	f-h , v-w-f
	No Audit	a-b , v+a+b-w-t	-b , v-w+b

The columns represent the strategies of the firm, while the rows represent the strategies of the social planner. The firm has two main strategies: the worker who attends the job interview will either be employed informally or

formally. In this context, informal employment refers to the worker being either fully excluded from social security and financial protections or employed partially, where only the minimum amounts are paid, resulting in missing the full contributions and losing rights in the future.

In the decision stages of the game, the parties have the option to implement social planner supervision based on their strategy profiles, while the company may choose to opt for informal employment. Specifically, in the first case (audit, formal), the strategy profiles of the parties are formed by pure strategies. In the second case (audit, informal), the strategy profiles of the parties are defined accordingly. The third case (no audit, formal) and the fourth case (no audit, informal) also represent distinct strategy profiles.

The social planner is depicted as an entity responsible for overseeing the state's public revenues and transfer expenditures, similar to social security or tax inspectors. Any shortfall identified through an audit by one authority can be immediately collected by another institution through penalties. Therefore, combining the roles of these two controllers into a single entity enhances the model's comprehensiveness. The social planner has two possible strategies: auditing and not auditing.

In the 2x2 social planner-employer game matrix, the benefit values are as follows: *v* represents the monetary value of the production level of the unregistered worker; *w* denotes the wage; *f* is the punishment (penalty); *t* refers to tax and social security premiums (legal obligations); *a* represents the monetary value of the productivity increase resulting from the worker's formal employment and payment of insurance premiums; *h* is the audit cost; and *b* stands for the incentives for registered employment provided to the employer. The constraint between these parameters is:

$$f>v>w>t>a>h>b>0$$

The benefit functions of the players, based on this information, lead to the following loss-gain clusters for the social planner:

$$U_{SP}= u(a,f,h,b)$$

For firm;

$$U_{F}= u(a,v,w,k,f,b)$$

As reflected in the utility function of the social planner (SP), SP's goal is to increase the number of registered workers, which in turn aims to enhance labor productivity (a) and overall production at both micro and macro levels. To achieve this, SP incurs audit costs (h), manages incentives (b), and imposes penalties (f) on companies that employ unregistered workers. The employer's behavior, as outlined in the company's utility function, is driven by several factors: the wage (w) paid to the workforce, the production level achieved in return for the wage, the legal obligations (t) associated with registered employment, the audit penalties (f) imposed if unregistered employment is chosen, and the incentives (b) offered by the government for registering workers.

In the first scenario, if the social planner chooses to conduct an audit and the firm opts for formal employment, the social planner's gain, which reflects the public interest (as the social planner makes decisions on behalf of the state), will be an increase in efficiency represented by *a*. However, since the audit incurs a cost of *h* for the public, the actual gain will be (a - h). It is also important to consider that countries offer certain tax and premium incentives to combat informal employment, which introduces an additional cost for the public. Thus, the net gain for the public will be (a - h - b).

On the other hand, since the firm has chosen formal employment, its gain will be (v + a + b - w - t), where *v* represents the monetary value of the production level under unregistered employment, and *a* is the increase in productivity from registered employment. In this case, the labor cost for the firm will be (w + t - b), with the remaining amount being the firm's profit. The productivity increase associated with the worker's formal employment, *a*, is the additional output generated when the worker is employed formally, and this is directly linked to *v*, the output level under informal employment. Mathematically, we can express the increase in productivity as $a = \frac{1}{X} . v$, where *x > 1* and *1/x* represents the productivity rate.

In the second case, if the social planner chooses to audit and the company opts for informal employment, the social planner's gain will be (f-h). This means that the social planner's profit is the punishment imposed on the company (f), minus the audit cost (h). On the other hand, if the company chooses informal employment and is audited, its earnings will be (v-w-f). In this case, the company's income from informal employment is (v-w), but it will be reduced by the penalty (f) imposed by the social planner during the audit.

In the third case, if the social planner decides not to audit and the company opts for formal employment, the social planner's gain will be an increase in efficiency

of "a", while the incentives provided to the formally employing company will result in a budgetary loss of "b". Since the audit cost is zero in this scenario, the economy will benefit by an additional amount of (a-b) from the state's perspective. However, this is only applicable if the company chooses formal employment. As the company prefers formal employment, its earnings will be (v+a+b-w-t), similar to the first case.

In the fourth case, when the social planner chooses not to audit and the company opts for informal employment, the social planner's gain will typically be zero. Since the workers are not registered, they will not work at full efficiency, resulting in no productivity-driven increase in production. With no subsequent inspection, there will be no punishment or audit cost. The company, therefore, will have minimized its labor costs and achieved the highest profit, which is (v-w). However, the company may not want all its workers to work informally. Some workers might be employed informally in addition to their registered positions, or all workers could be registered at the minimum wage, while the remainder of their agreed-upon wage is paid informally. This arrangement causes workers to forfeit social rights such as severance pay and retirement pensions, while allowing the company to calculate these amounts at lower levels and benefit from tax and premium incentives due to regular premium payments (Uçarı and Koç, 2017). As a result, the social planner's loss will be the incentives provided (-b), while the company's gain will be the lower labor costs, represented as (v-w+b). The model assumes that the company has a dual employment structure.

However, as observed, there are pure strategies in all four cases. Therefore, a Nash equilibrium cannot be formed using pure strategies, as the expected benefits and strategies of the parties differ from one another. Since no equilibrium can be achieved in this static game, it is necessary to explore whether a Nash equilibrium can be established using mixed strategies. To do so, the strategy options for the parties must be incorporated into the game model, and the matrix needs to be reorganized accordingly.

Table 2. Social Planner – Employer 2x2 Mixed Strategy Game Matrix

		Firm (Employer)		
		Formal	Informal	
Social Planner	Audit	a-h-b , v+a+b-w-t	f-h , v-w-f	p
	No Audit	a-b , v+a+b-w-t	-b , v-w+b	1-p
		q	1-q	

The "p" and "q" values in the new matrix represent the decision probabilities for the players. Specifically, "p" denotes the probability that the social planner will choose to audit, while "q" represents the probability that the company will opt for registered workers (or informal employment). These probability values, "p" and "q," range between 0 and 1. Therefore, the probability that the social planner will not conduct an audit is (1-p), and the probability that the company will choose unregistered employment is (1-q). With this in mind, when the benefit functions and loss-gain sets of the parties are adjusted, for the social planner:

$$U_{SP} = u(a,f,h,b,q)$$

for firm;

$$U_{F} = u(a,v,w,k,f,b,p)$$

In this new scenario, where there is no pure strategy equilibrium but a mixed strategy equilibrium, a player is indifferent between the pure strategies they randomize. To find the mixed strategy equilibrium of the game, the expected utility functions of the pure strategies must be equalized. Once this condition is met, the probabilities "p" and "q" can be determined through algebraic processes:

$$EU(\text{Audit}) = (a\text{-}h\text{-}b)q + (f\text{-}h)(1\text{-}q) \quad (1)$$

$$EU(\text{No Audit}) = (a\text{-}b).q - b(1\text{-}q) \quad (2)$$

When these two equations are equal to each other;

$$(a\text{-}h\text{-}b)q + (f\text{-}h)(1\text{-}q) = (a\text{-}b)q - b(1\text{-}q) \quad (3)$$

$$aq - hq - bq + f - fq - h + hq = aq - bq - b + bq \quad (4)$$

In the final equality, the identical or opposite values on both sides cancel each other out, resulting in equality;

$$-bq + f - fq - h = -b \quad (5)$$

If the constant values are moved to the right side of the equation and the "q" factor is factored out, the equation is represented as;

$$(b + f)q = f - h + b \quad (6)$$

This will result in the q value, which represents the probability of the company choosing to make an unregistered employment decision.

$$q = \frac{f-h+b}{f+b} \quad (7)$$

Equation (7), derived above, calculates the probability of the company choosing registered employment. In this equation, the ratio of the value obtained by subtracting the audit cost from the sum of punishment and incentive expenditures to the total of punishment and incentive expenses gives the probability "q." As the punishments (f), incentives (b), and audit costs (h) rise, the firm's probability of engaging in unregistered employment (1-q) will decrease. This is because increased audit costs reflect more frequent audits and a higher number of auditors employed by the public authority. Conversely, as audit costs decrease, the likelihood of unregistered employment increases, as the company anticipates fewer audits. A reduction in audit costs is associated with less frequent audits and fewer auditors employed by the public authority.

Similarly, the same method must be applied to determine the equilibrium of the company's strategies.

$$EU(Formal) = (v+a+b-w-t)p + (v+a+b-w-t)(1-p) \quad (8)$$

$$EU(Informal) = (v-w-f)p + (v-w+b)(1-p) \quad (9)$$

By setting the two functions equal to each other, equation (10) is derived;

$$(v+a+b-w-t)p + (v+a+b-w-t)(1-p) = (v-w-f)p + (v-w+b)(1-p) \quad (10)$$

By distributing (p) and (1-p) on both sides of the equation and performing algebraic operations, equation (11) is derived.

$$(f+b)p= -a+t \quad (11)$$

As a result of this equation, equation (12) is derived:

$$p = \frac{t+a}{f+b} \quad (12)$$

The "p" value derived from this process represents the probability that the social planner will conduct an audit. In other words, the likelihood of an audit is calculated by dividing the difference between legal obligations (t) and productivity increase (a) by the sum of the punishments (f) and incentives (b) given to the company. As the difference (t-a) decreases, the probability of the social planner auditing will increase. This means that when taxes and premiums are lower, and productivity is reduced, the social planner will have a higher tendency to audit. Conversely, if punishments and incentive rates are higher, the probability of the social planner conducting an audit will decrease, as it is expected that unregistered employment will not be used.

The "p" value derived from this process represents the probability that the social planner will conduct an audit. Specifically, the likelihood of an audit is calculated by dividing the difference between legal obligations (t) and the increase in productivity (a) by the sum of the punishments (f) and incentives (b) provided to the company. As the difference (t-a) decreases, the probability of the social planner auditing increases. This means that when taxes and premiums are lower, and productivity is reduced, the social planner is more likely to conduct an audit. Conversely, when punishments and incentives are higher, the probability of the social planner carrying out an audit decreases, anticipating that unregistered employment will not be used.

Nash Equilibrium Solution of the Model and Findings

To illustrate the application of the results obtained in the model and determine the Nash Equilibrium, the benefit functions of the players can be plotted on a graph by assigning values to the defined parameters. This approach generates the "best response curve" and helps identify the Nash Equilibrium (Goeree and Holt, 2004; Reeves and Wellmann, 2012). The most straightforward and effective method for determining Nash equilibrium in mixed strategy games is by deriving the best response curves.

To begin, the parameters must first be assigned values. Since our parameters are represented as S = {f, v, w, t, a, h, b} in descending order, their corresponding numerical values are assigned as S* = {17, 14, 11, 5, 3, 2, 1}. After distributing the parentheses in equation (1), equation (13) is derived:

$$EU(Audit) = (a-h-b)q + (f-h)(1-q) \quad (1)$$

$$EU(Audit) = aq - hq - bq + f - h - fq + hq \quad (13)$$

The new equation to be obtained at the end of the algebraic process is as follows:

$$EU(Audit) = (f-h) + q(a-b-f) \quad (14)$$

When the parameter values are replaced in equation (14) obtained above;

EU(Audit) = 15-15q (15)

The expected function of the social planner based on the control strategy is derived. If the same steps are applied to the scenario where the social planner does not conduct an audit, equation (2) should be used. Following the algebraic process in equation (2), the result is;

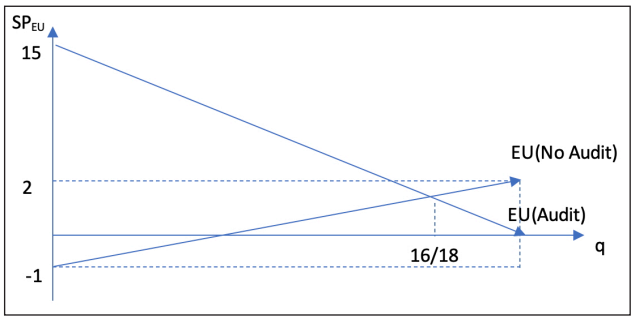
EU(No Audit)) = (a-b)q - b(1-q) (2)

EU(No Audit) = aq-bq-b+bq = aq-b (16)

By substituting the parameter values into the function, we get:

EU(No Audit) = 3q-1 (17)

The expected function is derived. Using the functions (15) and (17) obtained above, the graphical representation of the social planner based on the expected utility functions is as follows.



Graph 1. Social Planner's Response Functions

When Graph 1 is analyzed, the point where the expected utility functions intersect also corresponds to the point where the equation $q = \frac{f-h+b}{f+b}$ is derived. In other words, the probability at this point represents the likelihood of the company employing formal workers, with a probability of 16/18. Conversely, the probability of unregistered employment is 2/18. As shown, the company's response probability is determined from the social planner's utility functions, and the company positions itself and formulates a strategy based on the social planner's potential actions.

To calculate the company's strategy values based on the company's expected benefit function, the "p" values in equation (8) are extracted from the parentheses, and algebraic operations are performed to derive equation (18).

EU(Formal) = (v+a+b-w-t)p + (v+a+b-w-t)(1-p) (8)

EU(Formal) = v+a+b-w-t (18)

By substituting the values and parameters from the S* set into the derived equation, the expected benefit function is obtained as equation (19), which is a constant function.

EU(Formal) = 2 (19)

The expected utility function for the company's unregistered employment strategies is presented in equation (9) above. By applying this equation and removing the probability value "p" from the parentheses, we derive equation (20).

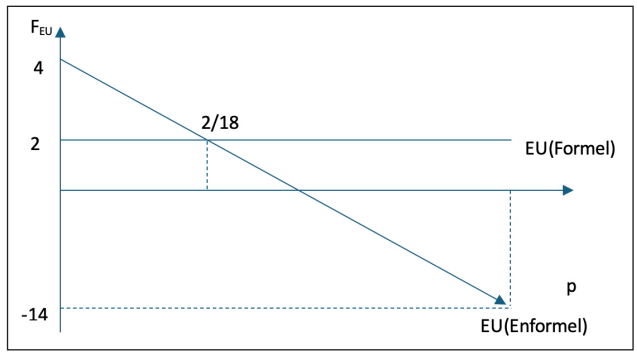
EU(Informal) = (v-w-f)p + (v-w+b)(1-p) (9)

EU(Informal) = (v-w)-p(f+b) (20)

By substituting the values from the S* set for the parameters, the expected utility function for informal employment (EU Informal) is obtained, as shown in equation (21):

EU(Enformel) = 4-18P (21)

In this way, the expected benefit functions for the firm's registered and unregistered employment strategies are derived. At this point, the company's response functions can be represented graphically:

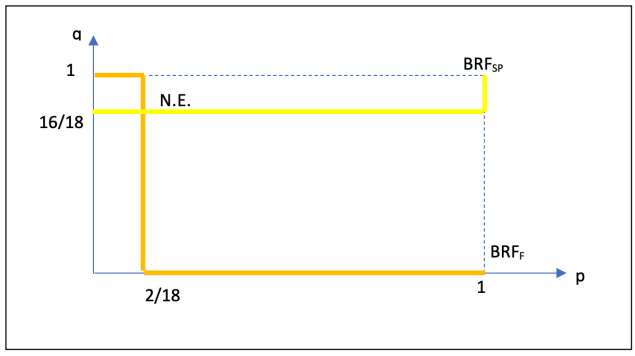


Graph 2. Firm's Response Functions

Graph 2 illustrates the graph derived from the company's expected utility functions. The intersection point of the company's response functions also determines the probability values of the strategies that the social planner will adopt. Specifically, this point corresponds to the equation $p = \frac{t-b}{f+b}$. Based on the values used in the model, the probability that the social planner will conduct an audit is 2/18. In other words, there is a 2/18 chance that the social planner will perform an audit.

After obtaining the expected utility functions for both parties and calculating the "p" and "q" probabilities, the best response curves for both players can be determined. These curves represent the strategies that will maximize each player's utility. To find the Nash equilibrium in the model, the best response functions of the players must be identified. Since the game model in this study is based on mixed strategies, finding the Nash equilibrium cannot be done in the same way as in pure strategy games. However, the best response functions provide a means to identify the Nash equilibrium. Therefore, the point where the "Best Response Curves" of the two parties intersect will indicate the Nash equilibrium (Goeree and Holt, 2004; Reeves and Wellmann, 2012).

As shown in Graph 3, the four pure strategy options (audit, no audit, formal employment, informal employment) do not constitute mutually best responses. As a result, there is no pure Nash equilibrium in the model. Similarly, combinations of the four mixed strategies do not form best responses. However, the intersection of the best response functions, at the point (2/18, 16/18), represents the Nash equilibrium, as it corresponds to the mutual best response Nash equilibrium.



Graph 3. Best Response Functions

From the perspective of the social planner, the optimal choice for the opposing player is to select a value within the range 0<q<1, p=2/18. In this scenario, the social planner remains indifferent to other potential choices. This means that the social planner will opt for an audit decision with a probability of 2/18 and is neutral regarding alternative options.

For the company, if the opposing player picks a value within the range 0<p<1, the ideal value for q would be 16/18. In this case, the decision regarding registered employment will occur with a probability of q=16/18. Consequently, the mixed strategy Nash equilibrium of the game is represented by the pair (2/18, 16/18).

CONCLUSION and POLICY IMPLACATIONS

The findings presented in this article provide insights into how social planners and employers respond to registered versus unregistered employment. Aligned with its objectives, the article models the strategies that social planners and employers adopt in response to employment regulations established by policymakers, as well as their reactions based on these strategies.

When evaluating the results derived from the implemented audit game, the strategic decision value for the social planner is expressed as. The primary goal of the social planner is to ensure that all jobs are registered and productive. Consequently, as the values of f (punishments) and/or b (incentives and rewards) increase, the probability value of p will decrease, leading to a reduced likelihood of the social planner conducting an audit.

In scenarios with high punishments, the social planner anticipates that employers will be deterred from hiring informal workers. Conversely, as the difference between (t-a) —the legal reserve value that includes taxes and insurance premiums— and productivity a increases, the probability of the social planner conducting an audit will rise. This is because low productivity relative to high tax and insurance payments makes informal employment more appealing to employers, ultimately decreasing income from social deductions. As a result, the need for audits becomes more pronounced.

Similarly, the primary objective of the firm is to achieve maximum efficiency from its workers at a low cost while maximizing profit. Consequently, the firm's decisions will influence whether it chooses to employ registered or unregistered workers. The value represents the probability of the employer's decision regarding employment.

As the values of punishments f and incentives b increase, the value of q will also rise, leading to a decreased probability of the employer hiring informal workers (represented by 1 - q). In other words, the employer will be more inclined to hire formal workers. However, if the value of h increases while punishments and incentives remain constant or are perceived as relatively low, then q will decrease. This results in an increase in (1 – q), thereby raising the likelihood that the employer will hire unregistered workers.

In essence, if the social planner reduces inspections due to rising audit costs, this situation may become more appealing for employers considering informal

employment. The social planner's decision to forego inspections could incentivize employers to pursue unregistered workers as a strategy for profit maximization and cost reduction.

The results yield two significant insights. First, if austerity measures implemented during an economic crisis are reflected in audit procedures, a reduction in audit frequency may lead to increased informality. As audit costs rise, the decreased frequency of audits due to these austerity measures will likely encourage employers to hire informally, as the likelihood of being caught by the social planner diminishes.

Second, an increase in public revenues from employment under fiscal policies during crisis periods can also incentivize employers to engage in informal hiring. High legal deductions, such as taxes on wages and social security premiums, raise labor costs for employers, potentially leading them to overlook productivity gains from their workers.

To reduce the frequency of audits and save on audit costs, the social planner should consider increasing penalties for informal employment. If policymakers aim for fewer audits due to limited resources and austerity measures, implementing higher punishments can serve as a deterrent. Additionally, since legal deductions can make informal employment more appealing to companies, high penalties should be employed as a precautionary measure.

Moreover, complementing high punishments with attractive incentives can enhance their deterrent effect. Improved incentives can lead to reduced informality and subsequently increase public revenues.

Conversely, it is evident that policymakers' decisions regarding punishments, incentives, taxes, and social security deductions influence employer behavior while sidelining worker productivity. Employers tend to prioritize their own benefit maximization in their decision-making processes. In other words, in response to the social planner's decisions, employers evaluate their choices based on the costs they will incur and the monetary benefits they anticipate.

As a result, when faced with the choice between informal employment and not hiring, employers often overlook the additional revenue that could be generated from increased workforce productivity. The primary motivation for employers in this decision-making process is to minimize costs and maximize profits, operating under the assumption that production levels will remain constant—meaning they do not expect any increase in productivity. Consequently, their main objective becomes achieving cost minimization and profit maximization.

Furthermore, the findings indicate that the variables established by social planners and policymakers —such as punishments, incentives, taxes, and social security premiums—should be implemented in conjunction with other factors that influence the model, including worker production levels, productivity increases, and control costs. Therefore, policymakers should adopt the optimal values derived from simulations designed to minimize unregistered employment in economic practice.

This article elucidates the behavioral decisions surrounding unregistered employment from the perspectives of both the company and the social planner (state) through mathematical methods based on game theory models. It highlights the necessity for the social planner to make optimal control decisions and implement rational punishments. Efforts should focus on reducing audit costs while developing alternative, lower-cost methods for oversight. Additionally, maintaining continuous control is crucial for preventing unregistered employment.

Following this study, it is anticipated that game theorists will create new models to calculate the optimal number of inspections and determine rational punishments. Furthermore, it is suggested that this model be expanded into a three-dimensional framework that incorporates the interactions between the worker, employer, and social planner, allowing for a revised Nash equilibrium based on this comprehensive approach.

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